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Abstract

We provide a complete characterization of intergenerational welfare state with education and pension under probabilistic voting where voters internalize the general equilibrium effects materializing in their life-span. We show that as public education is introduced in the economy through the political process of voting, it always increases (reduces) the accumulation of human capital (physical capital), but strikingly, has no effect on the political equilibrium of PAYG social security tax. On the other hand, the introduction of a politically determined PAYG social security most definitely reduces physical capital accumulation, however it will reduce the human capital accumulation if only if the public education is already present in the economy. Otherwise, it may lead to an increase in the human capital accumulation. We also demonstrate that the general equilibrium effects are crucial to sustain the social security program, and explain why the presence of PAYG social security may not provide sufficient incentive for public investment in education. Finally, we show that the simultaneous arrangement of public education and pension can increase the long-run growth if and only if the relative political weight of the old is small so that the pension program is thin, which makes the result of Boldrin and Montes (2005) study conditional on the intergenerational distribution of voting power in our political economy setup.

Keywords : Education, Social security, Probabilistic voting, Markov Perfect Equilibrium, Endogenous growth

JEL Classification: E6, H3, H52, H55, D90

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1 Introduction

Recently the concern of future solvency of social security program has invoked a heated public policy debate (see Feldstein (2005) Gonzalez-Eiras and Niepelt (2008) and Kaganovich and Zilcha (2012) for example). So far it is clear that a cut in the generosity of social security is a serious political issue and the history confirms that a cut is almost impossible. This guarantees that lowering the social security tax rate is a distant possibility, keeping the tax rate unchanged may be the second best while increasing the tax rate is probably the more realistic route to deal with the present social security problem. This is also supported by the estimates provided by the Social Security Administration in the US, which says that the cost of providing the benefits specified in the current law will rise to 14.5 percent in 2030, 15.4 percent in 2050 and 16.6 percent in 2075. Gonzalez-Eiras and Niepelt (2008) deals with a probabilistic voting mechanism with population growth in which they take into account the general equilibrium effects of the existing policy on the future outcome. Their model suggests that in response to the projected demographic transition, in the US, social security tax rates will gradually increase to 16%. However, even in this background, the idea of lowering the tax rate has not been considered by the researchers since this policy probably will have a very small chance to see success.[1] This is the story in one hand and on the other, there are theories that suggest that since education and pension are the two biggest as well as directionally opposite avenues of spending in any welfare state, it is imperative to study both the instruments in unison, and not separately.[2] As a result, our study seems very natural. This paper provides a complete welfare analysis of a political economy where both these two arms of intergenerational transfers are present, and thus allows us to explore the inter-connection between the two public programs such as whether the existence of social security could increase the incentive to expand the public investment in education, or as public education program provides more generous funding in education investment, whether the enhanced future productivity could help reduce the size of social security program.

The framework that we consider is a standard overlapping generations economy where agents live for three periods. We present a base line model which is a laissez-faire economy where education is privately provided through parental investment, and we then test it against different policy experiments step by step. Under the policy experiments, we first consider a situation

[1] As argued by Mulligan and Sala-i-Martin (1999) that the political clout of the old people has grown beyond the predicted by the evolution of the demographics. They interpret the data in favor of the old as a political power enjoyed by those citizen (also see Preston (1984), Lindert (1994)). As is mentioned in Song, Storesletten and Zilibotti (2012), the voters turnout in the US is falling however, the participation of the old is increasing. Further, they report that the share of vote by the old with 61+ age is expected to reach to 50% by the year 2050 in the OECD countries. These phenomena well explain that in the voting framework, the equations are changing rapidly.

[2] Some of the earlier studies that considered the link between public education and public pension are Pogue and Sgontz (1977), Richman and Stagner (1986), Becker and Murphy (1988), Cremer, Kessler and Pestieau (1992) among others.

where a public source of education is available but PAYG social security is absent. However this provision of public education is subject to voting on the issue of education tax. As the next policy experiment, we consider a political economy with a pension scheme in place; however the source of funding for education is now private. Thus the above two analyses deal with the economies where only one intergenerational transfer instrument is present. Finally, we integrate the instruments together and consider an economy that not only has both the two instruments, namely education subsidy and pension together but also lets the agents vote simultaneously for these two instruments. Our dynamic political economy framework is a probabilistic voting model at each period where the voting power is distributed intergenerationally. In this equilibrium, voters are allowed to sequentially choose their policies under rational expectation about the effect on future outcomes. Further, we rely on the Markov Perfect Equilibrium (henceforth MPE) in the first two experiments and particularly on Simultaneous Nash MPE (which we call SNMPE and define later) for the last one. The politico - economic equilibrium are founded on competitive equilibrium with subgame perfect tax rate and transfers since the voters are not bound by their past political decisions. This complete analysis provides some important results that are worth noting.

