Optimal Privatization and Entry in a Differentiated Mixed Oligopoly

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Abstract

We address two questions. First, does the excess entry result of pure oligopoly hold when firms face a substitute good produced by a public firm? Second, what would be the optimal ownership of the public firm? We find that excess entry still occurs, but the excessiveness is largely mitigated due to the presence of the public firm. On the ownership of the public firm, we find that partial privatization need not always be optimal. Depending on the substitutability of the two products, the social optimum may involve one or more private firms, and full or partial public ownership.

Keywords: Partial privatization, Mixed oligopoly, Entry

JEL Codes: L13, L33, D43

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1 Introduction

This paper attempts to bridge a gap between the mixed oligopoly literature (see De Fraja and Delbono, 1989; Matsumura, 1998) and the excess entry literature (see Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). The mixed oligopoly literature studies social welfare under strategic interactions between a public firm and one or more private firms. The entry literature, on the other hand, studies the effect of free entry on social welfare under pure oligopolistic competition (involving private firms). However, so far there has been no attempt to integrate these two literatures. We make a modest attempt to study social optimality of entry in a mixed oligopoly involving two differentiated (substitute) products. In our set-up one product is produced by a public firm, and the other product is produced by several private firms. It is the entry in the second market and the degree of public ownership in the first market that we wish to study. Two questions particularly hold our interest. First, does the *excess entry* result (for quantity competition) hold in a mixed oligopoly? Second, what would be the *optimal public ownership* of the public firm? The answer to the first question is not obvious, because unlike in a pure oligopoly, here the government can influence the entry via partial public ownership, though the influence is only indirect due to product differentiation. Nevertheless, it seems that the government might be able to reduce the excessiveness of entry. But then it may have to divest or retain too much of its ownership in the public firm. This takes us to the second question, which has gained importance in recent works on mixed oligopoly, and also in the empirical literature on privatization (See Djankov and Murrell, 2002). The mixed oligopoly literature generally finds partial privatization to be optimal. In this paper, we develop a formulation that will provide a range of possibilities vis-a-vis ownership of the public firm and the optimal entry of private firms. We will see that partial privatization need not always be optimal.

The literature on optimal entry has examined whether more entry means greater social welfare. Mankiw and Whinston (1986) provided a unified framework to study social optimality of entry both under quantity and quality competition. Under quantity competition (with homogenous good) they noted that free entry produces excessive entry relative to the social optimum, a result previously noted by other authors under specific examples. Under quality competition, the result is ambiguous and sensitive to consumer preferences for quality. Suzumura and Kiyono (1987) generalized the excessive entry result for the homogenous good to non-Cournot setups by using conjectural variations. A key reason for excessive entry in these models is fixed cost of entry. However, Lahiri and Ono (1988) and later Ghosh and Saha (2007) have shown that even without fixed cost the excessive entry result holds if firms have asymmetric costs. Recently, this literature has been extended to vertical oligopoly. Kuhn and Vives (1999) have observed that excess entry in the downstream market makes vertical integration socially beneficial, and Ghosh and Morita (2007) have found that free entry produces *less* than socially optimal entry. However, there is room for reexamining the excessive entry result in the context of differentiated products.

The mixed oligopoly literature may be seen to have evolved around three themes of research. The first theme concerns the effect of public ownership on social welfare. De Fraja and Delbono (1989) raised this question in the context of a Cournot oligopoly involving one public firm and several private firms. They had shown that social welfare of a mixed oligopoly can be lower than that of a pure oligopoly (where all firms are private).[1] Several subsequent contributions reexamined this question by introducing production subsidy for private firms (White, 1997), or allowing foreign-owned private firms (Fjell and Pal, 1996), or assigning Stackelberg leadership to the public firm (Fjell and Heywood, 2004). The second theme concerns partial privatization. Fershtman (1990) considered a Cournot duopoly and showed that partial public ownership (up to a critical level) helps the public firm achieve higher profit than its rival private firm. Matsumura (1998) showed that social welfare of a mixed duopoly is maximized only if the

[1]The reason is that with increasing marginal cost, social welfare maximization requires evenly distributing outputs across (identical) firms to the extent that price equals marginal cost; but public ownership of one firm creates asymmetry in that distribution inflicting loss in social welfare. This loss, depending on the number of private firms, can be so great that a pure oligopoly would have performed better.

public firm is partially public. Since then, several studies have found partial privatization as the optimal policy. See for instance Bennett and Maw (2000) and Saha and Sensarma (2004).[2] But Matsumura and Kanda (2005) showed that the partial privatization result of Matsumura (1998) does not hold if free entry of private firms is allowed; in this case 'no privatization' is optimal. The third theme of research considers quality competition. Anderson *et al* (1997) adopted a Dixit and Stiglitz (1977) type of monopolistic competition model, with one public firm and several private firms, each producing only one variety.[3] They showed that transforming the public firm into a (fully) private firm leads to welfare loss in the short run (fixed entry), but welfare gains in the long run (free entry) through increase in varieties.[4] Some of the recent works have adopted the differentiated product approach, instead of monopolistic competition.[5] These are Fujiwara (2007), Bennett and Maw (2003) and Barcena-Ruiz and Garzon (2003). Of these the first two focus on optimal privatization, which is found to be partial again.[6]

Thus, it appears that partial public ownership produces greater welfare than both full public ownership and full privatization. This is true at least in those models where the public ownership is made endogenous. It is also the case that none of these models examine the social optimality of entry.[7] Furthermore, in all of these models, the public firm's objective is to maximize the aggregate social welfare. Essentially, there is no difference between the public firm's objective function and the social planner's objective function. While this is easily acceptable in a single good model, it is less convincing in a multiple goods model. Here, even

[2]For example, Bennett and Maw (2000) determined optimal public ownership needed to induce post-privatization investment, and Saha and Sensarma (2004) studied optimal divestment of a public bank to accommodate superior private competitors.

[3]Another approach is to take the spatial framework. See for instance Cremer *et al* (1991), Matsumura and Matsushima (2003) and Kumar and Saha (2008). In these articles the choice of location is the key question.

[4]This result holds provided the consumer preference for variety is not too weak.

[5]Recent authors have used a demand structure similar to the one utilized by Singh and Vives (1984). The underlying preference of the consumer is given by a quadratic utility function, while in Anderson *et al* it is a CES utility function.

[6]Fujiwara (2007) reexamines Anderson *et al* and arrives at partial privatization. In Bennett and Maw (2003), the purpose of privatization is to induce post-privatization, but their attention is restricted to a two-firm setup. Barcena-Ruiz and Garzon (2003) also consider two firms and two products, but they study merger incentives of these two firms, one of them being public.

[7]Matsumura and Kanda (2005) only touches on this.

though the public firm's activity is confined to only one market, it tries to maximize the sum of social welfare arising from all (related) markets. But in reality a public firm may not have the same objective as the government or the social planner. It may distance itself from the grand objective of the government and focus only on the market it is operating in.

In this paper we adopt a symmetric two-product setup, where one product is produced by the public firm, whose existence is given, while the other product is produced by several private firms. The government's objective is to maximize aggregate social welfare (combining the two markets), but the public firm's objective is to maximize its product-specific social welfare; *i.e.* the social welfare arising from the product it is producing. If it is partially public then it maximizes a weighted sum of this product-specific welfare and profit. First we determine the socially optimal level of entry (of private firms) and the degree of private ownership in the public firm. Here we see that if the degree of substituteness of the two products is below a critical level, then optimal public ownership is partial and the optimal number of private firms is greater than one. Greater the degree of substituteness (but still below this critical level), greater is the share of private ownership in the public firm, and greater is the number of private firms entering the second market. Here, we have a mixed oligopoly with mixed ownership in the public firm. However, above this critical degree of substituteness, if both products are to be produced, the number of private firms should be restricted to just one, and private ownership in the public firm will continually fall, and eventually we reach a situation where the public firm should be entirely public. This is the case of a mixed duopoly between a private and a fully public firm. This suggests that the existing results of partial privatization as social welfare maximizing strategy should not be taken for granted, especially in the context of differentiated products. Full public ownership can also be optimal. It should depend on the degree of substituteness as well as the objective function of the public firm.

We then compare with the free entry outcome. Following Mankiw and Whinston (1986) we note that within the second market where private firms are producing a homogenous good and are engaged in quantity competition, entry indeed creates a *business stealing effect.* But because of the presence of a substitute good, it also creates a *substitute good effect* running counter to the business stealing effect. However, the substitute good effect is outweighed by the business stealing effect and therefore entry remains privately optimal even when socially it is not. Thus, we have excessive entry. This is easily shown for any given level of public ownership in the other market. But in our equilibrium, the public ownership will not remain unchanged. The government will adjust the optimal privatization. Here two opposite conclusions emerge. When the government chooses public ownership simultaneously with firms' entry, it ends up with too much of privatization and thereby encourages too much of entry. Essentially privatization becomes reactive and accommodating to private entry. So the problem of excess entry exacerbates. But if the government privatizes prior to firm entry, it would choose a much smaller degree of privatization and pre-empt much of the excessive entry. Pre-emptive privatization helps to mitigate the excess entry problem to a significant extent. Thus, in this case the government is able to control entry indirectly by varying the ownership of a substitute good producing firm, which is not possible in a pure oligopoly.

