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Abstract

This paper develops a dynamic, theoretical model of demand for money under decreasing marginal impatience (DMI). Given certain conditions, the steady state is shown to be saddle-path stable and unique. It is shown that, under DMI, an increase in income inequality increases the aggregate demand for money. Empirical evidence supporting this hypothesis is provided in the context of the U.S. economy.

1 Introduction

The standard Baumol-Tobin “square root” formula of demand for money predicts a negative relationship between household income level and the *ratio* of real balances to income. This reflects economies of scale in money holding, and, its implication is that in an unequal society characterized by heterogeneous agents with different income levels, the greater the degree of inequality, the lower is the aggregate demand for real money holding.

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Empirical studies do not however seem to confirm this prediction. Using annual data on income distribution and M1 money supply in the US for the period 1949-1988, Cover and Hooks (1993) have found that as income becomes less equally distributed, the total demand for real balances *increases*. Fair and Dominguez (1991) also report that younger people in their prime working years (and therefore with higher incomes) have a higher demand for money relative to their income than the people belonging to the older cohort with lower incomes. These studies indicate *diseconomies* of scale in money holding. What can be a rationale behind diseconomies in holding money? Barring some conjectures by Cover and Hooks, there seems to be no well-formulated alternative ‘theory’ as such that explains this. The purpose of this paper is two-fold: (a) to provide a theory of how an increase in income inequality may lead to an increase in the demand for real balances and (b) to present supporting empirical evidence in the context of the U.S. economy by way of building upon the previous work by Cover and Hooks.

Our theoretical model relies on recent work on variable rate of time preference, especially Das (2003). The standard dynamic models of asset accumulation with variable rate of time preference, e.g. Uzawa (1968), Lucas and Stokey (1984), Epstein and Heynes (1983) and Obstfeld (1990), assume increasing marginal rate of impatience - which we call IMI. That is, as an agent’s current consumption or utility increases, she discounts the future more. This is unintuitive and has been recognized so in the literature. As Obstfeld (1990, page 51) writes, “There is considerable disagreement over whether impatience to consume should increase or decrease as actual consumption rises. Koopmans suggests that a majority of economists would make an introspective argument in favor of *decreasing* marginal impatience” (italics added). Yet IMI is assumed on the ground that it ensures stability of steady state of the economy in which agents accumulate wealth. On the other hand, decreasing marginal impatience -

DMI, which is economically intuitive - is not used as it may lead to instability. However, the important point that has missed the emphasis it deserves is that DMI is *not* inconsistent with stability of accumulation of wealth either. As Das (2003) has recently shown, under mild restrictions, DMI does not conflict with stability of a macro economy.

Indeed, DMI is supported by many – not just on grounds of intuition but also on empirical grounds. See Koopmans (1986), Barro and Sala-I-Martin (1995), Lawrence (1991), Ogaki-Atkinson (1997), Samwick (1998).

Once we accept DMI it is straightforward to see how inequality may affect demand for money as an asset. The rich will have a lesser rate of discount than the poor. Equivalently, individual asset demand is an increasing and strictly convex function of current income. Hence a redistribution that causes the rich to have more income and the poor to have less will imply that the increase in asset holding by the former will outweigh the decrease in asset holding by the poor. Consequently the aggregate demand for asset (money) is greater.

While Das (2003) has analyzed DMI in the context of capital accumulation, in what follows, in Section 2, we develop a simple model of demand for money and show that, under DMI, an increase in income inequality leads to an increase in the aggregate demand for money. Section 3 presents our empirical analysis. Concluding remarks are made in Section 4.

2 The Theory

In order to avoid the general equilibrium consequences of other variables in the economy, especially the price level, consider a small open economy facing a given foreign price and a given exchange rate. Additionally, let money be the only asset present in the economy.

There are H infinitely lived dynastic households, all with identical tastes and preferences, but with different income levels. The income levels are denoted by y_1, y_2, \dots, y_H , which are

exogenously given. There is one good produced in the economy with constant returns. Labor is the only input. Perfect competition prevails. Therefore, real wage per unit of time is given. Different households have varying endowments of labor or skill, implying a non-degenerate distribution of income.

The final good is perfectly tradable, with its international price equal to one. Let the exchange rate, maintained by the monetary authority, be normalized to unity also. The aggregate money supply, the sum of domestic credit and foreign reserves, is thus perfectly elastic and perfectly accommodates the aggregate demand for money. Given the price level and the exchange rate, there is effectively no difference between nominal and real balances. Let the initial distribution of money balances among the H households, at time 0, be given by $m_{10}, m_{20}, \dots, m_{H0}$ respectively.

Following money-in-the-utility-function approach, households are assumed to derive utility from consumption of the tradable goods as well as from holding (real) money balances. They maximize discounted value of their lifetime utility subject to the budget constraint. Denoting consumption by c_{ht} and money balance by m_{ht} , the objective of the h -th household is given by

$$\text{Maximize } W_h = \int_0^\infty u(c_{ht}, m_{ht}) \exp \left[- \int_0^t \delta(u(c_{hv}, m_{hv})) dv \right] dt. \quad (1)$$

Note that, unlike in the standard dynamic problems, the discount rate is not constant. It depends on current utility. Apart from that we assume decreasing instead of increasing marginal impatience, the objective function is a generalization of Obstfeld (1990). The function δ denotes the instantaneous rate of time preference. The household maximizes (1) subject to

$$\dot{m}_{ht} = \frac{dm_{ht}}{dt} = y_h - c_{ht}; \quad m_{h0} \text{ given.} \quad (2)$$

The following three assumptions characterize the instantaneous utility function u and the instantaneous rate of time preference function δ .