According to Pogue and Sgontz (1977), PAYG social security creates incentives for public investment in education. Becker and Murphy (1988) also suggest that PAYG social security strengthens the political support among the current working agents for public investment in education. Their study connects education investment made by the parents with pension by considering this as a trade among generations; children receive education from their parents and in exchange pay for their old age benefits. In a relatively recent work, Rangel (2003) also focuses on the issue of sustainability when both forward and backward intergenerational goods are present and shows that backward intergenerational goods (BIGs), such as social security, play a crucial role in sustaining investment in forward intergenerational goods (FIGs) like education: without them investment is inefficiently low, but with them optimal investment is possible. The first observation that we present in this paper is in complete contrast to these above studies. We show that even with downward altruism, the presence of PAYG may not provide sufficient incentive for public investment in education. However, this result is somewhat in line with Soares (2006) and Kaganovich and Zilcha (2012). Soares (2006) in a calibrated OLG framework finds that infusing a PAYG system results in a political equilibrium with lower funding for public education compared to the case when social security is absent. This happens because when a PAYG scheme is present, the general equilibrium effects on factor prices dominates the incentive of the voters to increase the productivity of future workers, which as a result reduces the present value of the pension benefits (Kaganovich and Zilcha, 2012). Thus our first observation guarantees that the existence of PAYG may not increase the benefits from public education monotonically. This important result is due to the general equilibrium effect that the agents consider during their life time when choosing their tax rates optimally

in a political equilibrium. Further, when PAYG social security is present, we show how crucial the general equilibrium effects are to sustain the social security program, the channel which has also been emphasized by for e.g., Boldrin and Rustichini (2000) and Gonzalez-Eiras and Niepelt (2008).

Our second result conveys the message that under MPE, the introduction of public education scheme by probabilistic voting in a laissez-faire economy increases human capital but reduces physical capital accumulation. In addition to that, in the long run, when the production technology is more human capital intensive, it increases the long run growth rate too. This crucial result is a reconfirmation of the celebrated result of Glomm and Ravikumar (1992) where they show that the investment in education is higher in public arrangements than when the funding for education is private. It is interesting especially because we reconfirm their results through a probabilistic voting mechanism where voting power is distributed intergenerationally. We show that this qualitative result is quite robust in nature in the sense that it holds even in the presence of PAYG social security. Additionally we also find that when PAYG social security is already present, introducing public education has no effect on the equilibrium tax rate of PAYG social security, even though the public education program provides more generous funding in education investment than parental education. This finding has a strikingly strong implication. It implies that the existing wisdom that when PAYG pension is present, introducing a forward intergenerational good, namely publicly funded education, can be helpful since it enhances future productivity is no more correct. Keeping Gonzalez-Eiras and Niepelt (2008) in mind, if it is indeed needed to increase the pension tax rate in the future, we conjecture that even augmenting the level of education will not help in reducing the pension tax rate in a political equilibrium.

Further, we show that introducing PAYG social security with a positive pension tax in a laissez-faire economy reduces physical capital accumulation. Interestingly however, even with positive pension tax rate human capital may increase. On the other hand, if positive pension tax is introduced in an economy where publicly funded education is present, it will always lead to a reduction in both human and physical capital accumulation. The same interesting feature is reflected in terms of the tax rate too, that is, whenever the pension tax rate is positive, the PAYG social security will decrease the equilibrium tax rate of public education. This in fact guarantees that when publicly funded capital education is already present, introducing PAYG social security with a positive tax will hurt the long run growth.

As our final policy experiment, we introduce both public education and PAYG social security simultaneously in a laissez-faire economy. If we focus on the political weights for which the pension tax is positive, the presence of both the instruments always reduces the physical capital investment but human capital will increase if the relative weight to the old is small and thus the PAYG program is very thin. This is because in our model, the presence of PAYG discourages public investment in education and therefore the human capital production can be enhanced if

and only if the size of the PAYG program is small as a result of the low relative weight assigned to the old retirees, which, in turn guarantees that the effect of this on public investment in education is very limited. Indeed the result raises an interesting point. In Boldrin and Montes (2005), designing simultaneous existence of public education and PAYG pension is justified as a means to implement an intergenerational transfer scheme that supports complete market allocations when credit market is missing.[3] In our political economy setup, if the same question arises as to whether a simultaneous arrangement of these two-armed intergenerational transfers is justifiable for the long run growth, the answer would depend on the distribution of political power among the generations and thus making the Boldrin and Montes (2005) result "conditional" in our framework.[4]

Thus the present study can be seen as a political economy version of Rangel (2003) with two-armed intergenerational transfers. The additional focus besides providing a political structure is the consideration of the general equilibrium effect which turns out be crucial. This consideration of the general equilibrium effects is however missing from Rangel (2003). In comparison in our political economy setup, we take into account the fact that the intergenerational transfer policies affect the factor prices as well as the crucial variables in subsequent periods and an agent considers these effects that are realized in her life-time while voting. The present paper can also be seen from an angle where Gonzalez-Eiras and Niepelt (2008) appears as one of the special cases in which human capital and therefore one of the two (intergenerational) arms of the welfare state, namely education, is truncated from the analysis.[5] Naturally, in this light, the present

[3] Wang (2013) extends their study by endogenizing the imperfection of the credit market and shows that the result could hold even when borrowing constraints for education loan arise endogenously. In another interesting work, Andersen and Bhattacharya (2013) show that when the economy is dynamically efficient, the education-pension welfare packages that satisfy the Pareto criterion, that is, where all current and future generations have welfare at least as high as that attainable under complete education-loan markets, exist. The crucial observation in their paper is that the forward part (the borrowing part) is associated with welfare gains, while the backward part (the saving part), that is needed to compensate tax payers for financing education for the young, is associated with welfare losses. Further, they show that in the presence of an intergenerational human capital externality, the pension component of such a package can be entirely phased out eventually.

