Intertemporal Substitution and Indeterminacy of Fiscal Policy in a Growth Model of the Environment

Evangelos V. Dioikitopoulos* and Sugata Ghosh†

14 November 2012

Abstract

This paper develops an endogenous growth model of the environment where pollution increases with output, and tax revenues are used by an optimizing government to maintain environmental quality in addition to spending on infrastructure. We study the second-best (Ramsey) fiscal policies within a decentralized economy framework. We find that for sufficiently large values of the elasticity of intertemporal substitution, there can exist multiple solutions for the fiscal policy instruments (tax rate, and infrastructural-to-total spending), and also the other endogenous variables. More specifically, we derive a high-growth and a low-growth equilibrium that are associated with different combinations of optimal policy instruments.

JEL classification: D90, E21, E62, H31, O44, Q28.

Keywords: Endogenous Growth, Environmental Quality, Fiscal Policy, Multiple Equilibria, Elasticity of Intertemporal Substitution.

Acknowledgements: We are grateful to S. Kalyvitis, P. Konstantinou and participants at various conferences. The usual disclaimer applies.

*Department of Economics and Finance, Brunel University London, Uxbridge, Middlesex UB8 3PH, UK. e-mail: evangelos.dioikitopoulos@brunel.ac.uk.

†Corresponding author: Department of Economics and Finance, Brunel University London, Uxbridge, Middlesex UB8 3PH, UK e-mail: sugata.ghosh@brunel.ac.uk, tel. +44-(0)1895266887

1 Introduction

In this paper, we investigate the possibility and consequences of multiple equilibria that could arise when an optimizing government pursues Ramsey (second-best) fiscal policy to maximize agents' welfare in an endogenous growth model of the environment. Focusing on the role of the elasticity of intertemporal substitution (EIS), we analyze the properties of two stable long-run equilibria and provide alternative policy implications.

The impact of economic growth on environmental quality is well-known. As economies prosper with ever-increasing production and consumption possibilities, this leads to an undeniable strain on the environment, the quality of which is bound to suffer in the absence of natural resource regeneration and active environmental policy pursued by the government. However, the presence of these features, which we capture in our paper, raises the possibility of the existence of an environmental Kuznets curve (EKC), whereby high-growth countries, by virtue of superior clean-up policies, could enhance environmental quality.

Theoretical explanations for these phenomena can be provided in terms of threshold effects and multiple equilibria. John and Pecchenino (1994), in a model where agents accumulate both capital and environmental quality, show that multiple equilibria can arise as a result of the interaction between those two variables. Jones and Manuelli (2001) show that the relationship between pollution and growth depends on these two choices and could be non-monotonic when individuals vote over effluent charges and the direct regulation of technology. Mariani et al. (2010), in a model where life expectancy is endogenously determined by environmental quality, find that multiple equilibria may emerge: here, the equilibrium with the higher stock of human capital is also associated with better environmental conditions.

Complementing this literature, our paper relates to multiple balanced growth paths arising from a benevolent government optimally setting its fiscal policy instruments. In particular, Ramsey fiscal policy could lead to welfare-maximisation in two possible ways: raising the tax rate and shifting resources towards the environment (the *static* channel), or lowering the tax rate and altering the composition of spending towards infrastructure (the *dynamic* channel), which enchances the tax base, providing more expenditures for the environment. We derive conditions where indeterminacy of government policies towards welfare maximization result in two stable long-run growth equilibria.

Some other aspects of our work should be stressed in comparison with the existing literature on fiscal policy indeterminacy. Park and Philippopoulos (2004) consider a government choosing the second-best (Ramsey) tax policy, where the pursuit of such policy gives rise to indeterminacy. In a model with public consumption and production services, indeterminacy emerges due to a coordination failure between private and public sectors in the presence of external effects from these two types of government expenditure.[1] In our model, fiscal policy is used to finance infrastructure and the environment rather than the different categories of public spending.

Finally, to highlight the value added of our model, we focus on the role of the EIS. Elbasha and Roe (1996) note that environmental quality affects the time path of consumption, providing the value of the EIS is not equal to 1 or the utility function is non-separable. In particular, they analyze the importance of the EIS on the question of dynamic substitutability between growth and the environment through their linkages with welfare. They find that the higher the EIS, the higher is the growth rate of the economy, and that for relatively low values of the EIS (EIS < 1), growth and environmental quality go hand in hand in improving welfare as environmental awareness increases. We additionally argue that the value of the EIS determines the dynamic effects of taxation on welfare and activates a mechanism for multiple equilibria. We show that in the case of multiple equilibria, in the high growth regime, there exists a complementary relation among environmental care, growth and environmental quality, even with a high value of EIS. However, in low-growth economies, and with high EIS, we obtain the opposite outcome, which is consistent with Elbasha and Roe (1996).

Within the indeterminacy literature, Bennett and Farmer (2000) and Kim (2005) demonstrate, in models where the utility function is defined over consumption and leisure (but without considering the environment), that a low degree of externalities would produce indeterminacy

[1] See also Park and Philippopoulos (2003), who consider a similar set-up as this, but with state-contingent redistributive transfers; there, anticipation of large transfers leads to infinite possible equilibrium paths for fiscal policy and growth.

depending on the value of the EIS. In our case, we show how a large EIS can be sufficient for policy indeterminacy when utility is defined over consumption and environmental quality. The aforementioned *static* channel will cease to become effective when the EIS takes on a small value, because current environmental benefits will need to be sacrificed to achieve higher growth, particularly when the natural rate of regeneration is low; hence the high-tax, low-growth equilibrium drops out. Our results can be compared with Economides and Philippopoulos (2008), who, with a logarithmic utility function (implying unitary EIS), achieve a unique equilibrium.

