Motivating over Time: Dynamic Win Effects in Sequential Contests*

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Abstract

In this paper we look at motivation over time by setting up a dynamic contest model where winning the first contest yields an advantage in the second contest. The win advantage introduces an asymmetry into the competition that we find reduces the expected value to the contestants of being in the game, whilst it increases the efforts exerted. Hence it may seem that a win advantage is advantageous for an effort maximizing contest designer, whereas in expectation it will not be beneficial for the players. We also show that the principal should distribute a majority of the total prize mass to the second contest. With *ex ante* asymmetry, the effect of the win advantage on the effort in the second contest depends on how disadvantaged the laggard is. A large disadvantage at the outset implies that as the win advantage increases, total effort for the disadvantaged firm is reduced as the discouragement effect dominates the catching-up effect. If the inital disadvantage is not significant, then the catching-up effect dominates and the laggard increases its total effort.

Keywords: dynamic contest, win advantage, prize division

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1 Introduction

Many contest situations have the features that (i) contestants meet more than once, (ii) winning in early rounds gives an advantage in later rounds, and (iii) the prize structure is such that the prize value in each stage may differ. In this paper, we set up a model to study such contest situation.

A crucial insight from our analysis is that the win advantage in an early round introduces an asymmetry into the subsequent competition that we show reduces the expected value to the contestants of being in the game, whilst it increases the efforts exerted. Hence, a win advantage is advantageous for an effort-maximizing contest designer, whereas in expectation it will not be beneficial for the players. We also show that, with constant returns to effort, the principal should distribute all of the prize mass to the second contest to maximize total effort across the two contests. With decreasing returns to effort and two players we find that the lowest amount of the prize that will be left for the second contest is 81.6% of the total prize mass.

When we introduce *ex ante* asymmetry, the effect of the win advantage on the effort in the second contest depends on how disadvantaged the laggard firm is. A large disadvantage at the outset implies that, as the win advantage increases, total effort for the disadvantaged firm is reduced as the discouragement effect dominates the catching-up effect. If the initial disadvantage is small, on the other hand,then the catching-up effect dominates and the laggard firm increases its total effort.

There are numerous applications, both in business, in politics, in economics, and related to motivation in general, that have features resembling such contest situations. Consider for example an R&D contest, where the winner is awarded a patent, and where the major round of contest is preceded by a series of smaller rounds of research prizes distributed by a national research council or the like. In a quite different setting, students are subject to a number of tests throughout the year while still the final ranking may be based on an exam in the end. In an organization, promotion games may have the same multi-stage structure, and in many sports, teams meet repeatedly throughout the season.

Competitive situations provide the winners with various types of rewards, such as financial rewards and "laurels" from winning sports or other forms of competitions. It is of interest to investigate the effects that these rewards may have on the teams' or individuals' motivation, as "..Reward is one of the most important influences shaping behavior." (Elliott, *et al.*, 2000, p. 6159). Motivational aspects of competition, and in particular, in relation to intrinsic motivation, is dealt with by Reeve, *et al.* (1985) and Vansteenkiste and Deci (2003) from a behavioural perspective. In direct competition, winning affirms competence, whereas losing leads to demotivation. The experimental study of Reeve, *et al.* (1985) shows that winners feel more competent than losers, and that winning facilitates competitive performance and contributes positively to an individual's intrinsic motivation. The experimental study by Vansteenkiste and Deci (2003) confirms the findings of Reeve *et al.* (1985). They study the effects of a reward contingent on winning a given competition on the intrinsic motivation of the individuals involved, and they show that winners are more intrinsically motivated than losers. The higher intrinsic motivation of winning may be interpreted as providing an advantage for later stage games. Elliott, *et al.* (2000) investigate the physiological effects of financial rewards and penalties using functional MRI-scanning while the subjects were engaged in a series of gambling tasks, and the subjects' motivational responses to rewards was significant. The financial reward's effect on motivation is similar to the win advantage presented in the present model

Pullins (2001) provides a study of intrinsic and extrinsic motivation in sales force compensation. The design of firms' sales compensation schemes is a primary concern in businesses, and changes are often in response to increased emphasis on long-term relationships (Pullins, 2001). Contests are frequently used to provide motivation for a firm's sales force, and such sales contests exhibit many of the key features set out above. Casas-Arce and Martínez-Jerez (2009) consider tournaments over multiple periods with interim evaluations based on ranking that in effect introduces heterogenous ability in the context of a sales tournament. To win, one would need to outperform the advantage gained by the competitor. They show that having a strong lead in a contest weakens incentives to exert effort, whereas the laggard will increase his effort in an attempt to catch up as long as the performance gap is not too large. This is similar to our findings when the players are *ex ante* asymmetric. In our setting, however, *ex ante* symmetric players and dynamic win effects imply that the effort is increased in the first stage for both players, whereas the advantaged player in the second stage of the game will be encouraged to increase effort and the disadvantaged player will always reduce his effort.

Lim, *et al.* (2008) investigate how the optimal prize structure in sales contests should be, by use of two laboratorium experiments and a field experiment of a trained sales force. If we can assume that sales people are risk averse, Kalra and Shi (2001) show that the optimal prize structure would involve multiple winners. They furthermore show that prizes should be rank-ordered to provide incentives to exert more effort, as identical prizes would not provide the sales force with incentives to exert more effort than just to achieve the lowest rank that awards a prize. The empirical findings of Lim, *et al.* (2008) support the theoretical prediction of multiple prize winners, but demonstrate, contrary to the theoretical prediction, that there is no negative effect on sales effort for sales contests with identical prizes. Neither Lim, *et al.* (2008) nor Kalra and Shi (2001) consider dynamic win effects, and by introducing dynamics we show that the motivation to exert effort is affected by how the total prize mass is distributed over time.