[4] According to Docquier, Paddison and Pestieau (2007), the definition of optimality in Boldrin and Montes (2005) framework is restrictive where the study disregards the effect of externality in education. They show that on the efficiency ground, the case for public pension is weak. In this analysis, because of the externalities, allocations of human and physical in competitive equilibrium differ from the planner's and a possibility naturally arises where the laissez-faire equilibrium experiences higher physical capital accumulation but lower human capital accumulation compared to the planner's allocations. However, Bishnu (2013) shows that if the origin of non-optimality of human capital accumulation is the consumption externality (may be because consumption of others is more visible than their level of human capital), the possibility that the accumulation of human and physical capital in a laissez-fare differ from the planner's in opposite direction is not at all feasible. This observation not only has crucial implication on pension and education subsidy but also can justify government intervention in education even in the absence of education externalities.

[5] Using a similar framework as in Gozalez-Eiras and Niepelt (2008), Song (2011) focuses on within generation heterogeneity and analyzes the interaction between social security transfers and wealth inequality. He finds that higher inequality is associated with higher equilibrium social security tax rates if social security redistributes within cohorts. This paper too relies on probabilistic voting and uses MPE, however like Gozalez-Eiras and Niepelt (2008), ignores the education channel of transfers.

paper seems more complete and has the flexibility to compare all the situations, specifically both private and public channels of investment in education and PAYG social security.

Here we must mention some papers in politico-economic framework that are related to our study[6]. In Boldrin and Montes (1998), expenditure in public education and social security are decided by majority rule and they show that the optimal allocation is sustainable using trigger strategies. Boldrin and Rustichini (2000) models pay-as-you-go social security systems as the outcome of majority voting. Cooley and Soares (1999) and Boldrin and Rustichini (2000) argue that the existence of intergenerational redistribution schemes, such as public debt or social security, tends to crowd out capital, and thus reduces real wages and increases real returns to capital. This creates a redistribution in favor of the old who is the owner of the capital and against the middle-aged individuals who provide human capital in our context. Poutvaara (2006) studies both pension and public education simultaneously in which he focuses on heterogeneous agents and where decisions are taken by majority voting. In an open economy framework with fixed factor prices, Naito (2012) also focuses on the sustainability of public education and pension under probabilistic voting setup where the proportion of expenditure on one program to the total expenditure is artificially fixed at some level. However, none of the two papers mentioned above consider the general equilibrium effect (as we did in our probabilistic voting framework) or private funding of education. In a very recent study, Ono (2013) incorporates longevity as well as altruism and allows voting on both the pension and publicly funded education. Though the framework and the focus are different from ours, on the technical front, this is the only paper beside ours which considers voting on bi-dimensional policy issues (as in subsection 3.3 below). However, while the agents vote on the generosity (volume) of the public programs in their case, in our model, they vote on the level of the tax instruments which forms one of the main discussions in this paper.

By bringing both private and public sources of education (both separately and in unison), along with PAYG under the GE effect in an economy with altruistically motivated agents, the present paper distinguishes itself from the existing papers in the literature. To the best of our knowledge, this model is the first attempt to study simultaneous voting on the two instruments (tax rates) of intergenerational transfers in a probabilistic voting model. Moreover we have taken into account the general equilibrium effect while comparing the effect of public funding

[6]Some of the few studies that are not in a political economy framework and have not been mentioned yet are as follows. Kaganovich and Zilcha (1999) consider an economy with altruistically-motivated parents who invest in the human capital of their children. According to them, the link between the level of human capital of the children and the parents' retirement benefits through taxing their children is disregarded in each parental educational decision, but it is captured by the government's social optimization. They find that for some parameter values the optimal policy entails not only subsidizing education but also taxing labor income to finance retirement benefits. The role of both public and private investment in education in the relationship between PAYG social security funding and economic growth was analyzed by Kaganovich and Zilcha (1999). Zhang and Zhang (1998) and Pecchenino and Utendorf (1999) studied the impact of PAYG social security on incentives for private investment in education.

in education with the effect of private funding, both in the absence as well as presence of PAYG social security. Naturally, the two-dimensional policy arrangements makes calculations more complicated, however, we have been able to characterize the possible cases analytically. We have also presented complete comparisons of the crucial variables when the source of education differs.

The rest of the paper is organized as follows. Section 2 presents the baseline model, that is, the laissez-faire economy. In section 3, we present the three policy experiments where different instruments are politically determined through a probabilistic voting model. Section 4 deals with the welfare implication of the intergenerational transfers. While section 5 concludes, proofs of Lemmas and Propositions are presented in the Appendix.

2 Laissez-faire Economy

We consider an economy that consists of an infinite sequence of three-period lived overlapping generations, an initial old generation and an initial middle-aged generation. In each generation, there is a continuum of identical agents of measure one. Agents receive education when young, while they work and carry out decisions of consumption, saving and education investment for their children during middle age. When they are old, the agents retire and consume out of the total return on their savings. An agent who is working at period t that is a middle-aged agent is called a generation-t agent.