The next section describes the model. Section 3 characterizes the decentralized equilibrium. This is followed by the analysis of the Ramsey (second-best) fiscal policy, which is the focus of our interest. Finally, Section 5 concludes the paper.

2 The model

This section presents the set-up of our closed economy model. The main features are as follows: (a) agents derive utility from private consumption and environmental quality, which has the characteristics of a public good; (b) public infrastructure provides externalities in production; (c) production activities generate environmental pollution; (d) the government imposes a tax on polluting output and uses the collected tax revenues to finance infrastructure and environmental care; and (e) the optimising government pursues the second-best (Ramsey) policy by choosing its fiscal instruments, taking agents' consumption and accumulation decisions as given.

2.1 The representative agent

The economy is made up of a large number of identical agents, each of them being infinitely-lived household-producers, and seeking to maximize the present discounted value of their lifetime utility:

$$\int_0^\infty \frac{\left(C^v N^{1-v}\right)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \tag{1}$$

where C is the representative agent's consumption, and N is the stock of economy-wide natural resources, interpreted as an index for environmental quality. ν $(0 < \nu \leq 1)$ denotes the importance of private consumption relative to the environment (so that a decrease in ν implies that an agent's environmental awareness relative to consumption increases), and σ $(0 < \sigma \leq 1)$ represents the fact that marginal utility (with respect to consumption and natural resources) is constant. σ is an indicator of the degree of intertemporal substitution, which is given by $1/(1+\sigma)$, and is an important parameter in our model, as the value of it determines whether or not fiscal indeterminacy exists in the model. Also, ρ is the rate of time preference of agents.

Households save in the form of capital, and their flow budget constraint is given by:

$$\dot{K} = (1-\tau)Y - C \tag{2}$$

where a dot over a variable denotes a derivative with respect to time, K denotes physical capital, Y denotes output, and τ $(0 < \tau < 1)$ is a tax rate on output.

The production function of the single good in this economy is given by:

$$Y = AK^{a}G^{1-\alpha} \tag{3}$$

Here G refers to the public productive expenditures provided by the government *a la* Barro (1990), and α $(0 < a < 1)$ denotes the share of physical capital in the production function. Labour supply is inelastic and is normalised to unity.

The household acts competitively by taking prices, policy, and environmental quality as given. The latter is justified by the open-access and public good features of the environment. Consumer optimization leads to the first-order condition given by the Euler equation below:

$$\frac{\dot{C}}{C} = \frac{1}{1-\nu\left(1-\sigma\right)}\left[\left(1-\nu\right)\left(1-\sigma\right)\frac{\dot{N}}{N} + r - \rho\right] \tag{4}$$

Here r, the social return on capital, is given by

$$r = \alpha(1-\tau)\left(\frac{G}{K}\right)^{1-\alpha} \tag{5}$$

2.2 The environment

Following Economides and Philippopoulos (2008), we assume that the stock of environmental quality evolves over time as follows:

$$\dot{N} = \delta N + \theta E - P \tag{6}$$

where E is the flow of public services in reviving the environmental investment (specified in equation (8c) below), and P is the pollution flow (see below). $\delta(>0)$ and θ $(0 < \theta \leq 1)$ are parameters measuring, respectively, the regeneration rate of natural resources and the effectiveness of public environmental policy. Thus, natural resources can be renewed by regeneration and public policy.[2]

We further assume that pollution arises as a by-product of final output:

$$P = sY \tag{7}$$

where s $(0 < s < 1)$ is a technology parameter that quantifies the detrimental effect of economic activity on the environment.[3] Production, Y, therefore impacts positively on the evolution of environmental quality through providing a tax base for the finance of public environmental investment (see below) and negatively through the induced pollution.

[2]Environmental quality can grow at a positive rate in our model as we are concerned with renewable resources that can regenerate over time, via a combination of the natural replenishment process and proactive environmental policy. See Perman et al. (2003) for growth capacities of renewable natural resources.

[3]Pollution as a linear function of income is assumed in van der Ploeg and Withagen (1991), Ligthart and van der Ploeg (1994), Greiner (2005), Economides and Philippopoulos (2008), and Ray Barman and Gupta (2010), while it is modelled as a function of the capital stock by Gradus and Smulders (1993).

2.3 The government

On the revenue side, the government imposes taxes per unit of output at a rate, τ $(0 < \tau < 1)$, while on the expenditure side, it spends G on infrastructure and E on the environment.[4] Assuming a balanced budget, we have:

$$G + E = \tau Y \tag{8a}$$

Equivalently, we can write (8a) as:

$$G = b\tau Y \tag{8b}$$

$$E = (1 - b)\tau Y \tag{8c}$$

where $0 < b \leq 1$ is the fraction of tax revenue used to finance infrastructure and $0 \leq (1-b) < 1$ is the fraction that finances environmental investment. Thus, at each instant, government policy can be summarized by the two policy instruments, τ and b.

3 Decentralized competitive equilibrium

The decentralized competitive equilibrium (DCE) is characterized by the household-producer maximizing intertemporal utility subject to its flow private budget constraint, which ensures that product and factor markets clear. The government budget constraint is satisfied, but the government's policy instruments, τ and b, are taken as given by the agent in its optimizing exercise. The initial endowment of capital, $K(0) > 0$, and the initial environmental stock, $N(0) > 0$ are taken as given by households. In this section we solve for a DCE, which holds for any feasible policy and analyze its properties.