Assumption 1

- (a) The function $u(\cdot)$ is twice continuously differentiable; $u(0, 0) = 0$; $u_c, u_m > 0$; $u_{cc}, u_{mm} < 0$; $u_{cm} \geq 0$.
- (b) The the function $u(\cdot)$ is homogeneous in c and m , such that u_m/u_c is a function of the ratio of consumption to money balances.¹
- (c) $\lim_{x \rightarrow 0} u_x = \infty$ and $\lim_{x \rightarrow \infty} u_x = 0$ for $x = c, m$. That is, Inada conditions hold with respect to consumption and money balance.

Let $u_m(c_h, m_h)/u_c(c_h, m_h) \equiv g(m_h/c_h)$. Assumptions 1(a) and 1(b) together imply that $g' < 0$. We further assume that

- (d) The function $g(\cdot)$ is convex in m_h at any given value of c_h . This means that $g'' \geq 0$.²

Assumption 2

- (a) $\delta(0) = \bar{\delta} > 0$; $\delta'(u) < 0 \leq \delta''(u)$.
- (b) $\lim_{u \rightarrow \infty} \delta(u) = \underline{\delta} > 0$.³

The crucial assumption here is that $\delta' < 0$, which implies DMI.

¹More generally we can assume the utility function is homothetic, i.e. $u = K(F(c, m))$, where $K' > 0$ but not constant and $F(c, m)$ homogeneous such that $F_c, F_m > 0$, $F_{cc}, F_{mm} < 0$, $F_{cm} \geq 0$, $u_c, u_m > 0$, $u_{cc}, u_{mm} < 0$, $u_{cm} \geq 0$.

²For instance, a Cobb-Douglas or CES utility function or $u = Ac_h^\alpha + Bm_h^\alpha$ satisfies this property.

³Examples include $\delta = k + \exp(-\xi u)$ and $\delta = k + 1/(1 + u)^\xi$, where $k, \xi > 0$.

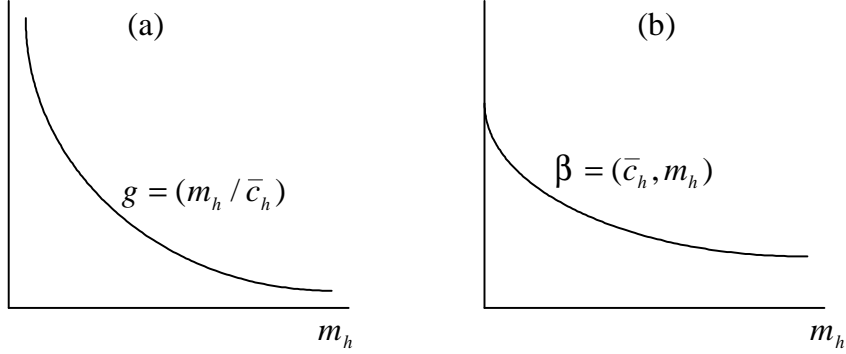


Figure 1: The $g(\cdot)$ and $\beta(\cdot)$ Functions

Assumption 1 implies that g as function of m_h , at any given value of c_h , has a shape as shown in Figure 1(a). It is convex to the origin and asymptotic to both axes.

Define $\beta(c_h, m_h) \equiv \delta(u(c_h, m_h))$. Then, by virtues of Assumptions 1 and 2,

$$\begin{aligned} \beta &> 0; \beta_c = \delta' \cdot u_c < 0; \beta_m = \delta' \cdot u_m < 0; \\ \beta_{cc} &= \delta' \cdot u_{cc} + \delta'' \cdot u_c^2 > 0; \beta_{mm} = \delta' \cdot u_{mm} + \delta'' \cdot u_m^2 > 0. \end{aligned} \tag{3}$$

Moreover, β , as a function of m_h , at any given value of c_h , has a shape as shown in Figure 1(b); that is, it has a finite, positive intercept on the vertical axis as well as a strictly positive lower bound.

The shapes of these curves imply that they intersect at least once. The following assumption ensures that they intersect exactly once and at the point of intersection the $g(\cdot)$ curve is steeper.

Assumption 3. At any given value of c_h , the $g(\cdot)$ function falls faster than the $\beta(\cdot)$ curve.

Indeed, we will see later that an intersection of these curves defines the steady state. In other words, DMI requires that the $\beta(\cdot)$ curve be downward sloping, while the uniqueness of the steady state is ensured by this curve not being too steep. Appendix A provides two

examples where Assumption 3 is met.

Define $\theta_{ht} \equiv \int_0^t \delta(u_h)dv = \int_0^t \beta(c_{hv}, m_{hv})dv$. Then

$$\dot{\theta}_{ht} = \beta(c_{ht}, m_{ht}) \text{ and } \theta_{h0} = 0. \quad (4)$$

This transformation enables us to restate the optimization problem as:

$$\text{Maximize } W_h = \int_0^\infty u(c_{ht}, m_{ht})\exp(-\theta_t)dt, \text{ subject to (2) and (4).}$$

Defining the current-value Hamiltonian as $H.\exp(\theta_{ht}) = u(c_{ht}, m_{ht}) + \mu_{ht}(y_h - c_{ht}) - \phi_{ht}\beta(c_{ht}, m_{ht})$, we have the following first-order conditions:

$$u_c(c_{ht}, m_{ht}) - \mu_{ht} - \phi_{ht}\beta_c(c_{ht}, m_{ht}) = 0 \quad (5)$$

$$\dot{\mu}_{ht} = -u_m(c_{ht}, m_{ht}) + \phi_{ht}\beta_m(c_{ht}, m_{ht}) + \mu_{ht}\beta(c_{ht}, m_{ht}) \quad (6a)$$

$$\dot{\phi}_{ht} = F_\phi(\phi_{ht}, m_{ht}, c_{ht}) \equiv \phi_{ht}\beta(c_{ht}, m_{ht}) - u(c_{ht}, m_{ht}) \quad (6b)$$

$$\dot{m}_{ht} = F_m(\phi_{ht}, m_{ht}, c_{ht}) \equiv y_h - c_{ht} \quad (6c)$$

$$\dot{\theta}_{ht} = \beta(c_{ht}, m_{ht}) \quad (6d)$$

$$\lim_{t \rightarrow \infty} H.\exp(-\theta_{ht}) = 0. \quad (6e)$$

Totally differentiating (5) with respect to time, simplifying and using eqs. (6a)-(6c), we get

$$\begin{aligned}
\dot{c}_{ht} &= F_c(\phi_{ht}, m_{ht}, c_{ht}) \\
&\equiv \frac{\beta_c(\cdot)u(\cdot) - \beta(\cdot)u_c(\cdot) + u_m(\cdot) - \phi_{ht}\beta_m(\cdot) + (y_h - c_{ht})[u_{cm}(\cdot) - \phi_{ht}\beta_{cm}(\cdot)]}{-u_{cc}(\cdot) + \phi_{ht}\beta_{cc}(\cdot)} \quad (7)
\end{aligned}$$