Providing incentives to exert effort for innovation purposes is important to secure profitability in a number of industries, and the use of contests is well-known (see, *e.g.*, Terwiesch and Xu, 2008, and Boudreau, *et al.*, 2011). Boudreau, *et al.* (2011) analyze empirically innovation contests for the development of software and demonstrate that an increased number of rivals in a single contest reduces the incentives to exert effort for all. However, there is also an opposing effect of adding rivals, as this implies that more firms pursuing parallel paths make it more likely that at least one rival will find an "extreme-value solution". For more uncertain problems, the latter effect tends to dominate, and adding more rivals produces positive incentives for effort. For less uncertain problems the former effect dominates. This is in line with the theoretical predictions of, e.g., Terwiesch and Xu (2008). For both Terwiesch and Xu (2008) and Boudreau, *et al.* (2011), the focus is on using contests to provide incentives in innovation processes with uncertainty. There is no dynamic effect of winning in an early round of the game, but their results that increasing the number of contestants has a positive impact on innovation effort are similar to ours. However, as is demonstrated below, introducing dynamics shows that more players imply more effort exerted in the early stages of the game, and also that most of the total prize mass should be awarded in the final stage of the game. Ofek and Sarvary (2003) demonstrate that if the current (technology) leader is more able with respect to R&D (*innovative advantage*), then it invests more to retain its leadership position. The innovative advantage is similar to our dynamic win advantage. There is an inversely Ushaped relationship between the degree of the advantage and the R&D effort. If, however, the advantage is based on the leader's ability to increase the probability of success due to factors such as brand recognition or channel relations (*reputation advantage*), then the leader invests less than the followers and the followers' R&D effort is an inverse-U relationship relative to the degree of the reputation advantage. This may result in the follower catching up and becoming the leader.

The dynamic contest approach suggested in the present analysis is also relevant for studying effort choices in markets with network externalities and in standards battles. In such settings, the outcome in any given stage of the game is typically path-dependent. The winner of a standards battle or the chosen platform for, *e.g.*, information technology applications, will often have an advantage over alternative platforms and standards when it comes to competing over applications based on the standard or platform, or when competing for customers in different market segments (see, *e.g.*, Katz and Shapiro, 1986, and Li, 2005). In R&D competition where a prototype needs to be developed, the winner of the prototype contest will normally have an advantage when entering the contest for the final developed product, and having won the first market segment in a market with network externalities makes it easier to win the second market segment due to already extensive use of one's product in the first market.

In political-science applications, there is typically an incumbency advantage in elections (see, *e.g.*, Konrad, 2009), and Schrott (1990) shows that there is a significant impact on the election outcome of winning TV debates by investigating election results in West Germany in four different elections. Consequently, there is an advantage from winning the first contest (the TV-debate) when it comes to the second contest (the election).

The central features of the above examples are the dynamic nature of the competitive situations, the potential for introduction of player heterogeneity between the contest periods, and the different sizes of prizes across the various stages of the contests. The situations generally fit the label of a multi-stage battle, as described in Konrad (2009) and Konrad and Kovenock (2009). But they have features that distinguish them from other multi-stage battles covered in the literature so far. For example, both the race and the tug-of-war, described by Konrad (2009), are situations where the number of stages is endogenous, whereas we are interested in a setting with a given number of stages of contests. Closer to our setting are the iterating incumbency fights studied by, for example, Mehlum and Moene (2008), but they have an infinite number of periods and thus no final period for allocation of a big prize. The analysis of Meyer (1992) is particularly close in spirit to ours; she discusses how a manager may benefit from creating a bias in favour of an early winner in a sequential promotion game between employees.

Whereas we in the present analysis emphasize dynamic *win* effects, whereby an early win creates a later advantage, Grossmann and Dietl (2009) and Clark and Nilssen (2011) discuss dynamic *effort* effects, whereby early efforts create later advantages. In Clark, *et al.* (2012), we carry out an analysis related to this one, where each stage contains an all-pay auction with a win advantage, whereas the present analysis is based on a Tullock contest at each stage.

The paper is organized as follows: In section 2, we present the model. In section 3, we analyze the optimal prize structure. In section 4, we consider extensions by introducing more players, the case of decreasing returns to effort, and *ex ante* asymmetry. In section 5, we present some concluding remarks.

2 A simple model

Consider two identical players, 1 and 2, who compete with each other in two interlinked sequential contests. In each contest a prize of size 1 is on offer, and the players compete by making non-refundable outlays. In the first contest, the players have a symmetric marginal cost of effort $x_{i,1}, i = 1, 2$, and the winner is determined by a Tullock contest success function:[1]

$$p_{1,1}(x_{1,1}, x_{2,1}) = \frac{x_{1,1}}{x_{1,1} + x_{2,1}} \tag{1}$$

where $p_{1,1}(x_{1,1}, x_{2,1})$ is the probability that player 1 wins the prize in the first contest, making $p_{2,1}(x_{1,1}, x_{2,1}) = 1 - p_{1,1}(x_{1,1}, x_{2,1})$ the probability that player 2 wins it. The linkage between the two contests occurs via a win advantage that reduces the marginal cost of effort of the first-contest winner in the second contest to $a \in (0, 1]$; the smaller is a, the larger is the win advantage. The loser of the first contest continues to the second contest with a marginal cost of effort of 1.