[4] Given the setup of the model, a tax levied on output boils down to taxing pollution. On the issue of abatement (i.e., public spending on the environment), see, among others, Smulders and Gradus (1996), Byrne (1997), and Managi (2006).

Combining (1)-(8), it is straightforward to show that the DCE is given by:

$$\frac{\dot{C}}{C} = \frac{(1-\nu)(1-\sigma)}{1-\nu(1-\sigma)}\frac{\dot{N}}{N} + \frac{1}{1-\nu(1-\sigma)}\left[a(1-\tau)A^{\frac{1}{a}}(b\tau)^{\frac{1-\alpha}{a}} - \rho\right] \tag{9a}$$

$$\frac{\dot{K}}{K} = (1-\tau)A^{\frac{1}{a}}(b\tau)^{\frac{1-\alpha}{a}} - \frac{C}{K} \tag{9b}$$

$$\frac{\dot{N}}{N} = \delta - [(s-\theta(1-b)\tau)]A^{\frac{1}{a}}(b\tau)^{\frac{1-\alpha}{a}}\frac{K}{N} \tag{9c}$$

Equations (9a)-(9c) summarize the dynamics of our economy.

Finally, the transversality condition for this problem is given by:

$$\lim_{t\to\infty}\frac{K(t)}{C(t)}e^{-\rho t} = 0 \tag{10}$$

The balanced growth path (BGP) is defined as a state where all the variables in the economy: consumption, private capital, public capital, and environmental quality grow at the same rate, i.e. $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{K_g}}{K_g} = \frac{\dot{N}}{N} \equiv \gamma$. Following usual practice, we will reduce its dimensionality to facilitate analytical tractability. We thus proceed by defining the following auxiliary stationary variables, $c \equiv \frac{C}{K}$ and $x \equiv \frac{K}{N}$. Then, it is straightforward to show that the dynamics of (9a)-(9c) are equivalent to the dynamics of the following system of equations:

$$\frac{\dot{c}}{c} = c - [(1-\frac{\alpha}{\sigma})(1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}} + \frac{\rho}{\sigma}] \tag{11a}$$

$$\frac{\dot{x}}{x} = [s-\theta(1-b)\tau]A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}x + (1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}} - c - \delta \tag{11b}$$

It follows that at the BGP, $\frac{\dot{c}}{c} = \frac{\dot{x}}{x} = 0$. Then (11b) and (11c) imply that the long-run ratios of consumption to private capital, and physical capital to environmental quality, are given by:

$$c - [(1-\frac{\alpha}{\sigma})(1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}} + \frac{\rho}{\sigma}] = 0 \tag{12a}$$

$$[s-\theta(1-b)\tau]A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}x+(1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}-c-\delta=0 \tag{12b}$$

Finally, substituting (12a) in (12b), x is determined by:

$$\Phi(x)\equiv[s-\theta(1-b)\tau]A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}x+\frac{\alpha}{\sigma}(1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}+\frac{\rho}{\sigma}-\delta=0 \tag{13}$$

Providing there exists a solution, $\hat{x}>0$, in (13), the balanced growth rate is then determined by (9a).

Proposition 1 *For exogenous tax rate and allocation of public expenditures, the long-run equilibrium exists and it is unique.*

Proof. $\Phi(x)$ is a linear function of x, and if it exists, it is unique. For some parameters and exogenous policy instruments, there exists an $\hat{x}$ which a fixed point of $\Phi(\hat{x})$ and is given by $\hat{x}=\frac{\frac{\alpha}{\sigma}(1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}+\frac{\rho}{\sigma}-\delta}{[\theta(1-b)\tau-s]A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}}}$. ■

4 Ramsey (second-best) fiscal policy

In this section, we consider an optimizing government that chooses its policy instruments appropriately to maximize the representative agent's utility. In other words, we endogenize the government's fiscal policy, as summarized by the time paths of the two policy instruments, $0<\tau<1$ and $0<b\leq 1$, by solving the Ramsey problem of the government. This is the second-best policy, and it is particularly important (and realistic) to capture this in a situation where the government (unlike an omniscient social planner) is unable to internalize the various externalities that exist in the system and/or does not have lump-sum tax/subsidy instruments at its disposal.

The benevolent government thus seeks to maximize the economy's welfare subject to the decentralized equilibrium conditions (i.e., the private budget constraint, the production function, the natural resource constraint, and the Euler equation), together with the government budget constraint. [5]

The first-order conditions of the government's Ramsey policy problem, i.e., the optimality conditions with respect to C, N, G, E, τ, respectively - where λ, μ, ξ are the costate variables associated with the private budget constraint, the natural resource constraint, and the government budget constraint - are as follows:

$$\frac{\dot{C}}{C} = -\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda} \tag{14}$$

$$(1-v)C^{v(1-\sigma)}N^{(1-v)(1-\sigma)-1} + \delta\mu = -\dot{\mu} + \rho\mu \tag{15}$$

$$\lambda(1-\tau) - \mu s + \xi\tau - \left(\frac{G}{K}\right)^{\alpha}\frac{\xi}{1-\alpha} = 0 \tag{16}$$

$$\xi = \theta\mu \tag{17}$$

$$\xi = \lambda \tag{18}$$

Combining (16)-(18) we get

$$\left(\frac{G}{K}\right)^{\alpha} = [1 - \frac{s}{\theta}](1-\alpha)A \tag{19}$$

and using (9c), (14) and (18)we get $\xi = \lambda = \theta\mu \Rightarrow \dot{\xi} = \dot{\lambda} = \theta\dot{\mu}$, and we obtain the growth rate