To derive the social optimal entry and privatization we assumed, following the excess entry literature that entry requires incurring fixed cost and the government takes into account the aggregate costs of entry. However, in many industrial organization models, the government is often made to care about the producer surplus, instead of profits, in which case the private entry costs would not matter for the socially optimal entry. In such cases, whether free entry produces too much entry or too little entry depends on the size of the entry cost. If the entry cost is such that under social optimum private firms make losses (alternatively positive profit), then under free entry we will witness too little (alternatively excess) entry.

These results might be helpful in understanding some of the effects of privatization witnessed throughout the world over the last two decades. In developing countries the goal of attracting private investment often assumes greater importance, not just for direct employment benefits, but also for its indirect effects on related markets. For example, privatization of telecom services (especially landline services) can trigger growth in the mobile phone industry. For complementary goods also privatization can be beneficial. Edgell and Barquin(1995) have shown that privatization of the Mexican national airline, Aeromexico, and nineteen hotels boosted the tourism sector in Mexico. There is also a large literature that has studied effects of privatization on firm performance. Though in our model we do not allow technology to change, in reality, that is often the main reason for privatization. In some cases, even partial privatization led to significant improvement in firm performance and productivity. See Estrin and Rosevear (1999) for a study on Ukraine, Dong *et al* (2006) for China and Gupta (2005) for India.

The remainder of the paper is organized as follows. Section 2 states the basic model, and presents the analysis of socially optimal privatization and entry. Section 3 considers the case of free entry and contrasts our results with Mankiw-Whinston (1986). Section 4 extends the analysis to the case of producer surplus, and not profit, being part of social welfare. Section 5 concludes.

2 The model

We consider two markets. While the first market is served by a public (or partially privatized) monopoly which we call firm 1, the second market has oligopoly with quantity competition and no government presence. There are potentially n private firms in the second market. The ownership of firm 1 is divided into two parts; fraction $\theta \in [0, 1]$ is privately owned and fraction $1 - \theta$ is government-owned. Two markets are characterized by a pair of symmetric (inverse) demand functions

$$p_1 = \alpha - \delta Q_2 - \beta Q_1 \tag{1}$$

$$p_2 = \alpha - \delta Q_1 - \beta Q_2 \tag{2}$$

where $\delta > 0$ (substitute goods) and $\beta > \delta$; p_i and Q_i denote the market price and quantity of good i ($i = 1, 2$). Following Singh and Vives (1984) it can be shown that such demand curves can be derived from a strictly concave preference function $u(Q_1, Q_2) = \alpha(Q_1 + Q_2) - \frac{\beta}{2}(Q_1^2 + Q_2^2) - \delta Q_1 Q_2$. All firms have identical marginal cost c and $c < \alpha$.

Our game is as follows.

- **Stage 1:** The government decides on the degree of privatization θ and the number of firms n entering the second market. n firms enter the second market by incurring an entry cost F and firm 1 is privatized.

- **Stage 2:** Firms play Cournot.

We will also consider a variant of this game, where entry will be determined by the zero profit condition. In that case entry will take place either simultaneously with the choice of θ or sequentially after the choice of θ. For the sequential choice case, output competition will be moved to stage 3.

The objective of the government is to maximize $W = SW_1 + SW_2$, the sum of social welfare in the two markets. For firm 1 the objective is to maximize a weighted sum of profit and social welfare: $z = \theta\pi_1 + (1-\theta)SW_1$. The measure of social welfare is the sum of consumer surplus and profit. It is noteworthy that the public firm is concerned only with its market-specific social welfare instead of aggregate social welfare. We feel that while the government takes a wholistic view by considering the social welfare of the two markets, the public firm need not do so. This separation of the objective function is consistent with the separation of decision making. It is also a natural extension from the homogenous good case.[8]

[8] However, Fujiwara (2007) assigns aggregate social welfare maximization as the objective of the public firm in a differentiated good context.

2.1 Optimal privatization and entry

Before we proceed to solve the game it may be helpful to emphasize that in a mixed oligopoly the existence of one (partially or wholly) public firm is given, and therefore the entry question concerns only the rest of the industry. Further, by segregating the operations of the public firm and the private firms in two markets (or two products) we wish to isolate the effect of public ownership which will commonly affect the private firms, from the *business stealing* effect of entry which will be occurring mainly within the second market. Of course, for the first effect the degree of substitutability is important. Greater the degree of substitutability, greater the effect of public ownership. Once we determine the socially optimal privatization for the public firm and the socially optimal level of entry in the second market, we will be able to see the optimal configuration of the mixed oligopoly and compare it with the free entry situation. Moreover, it can also be ascertained whether the second market witnesses 'excessive entry' under free entry and whether the government can contain it through public ownership in the first market.

There is also an empirical motivation for this exercise. In a differentiated oligopoly of our kind, profitability of the second market critically depends on the price prevailing in the first market. Consider a special situation where the entry cost F is such that only a monopolist can break even in the second market when the first market is served by a wholly public firm. In this case, the government may undertake privatization with the objective of *creating* the second market.[9]

In what follows we first derive the socially optimal privatization (in the first market) and entry (in the second market), and then contrast it with the free entry case, analyzing in particular the issue of *excessive entry*.

We solve the game by backward induction. A typical firm in the second market (say the

[9] This would be a case where entry is assumed to be exogenous. Apart from creating a level playing field for private firms, a variety of other motives could be important, such as promoting private entrepreneurship, or expressing commitment to private investments as were seen in Eastern Europe, China and Latin America.

i^{th} firm) chooses output by maximizing its post-entry profit $\pi_{2i} = (p_2 - c)q_{2i}$. The individual and aggregate reaction functions are

$$q_{2i} = \frac{\alpha - \delta Q_1 - c}{\beta(n+1)}, \tag{3}$$

$$Q_2 = n\left[\frac{\alpha - \delta Q_1 - c}{\beta(n+1)}\right]. \tag{4}$$

For market 1 we maximize $z = \pi_1 + (1-\theta)CS_1 = (\alpha - \delta Q_2 - c)Q_1 - \beta(1+\theta)\frac{Q_1^2}{2}$ and derive firm 1's reaction function as

$$Q_1 = \frac{\alpha - c - \delta Q_2}{\beta(1+\theta)}. \tag{5}$$

Solving (4) and (5) we get

$$Q_1^* = (\alpha - c)\left[\frac{\beta m - \delta}{\beta^2(1+\theta)m - \delta^2}\right] \tag{6}$$

$$Q_2^* = (\alpha - c)\left[\frac{\beta(1+\theta) - \delta}{\beta^2(1+\theta)m - \delta^2}\right]. \tag{7}$$

where $m = \frac{(1+n)}{n}$.[10]

It is noteworthy that Q_1 varies inversely with both θ and n, while Q_2 will vary directly with both of them. Their derivatives are going to be useful for subsequent analysis. Hence, we

[10]It is clear that if $\theta = \frac{1}{n}$ (so that $m = (1+\theta)$), the two aggregate outputs become equal. It can also be checked that if $\theta = 1$ and $n = 1$, we have the symmetric differentiated duopoly outcome, $Q_1 = Q_2 = \frac{(\alpha-c)(2\beta-\delta)}{4\beta^2-\delta^2}$. On the other hand, if $\delta = 0$, then we have the fully differentiated case. When $\theta = 0$ we have the case of the mixed differentiated oligopoly (one fully public firm competing with n private firms) with $Q_1 = \frac{(\alpha-c)(\beta m-\delta)}{\beta^2 m-\delta^2}$ and $Q_2 = \frac{(\alpha-c)(\beta-\delta)}{\beta^2 m-\delta^2}$.

collect them here by differentiating (6) and (7).

$$\begin{aligned}
\frac{\partial Q_1^*}{\partial \theta} &= -\frac{(\alpha-c)m\beta^2(\beta m-\delta)}{[\beta^2(1+\theta)m-\delta^2]^2} < 0, & (8)\\
\frac{\partial Q_2^*}{\partial \theta} &= \frac{(\alpha-c)\beta\delta(m\beta-\delta)}{[\beta^2(1+\theta)m-\delta^2]^2} > 0. & (9)\\
\frac{\partial Q_1^*}{\partial n} &= -\frac{(\alpha-c)\beta\delta[\beta(1+\theta)-\delta](m-1)^2}{[\beta^2(1+\theta)m-\delta^2]^2} < 0, & (10)\\
\frac{\partial Q_2^*}{\partial n} &= \frac{(\alpha-c)\beta^2[\beta(1+\theta)-\delta](1+\theta)(m-1)^2}{[\beta^2(1+\theta)m-\delta^2]^2} > 0. & (11)
\end{aligned}$$

Now we consider the stage 1 problem where the government decides on how much of the ownership of the public firm is to be divested in industry 1 and how many (private) firms should be permitted to enter the second industry. The government makes these two decisions so that the sum of social welfare (net of entry costs) from the two markets is maximized.