Eqs. (6b), (6c) and (7) essentially constitute the dynamic system for household h . Our small country assumption yields that the price level is exogenous and hence each household's dynamic system is independent of any other's.

2.1 Steady State

In the steady state, for all households, $\dot{c}_{ht} = \dot{\phi}_{ht} = \dot{m}_{ht} = 0$. From (6b) and (6c), we have

$$\phi_h = \frac{u(c_h, m_h)}{\beta(c_h, m_h)}; \quad c_h = y_h. \quad (8)$$

Substituting (8) into (7), using $\dot{c}_{ht} = 0$ and recalling the $g(\cdot)$ function, we obtain a simple relation:

$$\beta(y_h, m_h) = g\left(\frac{m_h}{y_h}\right). \quad (9)$$

This is a key equation, which determines the demand for money by a household in the steady state. It states intuitively that, in the steady state, the rate of discount is equal to the marginal rate of substitution of consumption for money balances.

Referring back to Figure 1 we now see that the intersection of the two curves defines the steady state level of m_h . Given the shapes of the $\beta(\cdot)$ and $g(\cdot)$ functions it is evident that an intersection occurs - i.e. a steady state exists. Furthermore, under Assumption 3, they intersect only once, i.e., there are no multiple steady states.

2.2 Stability

We now address the issue of stability of the steady state. Unfortunately, eqs. (6b), (6c) and (7) describe a 3×3 system, not amenable to two-dimensional geometry. However, under our assumptions, the steady state *is* saddle-path stable – which is proven next.

Totally differentiating (6b), (6c) and (7) and evaluating the derivatives at the steady state,

$$\begin{aligned}
 F_{\phi\phi} &= \delta > 0; F_{\phi m} = -u_m(1 - \phi\delta'); F_{\phi c} = -u_c(1 - \phi\delta'); \\
 F_{m\phi} &= 0; F_{mm} = 0; F_{mc} = -1; \\
 F_{c\phi} &= \frac{-u_m\delta'}{\phi\delta''u_c^2 - (1 - \phi\delta')u_{cc}}; F_{cm} = \frac{u_{mm}(1 - \phi\delta') - u_{mc}(\delta - u\delta')}{\phi\delta''u_c^2 - (1 - \phi\delta')u_{cc}}; \\
 F_{cc} &= \frac{-u_{cc}(\delta - u\delta') + \phi\delta''u_c u_m}{\phi\delta''u_c^2 - (1 - \phi\delta')u_{cc}} > 0
 \end{aligned}$$

Since the system has a single predetermined variable m , the existence of a unique saddle-path is guaranteed if the matrix

$$J = \begin{pmatrix} F_{\phi\phi} & F_{\phi m} & F_{\phi c} \\ F_{m\phi} & F_{mm} & F_{mc} \\ F_{c\phi} & F_{cm} & F_{cc} \end{pmatrix}$$

has exactly one negative eigen root. Given the expressions of various partials, we find

$$\begin{aligned}
 \text{Trace } J &= F_{\phi\phi} + F_{mm} + F_{cc} > 0 \\
 \text{Det } J &= F_{\phi\phi}F_{cm} - F_{c\phi}F_{\phi m} = \frac{(1 - \phi\delta')u_m \left(\frac{u_c u_{mm} - u_m u_{cm}}{u_c^2} - u_m \delta' \right)}{\phi\delta''u_c^2 - (1 - \phi\delta')u_{cc}}.
 \end{aligned}$$

Note that in the numerator of Det J , the term $\frac{u_c u_{mm} - u_m u_{cm}}{u_c^2}$ is the slope of the $g(\cdot)$ curve and the term $u_m \delta'$ is the slope of the $\beta(\cdot)$ curve. Given that the $g(\cdot)$ curve is steeper at the steady state it follows that $\frac{u_c u_{mm} - u_m u_{cm}}{u_c^2} - u_m \delta' < 0$. This implies Det $J < 0$. The trace is equal to the sum and the determinant equals the product of the roots. Hence Trace $J > 0$ and Det $J < 0$ imply that the matrix J has two positive roots and one negative root. The steady state is thus saddle-path stable.

Indeed, one can work out the following local-solution expression of m_{ht} . Letting $-\lambda_h$ denote the negative eigen root,

$$m_{ht} = (m_{h0} - m_h^*) \exp(-\lambda_h t). \quad (10)$$

The demand for money by a household increases or decreases monotonically over time as its initial money holding falls short of or exceeds the steady-state level.

The upshot here is that the often-used but counter-intuitive assumption of IMI – increasing marginal impatience – is *not* necessary for either stability or uniqueness. Neither is DMI inconsistent with stability or uniqueness of steady state. Coupled with the assumption that the time-preference as a function of m_h is steeper than marginal rate of substitution of consumption as a function of m_h , DMI implies that the steady state is stable and unique. Of course, without this assumption, there may be multiple steady states and ‘poverty traps’ in the presence of DMI; but these are separate issues.⁴

⁴There is an analogy of this to the case of diminishing marginal cost in the theory of an imperfectly competitive firm. Given that the marginal revenue curve is downward sloping, decreasing marginal cost may vitiate the second-order condition so that MR=MC doesn’t spell the profit maximization for say a monopoly. But, we don’t dismiss decreasing MC (i.e. scale economies of this sort) on this ground. Numerous studies model scale economies under imperfect competition, by assuming that MC decreases slower than the MR, so that the standard first-order condition of profit maximization is applicable.