[5] Note that E&P (2008) consider a logarithmic utility function ($\sigma = 1$), which is somewhat restrictive. Our methodology for capturing Ramsey fiscal policy allows us to derive equilibrium conditions under isoelastic utility functions, which is a contribution of this paper. As we shall see, we are able to derive interesting results by varying the elasticity of intertemporal substitution, and one of its key properties is that a large value of this parameter (i.e., small σ) leads to fiscal indeterminacy, which does not arise in E&P (2008) because $\sigma = 1$ there.

of consumption in the Ramsey environment as

$$\gamma \equiv \frac{\dot{C}}{C} = \frac{1}{\sigma}\left(\frac{1-v}{v}\theta\frac{C}{N} + \delta - \rho\right) \tag{20}$$

Using the Euler equation, the growth rate of consumption in the Ramsey economy has to be the same as that under the decentralized environment, and the following has to hold:

$$\frac{1}{\sigma}\left(\frac{1-v}{v}\theta\frac{C}{N} + \delta - \rho\right) = \frac{1}{\sigma}\left(\alpha(1-\tau)A^{\frac{1}{\alpha}}(b\tau)^{\frac{1-\alpha}{\alpha}} - \rho\right) \tag{21}$$

Also, combining $G = b\tau AK^{\alpha}G^{1-\alpha} \Rightarrow \left(\frac{G}{K}\right)^{\alpha} = Ab\tau$ with equation $\left(\frac{G}{K}\right)^{\alpha} = [1-\frac{s}{\theta}](1-\alpha)A$, we obtain

$$b\tau = [1-\frac{s}{\theta}](1-\alpha) \tag{22}$$

Then, the dynamic equilibrium of the Ramsey problem is characterized by the following 5 equations:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left(\alpha(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} - \rho\right) \tag{23}$$

$$\frac{\dot{K}}{K} = (1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} - \frac{C}{K} \tag{24}$$

$$\frac{\dot{N}}{N} = \delta - [s-\theta(1-b^R)\tau^R]A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}}\frac{K}{N} \tag{25}$$

$$\frac{1-v}{v}\theta\frac{C}{N} + \delta = \alpha(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} \tag{26}$$

$$b^R\tau^R = [1-\frac{s}{\theta}](1-\alpha) \tag{27}$$

We can express the stationary long-run equilibrium ($\dot{c} = \dot{x} = 0$) under the Ramsey fiscal policy as follows:

$$0 = \tilde{c} - [(1-\frac{\alpha}{\sigma})(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} + \frac{\rho}{\sigma}] \tag{28}$$

$$0 = [s-\theta(1-b^R)\tau^R]A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}}\tilde{x} + (1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} - \tilde{c} - \delta \tag{29}$$

$$\frac{1-v}{v}\theta\tilde{c}\tilde{x}+\delta=\alpha(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} \tag{30}$$

$$b^R\tau^R=[1-\frac{s}{\theta}](1-\alpha) \tag{31}$$

This is a system of 4 equations in 4 unknowns (τ^R, b^R, $\tilde{c}$, $\tilde{x}$), and represents the stationary solution for the Ramsey policy environment, from which γ, the economy's growth rate, can be computed. Solving this system we can derive the long-run second-best tax rate, τ^R, and allocation of revenues to infrastructure, b^R, summarized in the following Proposition:

Proposition 2 *The Ramsey second-best tax rate, τ^R, and allocation of government revenues to productive activities, b^R , are given by the following equations:*

$$F(\tau^R;s,\theta,\alpha,\sigma,A)\equiv\Omega[(s-\theta(\tau^R-[1-\frac{s}{\theta}](1-\alpha)))$$

$$\left(\frac{\alpha(1-\tau^R)\Omega-\delta}{\frac{1-v}{v}\theta[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]}\right)]+\frac{\alpha}{\sigma}(1-\tau^R)-\frac{\rho}{\sigma}-\delta=0$$

$$b^R=\frac{[1-\frac{s}{\theta}](1-\alpha)}{\tau^R}.$$

There can be a unique equilibrium or multiple equilibria depending on the following cases:

Case 1: If $\alpha\leq\sigma$ and $\gamma<\delta$, then the solution is unique.

Case 2: If $\alpha>\sigma$, $\gamma>\delta$ and $\delta>\check{\delta}\equiv-\frac{\alpha\frac{\rho}{\sigma}}{(1-\frac{\alpha}{\sigma})}$, then multiple solutions can exist.