The government wishes to maximize the sum of social welfare of the two markets ($W = SW_1 + SW_2$):

$$\begin{aligned}
W(\theta,n) &= \int_0^{Q_1} p_1(s_1)ds_1 - cQ_1 + \int_0^{Q_2} p_2(s_2)ds_2 - cQ_2 - nF\\
&= \frac{(1+2\theta)}{2}\beta Q_1^{*2} + \frac{(2m-1)}{2}\beta Q_2^{*2} - nF. & (12)
\end{aligned}$$

Substituting (6) and (7) in the above and differentiating it with respect to θ and n we get (see Appendix for derivations)

$$\begin{aligned}
\frac{\partial W}{\partial \theta} &= a_1\Big[-\theta\beta\{(2m-1)\beta^2-(3m-1)\beta\delta+(m-1)^2\beta^2\}\\
&\quad +\delta\{(2m-1)\beta^2-(3m-1)\beta\delta+\delta^2\}\Big] = 0, & (13)\\
\frac{\partial W}{\partial n} &= a_2\Big[(m-1)\{\beta[(1+2\theta)\beta^2-\delta\beta(2+3\theta)+\theta^2\beta^2]\}\\
&\quad -\delta\{(1+2\theta)\beta^2-\delta\beta(2+3\theta)+\delta^2\}\Big] - F = 0, & (14)
\end{aligned}$$

where $a_1 = \frac{(\alpha-c)^2(\beta m-\delta)\beta}{[m\beta^2(1+\theta)-\delta^2]^3} > 0$ and $a_2 = \frac{\beta(\alpha-c)^2(m-1)^2[\beta(1+\theta)-\delta]}{[m\beta^2(1+\theta)-\delta^2]^3} > 0$. We assume that the second order condition holds.[11]

From (13) we immediately obtain (by substituting $m = 1 + \frac{1}{n}$)

$$\theta = \frac{\delta}{\beta}\left[\frac{(1+\frac{2}{n})\beta^2 - (2+\frac{3}{n})\beta\delta + \delta^2}{(1+\frac{2}{n})\beta^2 - (2+\frac{3}{n})\beta\delta + \frac{\beta^2}{n^2}}\right]. \tag{15}$$

However, the equation for n resulting from (14) critically depends on whether F is zero or positive. We first consider the case of $F = 0$.

Case A: The special case of $F = 0$:

Typically in a homogenous good setting the combination of zero entry cost and constant marginal cost of production would make optimal entry infinitely large. This is so because socially optimal output will go to the competitive level. But as we see below that is not the case in the context of differentiated products. Driving output in the second market to the competitive level will reduce the social welfare of the first market. Therefore, even if there is no fixed cost, socially optimal entry will still be finite.

There is another reason why we should consider this case. The solution obtained here will be valid even if $F > 0$ when the government cares about producer's surplus instead of profit. In Section 4 we will consider such a case.

By setting $F = 0$ in equation (14) and simplifying it we obtain

$$\frac{1}{n} = \frac{\delta}{\beta}\left[\frac{(1+2\theta)\beta^2 - (2+3\theta)\beta\delta + \delta^2}{(1+2\theta)\beta^2 - (2+3\theta)\beta\delta + \theta^2\beta^2}\right]. \tag{16}$$

[11]

$$\frac{\partial^2 W(\theta,n)}{\partial\theta^2} < 0, \quad \frac{\partial^2 W(\theta,n)}{\partial n^2} < 0,$$
$$\Delta = \frac{\partial^2 W(\theta,n)}{\partial\theta^2}\frac{\partial^2 W(\theta,n)}{\partial n^2} - \left(\frac{\partial^2 W(\theta,n)}{\partial\theta\partial n}\right)^2 > 0.$$

Notice the symmetry between (15) and (16). If a symmetric solution to the government's problem exists, then it must be the case that $\theta = \frac{1}{n}$. Since $\theta \leq 1$, we must then have $n \geq 1$. We now derive the value of the symmetric optimal θ and determine when this symmetric solution holds. We then characterize the socially optimal (θ, n) for the whole range of $\delta \in (0, \beta)$.

Proposition 1. *When both goods are to be produced, there exists a pair* (θ, n)*,* $0 \leq \theta < 1$*,* $n \geq 1$ *that maximizes aggregate social welfare. The optimal* (θ, n) *is as follows.*

1. $\theta^* = \frac{\delta}{\beta} < 1, n^* = \frac{\beta}{\delta} > 2,\ if\ \delta < \frac{1}{2}\beta$

2. $\theta^* = \frac{\delta}{\beta}[\frac{3\beta^2 - 5\beta\delta + \delta^2}{3\beta^2 - 5\beta\delta + \beta^2}] < \frac{\delta}{\beta}, n^* = 1,\ if\ \frac{1}{2}\beta \leq \delta < 0{\cdot}697\beta$

3. $\theta^* = 0, n^* = 1,\ if\ 0{\cdot}697\beta \leq \delta < \beta$

For Proof refer Appendix.

Proposition 1 states that depending on the degree of substitutability between the two goods, the government can vary the two instruments (θ, n) to maximize the aggregate social welfare. In general optimal privatization is partial if the degree of substitutability is not high (as in cases (1) and (2)). Figure 1 depicts all the three cases of proposition 1. For convenience we will use the $(\theta, 1/n)$ plane to represent the three cases. When the degree of substitutability is low (*e.g.* Case 1), the government privatizes partially and also permits more than two firms to enter the second market. This is because even though the output rises in the second market, the reduction in output in the first market is not substantial and social welfare on the whole increases after privatization attaining a maximum at a $\theta \in (0, 1)$. Point a in the left panel of Figure 1 depicts this case.

When $\frac{1}{2} \leq \frac{\delta}{\beta} < 0{\cdot}697$, optimal privatization is partial but the government limits entry to only one firm (see the middle panel of Figure 1). Here, the interior solution actually brings down n below 1 and effectively to zero, and simultaneously reduces θ well below $\frac{\delta}{\beta}$. This means that social welfare will be greater if only one good is produced. The reason is that the degree

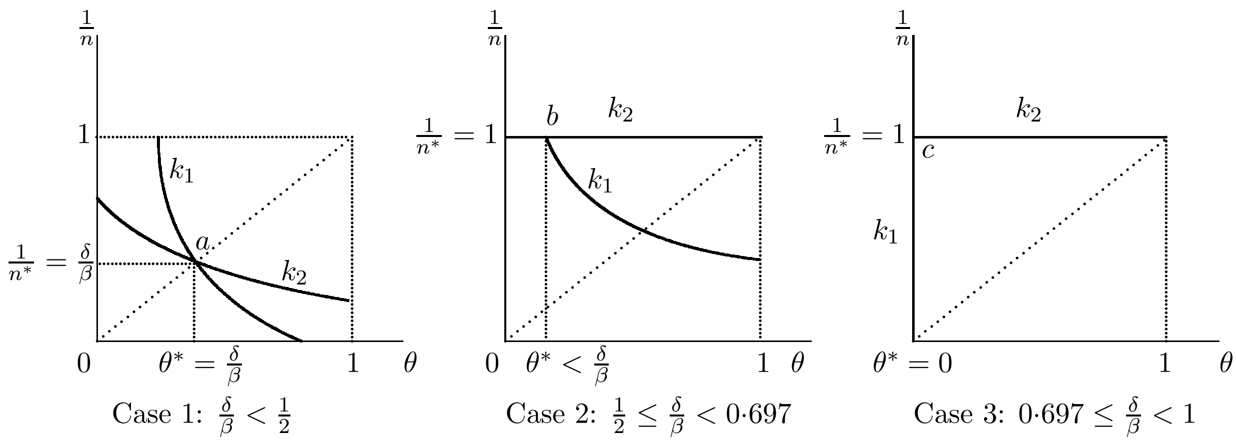
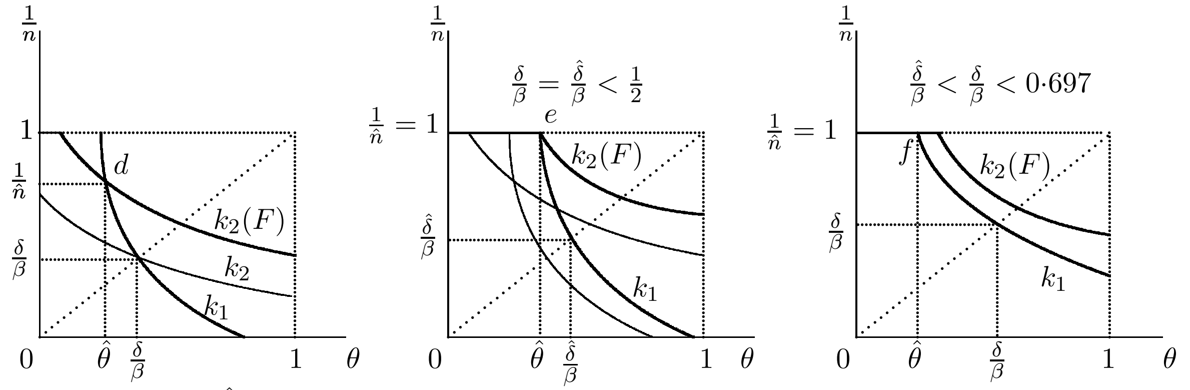