2.3 Effect of Income Distribution

Our primary interest lies in the effect of a change in income inequality. Towards this end, consider how an increase in the income of a household will affect its long-run demand for money. The crucial equation is the steady-state condition (9). At any given m_h , an increase in y_h tends to increase u_h and hence decrease β_h . Thus, in terms of Figure 2 the $\beta(\cdot)$ curve shifts in. The ratio m_h/y_h falls as y_h increases; g' being negative, the $g(\cdot)$ curve then shifts out. It follows that the household's demand for money increases. This is an 'obvious' prediction given that money is a normal good in the utility function. What is not so obvious is that, as both y_h and m_h increase, the value of $\beta(\cdot)$ decreases (by virtue of DMI). This implies that $g(\cdot)$ decreases in equilibrium – which, in turn, implies that the ratio m_h/y_h increases as y_h rises.⁵ In other words,

Proposition 1 There are diseconomies of scale in holding money in the sense that as income increases, not only the money holding increases, but also its ratio with respect to income increases.

The assumption of DMI is the key here. An equivalent way to state this proposition is that money demand is an increasing and a strictly convex function of household income.

We are now ready to analyze the effect of a change in income distribution. Let a mean-preserving spread in y_h define an increase in income inequality. Given that money demand is a convex function of income, Jensen's inequality implies immediately that

Proposition 2 An increase in income inequality leads to an increase in the aggregate

⁵To see this, note that

$$\frac{dg}{dy_h} = \frac{d\beta}{dy_h} = \frac{\partial\beta}{\partial y_h} + \frac{\partial\beta}{\partial m_h} \frac{dm_h}{dy_h} < 0 \Rightarrow \frac{d(m_h/y_h)}{dy_h} = \frac{1}{g'} \frac{dg}{dy_h} > 0.$$

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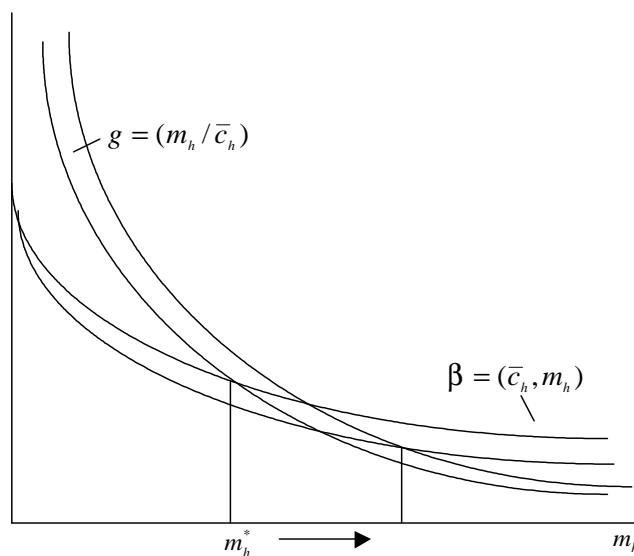


Figure 2: Increase in Income

demand for money in the long run.⁶

This is our main proposition. It is intuitive to see that DMI is the critical assumption behind it.

Proposition 2 relates to the long run. How does an increase in income inequality affect aggregate demand for money over the transition period, especially in the (immediate) short run? Given that the economy is initially (at $t = 0$) in a steady state, a change in income distribution doesn't affect m_{h0} . But it will affect money holding in subsequent periods. The question is: how does \dot{m}_{h0} change? If it is strictly convex or concave respectively with respect to y_h , then the aggregate demand for money in the short run increases or decreases respectively in response to an increase in income inequality. However, it is generally hard to ascertain the short-run effects, because they depend on the long-run effect as well as on the magnitude of the mean-preserving spread and how a change in income affects the speed of adjustment, λ_h .

⁶Formally, we can follow the procedure as given in Mas-Collell, Whinston and Green (1995, Example 6.D.2), or more generally we can appeal to the notion of the second-order stochastic dominance (Hadar and Russell (1969) and Mayer and Ormiston (1989)).

Formally, from (10), $\dot{m}_{h0} = -\lambda_h(m_{h0} - m_h^*)$ and hence

$$\frac{d\dot{m}_{h0}}{dy_h} = \lambda_h \frac{dm_h^*}{dy_h} - (m_{h0} - m_h^*) \frac{d\lambda_h}{dy_h} = \lambda_h \frac{dm_h^*}{dy_h} > 0,$$

since $m_h^* = m_{h0}$ originally. Totally differentiating this expression again with respect to y_h and using $m_h^* = m_{h0}$ as the initial condition,

$$\frac{d^2(\dot{m}_{h0})}{dy_h^2} = \lambda_h \frac{d^2 m_h^*}{dy_h^2} + 2 \frac{d\lambda_h}{dy_h} \frac{dm_h^*}{dy_h}. \quad (11)$$

The last expression (11) shows that whether or not money demand in the short-run is a convex function of income depends in part on the long-run effect and in part on how the speed of adjustment is affected. These effects are represented respectively by the first and the second term in the right-hand side. The first effect is positive: diseconomies of scale in holding money in the long run tend to imply such diseconomies in the short run as well, all other things remaining the same. The response of the speed of adjustment (along the saddle path) is a complex outcome of initial values – as well as signs and values of the derivatives of the time-preference function and the utility function up to the third degree. If $d\lambda_h/dy_h \geq 0$, then $d^2(\dot{m}_{h0})/dy_h^2 > 0$ unambiguously, implying an increase in the aggregate demand for money in response to an increase in income inequality. If $d\lambda_h/dy_h < 0$, then, in general, $d^2(\dot{m}_{h0})/dy_h^2 \geq 0$; but if $|d\lambda_h/dy_h|$ is not too large, $d^2(\dot{m}_{h0})/dy_h^2$ is positive again.