Denote by h_t the human capital of an individual belonging to generation t. The human capital of a generation $t+1$ agent is a function of educational expenditure e_t she makes when young and her parent's human capital h_t, the endowment of basic knowledge she is born with. We assume the human capital is produced by a constant-return-to-scale technology $h_{t+1} = h(e_t, h_t) = Be_t^{\beta} h_t^{1-\beta}$, $B > 0$, $\beta \in (0,1)$.[7] There is a single final good produced with a constant returns to scale production function $F(K_t, H_t)$, where K_t and H_t are aggregate physical capital and human capital at t. Defining $k_t \equiv K_t/N_t$ and $h_t \equiv H_t/N_t$ in which N_t is the population of generation-t agents, output per middle-aged agent at time t can be expressed as an intensive form $f(k_t, h_t) = F(k_t, h_t)$. We assume that f takes the Cobb-Douglas form, i.e., $f(k_t, h_t) = Ak_t^{\alpha} h_t^{1-\alpha}$, $A > 0$, $\alpha \in (0,1)$. Since the measure of the members of each generation is one, we know $N_t = 1$ and thus $k_t = K_t$ and $h_t = H_t$. The final good can either be consumed in the period it is produced, or it can be saved to provide capital in the following period. Capital is conveniently assumed to depreciate fully between periods. Young agents supply labor inelastically in competitive labor markets, earning a wage of $w_t = \partial f(k_t, h_t)/\partial h_t = (1-\alpha) Ak_t^{\alpha} h_t^{-\alpha}$ at time t; similarly, capital is traded in competitive capital markets, and earns a gross real return of R_{t+1} between t and $t+1$ where $R_t = \partial f(k_t, h_t)/\partial k_t = \alpha A k_t^{\alpha-1} h_t^{1-\alpha}$.

[7] Most of our results hold for a human capital specification that abstracts from generational human capital externalities, that is, where h_{t+1} depends on e_t only.

We assume a generation-t agent draws utility from $(c_t^m, c_{t+1}^o, h_{t+1})$, the terms in the parenthesis denoting consumption at middle age and old age, and the level of human capital of her children respectively. More specifically, the life-time utility for a generation$-t$ agent is

$$\mathcal{U} \equiv u(c_t^m) + \delta \left[u\left(c_{t+1}^o\right) + \phi u\left(h_{t+1}\right) \right] \tag{1}$$

where $\delta \in (0,1)$ is the standard discount factor and $\phi \in (0,1)$ represents the relative weight assigned to the utility that an old agent enjoys from her children's human capital, expressing parents' altruism towards their offspring.[8] Instantaneous utility function u is continuously differentiable, strictly increasing and strictly concave. An agent when middle-aged allocates her labor income among consumption, saving and investment in education for her children.[9] Saving s_t while middle-aged returns $s_t R_{t+1}$ in the next period when the agent is old. These imply that $c_t^m = w_t h_t - s_t - e_t$ and $c_{t+1}^o = s_t R_{t+1}$. Thus given the factor prices, human capital production technology $h_t = h\left(e_{t-1}, h_{t-1}\right)$ and the budget constraints stated above, when a generation$-t$ agent maximizes her utility (1) with respect to $\{s_t, e_t\}$, we arrive at the following first order conditions[10]:

$$u'\left(w_t h_t - s_t^L - e_t^L\right) = \delta R_{t+1} u'\left(R_{t+1} s_t^L\right) \tag{2}$$

$$u'(w_t h_t - s_t^L - e_t^L) = \delta \phi u'\left(h_{t+1}\right) \frac{\partial h\left(e_t^L, h_t\right)}{\partial e_t^L}. \tag{3}$$

While equation (2) represents the optimal intertemporal consumption allocation, that is the standard Euler's equation, (3) indicates that the marginal sacrifice in utility from investing in education of the descendents is equal to the marginal benefit from utility gain adjusted to the gain in the level of human capital of their descendents. To derive closed-form solutions of political equilibrium and economic growth, we must impose functional form restrictions on the utility function (1). Specifically, we assume that the utility function is logarithmic. Specifically by solving the above two first order conditions, we get the followings:

[8] Utility specification that represents altruism through the level of human capital of the next generation is very common and vastly used in the literature. Our specification is simple and standard, for example, as in Kaganovich and Zilcha (1999), Pecchenino and Utendorf (1999) Glomm and Kaganovich (2003, 2008) and in line with Glomm and Ravikumar (1992), De la Croix and Doepke (2003) among others.

[9] As a common treatment in the literature, e.g., the studies mentioned in the footnote 8, we, in this paper, do not bring a credit market that can fund education for the young. All the discussions here are on the two sources, private investment (made by parents) and public investment (made by the government), due to the following considerations. First, these two sources are the main sources of funds in any economy. Second, owning to the inalienability of human capital, future labor income cannot be collateralized and credit markets severely restrict any borrowing against future human capital for education purposes. A credit market for education loan even does not exist in most developing countries, e.g., China or trivially thin as in India.