Only efforts in the second contest determine the winner of the second contest prize, according to the same rule as in (1). Denote by $x_{1,2}(i)$ and $x_{2,2}(i)$ the efforts of player 1 and 2 in the second period given that player i has won the first contest. Based on these efforts, the probability that player 1 wins the second contest is

$$p_{1,2}(x_{1,2}(i), x_{2,2}(i)) = \frac{x_{1,2}(i)}{x_{1,2}(i) + x_{2,2}(i)} \tag{2}$$

The players determine their efforts in each contest as part of a subgame perfect Nash equilibrium in which their aim is to maximize own expected payoff. The model is solved by backward induction starting with contest 2 in which the expected payoffs are given by

$$\begin{aligned}
\pi_{i,2}(i) &= \frac{x_{i,2}(i)}{x_{i,2}(i) + x_{j,2}(i)} - a x_{i,2}(i), i = 1, 2, j \neq i \\
\pi_{j,2}(i) &= \frac{x_{j,2}(i)}{x_{i,2}(i) + x_{j,2}(i)} - x_{j,2}(i)
\end{aligned}$$

An internal equilibrium is characterized by the following first-order conditions for the first-contest winner and loser:

$$\begin{aligned}
\frac{\partial \pi_{i,2}(i)}{\partial x_{i,2}(i)} &= \frac{x_{j,2}(i)}{(x_{i,2}(i) + x_{j,2}(i))^2} - a = 0 \\
\frac{\partial \pi_{j,2}(i)}{\partial x_{j,2}(i)} &= \frac{x_{i,2}(i)}{(x_{i,2}(i) + x_{j,2}(i))^2} - 1 = 0
\end{aligned}$$

yielding equilibrium efforts in the second contest of[2]

$$x^*_{i,2}(i) = \frac{1}{(1+a)^2} \tag{3}$$

$$x^*_{j,2}(i) = \frac{a}{(1+a)^2} \tag{4}$$

[1] As axiomatized by Skaperdas (1996) and used in numerous contest applications; see, for example, Konrad (2009).

[2] A * denotes equilibrium value.

Hence, the winner of the first contest becomes more efficient at exerting effort in the second contest, and exerts more effort than the rival. This leads to a larger than one-half chance of winning the second contest, and the more efficient player also has a larger expected payoff:

$$\pi^*_{i,2}(i) = \left(\frac{1}{1+a}\right)^2 \tag{5}$$

$$\pi^*_{j,2}(i) = \left(\frac{a}{1+a}\right)^2 \tag{6}$$

At the beginning of the first contest, each player has an expected payoff of

$$\pi_{1,1} = p_{1,1}(1+\pi_{1,2}(1)) + (1-p_{1,1})\pi_{1,2}(2) - x_{1,1} \tag{7}$$

$$\pi_{2,1} = (1-p_{1,1})(1+\pi_{2,2}(2)) + p_{1,1}\pi_{2,2}(1) - x_{2,1} \tag{8}$$

The expected payoff from the first contest consists of three elements: (i) the probability that a player wins the first contest multiplied by the prize for the first contest, plus the expected profit from the second contest having won the first; (ii) the probability of losing the first contest multiplied by the expected payoff from the second contest having lost the first; and (iii) the first-period cost of effort. Seen from the first period, the players solve identical maximization problems. Writing out the expected payoff for player 1 in full, using (1),(3),(4),(5), and (6), gives

$$\begin{aligned}\pi_{1,1} &= \frac{x_{1,1}}{x_{1,1}+x_{2,1}}\left[1+\left(\frac{1}{1+a}\right)^2\right] + \left(1-\frac{x_{1,1}}{x_{1,1}+x_{2,1}}\right)\left(\frac{a}{1+a}\right)^2 - x_{1,1} \\ &= \left(\frac{a}{1+a}\right)^2 + \frac{x_{1,1}}{x_{1,1}+x_{2,1}}\left(\frac{2}{1+a}\right) - x_{1,1}\end{aligned} \tag{9}$$

Differentiating this expression with respect to the choice variable of player 1, $x_{1,1}$, gives

$$\frac{\partial \pi_{1,1}}{\partial x_{1,1}} = \frac{x_{2,1}}{(x_{1,1}+x_{2,1})^2}\left(\frac{2}{1+a}\right) - 1$$

At an interior symmetric equilibrium, we have $x^*_{1,1} = x^*_{2,1} \equiv \chi$, where χ solves

$$\frac{\chi}{(2\chi)^2}\left(\frac{2}{1+a}\right) - 1 = 0$$

implying

$$\chi = \frac{1}{2(1+a)},$$

so

$$x^*_{1,1} = x^*_{2,1} = \frac{1}{2(1+a)}. \tag{10}$$

Using (10) in (9), we find that the total expected value of the two-contest game to each player is

$$\pi^*_{1,1} = \pi^*_{2,1} = \frac{1}{2}\left(1 - \frac{a\,(1-a)}{(1+a)^2}\right). \tag{11}$$

Denote total expected efforts in equilibrium as X_1^* for contest 1 and X_2^* for contest 2. These magnitudes are given by

$$\begin{aligned}
X_1^* &= x_{1,1}^* + x_{2,1}^* = \frac{1}{1+a} \\
X_2^* &= 2\left[p_{1,1}\left(x_{1,1}^*, x_{2,1}^*\right) x_{1,2}^*(1) + (1 - p_{1,1}\left(x_{1,1}^*, x_{2,1}^*\right))x_{1,2}^*(2)\right] \\
&= \frac{1}{1+a} \\
X_1^* + X_2^* &= \frac{2}{1+a}
\end{aligned}$$

Total expected effort over the two contests is decreasing in a so that the larger the win advantage (*i.e.*, the smaller a) the more effort can be expected by each player. When $a = 1$, the model reverts to being of two independent Tullock contests each over a prize of size one, with marginal cost of effort equal to one for each contestant in each period. In this case, expected effort for each player is $\frac{1}{4}$ in each period. When a decreases below 1, an asymmetry is introduced into the second period contest, since one player will be advantaged in relation to the other. This encourages the advantaged player to increase effort in the second contest, whilst the disadvantaged player slackens off to save effort cost. In the first contest, the players compete not only for the prize at that contest stage, but also the right to be the advantaged player in contest two. Hence efforts in contest one are increased compared to the level that would occur with two independent contests. Notice that the expected efforts in each contest are equal.