Proposition 3 As there is a mean-preserving spread in income, the aggregate demand for money increases in the short run as long as either $d\lambda_h/dy_h \geq 0$, or $d\lambda_h/dy_h < 0$ but $|d\lambda_h/dy_h|$ sufficiently small.

There is however no particular theoretical bias for $d\lambda_h/dy_h$ to be positive or negative in sign. It is therefore likely that $|d\lambda_h/dy_h|$ is small and thus an increase in income inequality

increases the aggregate demand for money in the short run also.

We now subject the basic prediction of our model to empirical test.

3 Empirical Evidence

As said in the Introduction, Cover and Hooks (1993) seems to be the only existing empirical work that links demand for money to income inequality measures. Below we generally follow their empirical approach. However, unlike theirs, we have a longer annual time series for the U.S., 1947-2001 (compared to their data span of 1947-1988), and the possibility of cointegration between money demand variables is considered.

Our empirical analysis finds the effects of our inequality measures on aggregate demand for money to be positive, thus supporting our theoretical finding involving the assumption of decreasing marginal impatience and the earlier empirical finding by Cover and Hooks. The greater the inequality of income, the greater is the aggregate demand for money. Conversely, the less the inequality of income, the less is the aggregate demand for money.

Cover and Hooks used autoregressions on the differences of the money demand variables to investigate the relationship between income inequality and money demand. However, they did not entertain the possibility of cointegration of the non-stationary money variables possibly because of the short time series they were forced to work with (42 observations). We have 13 more observations so we decided to conduct Engle/Granger (1987) tests of cointegration to avoid possible misspecification of our money demand equations. Without testing for cointegration we cannot reasonably decide whether the money demand equations should be estimated in error correction form or as autoregressions on differences as in Cover and Hooks. Our cointegration test results presented below, however, indicate the absence of cointegration. Thus, the autoregressions on differences used by Cover and Hooks again appear appropriate

for the extended data set we examine here.

3.1 Variables

We consider both M1 and M2 measures of money stock. As the basic explanatory variables, the standard estimation of money demand includes a scale variable and an opportunity cost variable. We use real GDP and real aggregate consumption as alternative scale variables. Moody's AAA corporate interest rate is taken to represent the opportunity cost of holding money. Our focus being on the effect of income inequality on aggregate demand for money, we also include a measure of income inequality. The GINI coefficient of income across families and the lowest 60th quantile income are taken as alternative measures of income inequality.⁷ Data sources are described in Appendix B.

3.2 Unit-Root Tests

Our first task was to determine the stochastic order of our time series variables – by carrying out unit-root tests. The results are reported in Table 1, where

$LRM1_{CPI}$ ($LRM1_{GDP}$): log of real M1 when M1 is deflated by CPI (consumer price index) or the GDP deflator;

$LRM2_{CPI}$ ($LRM2_{GDP}$): similarly defined;

AAA : the interest rate variable;

$LRCO$: log of real consumption;

$LRGDP$: log of real GDP;

$LGINIF$: log of GINI coefficient of family income;

$Q60$: the lowest 60th quantile income level.

⁷There are other measures of income inequality available, such as Atkinson's. But interval of years for which it is available is much smaller – from 1970 onwards (*Bureau of Census*).

On the levels of the data we conducted (a) the augmented Dickey-Fuller (ADF) unit-root test (Dickey and Fuller (1979)), (b) the Phillips-Perron (PP) unit-root test (Phillips and Perron (1988)), and (c) the KPSS unit-root test (Kwiatkowski, Phillips, Schmidt, and Shin (1992)). In the case of the ADF and PP tests, the null hypothesis is that the series contains a unit-root and must be differenced in order to make it stationary ($I(1)$). In contrast, the KPSS test has as its null hypothesis trend (or mean) stationarity of the time series. For the ADF and PP tests the alternative hypothesis is either trend stationarity or mean stationarity depending on the apparent presence or absence of trend in the data. The alternative hypothesis assumed by the KPSS test is that of difference stationarity. In addition, the probability values of the test statistics under the null hypothesis are reported in parentheses below the test statistics. Confirmatory analysis of the chosen stochastic order is carried out by examining the stochastic order of the differenced data by means of the ADF and PP unit root tests.

As is evident from Table 1, the stochastic order of all variables appears to be $I(1)$. On the levels of the data, the ADF and PP tests accept the null hypothesis of a unit-root, while the KPSS test rejects the null hypothesis of trend stationarity and accepts the alternative hypothesis of difference stationarity. With respect to the first difference of the series, the alternative hypothesis of mean stationarity is accepted indicating that first-order differencing is sufficient to establish stationarity of all of the individual time series.

3.3 Cointegration Tests

Given that all of the time series are $I(1)$, we considered the possibility of cointegration among the series. Previous studies of money demand based on quarterly data both in the U.S. and internationally have generally found evidence of cointegration among some money demand variables. See, for instance Engle and Granger (1987), Dickey et al. (1991), Hafer and Jansen

(1991), and Hoffman and Rasche (1991) for the U.S. and Hoffman et al. (1995) for international comparisons of money demand in five industrial countries. In contrast to these studies using quarterly data, our study focuses on annual data because our income inequality measures are available only on an annual basis. Even in the quarterly studies when cointegration is found between one definition of money and scale variables, it need not be found between other measures of money and the same scale variables. For example, Engle and Granger (1987) find cointegration between M2 and GNP but not between M1, M3, or total liquid assets and GNP. Also see Ebrill (1988) and Hwang (2002) for similar findings on money demand for the U.S. and Korea. Sriram (2001) provides a recent comprehensive review of the cointegration research on money demand.