Figure 1: Optimal (θ, n) when $F = 0$

of substitutability is stronger now, and a higher output in the second market increases the welfare in the second market but reduces welfare in the first market. But if both markets are to be operative, which we assume is the government's preference, n must be restricted to 1 and in turn θ is to be adjusted appropriately for $n = 1$. Clearly this is a corner solution and social welfare on the whole is lower compared to case 1. Moreover as only one firm has to be accommodated in the second market the government also privatizes to a significantly lesser degree as compared to case 1. The middle panel of Figure 1 depicts this case. Finally when the degree of substitutability is very high (as in Case 3), the government may not privatize at all. The welfare reduction in the first market is so severe that the government sets $\theta = 0$ and allows only one private firm to enter the second market. The right most panel of Figure 1 depicts this case.

Case B: Positive fixed cost: Solution to this problem is obtained by substituting for θ from equation (13) into equation (14) to get the optimal n, which we denote as $\hat{n}$. When $\hat{n}$ is plugged back into equation (13), we get $\hat{\theta}$. Starting from a (hypothetical) situation of complete inoperation, the second market begins to experience entry as privatization takes place in the first market. Then as privatization increases, the entry process gathers momentum. With more firms entering the second market social welfare increases there. But with the rising aggregate cost of entry the optimal number of firms has to be less than the previous case. Proposition 2 states this formally.



Case 4: $\frac{\delta}{\beta} < \frac{\hat{\delta}}{\beta} < \frac{1}{2}$ Case 5: Effect of greater δ Case 6: Effect of greater δ

Figure 2: Optimal (θ, n) when $F > 0$

Proposition 2. *(a) When $F > 0$ there exists a critical value of δ, say $\hat{\delta}$, $(< \frac{1}{2}\beta)$ such that at all $\delta < \hat{\delta}$, social welfare is maximized by $(\hat{\theta}, \hat{n})$, $\hat{\theta} < \frac{\delta}{\beta}$, $\hat{n} \geq 1$. (b) For $\hat{\delta} \leq \delta < 0{\cdot}697\beta$, the social optimum is $\hat{\theta} = \frac{\delta}{\beta}\left[\frac{3\beta^2 - 5\beta\delta + \delta^2}{3\beta^2 - 5\beta\delta + \beta^2}\right]$ and $\hat{n} = 1$. For $0{\cdot}697 \leq \frac{\delta}{\beta} < 1$, $\hat{\theta} = 0, \hat{n} = 1$.*

For Proof refer Appendix.

Proposition 2 states that when $\frac{\delta}{\beta} < \frac{1}{2}$, the optimal (θ, n) is smaller as compared to the case of $F = 0$, because now there is a social marginal cost of entry that must be taken into account. with smaller n social welfare of the first market can be raised by privatizing less. Hence, θ will also be smaller.

Figure 2 depicts the optimal (θ, n) as $\frac{\delta}{\beta}$ changes. Consider first the case where $\frac{\delta}{\beta} < \frac{\hat{\delta}}{\beta} < \frac{1}{2}$ as shown in the left most panel of Figure 2. The optimal solution is at d where $\hat{\theta} < \frac{\delta}{\beta}$ and $\hat{n} < \frac{\beta}{\delta}$. As $\frac{\delta}{\beta}$ changes, both k_1 and k_2 shift out. There is a critical value of δ, namely $\hat{\delta} < \frac{1}{2}\beta$ where the intersection between k_1 and k_2 is at e as shown in the middle panel of Figure 2 where $\hat{n} = 1$. For all $\frac{\delta}{\beta} > \frac{\hat{\delta}}{\beta}$, $\hat{n} = 1$ and $\hat{\theta} \in (0, 1)$ or $\hat{\theta} = 0$ depending on $\frac{\delta}{\beta}$. If $\frac{\hat{\delta}}{\beta} < \frac{\delta}{\beta} < 0{\cdot}697$, then $\hat{\theta} \in (0, 1)$ as is shown in the rightmost panel of Figure 2. If $0{\cdot}697 \leq \frac{\delta}{\beta} < 1$, then $\hat{\theta} = 0$, which we already shown in Proposition 1.

When there is an interior solution, one can readily determine the following comparative statics properties of $\hat{n}$ and $\hat{\theta}$:

$$\begin{aligned}
\frac{\partial \tilde{n}}{\partial F} &= \frac{\partial^2 SW}{\partial \theta^2}\left(\frac{1}{\Delta}\right) < 0, \\
\frac{\partial \tilde{\theta}}{\partial F} &= -\frac{\partial^2 SW}{\partial \theta \partial n}\left(\frac{1}{\Delta}\right) < 0 \ \ if \ \ \frac{\partial^2 SW}{\partial \theta \partial n} > 0.
\end{aligned}$$

As the two goods are substitutes, θ and n should be complements to each other and therefore, the effect of an increase in the fixed cost of entry in the second market should reduce optimal privatization. Therefore, compared to the benchmark case, positive F induces the government to choose a smaller θ and also a smaller n. Fewer firms will be permitted in the second industry and in the first industry the public firm will be privatized by a lesser degree.

We now compare our results with the existing literature. To the best of our knowledge there is no previous work that has simultaneously considered optimal entry and privatization. Many authors also abstract from fixed cost. Though several articles do allow for fixed costs, such as De Fraja and Delbono (1989) and Matsumura (1998), the number of firms is treated there as exogenous. Only a handful of articles, such as Anderson *et al* (1997), Matsumura and Kanda (2005) and Fujiwara (2007) consider the free entry case, and the last two of these also study optimal privatization. But none of these study optimal entry. On the question of optimal privatization, most authors find privatization to be partial assuming homogenous good and exogenous entry. Matsumura and Kanda (2005) have shown that with free entry and homogenous good full public ownership will be optimal. For differentiated goods, Fujiwara (2007) has shown that under both exogenous entry and free entry, privatization should be partial.

We extend this literature by allowing for a fixed cost of entry and by considering socially optimal entry along with privatization. We also allow free entry (in the next section), but we restrict our model only to two products. This is done in order to study entry in the context of quantity competition.[12] Moreover, our public firm has a narrower social objective than what has been assumed in the existing models of differentiated oligopoly. We see that the presence of fixed cost affects the nature of the socially optimal solution. Nevertheless, regardless of the fixed cost, partial privatization is indeed optimal under certain circumstances, as have many authors found in their models. But that is not the only possibility we see. If the degree of substitutability of the two products (δ) is sufficiently high, *no privatization* can also be optimal, which contradicts the finding of Fujiwara (2007). Moreover, we also provide a new insight in terms of optimal entry, and thereby suggest optimal configurations of mixed oligopoly at different values of δ. While at low values of δ we should expect one partially public firm to compete with several private firms, at higher values we might see one fully public firm competing with only one private firm.

3 Free entry

Now we consider the free entry case and compare the number of private firms entering the second industry with the socially optimal entry derived in the previous section. It is well known from Mankiw and Whinston (1986) that with quantity competition and homogenous good free entry equilibrium produces *excessive entry* compared to the social optimum. It is, however, not clear if this result gets modified in the presence of a substitute good, especially when the substitute good is produced by a public firm. It seems that the government may be able to contain the extent of entry via public ownership of the public firm which produces a rival good. This instrument of indirect regulation is not available in a pure oligopoly, and therefore, one hopes to see that mixed oligopoly will perhaps bring down the excessiveness of entry. In this section we investigate this prospect.

Our game is modified to restrict the government's choice only to θ. But with respect to

[12] When multiple products or varieties are considered and each firm is given monopoly over one product, entry inevitably leads to quality competition, which we did not wish to pursue here.

the timing of this choice we consider two possibilities. In one, θ is chosen simultaneously with the timing of entry. In the other, θ is chosen prior to entry. The number of firms entering the second market is given by the zero profit condition. In the sequential case, stage 2 of the game witnesses entry and output competition takes place in stage 3.