Whilst the relationship between the win advantage a and total expected effort in equilibrium is monotonic, the same is not the case for the total expected value of the two contest games given in (11) and graphed in Figure 1.

Figure 1 here

With two independent contests ($a = 1$) each contest would have a value of $\frac{1}{4}$ to each player, and hence $\frac{1}{2}$ in total per player. It is apparent that introducing a win effect in the first contest will initially cause the expected value of the contest to fall for high values of a (i.e. a small win advantage), and that this value will increase as the win advantage becomes larger (smaller values of a). From (11) it is easy to verify that $\pi_{1,1}^*$and $\pi_{2,1}^*$ reach a minimum value of $\frac{7}{16}$ at $a = \frac{1}{3}$. As the win advantage becomes bigger, total expected effort increases as discussed above; why then does the expected value initially decrease and then increase as the win advantage gets larger? The explanation lies in the total expected cost of effort. When a is small, there is a large advantage to winning the first contest and this winner exerts a lot of effort in contest two. However, the effort cost of this player decreases as a gets smaller. The total expected cost of effort for each player is given as

$$\begin{aligned}
EC_1^* &= EC_2^* = x_{1,1}^* + p_{1,1}\left(x_{1,1}^*, x_{2,1}^*\right) a x_{1,2}^*(1) + (1 - p_{1,1}\left(x_{1,1}^*, x_{2,1}^*\right))x_{1,2}^*(2) \\
&= \frac{1}{2}\left(1 + \frac{a\,(1-a)}{(1+a)^2}\right),
\end{aligned}$$

and is also depicted in Figure 1. This expression reaches a maximum at $a = \frac{1}{3}$; as a is initially reduced from 1, the extra effort induced by the first contest winner occurs at quite a high cost, and as a falls further the extra effort costs less and less at the margin, until the cost effect dominates and more effort actually costs less.

The win advantage introduces an asymmetry into the competition that reduces the expected value to the contestants of being in the game, whilst it increases the efforts exerted. Hence it may seem that a win advantage is advantageous for an effort maximizing contest designer, whereas in expectation it will not be beneficial for the players. A winning experience might in this sense be thought of as negative, although the player that actually ends up winning the first round will have an increase in expected payoff in the second contest.

3 Optimal prize distribution

Given that the contests are interconnected by a win advantage effect, a contest designer might wish to exploit this in order to maximize effort. Whilst the amount of the win advantage will probably be out of the designer's control, another variable is at his disposal, namely the prize distribution between the two contests. In the previous section, the prize was assumed to be distributed in two equal amounts. Here, we consider a designer (principal) who wishes to maximize expected total effort in the contest by choosing a distribution of the total prize mass across the two contests. To maintain comparability with the previous section, we assume that the principal distributes a total prize mass of 2, saving an amount M for the second contest and awarding a prize of $2-M$ in the first.

With this prize distribution, the expected payoffs in the second contest will be

$$\begin{aligned}
\pi_{i,2}(i) &= \frac{x_{i,2}(i)}{x_{i,2}(i)+x_{j,2}(i)}M - a x_{i,2}(i), i=1,2, j\neq i \\
\pi_{j,2}(i) &= \frac{x_{j,2}(i)}{x_{i,2}(i)+x_{j,2}(i)}M - x_{j,2}(i)
\end{aligned}$$

Straightforward calculations give the following equilibrium values for efforts and payoffs in the second contest (equilibrium denoted by $\widehat{\ }$):

$$\begin{aligned}
\widehat{x}_{i,2}(i) &= \frac{1}{(1+a)^2}M \\
\widehat{x}_{j,2}(i) &= \frac{a}{(1+a)^2}M \\
\widehat{\pi}_{i,2}(i) &= \frac{1}{(1+a)^2}M \\
\widehat{\pi}_{j,2}(i) &= \left(\frac{a}{1+a}\right)^2 M
\end{aligned}$$

At the beginning of the first contest, each player maximizes:

$$\begin{aligned}
\pi_{1,1} &= \frac{x_{1,1}}{x_{1,1}+x_{2,1}}\left(2-M+\frac{1}{(1+a)^2}M\right) \\
&\quad + \left(1-\frac{x_{1,1}}{x_{1,1}+x_{2,1}}\right)\left(\frac{a}{1+a}\right)^2 M - x_{1,1} \\
&= \left(\frac{a}{1+a}\right)^2 M + 2\frac{x_{1,1}}{x_{1,1}+x_{2,1}}\left(1-\frac{a}{1+a}M\right) - x_{1,1}
\end{aligned}$$

Equilibrium values for the first contest can easily be found to be[3]

$$\begin{aligned}\widehat{x}_{1,1} &= \widehat{x}_{2,1} = \frac{1}{2}\left(1 - \frac{a}{1+a}M\right), \text{ and} \\ \widehat{\pi}_{1,1} &= \widehat{\pi}_{2,1} = \frac{1}{2}\left(1 - \frac{a\,(1-a)}{(1+a)^2}M\right). \end{aligned} \tag{12}$$

Total effort in the contest is the sum of the efforts expended in contest one, $\Omega_1 = \widehat{x}_{1,1} + \widehat{x}_{2,1}$, and contest two, $\Omega_2 = \widehat{x}_{i,2}(i) + \widehat{x}_{j,2}(i)$, so that

$$\begin{aligned}\Omega_1 &= 1 - \frac{a}{1+a}M \\ \Omega_2 &= \frac{1}{1+a}M \\ \Omega_1 + \Omega_2 &= 1 + \frac{1-a}{1+a}M\end{aligned}$$

Note that effort in contest one is decreasing in M (the second round prize), while effort in contest two increases in it. The latter effect is the stronger as can be seen from the expression for the sum of efforts, which is always increasing in M. Hence, to maximize total effort in the two contests, the principal should distribute all of the prize mass in the second contest. In the first contest, participants compete to be the advantaged player in the second contest with no instantaneous prize. In the second contest, an asymmetry is introduced which would tend to reduce effort compared to a symmetric contest, and to mitigate this effect the principal should award a large prize here. This prize distribution, however, minimizes the expected value of the two-contest game for the players as is seen from (12).