In our context, since the annual time series we are dealing with is quite short (53 observations), we decided to forego the system-based Johansen tests (1988, 1991) which are very expensive in terms of degrees of freedom and, thus, subject to substantial loss of power. Instead we used the original single-equation, residual-based tests proposed by Engle and Granger (1987) with response surface critical values provided by MacKinnon (1991). Table 2 reports the results of the Engle/Granger cointegration tests on the various possible cointegrating (long-run equilibrium) relationships involving the four logarithmic real money measures and the two inequality measures. The test statistic reported in Table 2 is the ADF t-statistic on the lagged residual of the potentially cointegrated equation along with the number of augmenting terms chosen by the Schwartz Information Criterion. The null hypothesis of the test is no cointegration whereas the alternative hypothesis is cointegration. As one can see from Table 2 *none* of the eight money demand functions in levels are cointegrated. Therefore, we proceed to estimate the various money demand equations in autoregressive, differenced form as originally proposed by Cover and Hooks (1993). These estimation results are reported in the next subsection.

3.4 Estimation of the Model in Autoregressive Form

Assuming the absence of cointegration among the time series, we estimated the model in the “ARX” form, meaning in terms of short-term dynamic equations in differenced form with explanatory variables (X). The results are reported in Table 3 and Table 4 respectively for M1 and M2. The differenced dependent variable, the annual growth rates of real M1 or real M2, is regressed on one or more lags of itself plus current and lagged “X-variables” in differenced form. The one-period lagged value is denoted by $GRM(-1)$. The X-variables consist of (a) the annual growth of real GDP (GY) or that of real consumption (GC), (b) the annual change in the AAA corporate interest rate ($DRATE$), and (c) the annual growth of the family Gini index ($DLGINIF$) or the change in the lowest 60th quantile for families ($DQ60$). Additional autoregressive lags of the dependent variable (of orders higher than one) were tried and found to be statistically insignificant. In addition, additional lags of GY , GC , $DRATE$, $DLGINIF$, and $DQ60$ were found to be statistically insignificant when added to the ARX equations.

Several diagnostic tests on the residuals were run to determine if the least squares standard errors should be adjusted for serial correlation and/or heteroscedasticity. With respect to serial correlation, the Breusch (1978) and Godfrey (1978) LM test with two lags was examined. The null hypothesis is no serial correlation while the alternative hypothesis is serial correlation. For heteroscedasticity, two tests were conducted, White’s (1980) test (with no cross-product terms) and a t -test of the significance of trend in a regression of the squared least squares residuals on a constant term and a time index. We found the latter often to be more informative and so we report it here. The null hypothesis of no heteroscedasticity in time is accepted when the t -statistic in the squared-residuals equation is statistically insignificant, otherwise the alternative of heteroscedasticity is accepted. For the Bruesch-Godfrey LM test the probability value is reported in parentheses below the test statistic. For the t -test of time-dependent

heteroscedasticity, the probability value is reported in parentheses below the test statistic. In most cases, there was at least moderate amounts of serial correlation in the residuals of the ARX equations and most frequently severe, time-dependent heteroscedasticity. As a result we choose to report the Newey-West (1987) heteroscedasticity-autocorrelation-consistent (HAC) standard errors for the least squares estimates in Tables 3 and 4.

For real M1 in Table 3, we see that the inequality measures add significantly to the explanatory power of the demand equations with strongly significant coefficients of expected sign and increased adjusted R-square measures. (See equations (2), (3), (5), and (6).) Given our theory presented in the earlier section, the expected sign of the annual growth in the family GINI coefficient ($DLGINIF$) is positive while that of the annual change in the lowest 60th quantile ($DQ60$) is negative. With respect to real M2 in Table 4, we see that when real GDP is used as the “scale variable” the effects of the inequality measures are statistically significant and of the expected sign. (See equations (2) and (3).) However, when real consumption is used as the scale variable, the direction of effect of the inequality measures is as expected but they are not statistically significant, although $DQ60$ does increase slightly the adjusted R-square of the model. (See equations (5) and (6).) Thus, one might say that the short-run dynamic effects of inequality have more significant effects for the narrow definition of money (M1) than for the broad definition of money (M2), although, in both cases the effects are of the expected sign given the DMI hypothesis.

4 A Sum-Up

The paper has ‘applied’ the notion of decreasing marginal impatience to the case of demand for money. In the first part of the paper we have shown that, under decreasing marginal impatience, individual demand for money is an increasing, convex function of income. This

implies that aggregate demand for money increases with income inequality. The second part of the paper has conducted an empirical exercise of this prediction in the context of the U.S. economy. The conclusion of this exercise is shown to agree with our theoretical result. Needless to say, more empirical work is warranted with respect to other countries. Paucity of contiguous time-series data on income inequality measures is however a constraining factor in case of many countries. But the availability of such data is improving over time. Hopefully then it will be possible in the near future to comprehensively address the important issue of how income and wealth distribution may affect the demand for money – as well as other assets – in an economy.