Proof. *See Appendix A.* ▪

Proposition 2 shows that there can be unique or multiple second-best taxes and allocation of government revenues to productive activities that maximize the welfare in the economy. In particular, Case 1 generalizes the outcome of Economides and Philippopoulos (2008) where the second-best tax rate that maximizes welfare is unique. If the intertemporal elasticity of substitution is sufficiently low, i.e., σ is sufficiently high (worth noting that $\sigma=1$ in the E&P paper) and the growth rate of the economy is lower than the rate of regeneration of the environment, then there exists only one tax rate and share of government expenditures that maximizes welfare. In the other case, where the elasticity of substitution is high and the growth rate of the economy is sufficiently higher than the natural rate of regeneration (Case 2), there can be two ways in which the government can use the policy instruments to maximize welfare, resulting in policy indeterminacy and different growth regimes.[6]

Intuitively, there are two ways in which the government can increase the welfare of agents. The direct way (static channel) by which agents can benefit in terms of welfare is if the government increases expenditures on the environment, by implementing higher taxes (to finance its expenditures) and by altering the allocation of existing revenues from infrastructure towards abatement technology, increasing environmental quality and welfare. However, it is also true that an increase in taxes distorts private savings and, in turn growth, and the allocation of expenditure from infrastructure to abatement reduces the source of endogenous growth in this economy. To this end, alternatively, the government can switch the allocation of expenditure to infrastructure which dynamically increases the growth rate and in turn, with a higher tax base can finance public expenditures in the environment increasing welfare (dynamic channel). The latter channel prevails, and becomes the only one that maximizes welfare (as in E&P), if the cost of intertemporal substitution of lower environmental quality today for higher tomorrow is relatively low (low elasticity of intertemporal substitution), and when the rate of natural regeneration is high so as to accommodate the polluting effect of higher growth (Case 1). In the other case, both channels can provide the maximum welfare, as the static route provides higher utility today but lower growth tomorrow, while the dynamic channel provides lower utility today (if the regeneration rate if lower than the growth rate) and higher growth tomorrow. In turn, the government can increase welfare by making choice of either two channels resulting in policy indeterminacy and different growth regimes.

To highlight the above intuition that underlines our theoretical results, we provide some numerical examples. We use the following baseline parameter values: $\alpha = 0.5$, $A = 1.8$, $\delta = 0.015$, $\rho = 0.04$, $\theta = 1$, $s = 0.5, v = 0.1$ along the lines of our parametric conditions and those used by E&P to solve the model numerically. Table 1 shows that under Case 2 of Proposition

[6] Although it is feasible to have more cases where uniqueness or multiplicity can exist, we focus on cases that are comparable with the aforementioned literature.

1, $\sigma(= 0.1 - 0.3) < \alpha(= 0.5)$, multiplicity arises.[7] In this case, lower σ would imply that the cost of substituting the future for present consumption is higher; so the less we can expect to sacrifice current in favour of future consumption. On the other hand, for sufficiently low elasticity ($\sigma > 0.5$), the dynamic channel becomes more pronounced and a sacrifice of current consumption (and natural resources) leads to higher future consumption, savings, environmental quality and growth.[8] So, it is interesting that both equilibria are associated with different comparative statics properties, which necessitate the implementation of different public policies.

Table 1: Changes in σ and the Ramsey Equilibrium

σ	b^R	τ^R	γ
0.1	(0.284)0.278	(0.878) 0.896	(0.0937)0.0206
0.3	(0.331) 0.282	(0.753) 0.886	(0.199)0.0203
0.7	0.288	0.867	0.020
0.9	0.291	0.858	0.019

For the unique equilibrium case, we can analyse the movement of the various endogenous variables with a rise in σ: there is a lower growth rate, a lower tax rate, a higher proportion of spending on infrastructure to total spending, a higher consumption-to-capital ratio, and the ratio of environment to capital is also higher. What happens is that as σ is higher, there is a direct effect on utility *per se*, but it is not clear if this effect is positive or negative. Note that higher σ would imply that the cost of substituting the future for present consumption is higher; so the less we can expect sacrificing current in favour of future consumption; so the saving rate of the economy is low in that case, and so will the rate of growth be. The effect of higher σ on welfare via the growth rate is thus unambiguously negative. In this set-up, the government chooses both τ and b optimally, and does so to maximise welfare, while at the same time, balancing its budget. Since a higher σ leads to lower growth, the government could actively seek to trigger the growth channel, and thereby the tax base and overall tax proceeds. It

[7] Both equilibria are stable in accordance with the stability analysis conducted in Appendix B.

[8] Note that in Table 1, $\gamma > \delta$, which is a non-trivial case. For an example where $\gamma < \delta$ and our sufficient but not necessary condition for uniqueness (Case 1) holds, see E&P. Also, the effect of EIS ($1/\sigma$) on the growth rate is positive (for the low-growth equilibrium), which is consistent with the analysis of Elbasha and Roe (1996).

could do this by channeling resources towards infrastructure (i.e., higher b). To mitigate adverse growth effects, the government needs also to lower the tax rate to increase the net-of-tax saving, but this, together with the lower growth rate will have a negative impact on the government's budget, which is counteracted by b being higher. However, note that welfare is also affected by the C/K and the N/K ratios. With a low substitution elasticity, the propensity for current consumption (at the expense of future consumption) is higher, which leads to a higher C/K ratio, and ties in well with a lower growth rate. Given that C and N are multiplicatively linked in the utility function, and hence complementary, the higher is σ, the lower is the sacrifice in terms of the environment, the higher is the N/K ratio in the current period. A lower tax rate and lower growth rate, leading to a lower tax base requires the composition of spending to shift more towards infrastructure relative to the environment to balance the government's budget, which is reflected in the value of b being higher in equilibrium.