First consider the zero profit condition

$$\pi_{2i} = (p_2 - c)q_{2i} - F = \beta(\alpha - c)^2 \left[\frac{(1+\theta)\beta - \delta}{(1+\theta)(n+1)\beta^2 - n\delta^2} \right]^2 - F = 0.$$

This yields

$$n(\theta) = \frac{\beta(1+\theta)\left[1 - \beta\sqrt{\frac{F}{\beta(\alpha-c)^2}}\right] - \delta}{\sqrt{\frac{F}{\beta(\alpha-c)^2}}\left[\beta^2(1+\theta) - \delta^2\right]} \tag{17}$$

from which we note

$$\frac{\partial n}{\partial \theta} \quad = \quad \frac{\beta^2\delta^2}{[\beta^2(1+\theta) - \delta^2]^2} > 0 \quad \forall\ \theta \in [0,1]. \tag{18}$$

Next, we show that given any θ, free entry is excessive relative to the social optimum, provided the interior solution holds. This is essentially a restatement of the Mankiw-Whinston (1986) result in the present context.

Lemma 1. *Given any θ free entry produces excessive entry relative to the social optimum.*

Proof: Recall that

$$W = \int_0^{Q_1} p_1(s_1)ds_1 - cQ_1 + \int_0^{Q_2} p_2(s_2)ds_2 - cQ_2 - nF.$$

Differentiating this with respect to n and assuming that an interior solution exists we have

$$\begin{aligned}\frac{\partial W}{\partial n} &= (p_1 - c)\frac{\partial Q_1}{\partial n} - \delta Q_1 \frac{\partial Q_2}{\partial n} + (p_2 - c)[q_2 + n\frac{\partial q_2}{\partial n}] - \delta Q_2 \frac{\partial Q_1}{\partial n} - F = 0, \\ &= \left[(p_1 - c)\frac{\partial Q_1}{\partial n} - \delta Q_1 \frac{\partial Q_2}{\partial n}\right] + \left[(p_2 - c)n\frac{\partial q_2}{\partial n} - \delta Q_2 \frac{\partial Q_1}{\partial n}\right] + \pi_2 = 0. \end{aligned} \tag{19}$$

The first bracketed term in the above equation captures the effect of n on the social welfare of the first market. The next two terms capture the effect of n on the social welfare of the second market. In the homogenous good context, the first term would be absent. We have already determined that $p_1 > c$, $\frac{\partial Q_1}{\partial n} < 0$ and $\frac{\partial Q_2}{\partial n} > 0$, and therefore the first term is clearly negative. The second bracketed term consists of two terms, of which the first term is again clearly negative as $p_2 > c$ and $\frac{\partial q_2}{\partial n} < 0$. This is what Mankiw-Whinston (1986) call the *business stealing effect*. Then we have an additional term due to the substitute good $(-\delta Q_2 \frac{\partial Q_1}{\partial n})$ which is positive. Let us call it the *substitute good effect*. An increase in n reduces Q_1 and thus boosts the demand for Q_2 exerting a favorable effect on social welfare countering the business stealing effect. If the business stealing effect outweighs the substitute good effect and thus the second bracketed term is negative, we can conclude that at the social optimum $\pi_2 > 0$.

Substituting relevant expressions we check that

$$(p_2 - c)n\frac{\partial q_2}{\partial n} > \delta Q_2 \frac{\partial Q_1}{\partial n} \tag{20}$$

for any $\theta \in [0, 1]$ if and only if $\delta < \frac{1}{\sqrt{2}}\beta$.[13] Since $\delta < \frac{1}{2}\beta$ is the upper limit for n to have an interior solution (1 or greater) when $F = 0$ and θ is arbitrarily given, we can safely conclude that in the interior optimum (for θ given) the business stealing effect dominates the substitute good effect and therefore $\pi_2 > 0$ at the social optimum, a key requirement for the excessive

[13] Note that

$$(p_2 - c)n\frac{\partial q_2}{\partial n} - \delta Q_2 \frac{\partial Q_1}{\partial n} = -\frac{(\alpha - c)^2[\beta(1+\theta) - \delta]^2 \beta(m-1)}{[\beta^2(1+\theta)m - \delta^2]^3}\left[\beta^2(1+\theta) - 2\delta^2\right] < 0.$$

entry result.

Next we need to show that π_2 is inversely related to n. From $\pi_2 = (p_2 - c)q_2 - F$ we obtain

$$\frac{\partial \pi_2}{\partial n} = \left[(p_2 - c)\frac{\partial q_2}{\partial n} - q_2\beta\frac{\partial Q_2}{\partial n}\right] - q_2\delta\frac{\partial Q_1}{\partial n}. \tag{21}$$

The first term (bracketed) is negative; but the last term which appears due to the substitute good throws some ambiguity. However, from (10) and (11) we see that $\frac{\partial Q_2}{\partial n} > |\frac{\partial Q_1}{\partial n}|$, and since $\beta > \delta$ the last term is always dominated. Therefore, $\frac{\partial \pi_2}{\partial n} < 0$.

Now since the free entry equilibrium determines n such that $\pi_2 = 0$, this n must be greater than the social optimum, given any θ. □

Of course, in our model θ will not remain unchanged as the government will optimize on θ depending on the timing of its decision. First consider the simultaneous case, when the privatization decision coincides with entry. From (19) we obtain

$$\frac{\partial W}{\partial \theta} = (p_1 - c - \delta Q_2)\frac{\partial Q_1}{\partial \theta} + (p_2 - c - \delta Q_1)\frac{\partial Q_2}{\partial \theta} = 0. \tag{22}$$

The explicit form of this equation has already been derived in equation (13).

In the simultaneous move case, the equilibrium (θ, n) solve (13) and (17). Let this solution be denoted as (θ^e, n^e). As the following proposition shows, relative to the social optimum free entry induces greater privatization and greater entry. In that sense, the government's attempt to optimize on the public ownership in the rival market exacerbates the problem of excessive entry. The reason is that unable to prevent the scale of entry, the government tries to improve the overall welfare by distorting public ownership in the related market, and the distortion goes in the direction of excessive privatization essentially to accommodate excessive entry in the other market.

But if the government could choose θ prior to entry, it would be able to pre-empt much of the excessive entry. So in the sequential case privatization would be far less and the problem of excessive entry would be significantly reduced. It is even possible that either privatization or entry, one of the variables, is brought back to the socially optimal level. Formally, in stage 1 the government chooses θ to maximize the total social welfare $W(\theta, n(\theta))$. Optimal privatization and subsequent entry which we denote as $\tilde{\theta}$ and $\tilde{n}$ must satisfy the following first order condition

$$\frac{\partial W(\theta, n(\theta))}{\partial \theta} = \frac{\partial W(\theta, n(\theta))}{\partial \theta} + \frac{\partial W(\theta, n(\theta))}{\partial n} \frac{\partial n}{\partial \theta} = 0 \tag{23}$$

and the zero profit condition (17), where the expressions of $\frac{\partial W}{\partial \theta}$ and $\frac{\partial W}{\partial n}$ can be obtained from (13) and (14) respectively.

Proposition 3. *When privatization and entry take place simultaneously, free entry produces excessive privatization and excessive entry relative to the social optimum. However, by privatizing prior to entry the government can reduce the excessiveness of both entry and privatization. That is to say,* $\theta^e > \hat{\theta}$ *and* $n^e > \hat{n}$, *and* $\tilde{\theta} < \theta^e$, $\tilde{n} < n^e$, *where* $(\hat{\theta}, \hat{n})$ *refers to the social optimum.*

For Proof refer Appendix.

The above proposition may be better understood with the help of Figure 3. We have drawn two iso-social welfare curves on the $(\theta, \frac{1}{n})$ plane with $W_2 > W_1$. The inner the location of the curve, the higher the level of social welfare. Obviously the highest social welfare is achieved at $(\hat{\theta}, \frac{1}{\hat{n}})$, where the iso-welfare curve reduces to a single point. The $k_2(F)$ curve represents the locus of all points at which the derivative of social welfare with respect to $1/n$ (or equivalently n) is zero. We know on this locus (given any θ) $\pi_2 > 0$. Therefore, the zero profit curve must be located below $k_2(F)$ corresponding to appropriately higher n (or lower $\frac{1}{n}$). The simultaneous case solution is given by $(\theta^e, \frac{1}{n^e})$, which not only must be South-East of the socially optimal point, but must also correspond to a lower level of social welfare (clearly

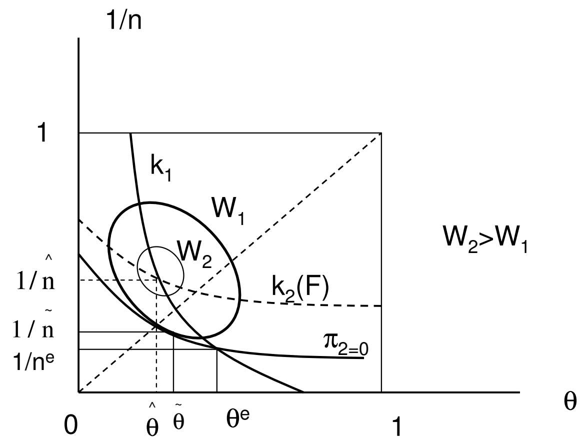


Figure 3: Free entry

below W_1 in the graph). Thus we see both excess privatization and excess entry compared to the social optimum. However, this excess entry is largely pre-empted when the government is able to move first and choose $\tilde{\theta}$ prior to the response of the private firms. By privatizing conservatively, the government is able to restrict the excessiveness of entry in the second market and also improve the overall welfare (to W_1 in the graph). It is quite possible that optimal privatization can come down to the socially optimal level.