[10] Superscript L represents the optimal and equilibrium values of the concerned variables in the laissez-faire economy. Similarly, the superscripts G, P and X are used to represent the optimal and equilibrium values of the concerned variables in the political economy of public education, political economy of pension and political economy of two-armed intergenerational transfers analysis as in subsections 3.1, 3.2, and 3.3 respectively.

$$e_t^L = \frac{\beta\delta\phi}{1+\delta+\beta\delta\phi} w_t h_t \tag{4}$$

and

$$s_t^L = \frac{\delta}{1+\delta+\beta\delta\phi} w_t h_t. \tag{5}$$

Using the above along with the equilibrium factor prices and the fact that the general-equilibrium condition $k_{t+1} = s_t$ holds at every t, we obtain a two-dimensional first-order dynamical system of this economy, specifically, $k_{t+1}^L = \delta(1-\alpha) A k_t^\alpha h_t^{1-\alpha} / (1+\delta+\beta\delta\phi)$ and $h_{t+1}^L = B\left[\beta\delta\phi A(1-\alpha)/(1+\delta+\beta\delta\phi)\right]^\beta k_t^{\alpha\beta} h_t^{1-\alpha\beta}$. Given k_0 and h_0, all the dynamic competitive equilibria are characterized by the sequences of $\left\{k_t^L, h_t^L\right\}$ that satisfy the two equilibrium paths expressed above. Given this dynamical system, we complete our characterization for this laissez-faire economy by focusing at the steady state equilibrium and thus we have the following lemma:

Lemma 1 *In the laissez-faire economy, there exists an unique steady state with balanced growth where the human and physical capital grow at a same constant rate.*

3 Political Economy

In this section we consider the same framework as before but additionally introduce two oppositely directed policies, namely public education and social security sequentially to investigate the roles that they play in an economy. We bring these two intergenerational welfare states through a political process which runs under a probabilistic voting framework (Lindbeck and Weibull, 1987), and the economy politically determines the size of these programs to be implemented.

In particular, we investigate three political economies. The choices are naturally not arbitrary, rather it helps providing a complete understanding of all possible scenarios. The first scenario is the Political economy of public education presented in subsection 3.1. In this economy, the only political decision made by the agents is determining the level of public investment in education. That is, the government collects tax revenue from the middle-aged agents and channelizes it to the present young to provide funds for their education. The second scenario is the Political economy of pension which is presented in subsection 3.2. This economy deals with another instrument of welfare state but it is a backward intergenerational good, namely social security. However, in this case, we keep the source of education funding private as in the laissez-faire economy, but introduce public pension benefit in the form of politically determined pay-as-you-go social security. The last two scenarios bring out the effects of the instruments when they are present in an economy in isolation. Thus, as a last policy experiment, in the Political economy of two-armed intergenerational transfers, we bring both of the two instruments

together through a political process that simultaneously determines these two intergenerational goods. With these three policy environments and the laissez-faire economy presented above, we not only can explore the political equilibrium of public education and PAYG social security, but can also examine their roles in all possible scenarios. In particular we can investigate the political equilibrium and the roles of public education or PAYG social security when the other good is existent and nonexistent respectively.

3.1 Political Economy of Public Education

Now in this subsection, we consider a modification of the laissez-faire economy. Here there is no social security; however the agents politically determine the size of its public education program. Suppose the government imposes a proportional tax rate θ_t at each t on the income earned by each of the generation$-t$ agents when they are middle-aged to finance the public education program for the present young. The fiscal program of subsidy needs to satisfy the period-wise balanced budget condition, i.e., $e_t = \theta_t w_t h_t$. The tax rate θ_t is determined by a repeated political process (to be discussed below in detail) at the beginning of each period. After the political process of voting is complete and the education tax rate is set, agents make their decisions on consumption and savings. Given the factor prices, education policies and the human capital production technology, a generation-t agent's optimization problem now is to maximize (1) subject to $c_t^m = (1-\theta_t)w_t h_t - s_t$ and $c_{t+1}^o = s_t R_{t+1}$. Here, when public education is present, we keep parents away from investing in children's education for simplicity. If we allow parents to invest in children's education as in section 2, then $e_t^L - \theta_t w_t h_t = [\beta\delta\phi/(1+\delta+\beta\delta\phi) - \theta_t]\, w_t h_t$ will be the private, top-up education investment over and above what is provided by the government. As will be shown below, in equilibrium, $\theta_t w_t h_t$ is larger than e_t^L. That is the private investment in education is optimally driven to the zero corner. Adding parental investment hence would only increase the burden of notations without changing any result of the model. Then given θ_t, the first order condition with respect to s_t is given by

$$u'\left[(1-\theta_t)w_t h_t - s_t^G\right] = \delta R_{t+1} u'\left(R_{t+1} s_t^G\right)$$

which ensures the optimal saving

$$s_t^G = \frac{(1-\theta_t)\delta w_t h_t}{1+\delta}. \tag{6}$$

Next, we solve the political equilibrium under a repeated voting process where only middle-aged and old participate. Abiding by the standard practice as discussed earlier, we disallow the young's participation in the voting process due to age-restriction. At the beginning of each period, the contemporaneous tax rate is determined by a candidate who is democratically elected by all the current voters. We assume that the size of the program is determined in a

probabilistic-voting framework. Under this political setting, there are two political candidates who are competing in an election. When deciding on which candidate to support, voters anticipate the effects of the candidate's policy platform on equilibrium prices, future's political decisions and their own welfare. As we mentioned earlier we focus on the MPE where agents can form perfect foresight on the policies which depend on the set of state variables of the economy. For rest of the analysis, we use the notation S^t to denote the set of state variables in period t, i.e., $S^t \equiv \{k_t, h_t\}$. Since winning the election is the only aim of the candidates, in the probabilistic-voting Nash-equilibrium, the two candidates seeking to maximize their vote shares propose the same policy platform. This policy platform maximizes a weighted average of the welfare of all voters, in which the weights assigned to different groups of voters reflect the size or the political power of different generations.