Table 1. Unit-Root Tests

Variable	Level Data			First-Differenced Data	
	ADF	PP	KPSS	ADF	PP
<i>LRM1_{CPI}</i>	-3.111015 (0.1144)	-2.225194 (0.4663)	0.191939 (.01<p<.05)	-5.054238 (0.0001)	-4.798445 (0.0002)
<i>LRM1_{GDP}</i>	-2.543272 (0.3071)	-2.126130 (0.5198)	0.179340 (.01<p<.05)	-4.792491 (0.0002)	-4.520944 (0.0006)
<i>LRM2_{CPI}</i>	-2.078486 (0.5455)	-0.1775376 (0.7029)	0.418494 (p<.01)	-4.566507 (0.0005)	-4.6258975 (0.0004)
<i>LRM2_{GDP}</i>	-2.427808 (0.3616)	-2.407527 (0.3717)	0.213855 (.01<p<.05)	-4.379209 (0.0009)	-4.277041 (0.0013)
<i>AAA</i>	-1.324645 (0.87091)	-1.056394 (0.9268)	1.133318 (p<.01)	-5.295410 (0.0000)	-5.295410 (0.0000)
<i>LRCON</i>	-2.022954 (0.5755)	-1.791133 (0.6953)	0.192711 (.01<p<.05)	-6.114262 (0.0000)	-6.026037 (0.0000)
<i>LRGDP</i>	-2.280997 (0.4367)	-2.182546 (0.4893)	0.130901 (.05<p<.10)	-6.964347 (0.0000)	-7.074055 (0.0000)
<i>LGINIF</i>	-1.726560 (0.7257)	-1.53799 (0.8039)	0.222666 (p<.01)	-7.896260 (0.0000)	-7.912525 (0.0000)
<i>Q60</i>	-1.802647 (0.6897)	-1.625612 (0.7697)	0.205754 (.01<p<.05)	-7.966956 (0.0000)	-7.984473 (0.0000)

Note: These unit root “t-tests” were conducted in EVIEWS©, Version 4.1 (2002). The lag length for the ADF test was determined by minimizing the Schwartz Information Criterion (SIC). The Newey-West bandwidth and Barlett kernel were used for the PP test (the default in EVIEWS). The Andrews bandwidth and Parzen kernel were used for the KPSS test. The results of the PP tests were not sensitive to the choice of bandwidth and kernel. The bandwidths of the KPSS tests were very wide when using the Newey-West bandwidth and Bartlett kernel, hence, we choose to obtain a more reasonable bandwidth by using the Andrews bandwidth and Parzen kernel. P-values of the statistics are reported in parentheses below the statistics.

Table 2. ENGLE/GRANGER Cointegration Tests

System	ADF t -statistic on RESID(-1) (No intercept, no trend)
$LRM1_{GDP}$ Vs. $C, LRGDP, AAA, LGINIF$	-2.830525
$LRM1_{GDP}$ Vs. $C, LRGDP, AAA, Q60$	-2.669145
$LRM1_{CPI}$ Vs. $C, LRCON, AAA, LGINIF$	-2.958397
$LRM1_{CPI}$ Vs. $C, LRCON, AAA, Q60$	-2.949903
$LRM2_{GDP}$ Vs. $C, LRGDP, AAA, LGINIF$	-3.066723
$LRM2_{GDP}$ Vs. $C, LRGDP, AAA, Q60$	-3.065637
$LRM2_{CPI}$ Vs. $C, LRCON, AAA, LGINIF$	-3.143516
$LRM2_{CPI}$ Vs. $C, LRCON, AAA, Q60$	-3.064674

10% critical value = -3.964; 5% critical value = -4.302; 1% critical value = -4.981.

Note: Critical values were obtained from MacKinnon (1991).

These unit-root “t-tests” were conducted in EViews©, Version 4.1 (2002). The lag length for the ADF Engle/Granger test was determined by minimizing the Schwartz Information Criterion.

Table 3. Money Demand Equations for Real M1, ARX Form (1947-2000)

	Equation					
	(1) <i>GRM1_{GDP}</i>	(2) <i>GRM1_{GDP}</i>	(3) <i>GRM1_{GDP}</i>	(4) <i>GRM1_{CPI}</i>	(5) <i>GRM1_{CPI}</i>	(6) <i>GRM1_{CPI}</i>
Constant	0.008684 (0.007524)	0.002959 (0.008371)	0.001952 (0.008120)	-0.005857 (0.010718)	-0.010658 (0.010865)	-0.012186 (0.010652)
<i>GY</i>	0.030310 (0.155420)	0.159328 (0.179449)	0.184659 (0.174165)	-	-	-
<i>GC</i>	-	-	-	0.413125* (0.242507)	0.507429** (0.249604)	0.543838** (0.007969)
<i>DRATE</i>	-0.026544*** (0.007173)	-0.024121*** (0.007513)	-0.024165*** (0.007640)	-0.029519*** (0.007173)	-0.026782** (0.007864)	-0.026580*** (0.007969)
<i>GRM(-1)</i>	0.308389*** (0.113359)	0.283302** (0.109587)	0.273284** (0.112952)	0.205543* (0.117606)	0.205013* (0.108174)	0.197466* (0.109630)
<i>DLGINIF</i>	-	0.011657** (0.005579)	-	-	0.010630** (0.004990)	-
<i>DQ60</i>	-	-	-0.017486** (0.006984)	-	-	-0.016526*** (0.006328)
<i>R</i> ²	0.416162	0.479565	0.441076	0.470646	0.485123	0.491698
<i>SE</i>	0.028948	0.028447	0.028324	0.030107	0.029693	0.029503
<i>LM(2)</i>	2.788711 (0.247993)	3.202357 (0.201659)	2.895602 (0.235087)	3.312292 (0.190873)	3.189255 (0.202984)	2.843217 (0.241326)
<i>ê²vs.c, t</i>	3.108737*** (0.0031)	3.831194*** (0.0004)	3.799459*** (0.0004)	3.517516*** (0.0009)	3.839698*** (0.0003)	3.874077*** (0.0003)
Method	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC

***Significant at 1%; ** Significant at 5%; *Significant at 10%

Note: HAC Standard errors are reported in parentheses below the coefficient estimates. Probability values for the LM(2) and $\hat{e}^2 vs. c, t$ statistics are reported in parentheses below the statistics. The HAC standard errors are based on Newey-West (1987) and their automatic bandwidth selection. All of these statistics were computed in EVIEWS© 4.1.