It is also worth analyzing the response of Ramsey policy instruments to changes in environmental awareness, as in Elbasha and Roe (1996) and E&P (2008), but for different growth regimes. In Table 2, we study how an increase in environmental awareness (increase in $1-\nu$) affects the policy instruments and the growth rate, for a given value of σ, $\sigma = 0.15$. [9] Consider an increase in $1-\nu$, which makes the utility effect of increasing environmental quality higher. In such a case, it appears intuitively that agents can benefit in terms of welfare if the government increases expenditures on the environment, either by implementing higher taxes (to finance its expenditures) or by altering the allocation of existing revenues from infrastructure towards abatement technology, which is the government's direct channel for increasing welfare. However, as noted earlier, an increase in taxes distorts private savings and, in turn, growth, and the allocation of expenditure from infrastructure to abatement reduces the source of endogenous growth in this economy. Under this regime, the combination of higher τ and lower b will reduce the tax base in the economy, and the financing of expenditures for the environment will create a virtuous cycle of low growth (Table 2a).

[9] The results hold for any value of σ for which multiple equilibria exist, $\sigma(= 0.1 - 0.3)$, following Table 1. In Appendix B we provide solutions for all the endogenous variables, $\tilde{c}$, $\tilde{x}$, τ^R, b^R, γ for both Table 1 and Table 2.

Instead, if the government pursues the alternative strategy of lowering τ and increasing b, the indirect channel to raise welfare via a higher growth rate is activated, and higher welfare is attained via the higher tax base and growth channel. However, pursuit of this strategy implies that both $\frac{C}{K}$ and $\frac{N}{K}$ decrease, though the possibility of welfare improvement via future consumption and natural resources is enhanced via the growth channel (Table 2b).

The possibility of the existence of twin mechanisms at the disposal of the government highlights a coordination problem that can exist between the preferences of the agents for higher environmental quality and the feasibility constraint driven by the budget constraint of the government. In particular, because the government has two available policy instruments, it can either use higher taxes and switch revenues towards the environment that increase environmental quality and welfare at the expense of lower growth, or use lower taxes but allocate more revenues to infrastructure that dynamically creates a higher tax base which finances both types of expenditures and increases welfare and economic growth. Both are welfare maximizing policies, and this policy indeterminacy results in different growth regimes. So, for any level of environmental awareness, the tax rate has to be lower in highly growing economies because higher growth implies higher tax base for the financing of public expenditures. Interestingly, both regimes are associated with a growth rate that is higher than the regeneration rate of the environmental stock, $\gamma > \delta$, which simulates a real world economy, unlike the case considered by E&P, where - with the assumption of the polluting effect of the environment exceeding the rate for the government's cleanup policy - the equilibrium under a logarithmic utility function is unique.

It is important also to note that, because the second regime operates through the dynamic channel of higher growth and higher tax base to finance expenditures on the environment, it becomes feasible only when the intertemporal elasticity of substitution between consumption and environmental quality is high enough (lower σ). If this channel is not feasible, σ is relatively high, then there exists only one combination and tax rate that can maximize welfare and the Ramsey tax rate is unique.

Table 2: Changes in $1-v$ under Multiple Equilibria ($\sigma = 0.15$)

2a "low" growth regime

$1-v$	b^R	τ^R	γ
0.7	0.2827	0.8841	0.0461
0.8	0.2807	0.8904	0.0289
0.9	0.2797	0.8936	0.0205

2b "high" growth regime

$1-v$	b^R	τ^R	γ
0.7	0.2868	0.8716	0.0799
0.8	0.2890	0.8647	0.0984
0.9	0.2902	0.8612	0.1079

5 Concluding remarks

In this paper, we considered an infinite-horizon endogenous growth model of the environment, and explored the implications of the second-best (Ramsey) fiscal policy pursued by an optimising government, highlighting in the process a key role played by the elasticity of intertemporal substitution. In our set-up, the representative household-producer derived utility from private consumption and environmental quality, externalities in production were generated by public infrastructure, and environmental pollution was a by-product of output. The income taxes generated by the government were used to provide infrastructural and environmental benefits to the citizens, while balancing its budget, and it chose its policy instruments to maximise agents' welfare. Our methodology enabled us to derive the government's Ramsey policy where the utility function is isoelastic (and, therefore, the elasticity of substitution is different from unity), unlike much of the literature which considers logarithmic utility functions. With plausible parameter values, we find that for cases where the elasticity is large, there exist two sets of optimising policy instruments (the income tax rate, and the ratio of infrastructural-to-total public spending) that maximize welfare in the economy, which result in policy indeterminacy, and different growth regimes. Here, a rise in environmental awareness unleashes both a direct and an indirect effect on welfare; the direct effect calling for a higher tax rate and an allocation of expenditures from infrastructure to the environment, and the indirect effect, which is the dynamic (growth-enhancing) channel, implying the opposite: as a result, both low-growth and high-growth equilibria are feasible. However, if the elasticity of substitution is small, the dynamic channel ceases to exist as the current sacrifice of environmental benefits is unlikely to be compensated via increased growth and, therefore, a unique equilibrium ensues. In this case, a Ramsey government can lower the tax rate and allocate spending towards infrastructure to bolster private and public savings to counter the effects of a falling growth rate. We also study the effects of an increase in environmental awareness. For large values of the EIS, we find that here, too, multiple equilibria emerge: the low (high) growth regime is associated with a high (low) tax rate and a relatively small (large) ratio of infrastructural-to-total spending. Our study has important implications for research on the choice of fiscal policy instruments in a growing economy where household utility is shaped by the quality of the environment.