By the foregoing discussion, the political decision on the equilibrium education policy can be derived by an exercise that maximizes the weighted sum of the indirect utilities of two generations, i.e.,

$$\max_{\theta_t \in [0,1]} \mathcal{W}\left(\mathcal{S}^t; \theta_t\right) = \omega V_t^o\left(\mathcal{S}^t; \theta_t\right) + V_t^m\left(\mathcal{S}^t; \theta_t\right), \tag{7}$$

where V_t^m and V_t^o respectively denote the welfare of the middle aged and the old at period t given the state S^t. The parameter ω is the political weight assigned to the old relative to a middle-aged by the political candidates. In line with the explanation by Song, Storesletten and Zilibotti (2012), this relative weight captures the relative political clout of each generation, reflecting, on one hand, its relative size and on the other hand, exogenous group-specific characteristics, such as the voting turnout or the salience of the fiscal policy for that group relative to other issues. In our model, the young receive education, but (7) shows that this particular generation is not allowed to vote and thus is completely dependent on others for investment in education. It should be noticed that although the young have no role in the voting process, the middle-aged have an incentive to invest in education for the future generation since they directly derive utility out of the level of education accumulated by their descendents. Not only that, agents also could acquire higher returns on their saving in the next period through general equilibrium effects.

To characterize the political decision on the public education investment, we first consider the welfare effect of this equilibrium education tax θ_t on various groups of voters, i.e., the middle-aged workers and the old retirees. Evidently, the education tax θ_t imposed in period t has no welfare effect on the current old, i.e., $\partial V_t^o / \partial \theta_t = 0$. This follows directly from the fact that all the variables, i.e., h_t, R_t and s_{t-1}, in the utility function of the old in period t, $V_t^o = u\left(R_t s_{t-1}\right) + \phi u\left(h_t\right)$, are pre-determined in t. For the middle aged, the welfare effect of

education tax is more complex. Differentiating $V_t^m(\cdot)$ with respect to θ_t yields

$$\frac{\partial V_t^m}{\partial \theta_t} = \underbrace{-u'\left(c_t^m\right) w_t h_t}_{\mathcal{A}} + \underbrace{\delta\phi u'\left(h_{t+1}\right)\frac{\partial h_{t+1}}{\partial \theta_t}}_{\mathcal{B}} + \delta s_t^G u'\left(c_{t+1}^o\right)\left(\underbrace{\frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \theta_t}}_{\mathcal{C}} + \underbrace{\frac{\partial R_{t+1}}{\partial h_{t+1}}\frac{\partial h_{t+1}}{\partial \theta_t}}_{\mathcal{D}}\right). \quad (8)$$

Note that the effect of θ_t on the savings of the middle aged cancels out by the envelope theorem. The first negative term A reflects the cost of investment in public education. The second term B captures the positive effect of public education through tax enjoyed by the parental generation because of altruism. As afore-discussed, the last two terms C and D reflect the general equilibrium effect of public education tax on the rate of interest through the channel of physical and human capital respectively. On one hand, by directing funds as a forward intergenerational good to the next generation, the education tax θ_t reduces private savings in period t and consequently also reduces the physical capital investment, which eventually leads to an increase in the rate of interest in the next period. That is, $\partial R_{t+1}/\partial k_{t+1} < 0$ along with $\partial k_{t+1}/\partial \theta_t < 0$. On the other hand, channelizing more funds towards the education of the next generation necessarily increases the level of human capital of the descendents, which in turn, also increases the rate of interest, i.e., $\partial R_{t+1}/\partial h_{t+1} > 0$ along with $\partial h_{t+1}/\partial \theta_t > 0$.[11] The aggregate general equilibrium effect of investment in public education is thus positive for the middle-aged workers with $C > 0$ and $D > 0$.

Under MPE, the equilibrium tax rate is the fixed point in $\theta_t\left(\mathcal{S}^t\right) = \arg\max_{\theta_t \in [0,1]} \mathcal{W}\left(\mathcal{S}^t; \theta_t\right)$. We first substitute the factor prices, the equilibrium condition for market clearing and the private optimal savings s_t^G given by (6) into $\mathcal{W}\left(\mathcal{S}^t; \theta_t\right)$. Then, by omitting the terms independent of policy parameter θ_t, the political objective function $\mathcal{W}\left(\mathcal{S}^t; \theta_t\right)$ reduces to[12]

$$\mathcal{W}\left(\mathcal{S}^t; \theta_t\right) \simeq (1+\alpha\delta)\ln(1-\theta_t) + \delta\beta\left(1-\alpha+\phi\right)\ln\theta_t.$$

By solving the first order condition of the above probabilistic-voting problem, i.e., by setting $\partial \mathcal{W}\left(\mathcal{S}^t, \theta_t\right)/\partial \theta_t = 0$, we finally have the following lemma:

Lemma 2 *In a political economy with public education determined in a probabilistic-voting setting, there exists an unique interior education tax rate θ^G under MPE and is given by*

$$\theta^G \equiv \frac{\phi\beta\delta + \delta\beta\left(1-\alpha\right)}{\Omega} \in (0,1)\,\forall t$$

where $\Omega = 1 + \alpha\delta + \beta\delta\left(1-\alpha\right) + \beta\delta\phi > 1$.