4 Extensions

In this section, we consider extensions to the basic model, where we first analyze the effect of the number of competitors on the results, then consider decreasing returns to effort in the contests, and finally the role played by the assumption that contestants are symmetric at the outset.

4.1 More players

Extending the basic model to the case of $n \geq 2$ participants is straightforward. For the case of $M = 1$, the following equilibrium values can be determined (with calculations in the Appendix) for the expected value of the game to each player ($\pi_s^*(n), s = 1, ..., n$) and the expected total cost of effort to each player ($EC_s^*(n)$):

$$\pi_s^*(n) = \frac{2}{n^2}\left[1 - a\,(1-a)\left(\frac{n-1}{n+a-1}\right)^2\right]; \tag{13}$$

$$EC_s^*(n) = \frac{2}{n^2}\left[(n-1) + a\,(1-a)\left(\frac{n-1}{n+a-1}\right)^2\right]. \tag{14}$$

[3] Note that equilibrium efforts are non-negative, since $a \leq 1$ and $M \leq 2$.

A similar picture emerges as in the case of $n = 2$ with respect to how a affects these magnitudes. It is easily verified that $\pi_s^*(n)$ is convex in a, reaching a minimum at $a = \frac{n-1}{2n-1} \in \left[\frac{1}{3}, \frac{1}{2}\right)$, while $EC_s^*(n)$ is concave in a, reaching a maximum at the same value. One can also verify that $\frac{\partial \pi_s^*(n)}{\partial n} < 0$, so that more competitors reduce the expected value of the game to each player.

Total effort by all n competitors in contests 1 and 2 can be determined from (A1) through (A3) in the Appendix as

$$X_1^*(n) = \frac{n-1}{n+a-1}\left\{1 + (1-a)\frac{n-2}{n}\left[1 - \frac{a(n-1)}{n+a-1}\right]\right\}, \tag{15}$$

$$X_2^*(n) = \frac{n-1}{n+a-1}. \tag{16}$$

We have that $\frac{\partial X_1^*(n)}{\partial a} < 0$, and $\frac{\partial X_2^*(n)}{\partial a} < 0$, as is the case for $n = 2$ above. Further, $\frac{\partial X_1^*(n)}{\partial n} > 0$, and $\frac{\partial X_2^*(n)}{\partial n} > 0$, so that total effort increases in the number of competitors. Even though effort per player decreases in each period as more rivals are added, total effort increases since there are more players.

Comparing $X_1^*(n)$ and $X_2^*(n)$, we see that $X_1^*(n) > X_2^*(n)$ for $n > 2$, since the square-bracketed term in (15) is positive for any $a \in (0, 1]$. The case of $n = 2$ is thus special in that the contestants even out their efforts in each contest with an equal prize; this occurs since the game is completely symmetric and each player has a one-half chance in equilibrium of being the advantaged player in the second contest. Players exert more effort in the first contest when there are more than two competitors. The game is still symmetric, but it is rational to move effort to the first contest since, at a symmetric situation, the probability of winning the first contest is $\frac{1}{n}$, so that a unilaterally increased effort gives a larger chance of beating $n - 1$ rivals. This way, an asymmetry in the incentives between contests arises, causing effort to shift to the early contest.

Calculations in the Appendix show that, when the principal can divide the prize mass into $2 - M$ and M between the two contests, the resulting total expected effort from all n participants is linearly increasing in M, and hence a corner solution obtains in which $M = 2$ maximizes total expected effort. As in the case with $n = 2$, this minimizes the expected value of the game to the players.

4.2 Decreasing returns to effort

Suppose that, in the two contestant model, there are decreasing returns to effort in each contest, so that the probability that player 1 wins contest $t = 1, 2$ is given by

$$p_{1,t} = \frac{x_{1,t}^r}{x_{1,t}^r + x_{2,t}^r},$$

where $r \in (0, 1)$ represents the elasticity of the odds of winning. With this restriction on r, there are decreasing returns to effort in each contest. This means that a principal may now have some benefit from dividing the prize mass between the two contests, in contrast to the case of $r = 1$ (constant returns) analyzed above. With a prize mass division of $2 - M$ in the first contest, and M in the second, one can calculate the following expected total efforts in each contest in equilibrium (a $\widetilde{}$ represents equilibrium values):

$$\begin{aligned}\widetilde{X}_1 &= \frac{r}{4^{1-r}}\left(\frac{1+a^r}{1+a^r(1-M)}\right)^{1-2r}\\ \widetilde{X}_2 &= \frac{r(1+a)}{a^{1-r}(1+a^r)^2}M\end{aligned}$$

Note that $\widetilde{X}_1 > 0$ because $1+a^r(1-M) > 0$, since $a^r < 1$ and $M \leq 2$.

It is immediately apparent that total effort in contest 2 is linear in M. For total effort in contest 1 we have that

$$\frac{d\widetilde{X}_1}{dM} = \frac{r}{4^{1-r}}(1-2r)\frac{a^r}{1+a^r}\left(\frac{1+a^r}{1+a^r(1-M)}\right)^{2(1-r)}.$$

Since $1+a^r(1-M) > 0$, we have that $\frac{d\widetilde{X}_1}{dM} > 0$ for $r < \frac{1}{2}$. Furthermore,

$$\frac{\partial^2\widetilde{X}_1}{\partial M^2} = \frac{r}{2^{1-2r}}(1-r)(1-2r)\left(\frac{a^r}{1+a^r}\right)^2\left(\frac{1+a^r}{1+a^r(1-M)}\right)^{3-2r}$$

is positive for $r < \frac{1}{2}$.