Appendix A: Proof of Proposition 2

We first solve for τ the system of equations (25)-(28). After some algebra we obtain the implicit solution of Ramsey tax rate as a function of parameters:

$$\Omega\left[(s-\theta(\tau^R-[1-\frac{s}{\theta}](1-\alpha)))\left(\frac{\alpha(1-\tau^R)\Omega-\delta}{\frac{1-v}{v}\theta[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]}\right)+\frac{\alpha}{\sigma}(1-\tau^R)\right]-\frac{\rho}{\sigma}-\delta=0\equiv F$$

where $\Omega\equiv A^{\frac{1}{\alpha}}([1-\frac{s}{\theta}](1-\alpha))^{\frac{1-\alpha}{\alpha}}$.

Before analysing the properties of F we write down conditions that satisfy positive solutions for the endogenous variables. In particular,

Conditions for positive $\tilde{c}$ and $\tilde{x}$.

In order for (27) to hold for positive values of $\tilde{c}$ and $\tilde{x}$, $\alpha(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}}>\delta$ has always to hold. Then, from (27) for $\tilde{x}>0$ (and $\tilde{c}>0$) we have that:

If $a < \sigma$, then $\tilde{x} > 0$, as $(1-\frac{\alpha}{\sigma})(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} + \frac{\rho}{\sigma} > 0$

If $a > \sigma$, then for $\tilde{x} > 0$ we need that $(1-\frac{\alpha}{\sigma})(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} < \frac{\rho}{\sigma}$, which also guarantees $\tilde{c} > 0$.

Thus, if $a < \sigma$, then $\tilde{x} > 0$ and $\tilde{c} > 0$

If $a > \sigma$, then for $\tilde{x} > 0$ and $\tilde{c} > 0$, we need $(1-\frac{\alpha}{\sigma})(1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} < \frac{\rho}{\sigma}$.

Properties of function F:

1. From the above conditions, if $\tilde{c} > 0$ and $\tilde{x} > 0$, then F is a continuous function in $\tau \in (0,1)$.

2. $\lim_{\tau^R \to 0} F = \Omega\left[\frac{(s+\theta([1-\frac{s}{\theta}](1-\alpha)))v}{(1-v)\theta}\left(\frac{\alpha\Omega-\delta}{[(1-\frac{\alpha}{\sigma})\Omega+\frac{\rho}{\sigma}]}\right)+\frac{\alpha}{\sigma}\right]-\frac{\rho}{\sigma}-\delta$

3. $\lim_{\tau^R \to 1} F = -\Omega\left[\frac{(s-\theta(1-[1-\frac{s}{\theta}](1-\alpha)))v}{(1-v)\theta}\left(\frac{\delta\sigma}{\rho}\right)\right]-\frac{\rho}{\sigma}-\delta$

4. $$\frac{\partial F}{\partial \tau^R} = -\underbrace{\left\{\frac{\Omega v}{1-v}\left[\left(\frac{\alpha(1-\tau^R)\Omega-\delta}{[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]}\right)\right]\right\}}_{positive}-$$
$$-\left\{\Omega\left[(s-\theta(\tau^R-[1-\tfrac{s}{\theta}](1-\alpha)))\frac{\Omega v}{(1-v)\theta}\left(\frac{\alpha\frac{\rho}{\sigma}+\delta(1-\frac{\alpha}{\sigma})}{[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]^2}\right)\right]\right\}-\frac{\alpha}{\sigma}\Omega$$

5. $$\frac{\partial^2 F}{\partial(\tau^R)^2} = -\underbrace{\left\{\frac{\Omega v}{1-v}\left[\left(\frac{\alpha\frac{\rho}{\sigma}+\delta(1-\frac{\alpha}{\sigma})}{[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]^2}\right)\right]\right\}}_{<0}-$$
$$-\left\{\Omega\left[(s-\theta(\tau^R-[1-\tfrac{s}{\theta}](1-\alpha)))\Omega\frac{1}{\frac{1-v}{v}\theta}\left(\frac{\alpha\frac{\rho}{\sigma}+\delta(1-\frac{\alpha}{\sigma})}{[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]^2}\right)\right]\right\}-$$
$$-\left\{\Omega\left[(s-\theta(\tau^R-[1-\tfrac{s}{\theta}](1-\alpha)))\Omega\frac{1}{\frac{1-v}{v}\theta}\left(\frac{\alpha\frac{\rho}{\sigma}+\delta(1-\frac{\alpha}{\sigma})}{[(1-\frac{\alpha}{\sigma})(1-\tau^R)\Omega+\frac{\rho}{\sigma}]^2}\right)\right]\right\}$$

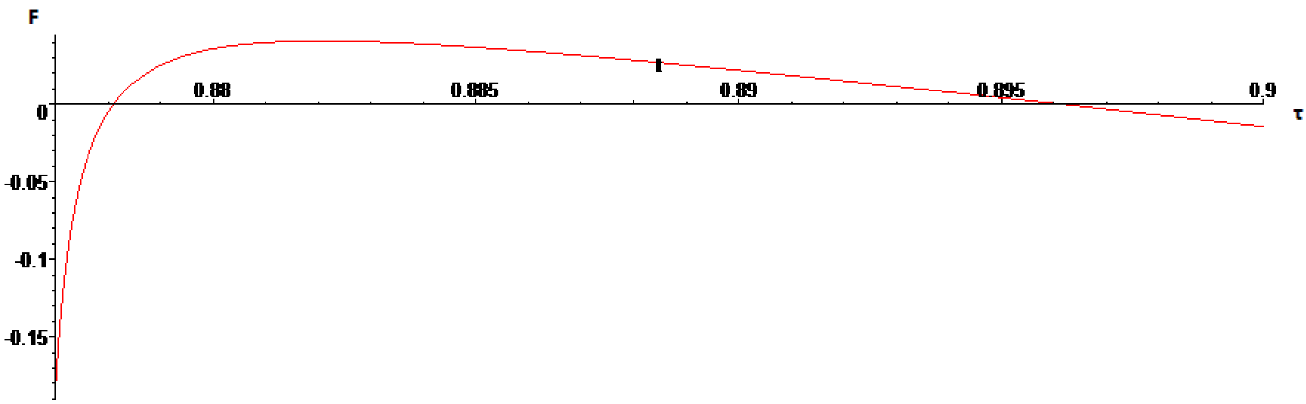
Case 1: $a < \sigma$ and $\gamma > \delta$.