[11] Note that under our specific functional form of the human capital production, $\partial h_{t+1}/\partial \theta_t = B\beta e_t^{\beta-1} h_t^{1-\beta} w_t h_t$.

[12] In all that follows, we will use the notation $\simeq$ to denote the effective value function that contains the relevant fiscal parameter but not the other irrelevant terms.

Lemma 2 shows that, under the logarithm utility, the equilibrium education tax is independent of the states of the economy and thus is constant over time. Using the equilibrium conditions and the balanced budget program of the government, the two-dimensional first-order dynamical system of this economy can be written as $k_{t+1}^G = \left[\delta A(1-\theta^G)(1-\alpha)/(1+\delta)\right] k_t^\alpha h_t^{1-\alpha}$ and $h_{t+1}^G = B\left[\theta^G A(1-\alpha)\right]^\beta k_t^{\alpha\beta} h_t^{1-\alpha\beta}$. Given k_0 and h_0, all dynamic, competitive equilibria are characterized by sequences of $\left\{k_t^G, h_t^G\right\}$ that satisfy the above two equilibrium paths.

3.2 Political Economy of Pension

We now proceed to consider an economy that politically implements only a social security program. Just as in a laissez-faire economy discussed in section 2, a proportional tax τ_t is imposed at period t on the wage income earned by a generation$-t$ agent. The total tax revenue is then collected and used up to provide pay-as-you-go social security b_t to the old generation at t. The government's budget balance requirement for this program ensures $b_t = \tau_t w_t h_t$. The social security tax rate, τ_t, is determined in the same political setting as in section 3.1. Given the factor prices, human capital production technology, the social security program and the budget constraints, a generation$-t$ agent's optimization problem now involves maximizing (1) subject to $c_t^m = (1-\tau_t) w_t h_t - s_t - e_t$ and $c_{t+1}^o = R_{t+1} s_t + b_{t+1}$. Thus given τ_t, the first order conditions with respect to (e_t, s_t) are given by

$$u'\left[(1-\tau_t) w_t h_t - s_t^P - e_t^P\right] = \delta\phi u'(h_{t+1}) \frac{\partial h_{t+1}}{\partial e_t}$$

and

$$u'\left[(1-\tau_t) w_t h_t - s_t^P - e_t^P\right] = \delta R_{t+1} u'\left(R_{t+1} s_t^P + b_{t+1}\right).$$

Solving the above two simultaneously results in the following equilibrium values of e_t^P and s_t^P:

$$e_t^P = \beta\delta\phi \frac{(1-\tau_t) w_t h_t + b_{t+1}/R_{t+1}}{1+\delta+\beta\delta\phi} \tag{9}$$

and

$$s_t^P = \frac{\delta(1-\tau_t) w_t h_t - (1+\beta\delta\phi) b_{t+1}/R_{t+1}}{1+\delta+\beta\delta\phi}. \tag{10}$$

As in section 3.1, we next consider the welfare effects of the social security tax τ_t on the middle-aged workers and the old retirees. Differentiating the utility of the old with respect to τ_t yields $\partial V_t^o/\partial\tau_t = u'(c_t^o) w_t h_t > 0$. Since the old benefit from the social security program without bearing any cost, it is evident that they always prefer a tax rate that is as high as possible. Compared to the political economy of public education, in which the utility of the present old is not at all affected by the tax rate on education, either directly or through the general equilibrium effect, here in the presence of backward intergenerational goods, e.g., PAYG

social security, the old will have a role to play in the political decision making process.

By using the government's balanced budget condition $b_t = \tau_t w_t h_t$, market clearing condition $k_{t+1} = s_t$, and the factor prices, the welfare effect of pension tax τ_t on a generation$-t$ agent is[13]

$$\frac{\partial V_t^m}{\partial \tau_t} = \underbrace{-u'\left(c_t^m\right) w_t h_t}_{\mathcal{E}} + \delta u'\left(c_{t+1}^o\right) \left(\underbrace{\tau_{t+1} w_{t+1} \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{F}} + \underbrace{w_{t+1} h_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t}}_{\mathcal{G}} + \mathcal{H} \right) \tag{11}$$

where

$$\mathcal{H} = s_t^P \left(\underbrace{\frac{\partial R_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \tau_t}}_{\mathcal{H}^1} + \underbrace{\frac{\partial R_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{H}^2} \right) + \tau_{t+1} h_{t+1} \left(\underbrace{\frac{\partial w_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \tau_t}}_{\mathcal{H}^3} + \underbrace{\frac{\partial w_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{H}^4} \right).$$

The first negative term E reflects the direct cost of social security contributions. The social security tax affects the welfare of the economy through its effect on the next generation's human capital. This effect can be derived from the expression of equilibrium e_t^P (equation (9)) and also by using the equilibrium prices as follows:

$$\frac{\partial h_{t+1}}{\partial \tau_t} = \frac{\beta h_{t+1}}{\left(1-\tau_t\right) w_t h_t + b_{t+1}/R_{t+1}} \underbrace{\left\{ \frac{\left(1-\alpha\right)}{\alpha} \frac{\partial \left(\tau_{t+1} k_{t+1}\right)}{\partial \tau_t} - w_t h_t \right\}}_{\mathcal{I}}. \tag{12}$$