Table 4. Money Demand Equations for Real M2, ARX Form (1947-2000)

	Equation					
	(1) <i>GRM2_{GDP}</i>	(2) <i>GRM2_{GDP}</i>	(3) <i>GRM2_{GDP}</i>	(4) <i>GRM2_{CPI}</i>	(5) <i>GRM2_{CPI}</i>	(6) <i>GRM2_{CPI}</i>
Constant	0.008684 (0.007524)	0.002959 (0.008371)	0.001952 (0.008120)	-0.005680 (0.008608)	-0.007233 (0.007881)	-0.009296 (0.007090)
<i>GY</i>	0.030310 (0.155420)	0.159328 (0.179449)	0.184659 (0.174165)	-	-	-
<i>GC</i>	-	-	-	0.802116*** (0.235999)	0.818775*** (0.236292)	0.857975*** (0.226062)
<i>DRATE</i>	-0.026544*** (0.007173)	-0.024121*** (0.007513)	-0.024165*** (0.007640)	-0.017015*** (0.004102)	-0.015906 (0.003737)	-0.015024*** (0.003743)
<i>GRM(-1)</i>	0.308389*** (0.113359)	0.283302** (0.109587)	0.273284** (0.112952)	0.142180 (0.156866)	0.159208 (0.151494)	0.163921 (0.157759)
<i>DLGINIF</i>	-	0.011657** (0.005579)	-	-	0.004261 (0.007790)	-
<i>DQ60</i>	-	-	-0.017486** (0.006984)	-	-	-0.0101953 (0.010609)
\bar{R}^2	0.416162	0.436196	0.441076	0.464951	0.459379	0.473403
<i>SE</i>	0.028948	0.028447	0.028324	0.025464	0.025596	0.025262
<i>LM(2)</i>	2.788711 (0.247993)	3.202357 (0.201659)	2.895602 (0.235087)	12.93132*** (0.001556)	13.50275*** (0.001169)	14.10363*** (0.000866)
$\hat{e}^2 vs. c, t$	3.108737*** (0.0031)	3.831194*** (0.0004)	3.799459*** (0.0004)	-0.059647 (0.9529)	0.388594 (0.6992)	0.834005 (0.4082)
Method	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC	LS with Newey- West HAC

*** Significant at 1%; **Significant at 5%; *Significant at 10%

Note: See footnote for Table 3.

Appendices

A Examples wherein Assumption 3 is met

This assumption is equivalent to

$$|g'(m_h)| > |\beta'(m_h)| \Leftrightarrow \frac{u_m u_{mc}}{u_c} - u_{mm} > -\delta' \cdot u_c u_m. \quad (\text{A1})$$

One example is where $u = Ac_h^\alpha m_h^{1-\alpha}$ and $\delta(u) = k + \frac{1}{\xi(1+u)^\xi}$, with parametric restrictions,

(i) $\xi = \frac{1}{1-\alpha}$ and (ii) $y_{\min} > \left(\frac{\alpha}{A^{1-\alpha}}\right)^{\frac{1}{\alpha(2-\alpha)}}$, y_{\min} being the minimum income across households.

The proof is as follows.

With the above functional forms of $u(\cdot)$ and $\delta(\cdot)$, the condition (A1) reduces to

$$\frac{c_h^{1-\alpha} (1 + Ac_h^\alpha m_h^{1-\alpha})^{1+\xi}}{m_h^{2-\alpha}} > A\alpha.$$

Substituting $\xi = \frac{1}{1-\alpha}$, this inequality reduces to

$$c_h^{1-\alpha} \left(\frac{1}{m_h^{1-\alpha}} + Ac_h^\alpha \right)^{\frac{2-\alpha}{1-\alpha}} > A\alpha.$$

This is satisfied for any value of m_h if $A^{\frac{1}{1-\alpha}} c_h^{\alpha(2-\alpha)} > \alpha$. In the steady state, $c_h = y_h$. Hence $\min c_h = \min y_{\min}$, and the last inequality is met if $A^{\frac{1}{1-\alpha}} y_{\min}^{\alpha(2-\alpha)} > \alpha$. This is equivalent to our condition (ii).

Our second example is where $u = Ac_h^\alpha + Bm_h^\alpha$ and $\delta = k + \exp(-\xi u)$. In this case the condition (A1) is same as

$$\frac{1-\alpha}{A\alpha\xi} c_h^{1-\alpha} \exp(A\xi c_h^\alpha) \frac{\exp(B\xi m_h^\alpha)}{m_h} > 1.$$

The function $\frac{\exp(B\xi m_h^\alpha)}{m_h}$ attains its minimum value at $m_h = (1/B\xi\alpha)^{1/\alpha}$. Substituting this value of m_h and using $c_h = y_{\min}$, we have

$$\frac{1-\alpha}{A\alpha\xi} (B\xi\alpha)^{\frac{1}{\alpha}} y_{\min}^{1-\alpha} \exp\left(\frac{1}{\alpha} + A\xi y_{\min}^\alpha\right) > 1.$$

If this inequality is met, the $g(\cdot)$ curve is steeper than the $\beta(\cdot)$ curve.

B Data Sources

For the macroeconomic and finance data from 1959 - 2002 we used the *Economic Report of the President, 2002*. To convert the M1 and M2 money measures to real terms, we used the GDP deflator when the real GDP scale variable was included in the demand equation and used the CPI deflator when the real consumption scale variable was included in the demand equation. The real measures of GDP and consumption were available directly. The Aaa bond rate was used as the interest rate for all of the demand specifications. To extend the macroeconomic and finance data from 1958 back to 1947 we used the following data sources:

Variable	Sources
M1, M2	Gordon (1998)
AAA bond rate	<i>Economic Report of the President</i> , 2002
Nominal GDP	<i>Survey of Current Business</i> , 2002
Real GDP (\$1996)	<i>Survey of Current Business</i> , 2002
Real Consumption (\$1996)	<i>Survey of Current Business</i> , 2002
GDP Deflator	<i>Survey of Current Business</i> , 2002
CPI	<i>Bureau of Labor Statistics</i>
GINI Coefficients	<i>Bureau of Census</i>
Quantile Shares	<i>Bureau of Census</i>

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