If $a < \sigma$ and $\gamma = (1-\tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} - \tilde{c} > \delta$, then a positive $\tilde{x}$ exists if $(s-\theta(1-b^R)\tau^R) > 0$ (see, 26). In this case all three components of $\frac{\partial F}{\partial \tau^R}$ are negative implying $\frac{\partial F}{\partial \tau^R} < 0$, which shows that F is a monotonically decreasing function within the defined domain of τ^R. In turn, if an equilibrium exists it is unique. Thus, this is a sufficient condition for uniqueness of equilibrium.

Case 2: If $\alpha > \sigma$ and $\gamma > \delta$ and $\delta > \check{\delta} \equiv -\frac{\alpha\frac{\rho}{\sigma}}{(1-\frac{\alpha}{\sigma})}$.

Then, the sign of the derivative of $\frac{\partial F}{\partial \tau^R}$ is ambiguous and, in turn, non-monotonic. In this case there can exist multiple solutions for τ^R. The following graph illustrates the graphical solution of F using the parameter values of Table 1.

Figure 1: Existence of Multiple Equilibria



Parameter values: $\alpha = 0.5$, $A = 1.8$, $\delta = 0.015$, $\rho = 0.04$, $\theta = 1$, $s = 0.5$, $\sigma = 0.1$, $v = 0.1$

Appendix B: Stability Analysis

In this section we conduct stability analysis of the dynamic system (20) and (25) by evaluating the determinant of the system with the endogenous steady state solution of the Ramsey equilibrium given by the system of equations (28)-(31).

Linearizing the system comprised by (20) and (25) we obtain the following Jacobian matrix

$$\begin{bmatrix} \dot{c} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} J_{cc} & J_{cx} \\ J_{xc} & J_{xx} \end{bmatrix} \begin{bmatrix} c - \tilde{c} \\ x - \tilde{x} \end{bmatrix}$$

where the elements of the Jacobian matrix, J, evaluated at the long run are:

$J_{cc} \equiv \frac{\vartheta \dot{c}}{\vartheta c} = 2\tilde{c} - [(1 - \frac{\alpha}{\sigma})(1 - \tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} + \frac{\rho}{\sigma}]$

$J_{cx} \equiv \frac{\vartheta \dot{c}}{\vartheta x} = 0,$

$J_{xc} \equiv \frac{\vartheta \dot{x}}{\vartheta c} = -\tilde{x} < 0$

$J_{xx} \equiv \frac{\vartheta \dot{z}}{\vartheta x} = 2[s - \theta(1 - b^R)\tau^R]A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}}\tilde{x} + (1 - \tau^R)A^{\frac{1}{\alpha}}(b^R\tau^R)^{\frac{1-\alpha}{\alpha}} - \tilde{c} - \delta$

The determinant of J, $det(J) = J_{cc}J_{xx} - J_{cx}J_{xc}$ has ambiguous sign. Due to the complexity involved in the computation of these signs, we provide numerical results for the determinant of the Jacobian matrix. The following tables (T1 and T2) are extensions of Table 1 and Table 2, respectively, reporting additionally the values of $\tilde{c}$, $\tilde{x}$ and $det(J)$. Our findings, using plausible parameter values, illustrate that we obtain two equilibria that are both stable, as confirmed by the negative sign of the determinant.

Table T1. Changes in σ and the Ramsey Equilibrium

σ	b^R	τ^R	γ	$\tilde{c}$	$\tilde{x}$	$det(J)$
0.1	(0.284)0.278	(0.878) 0.896	(0.0937)0.0206	(0.005)0.063	(0.758)0047	$(-0.00039) - 0.00035$
0.3	(0.331) 0.282	(0.753) 0.886	(0.199)0.0203	(0.0001)0.0718	(68.20)0.048	$(-0.00002) - 0.00038$
0.7	0.288	0.867	0.020	0.087	0.049	-0.0004
0.9	0.291	0.858	0.019	0.095	0.0493	-0.00041

Table T2: Changes in $1-v$ under Multiple Equilibria ($\sigma = 0.15$)

2a "low" growth regime

$1-v$	b^R	τ^R	γ	$\tilde{c}$	$\tilde{x}$	$det(J)$
0.7	0.2827	0.8841	0.0461	0.0476	0.2870	-0.0014
0.8	0.2807	0.8904	0.0289	0.0597	0.1228	-0.0008
0.9	0.2797	0.8936	0.0205	0.0656	0.0475	-0.0003

2b "high" growth regime

$1-v$	b^R	τ^R	γ	$\tilde{c}$	$\tilde{x}$	$det(J)$
0.7	0.2868	0.8716	0.0799	0.024	0.6588	-0.0015
0.8	0.2890	0.8647	0.0984	0.011	0.8978	-0.0009
0.9	0.2902	0.8612	0.1079	0.004	1.031	-0.0004

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