Thus we see that in a mixed oligopoly the government can still influence entry even if it does not directly regulate it. By adjusting the public ownership in one market, it can orient entry in a related market in the direction of social optimum. But in order to do so, it must act prior to entry; otherwise the problem of excessive entry will be exacerbated and the equilibrium outcome will be further away from the social optimum. Nevertheless, privatization will remain partial and consistent with similar finding in the literature, though the underlying forces of partial privatization are different here.

4 Producer surplus as part of social welfare

So far we have assumed that social welfare consists of consumer surplus and profit, which means that the entry cost is to be subtracted from social welfare. This is the approach taken by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). One justification for taking this approach is that the entry costs will not be subsidized by the government. However, it is an equally common practice to make producer surplus, instead of profit, a part of social welfare, in which case the social optimum will not depend on the entry cost. However, an immediate implication is that the government should be prepared to subsidize the entry cost, if social optimum so requires. This means, free entry may produce *less* entry. In fact, whether there will be 'excess entry' or 'less entry' depends on the size of the fixed cost.

Since formally the analysis of the social optimum of this case is identical to that of no fixed cost, we take (θ^*, n^*) as defined in Proposition 1, as the solution to the government's optimization problem. For $\delta/\beta < 1/2$ we have a unique symmetric solution, $\theta^* = \frac{1}{n^*} = \frac{\delta}{\beta}$. Now define $F^*(\delta)$ as the critical value of F such that $\pi_2(\theta^*, n^*; \delta) = 0$. That is, the private firm's profit is zero at the social optimum if its fixed cost is F^* given any δ. Since profit is declining in n and F, under free entry we must have 'excess entry' if $F < F^*(\delta)$ and 'less entry' if $F > F^*(\delta)$, assuming that θ is chosen simultaneously with entry. In the sequential case where θ is chosen prior to entry, some of the social inefficiency will be mitigated by inducing greater (less) entry, if $F > F^*$ $(F < F^*)$. But this inducement will have diametrically opposite effects on optimal privatization. If $F > F^*$ $(F < F^*)$ privatization will rise above (fall below) the socially optimal level. Since these results can be derived in a straight-forward way, we omit the formal proof.

Proposition 4. *When the government cares about producer's surplus instead of profit, there exists a critical value of F, say $F^*(\delta)$, such that free entry produces less entry at $F > F^*(\delta)$, excess entry at $F < F^*(\delta)$ and just socially optimal entry at $F = F^*(\delta)$. In the event of less*

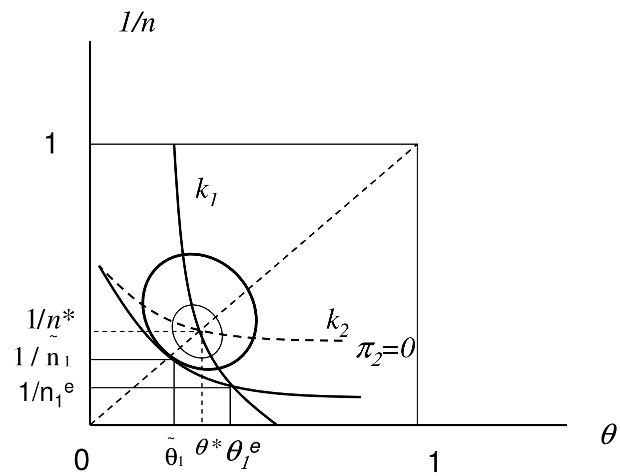


Figure 4a: The case of SW= CS + PS

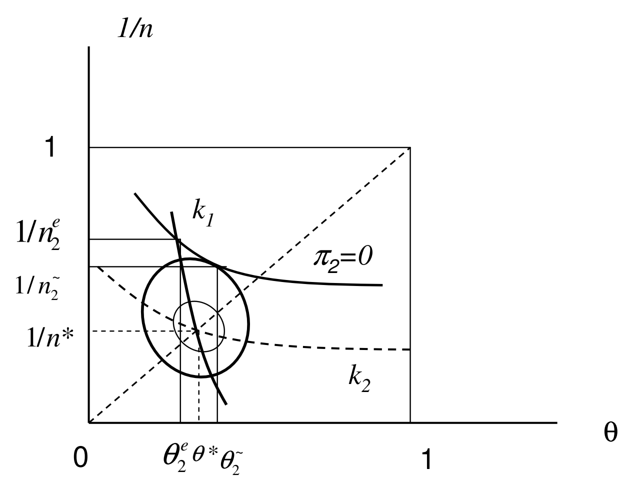


Figure 4b: The case of SW=CS+ PS

(excess) entry, by choosing θ prior to entry the government can induce (restrict) greater entry by privatizing more (less).

We illustrate these results in Figures ($4a$) and ($4b$). In Figure ($4a$) the size of the entry cost is assumed to be small; so in the social optimum private firms still make positive profit. Free entry in this case will lead to excess entry. In Figure ($4b$), we assume that the entry cost is relatively large, so that private firms will be incurring loss under the social optimum. Under free entry then, fewer firms will enter, a reversal of the excess entry result, primarily because the government does not care about the entry cost.

Both figures depict the simultaneous as well as the sequential case. As before, k_1 and k_2 represent the two first-order conditions (13) and (14) and $\pi_2 = 0$ represents the zero profit condition. $(\theta^*, \frac{1}{n^*})$ denotes the socially optimal privatization level and the number of firms entering the second market. At the socially optimal solution, firms make positive profit. With free entry and simultaneous choice of privatization coinciding with entry, the solution is given by $(\theta_1^e, \frac{1}{n_1^e})$ with $\theta_1^e > \theta^*$ and $n_1^e > n^*$. This is a case of excess entry. With prior choice of θ the excess entry scenario is largely mitigated as we have $\tilde{\theta}_1 < \tilde{\theta}_1^e$ and $\tilde{n}_1 < n_1^e$.

When the fixed cost is substantially large, the scenario may change to Figure (4b). In this case, under simultaneous choice of θ the free-entry solution is $(\theta_2^e, \frac{1}{n_2^e})$. Since $n_2^e < n^*$, we have less entry. Consequently the government also privatizes less as compared to the socially optimal level. With a very high fixed cost of entry only a small number of firms can be sustained and the excessive entry result holds no longer. As less than efficient entry occurs, under the sequential case the government will over-privatize to induce more firms to enter the second market. This is depicted in figure ($4b$) where $\tilde{\theta}_2 > \theta_2^e$ and $\tilde{n}_2 > n_2^e$. However, inspite of over-privatization, the level of entry may still be below the socially optimal level as can be seen from the figure where $\tilde{n}_2 < n^*$. So the problem of insufficient entry may persist even under pre-emptive privatization.

5 Conclusion

In this paper we make an attempt to bridge the gap between the mixed oligopoly literature and the excess entry literature. By considering a model of differentiated products where one good is produced by a government owned firm and the other good is produced by many private firms we deduce the optimal degree of privatization and the optimal number of firms entering the second market so as to maximize aggregate social welfare. Contrary to the existing literature, which generally suggest partial privatization as the welfare maximizing strategy, our analysis shows that partial privatization need not always be optimal. In fact, both optimal privatization and optimal entry depend on the degree of substitutability between the two goods. An important policy implication of our analysis is that while privatizing in one market the government may not want to liberalize the other market completely, as more than the optimal number of firms may enter the second market. Therefore privatization with entry regulation may be an optimal strategy for the government, rather than privatizing alone or regulating entry alone.

Next we show that under free entry there now exists a new effect called the *substitute good effect* in addition to the commonly observed *business stealing effect*. Excessive entry arises if the business stealing effect outweighs the substitute good effect. Unlike models of pure oligopoly, in our model the government has an additional instrument of privatization through which it can control the effect of excessive entry, though, the timing of the choice of privatization is crucial in this case. Simultaneous choice of privatization increases social inefficiency. We also consider the case where producer surplus, instead of profit, constitutes social welfare. In this case there could be excess entry or less entry depending on the size of the entry cost.