This means that $\widetilde{X}_1 + \widetilde{X}_2$ is increasing and convex in M for $r \in \left(0, \frac{1}{2}\right)$, so that it is optimal to set $M = 2$ in this case to maximize expected effort. When $r = \frac{1}{2}$, $\widetilde{X}_1 + \widetilde{X}_2$ increases linearly in M and the same corner solution is optimal. For cases where $r \in \left(\frac{1}{2}, 1\right)$, we can have an interior solution for M characterized by the first-order condition

$$\frac{d\left(\widetilde{X}_1+\widetilde{X}_2\right)}{dM} = \frac{r}{4^{1-r}}(1-2r)\frac{a^r}{1+a^r}\left(\frac{1+a^r}{1+a^r(1-M)}\right)^{2(1-r)} + \frac{r(1+a)}{a^{1-r}(1+a^r)^2} = 0.$$

Solving for M gives the optimal choice of

$$\overline{M} = \frac{1+a^r}{a^r}\left[1-\frac{1}{2}\left(\frac{a(1+a^r)(2r-1)}{1+a}\right)^{\frac{1}{2(1-r)}}\right].$$

Some permissible parameter combinations of a and r give $\overline{M} > 2$, and hence the optimal choice of M for $r \in \left(\frac{1}{2}, 1\right)$ is

$$\widetilde{M} = \min\{\overline{M}, 2\}$$

The internal solution only obtains for high values of a. For example, for $a \in \left(0, \frac{3}{4}\right)$, we have that $\overline{M} > 2$, whilst for $a = \frac{4}{5}$, $\overline{M} < 2$ only for $r \in (0.69, 0.76)$. Further inspection of $\overline{M}$ shows that it reaches its minimum value in the limit case in which $a = 1$ and $r \to 1$, at $\overline{M} = 2 - \frac{1}{e} \approx 1.632$. We can conclude that this is the lowest amount of the prize that will be left to the second contest, implying that at least 81.6% of the total prize mass will be distributed in the second contest.

4.3 *Ex-ante* asymmetry

Suppose that player 1 has an initial cost advantage over player 2 at the beginning of contest one. Specifically, let the initial marginal cost of effort for player 1 be $y < 1$, falling to ay in the second contest if he wins the first. Player 2 has initial marginal cost of 1. Suppose further that a prize of 1 is available in each contest. The equilibrium efforts in the first contest are naturally no longer symmetric:

$$\begin{aligned}
x_{1,1} &= \frac{(1+2ay+y^2)\left[2(1+ay)-a^2(1-y^2)\right]^2}{(1+ay)^2(1+y)^2\left[2(1+ay)-(y+a^2)(1-y)\right]^2} \\
x_{2,1} &= y\frac{(1+2ay+y^2)\left[2(1+ay)-a^2(1-y^2)\right]^2}{(1+ay)^2(1+y)^2\left[2(1+ay)-(y+a^2)(1-y)\right]^2}.
\end{aligned}$$

The leader (player 1) exploits his initial advantage by having more effort in the first contest: $x_{1,1} > x_{2,1}$. Moreover, $\frac{dx_{1,1}}{da} < 0$, $\frac{dx_{1,1}}{dy} < 0$,and $\frac{dx_{2,1}}{dy} > 0$, while the sign of $\frac{dx_{2,1}}{da}$ is ambiguous (positive for small y, negative for large). Expected efforts in the second contest for the two players are given by

$$\begin{aligned}
Ex_{1,2} &= \frac{2-a^2+3ay+3a^2y^2+ay^3}{(1+ay)^2(1+y)\left[2(1+ay)-(y+a^2)(1-y)\right]}; \\
Ex_{2,2} &= \frac{a(2-a^2)+2a^2y+(1+a^3)y^2+2ay^3+y^4}{(1+ay)^2(1+y)\left[2(1+ay)-(y+a^2)(1-y)\right]}.
\end{aligned}$$

From this we have that $\frac{dEx_{1,2}}{da} < 0$, and $\frac{dEx_{1,2}}{dy} < 0$, whereas the effects of a and y on $Ex_{2,2}$ are ambiguous. The effects of a and y on the total effort of player 1 are thus monotonic, whereas the relationship for player 2 is more complicated. Let the sum of the efforts in the two periods for player 2 be given by z, where

$$\begin{aligned}
z = &\frac{y(1+2ay+y^2)\left[2(1+ay)-a^2(1-y^2)\right]^2}{(1+ay)^2(1+y)^2\left[2(1+ay)-(y+a^2)(1-y)\right]^2} \\
&+\frac{a(2-a^2)+2a^2y+(1+a^3)y^2+2ay^3+y^4}{(1+ay)^2(1+y)\left[2(1+ay)-(y+a^2)(1-y)\right]}
\end{aligned}$$

Figure 2 depicts the areas in which $\frac{\partial z}{\partial a}$ is positive and negative.

Figure 2 here

For low values of y, to the left of the curve in the Figure, player 2 is at a large disadvantage at the outset; in this area, when a falls, meaning that the winner of the first contest gains an even larger advantage in the second contest, player 2 reduces effort. The chances are large that player 1 will win the first contest, but player 2 might win leading to him evening out some of the initial disadvantage; this encourages effort. On the other hand, there is a large chance that player 1 will win the first contest, and as a falls, gain an even larger advantage in the second; this discourages effort. The latter effect dominates to the left of the line in Figure 2. For quite large values of y, player 2 is not so disadvantaged initially; if a falls in this area, this player will react by increasing total effort, enticed by the possibility of catching up player 1's initial, relatively small advantage.

5 Conclusions

We have analyzed a simple, dynamic contest, in which the winner of the first contest gains an advantage over the losing player in terms of reduced cost of effort in the second contest. The goal has been to shed light on an issue which is prevalent in a number of management, marketing, economics and political-science applications. Our results can add to the understanding of, among other things, how sales force compensation schemes should be designed to increase sales effort incentives, and how R&D contests should be designed to maximize effort. The research is also related to research in psychology on (intrinsic and extrinsic) motivation in competitive environments.

Our results should be of particular interest to personnel managers. Personnel such as sales force and many others are involved in situations of intense internal competition, where employees are measured against each other. As we show here, any gains to early winners in such internal competitions are to the advantage of the personnel manager. Furthermore, if the manager can put the main prize mass at later stages, this would maximize her benefit from the situation. This calls, for example, for the use of promotions in sales-force management: it is when there is a sense among the sales force that a promotion of one of them is the climax of a sales season, or any other period of intense internal competition with dynamic win advantages, that the sales-force manager gets the most out of her employees.

A Appendix: The case of $n \geq 2$ players

Using the n-player equivalent of (1) and (2), with a prize of 1 in each round and player i as the winner of the first contest, efforts in contest 2 for the winner of contest 1 and the $n-1$ losers are

$$\begin{aligned} x_{i,2}(i) &= \frac{n-1}{n+a-1}\left(1-\frac{a\,(n-1)}{n+a-1}\right); \qquad \text{(A1)} \\ x_{j,2}(i) &= \frac{a\,(n-1)}{(n+a-1)^2},\ j\neq i. \qquad \text{(A2)} \end{aligned}$$

Expected profits for contest 2 are then

$$\begin{aligned} \pi_{i,2}(i) &= \left(1-\frac{a\,(n-1)}{n+a-1}\right)^2 \\ \pi_{j,2}(i) &= \left(\frac{a}{n+a-1}\right)^2,\ j\neq i \end{aligned}$$

These values are then used in the expected profit for the first contest for player $s = 1, ..., n$

$$\pi_{s,1} = \frac{x_{s,1}}{x_{s,1}+\sum_{v\neq s} x_{v,1}}\left(1+\pi_{s,2}(s)\right)+\left(1-\frac{x_{s,1}}{x_{s,1}+\sum_{v\neq s} x_{v,1}}\right)\pi_{j,2}(s)-x_{s,1},\ j\neq s$$

Maximizing this expression with respect to $x_{s,1}$ and computing a symmetric equilibrium yield

$$x^*_{s,1} = \frac{n-1}{n\,(n+a-1)}\left[1+(1-a)\,\frac{n-2}{n}\left(1-\frac{a\,(n-1)}{n+a-1}\right)\right] \qquad \text{(A3)}$$

From these equations, the expressions in (13) and (14) in the text can be derived.

Suppose now that there are prizes of total value 2, divided into $2 - M$ and M. Given that i wins the first contest, efforts in contest 2 are

$$\begin{aligned} x_{i,2}(i) &= M\frac{n-1}{n+a-1}\left(1-\frac{a\,(n-1)}{n+a-1}\right), \text{ and} \\ x_{j,2}(i) &= M\frac{a\,(n-1)}{(a+n-1)^2},\ j \neq i, \end{aligned}$$

with total effort in contest 2 equal to

$$X_2 = M\frac{n-1}{n+a-1},$$

and expected payoffs in contest 2 equal to

$$\begin{aligned} \pi_{i,2}(i) &= M\left(1-\frac{a\,(n-1)}{n+a-1}\right)^2 \\ \pi_{j,2}(i) &= M\left(\frac{a}{n+a-1}\right)^2,\ j \neq i \end{aligned}$$

For player k, the expected profit at contest 1 is

$$\pi_{k,1} = \frac{x_{k,1}}{x_{k,1}+X_{-k,1}}(2-M+\pi_{k,2}(k)) + \left(1-\frac{x_{k,1}}{x_{k,1}+X_{-k,1}}\right)\pi_{k,2}(i) - x_{k,1},\ k \neq i,$$

where $X_{-k,1}$ is the total effort in contest 1 of player k's rivals. Maximizing this expression with respect to k's effort, using the continuation payoffs above, yields a symmetric equilibrium effort in the first contest per player of[4]

$$x_{s,1} = \frac{n-1}{n^2}\left\{2 - Ma\left[1+\frac{(1-a)\left((n-1)^2+a\right)}{(n+a-1)^2}\right]\right\}, s = 1, 2, ..., n \tag{A4}$$

Total effort in contest 1 is hence $X_1 = nx_{s,1}$. Aggregate efforts over both contests are

$$X_1 + X_2 = \frac{n-1}{n}\left(2 + M\frac{1-a}{(n+a-1)^2}\left[(1-a)\,n^2 - (1-4a)\,n - 2a\right]\right), \tag{A5}$$

which is linear in M. It is easily checked that the square-bracketed term in (A5) is positive for feasible values of n and a. Hence, total effort increases in M, and the effort-maximizing choice of this variable is $M = 2$, as claimed in the text. Inserting equilibrium efforts into the payoff functions of the players yields the equilibrium expected payoff

$$\Pi(M,n) = \frac{2}{n^2}\left[1 - Ma\,(1-a)\left(\frac{n-1}{n+a-1}\right)^2\right],$$

which is clearly decreasing in M.

[4]Note that the equilibrium effort in the first contest is non-negative, since $M \leq 2$ and the term in square brackets in (A4) is no greater than $\frac{1}{a}$ for feasible values of n and a.