Our insights in this paper may be helpful in understanding why governments in reality are sometimes hesitant (or otherwise) to privatize, even when its direct benefits are obvious. Secondary effects in related markets can be a matter of concern (or jubilation) and thus they can inhibit (or boost) privatization.

Appendix

A0. Some important derivations:

Utilizing (6) and (7) we obtain the following derivations:

$$
\begin{aligned}
p_1^* &= \frac{\alpha[\beta\theta(\beta m-\delta)] + c[\beta^2 m + \delta\beta\theta - \delta^2]}{\beta^2(1+\theta)m - \delta^2}, & (24)\\
p_2^* &= \frac{\alpha[\beta(m-1)\{\beta(1+\theta)-\delta\}] + c[\beta^2(1+\theta) + \delta\beta(m-1) - \delta^2]}{\beta^2(1+\theta)m - \delta^2}, & (25)\\
\pi_1^* &= \theta\beta\frac{(\alpha-c)^2(\beta m-\delta)^2}{[\beta^2(1+\theta)m-\delta^2]^2} = \theta\beta Q_1^{*2}, & (26)\\
n\pi_2 * &= (m-1)\beta\frac{(\alpha-c)^2[\beta(1+\theta)-\delta]^2}{[\beta^2(1+\theta)m-\delta^2]^2} = (m-1)\beta Q_2^{*2}, & (27)\\
CS_1^* &= \frac{\beta}{2}(\alpha-c)^2\left[\frac{\beta m-\delta}{\beta^2(1+\theta)m-\delta^2}\right]^2 = \frac{\beta}{2}Q_1^{*2}, & (28)\\
CS_2^* &= \frac{\beta}{2}(\alpha-c)^2\left[\frac{\beta(1+\theta)-\delta}{\beta^2(1+\theta)m-\delta^2}\right]^2 = \frac{\beta}{2}Q_2^{*2}. & (29)
\end{aligned}
$$

A1. Derivations of (13) and (14):

The two first order conditions of maximizing $W(\theta, n)$ are:

$$
\begin{aligned}
\frac{\partial W(\theta,n)}{\partial\theta} &= \frac{\partial SW_1(\theta,n)}{\partial\theta} + \frac{\partial SW_2(\theta,n)}{\partial\theta} = 0, & (30)\\
\frac{\partial W(\theta,n)}{\partial n} &= \frac{\partial SW_1(\theta,n)}{\partial n} + \frac{\partial SW_2(\theta,n)}{\partial n} = 0. & (31)
\end{aligned}
$$

Differentiating SW_1 with respect to θ one gets

$$\frac{\partial SW_1}{\partial\theta} = -\frac{(\alpha-c)^2(\beta m-\delta)\beta}{[\beta^2 m(1+\theta)-\delta^2]^3}\{(\theta\beta^2 m+\delta^2)(\beta m-\delta)\} < 0.$$

Similarly differentiating SW_2 with respect to θ one obtains

$$\frac{\partial SW_2}{\partial\theta} = \frac{(\alpha-c)^2(\beta m-\delta)\beta}{[\beta^2 m(1+\theta)-\delta^2]^3}\{\beta\delta[\beta(1+\theta)-\delta](2m-1)\} > 0.$$

Adding these two expressions and rearranging terms equation (13) is obtained.

Similarly differentiate SW_1 and SW_2 with respect to n and obtain

$$\frac{\partial SW_1}{\partial n} = -\frac{(\alpha-c)^2[\beta(1+\theta)-\delta](m-1)^2\beta}{[\beta^2 m(1+\theta)-\delta^2]^3}\{\beta\delta(1+2\theta)(\beta m-\delta)\} < 0, \tag{32}$$

$$\begin{aligned}\frac{\partial SW_2}{\partial n} &= -\frac{(\alpha-c)^2[\beta(1+\theta)-\delta](m-1)^2\beta}{[\beta^2 m(1+\theta)-\delta^2]^3}\{(m-1)(1+\theta)\beta^2[(1+\theta)\beta-\delta] \\ &\quad +\delta^2\beta(1+\theta)-\delta^3\} > 0.\end{aligned} \tag{33}$$

Adding these two equations and rearranging terms we get (14).

A2. Proof of Proposition 1:

Proof: Consider (13) and (14). Note that the sign of (13) and (14) depends on the bracketed term only as the first term in both the equations are always positive. For convenience we will express the solution in terms of θ and m noting that $m = 1 + \frac{1}{n}$. Denote

$$\begin{aligned} k_1(\theta, m) &= -\theta\beta[(2m-1)\beta^2 - (3m-1)\beta\delta + (m-1)^2\beta^2] \\ &\quad +\delta[(2m-1)\beta^2 - (3m-1)\beta\delta + \delta^2] \end{aligned} \tag{34}$$

$$\begin{aligned} k_2(\theta, m) &= (m-1)\beta[(1+2\theta)\beta^2 - (2+3\theta)\beta\delta + \theta^2\beta^2] \\ &\quad -\delta[(1+2\theta)\beta^2 - (2+3\theta)\beta\delta + \delta^2] \end{aligned} \tag{35}$$

(1) Note that $\theta^* = \frac{\delta}{\beta}$ and $n = \frac{\beta}{\delta}$; *i.e* $m = \frac{\beta+\delta}{\beta}$ is the only solution satisfying $k_1(.) = 0$ and $k_2(.) = 0$. So to confirm that it is a maximum we check the second order condition. For an interior (θ, m), as is the case here, the second order condition for a maximum is $\frac{\partial k_1}{\partial \theta} < 0$, $\frac{\partial k_2}{\partial n} < 0$, and $\frac{\partial k_1}{\partial n}\frac{\partial k_2}{\partial \theta} - \frac{\partial k_1}{\partial \theta}\frac{\partial k_2}{\partial n} > 0$ at the optimal solution. Evaluating these derivatives at the

optimal solution we have ;

$$\begin{aligned}
\frac{\partial k_1}{\partial \theta} &= -\beta(\beta^2 - 2\delta^2) < 0 \; if \; \frac{\delta}{\beta} < \frac{1}{\sqrt{2}} \\
\frac{\partial k_2}{\partial n} &= -\frac{\delta^2}{\beta}(\beta^2 - 2\delta^2) < 0 \; if \; \frac{\delta}{\beta} < \frac{1}{\sqrt{2}} \; and \\
\frac{\partial k_1}{\partial n}\frac{\partial k_2}{\partial \theta} - \frac{\partial k_1}{\partial \theta}\frac{\partial k_2}{\partial n} &= \beta^2\delta^2(\beta^2 - 4\delta^2) > 0, \; if \frac{\delta}{\beta} < \frac{1}{2}.
\end{aligned}$$

Therefore if $\frac{\delta}{\beta} < min[\frac{1}{2}, \frac{1}{\sqrt{2}}]$, the solution $\theta^* = \frac{\delta}{\beta}$ and $n^* = \frac{\beta}{\delta}$ maximizes aggregate social welfare.

(2) Now consider $\frac{\delta}{\beta} \geq \frac{1}{2}$. The possible candidates for solution are corner solutions as the interior solution does not yield a maximum. We will first show that neither $\theta = 0, m = 1$ nor $\theta = 1, m = 2$ can be optimal. Evaluating k_1 and k_2 at $\theta = 0, m = 1$ we have $k_1 = \delta(\beta - \delta)^2 > 0$ and $k_2 = -\delta(\beta - \delta)^2 < 0$. Clearly $\theta = 0, m = 1$ cannot be optimal. Similarly for $\theta = 1, m = 2 (or \; n = 1)$, $k_1 = -(3\beta^2 - 5\beta\delta)(\beta - \delta) - (\beta^3 - \delta^3) < 0$ and $k_2 = (3\beta^2 - 5\beta\delta)(\beta - \delta) + (\beta^3 - \delta^3) > 0$ which implies that $\theta = 1, m = 2$ cannot be optimal. So the corner solutions must be asymmetric. We need to consider three possibilities: (i) $\theta = 0, 1 < m < 2$ and symmetrically $0 < \theta < 1, m = 1$, (ii) $0 < \theta < 1, m = 2$ and (iii) $\theta = 0, m = 2$.

(i) Suppose that $m = 1$. Evaluating k_1 at $m = 1$ we have $k_1 = \delta(\beta - \delta)^2 - \theta\beta(\beta^2 - 2\beta\delta)$. For $0 < \theta < 1$ to be a solution $k_1 = 0$. But for $\frac{\delta}{\beta} > \frac{1}{2}$, $k_1 > 0$, $\forall \; \theta \in (0, 1)$. Therefore $0 < \theta < 1, m = 1$ cannot be an optimal solution. By similar reasoning $\theta = 0, 1 < m < 2$ cannot be an optimal solution.