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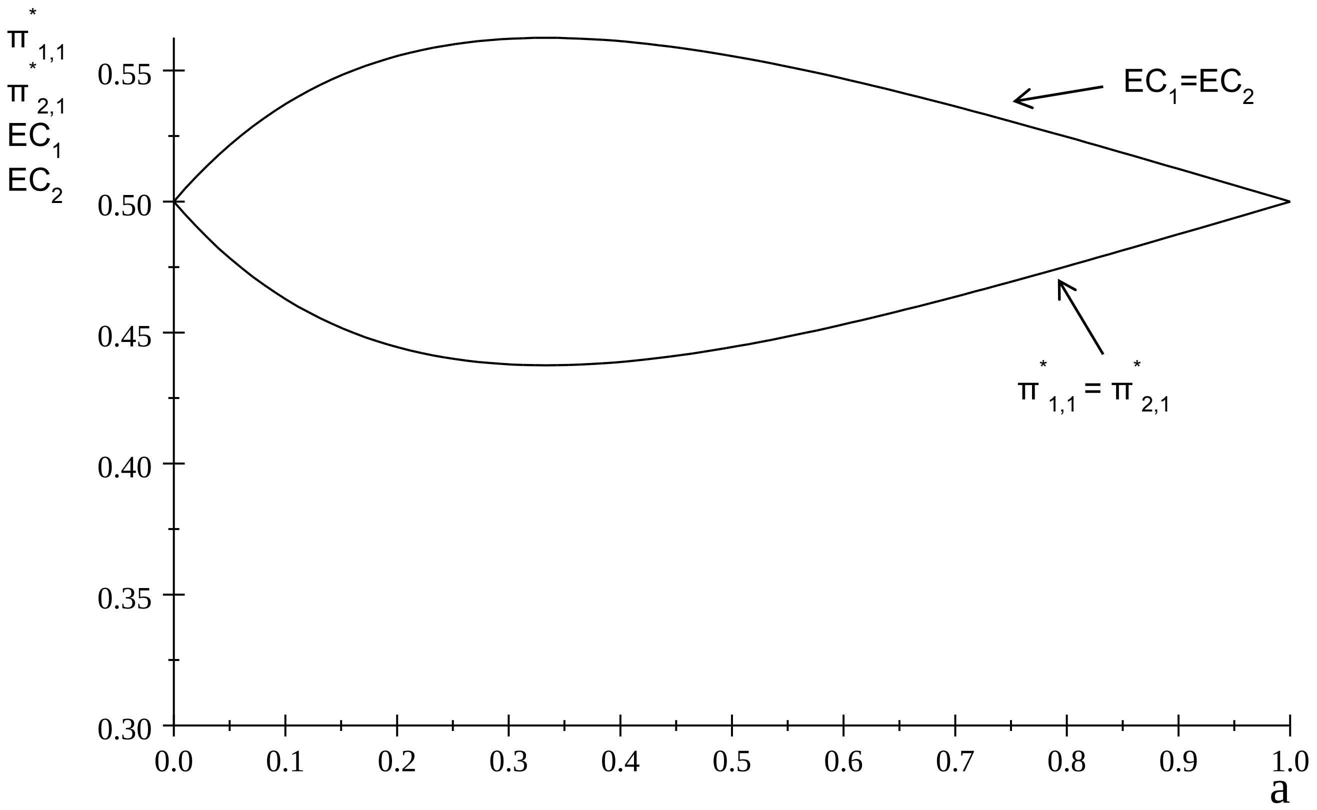

FIGURE 1

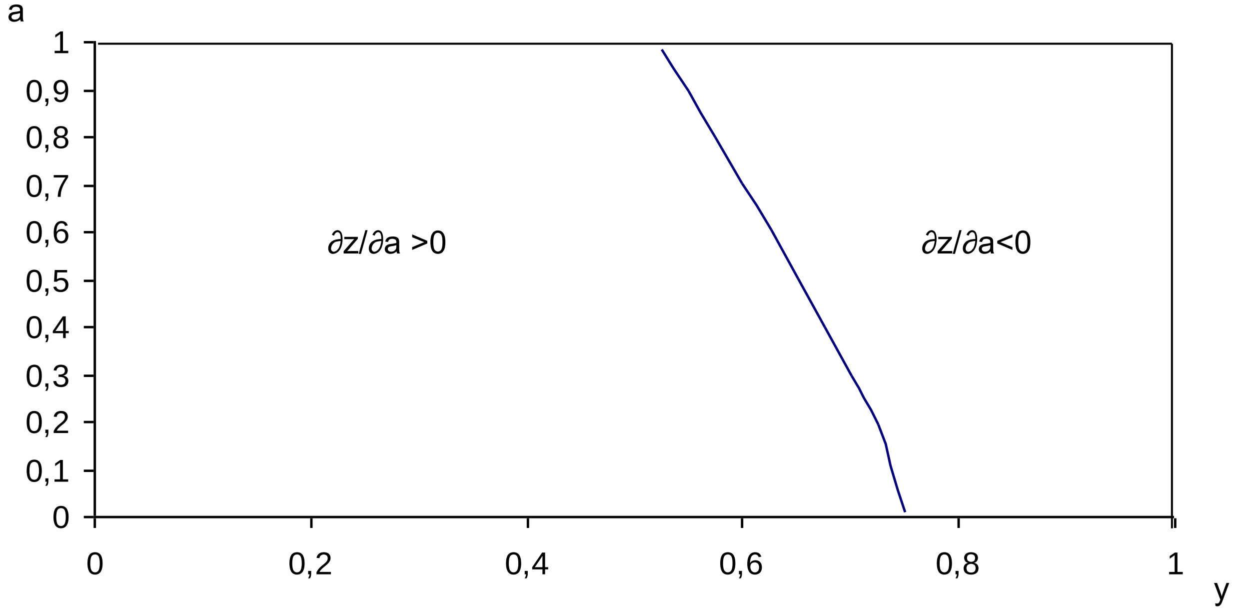

FIGURE 2