(ii) Now consider $m = 2$. From (**??**), $k_1 = \delta[3\beta^2 - 5\beta\delta + \delta^2] - \theta\beta[3\beta^2 - 5\beta\delta + \beta^2]$. If $3\beta^2 - 5\beta\delta + \delta^2 > 0$ which occurs for $\delta < 0{\cdot}697\beta$, $\exists \; \theta \in (0, 1)$ such that $k_1 = 0$. In fact $\theta^* = \frac{\delta}{\beta}[\frac{3\beta^2 - 5\beta\delta + \delta^2}{3\beta^2 - 5\beta\delta + \beta^2}] < \frac{\delta}{\beta}$. The only thing remaining is to show that at this optimal θ setting $m = 2$ is optimal. This boils down to showing that $k_2 > 0$ at $m = 2$. Suppose not; *i.e.* $k_2 \leq 0$. If $k_2 = 0$, then $1 < m < 2$. But then we have shown that there is no asymmetric interior solution. Similarly if $k_2 < 0$, then $m = 1$. This has been ruled out (see (i)). Therefore $k_2 > 0$

implying that for $\frac{1}{2} \leq \frac{\delta}{\beta} < 0{\cdot}697$, the optimal solution is given by (2).

(3) To complete the proof we now consider $\delta \geq 0{\cdot}697\beta$. First consider $0{\cdot}697\beta \leq \delta < 0{\cdot}8\beta$. In this interval $3\beta^2 - 5\beta\delta + \delta^2 < 0$ and $3\beta^2 - 5\beta\delta + \beta^2 > 0$. Evaluating k_1 at $m = 2$ we have $k_1 = \delta[3\beta^2 - 5\beta\delta + \delta^2] - \theta\beta[3\beta^2 - 5\beta\delta + \beta^2] < 0$ which implies that $\theta = 0$ must be optimal.

Finally, if $0{\cdot}8\beta < \delta < \beta$, we have $3\beta^2 - 5\beta\delta + \delta^2 < 0$ and $3\beta^2 - 5\beta\delta + \beta^2 < 0$. At $m = 2$ we have $k_1(\theta = 0) = \delta[3\beta^2 - 5\beta\delta + \delta^2] < 0$ and $K_1(\theta = 1) = \delta(3\beta^2 - 5\beta\delta + \delta^2) - \beta(4\beta^2 - 5\beta\delta) < 0$ which implies that the optimal solution must be $\theta = 0$ or $\theta = 1$. But we have already shown that $\theta = 1, m = 2$ cannot be a solution. Thus $\theta = 0$ and $m = 2$ $(n = 1)$. This completes the proof. □

A3. Proof of Proposition 2:

Proof: (a) We will first prove that $\exists(\hat{\theta}, \hat{n})$ that maximizes social welfare. We can rewrite (15) as,

$$\theta(n) = \frac{n\delta}{\beta} \frac{[(n+2)\beta^2 - (2n+3)\beta\delta + n\delta^2]}{[n(n+2)\beta^2 - n(2n+3)\beta\delta + \beta^2]}. \tag{36}$$

Therefore as $n \to 0$, $\theta(n) \to 0$. Define

$$\begin{aligned}\Lambda(n) \equiv\ & \frac{\beta(\alpha-c)^2(m-1)^2[\beta(1+\theta)-\delta]}{[m\beta^2(1+\theta)-\delta^2]^3}\Big[\theta^2(m-1)\beta^3 + \theta\beta\{2(m-1)\beta^2 - (3m-1)\beta\delta + 3\delta^2\} \\ & + \{(m-1)\beta^3 - (2m-1)\beta\delta + \delta^2(2\beta-\delta)\}\Big] - F. \end{aligned} \tag{37}$$

As $n \to 0$, $\Lambda(n) \to \frac{(\alpha-c)^2(\beta-\delta)(\beta-2\beta\delta)}{\beta^3} - F > 0$, for $\delta < \frac{1}{2}\beta$ and F not too large. As $n \to \bar{N}$, with $\lim_{n\to\bar{N}} \frac{1}{n} \to 0$, $\theta \to \min[\frac{\delta(\beta-\delta)^2}{\beta^2(\beta-2\delta)}, 1]$, $m \to 1$ and therefore $\Lambda(n) \to -F < 0$. $\Lambda(n)$ is continuous in n. By the Intermediate Value Theorem $\exists\hat{n} \in [0, \bar{N}]$ such that $\Lambda(\hat{n}) = 0$. From (**??**), $\hat{\theta} \equiv \hat{\theta}(\hat{n})$.

To complete the proof we now show that $\exists\frac{\hat{\delta}}{\beta} < \frac{1}{2}$ such that $\hat{n} = 1$. As $\frac{\delta}{\beta}$ increases, both k_1 and k_2 move outward. So whether the point d in Figure 2 (left panel) will move up or down or stay constant depends on the rate of change of k_1 and k_2 with δ. Analytically comparing these derivatives is difficult. We will argue that the point d must move north or northeast as $\frac{\delta}{\beta}$ increases. Suppose not; *i.e.* the point d stays constant or moves south or southeast. If this was the case then $\frac{1}{n}$ will either be constant or will decrease. If $\frac{1}{n}$ was constant then at some $\frac{\delta}{\beta}$, $\hat{\theta} = \frac{\delta}{\beta}$. But then $\hat{\theta} = \frac{\delta}{\beta}$ cannot be a solution when $F > 0$ (see (a)). Now suppose that $\frac{1}{n}$ decreases; *i.e.* n increases. This implies that $m = 1 + 1/n$ decreases. Consequently at some n, $\hat{\theta} = \frac{\delta}{\beta}$ which cannot be an optimal solution. Therefore, $\frac{1}{n}$ must increase. At some $\frac{\hat{\delta}}{\beta}$, the point of intersection is at e where $\hat{n} = 1$. This must occur for a $\frac{\hat{\delta}}{\beta} < \frac{1}{2}$. If not, then for $\frac{\delta}{\beta} \in (0{\cdot}697, 1)$, $\hat{\theta} = 0$ (see (b)) contradicting that $\hat{\theta} \in (0, 1)$ as is the case.

Since $F > 0$, the first expression of (14) must be positive. Specifically $k_2 > 0$ implying that m must be greater as compared to the case of $F = 0$. Since $m = 1 + 1/n$, n will be smaller. Consequently, as fewer firms have to be accommodated in the second market, the government also privatizes to a lesser degree.

(b) For $\frac{1}{2} \leq \frac{\delta}{\beta} < 0{\cdot}697$, $\hat{\theta} \in (0, 1)$ and $\hat{n} = 1$. To see this note that from Proposition 1 where $F = 0$, $n = 1$ when $\frac{1}{2} \leq \frac{\delta}{\beta} < 0{\cdot}697$. So when $F > 0$, n cannot be greater than 1. Moreover since F is such that firms earn a positive profit when they enter, $n \nless 1$. Hence the optimal $n = 1$. Optimal θ is derived from (**??**). A similar argument proves that when $0{\cdot}697 \leq \frac{\delta}{\beta} < 1$, $\hat{\theta} = 0$ and $\hat{n} = 1$. □

A4. Proof of Proposition 3:

Proof: Consider equations (13) and (17). Without loss of generality take an arbitrary combination $(\hat{\theta}, n^e(\hat{\theta}))$. By definition, $n^e(\hat{\theta})$ satisfies equation (17) if $\theta = \hat{\theta}$. But from (13) we know that $\frac{\partial W}{\partial \theta} = 0$ if both $\theta = \hat{\theta}$ and $n = \hat{n}$ and $\frac{\partial W}{\partial \theta} > 0$ if $\theta = \hat{\theta}$, but $n > \hat{n}$. By Lemma 1, we also know that $n^e(\hat{\theta}) > \hat{n}$. Therefore, at $(\hat{\theta}, n^e(\hat{\theta}))$, social welfare can improve, if θ is raised. I particular, θ needs to be raised such that $\pi_2 = 0$ is maintained, which requires raising n as well. Thus we will have $\theta^e > \hat{\theta}$ and $n^e > \hat{n}$.

Next, consider the sequential case. Here the optimal solution satisfies equations (**??**) and (17). From Lemma 1, it follows that since zero profit implies excess entry, at all (θ, n) satisfying (17) we must have $\frac{\partial W}{\partial n} < 0$. In addition we know such combinations of (θ, n) generates positively sloped locus. Therefore, in (**??**) we must have $\frac{\partial W(\theta, n(\theta))}{\partial \theta} > 0$. That is to say, $\tilde{\theta} < \theta^e$. From (17) it immediately follows that $\tilde{n} < n^e$. $\square$

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