According to (9), the social security affects the human capital accumulation by changing the present lifecycle income through three channels: 1) by directly reducing the wage income of the middle aged, it crowds out parental investment in children's education; 2) by offering the lump-sum transfer b_{t+1} to the agents when old, it increases agents' income available for education investment; 3) finally, its effect on interest rate changes the present income and hence education investment via modifying the way the future income is discounted to arrive at its present value. As will be shown below, in equilibrium τ_{t+1} is independent of τ_t. This together with the fact that $\partial k_{t+1}/\partial \tau_t < 0$, results in the negativity of the total effect of the last two terms inside the parenthesis of equation (12). As a result I as well as $\partial h_{t+1}/\partial \tau_t$ is negative. The second term F captures the negative effect of social security tax which reduces the level of education of the next generation and consequently the transfer benefits that the present middle aged generation receives when old. The third term G captures the effect of the current tax choice on the future tax outcome and will be cancelled out if the equilibrium tax rate is independent of the states of the economy.

The last term H reflects the general equilibrium effect of social security tax through the

[13] By the envelope theorem, the effect of τ_t^P on the savings and education investment of the middle aged cancels out.

channel of physical and human capital on the factor prices. By shifting income from the middle-aged to the old, the PAYG pension reduces savings as well as the physical capital investment. The effect of pension tax on the physical capital investment is thus negative i.e., $\partial k_{t+1}/\partial \tau_t$. Here the movements of capital in terms of backward intergenerational goods not only decreases the level of physical capital in the future, thus increasing the rate of interest, but also decreases the level of investment in human capital for the next generation, consequently reducing human capital in the future. This in turn leads to an opposite effect (compared to physical capital) on the rate of interest. We see a similar thing happening with the other factor price, namely wages. While wages fall owing to the fact that there is a loss in physical capital due to backward transfer, we will also expect the price of human capital to be augmented since human capital also goes down in the future. Thus, unlike the previous case of political economy of public education, there is always a tension between the two opposite effects of two types of capital on the factor prices. Therefore H^1 and H^4 are positive and H^2 and H^3 are negative, with the result that the sign of the aggregate general equilibrium effect of the social security tax, H, is ambiguous. However since E and F are negative, if the social security can be maintained in equilibrium, the aggregate general equilibrium effect of the social security tax must be positive. Indeed this particular result has support from the existing literature. As emphasized by, for e.g., Boldrin and Rustichini (2000) and Gonzalez-Eiras and Niepelt (2008), the general equilibrium effect is very crucial in sustaining the social security program in equilibrium. Our analysis in a different framework, in fact, reconfirms the necessity of the general equilibrium effect that keeps PAYG program sustainable.

In equilibrium, as usual, political candidates who maximize their respective vote shares propose the same policy platform and maximize the combination of the welfare of all voters, which is given by $\mathcal{W}(\mathcal{S}^t;\tau_t) = \omega V_t^o(\mathcal{S}^t;\tau_t) + V_t^m(\mathcal{S}^t;\tau_t)$. First by substituting the factor prices, private optimal savings and education investment (expressions (10), (9)), and imposing the equilibrium condition $k_{t+1} = s_t$ in $\mathcal{W}(\mathcal{S}^t;\tau_t)$, and then by omitting the terms independent of policy parameter τ_t, the political objective function $\mathcal{W}(\mathcal{S}^t;\tau_t)$ reduces to

$$\mathcal{W}\left(\mathcal{S}^t;\tau_t\right) \simeq \omega \ln\left[\alpha + \tau_t(1-\alpha)\right] + \Omega \ln\left(1-\tau_t\right).$$

Following Gonzalez-Eiras and Niepelt (2008), we here make a conjecture that future equilibrium policy τ_{t+1} is independent of current political choice of τ_t, which will be verified to be indeed the case. Solving the first order condition of the probabilistic-voting problem, i.e., by setting $\partial W(\mathcal{S}^t;\tau_t)/\partial\tau_t = 0$, we have

Lemma 3 *In a political economy with PAYG pension determined in a probabilistic-voting setting, there exists an unique social security tax rate τ^P under MPE and is given by*

$$\tau^P = \frac{\omega - \alpha\Omega/(1-\alpha)}{\omega + \Omega} < 1 \ \forall t.$$

Under logarithmic utility, the equilibrium social security tax is independent of the states of the economy, verifying the conjecture we made above. Notice that while the tax rate on education in the previous case of political economy of public education is bounded between zero and unity, in case of pension tax there is no guarantee that it will always be non-negative.[14] By using the government's balanced budget condition $b_t = \tau_t w_t h_t$, and the market clearing condition $k_{t+1} = s_t$, the two-dimensional first-order dynamical system of the economy can be written as $k_{t+1}^P = \alpha\delta A k_t^\alpha h_t^{1-\alpha} / [\alpha\delta + \omega(1 + \alpha\delta + \beta\delta\phi)/\Omega]$ and $h_{t+1}^P = B k_t^{\alpha\beta} h_t^{1-\alpha\beta} \{A\omega\beta\delta\phi / [(\alpha\delta + \omega(1 + \alpha\delta + \beta\delta\phi)/\Omega)(\omega + \Omega)]\}^\beta$. Given k_0 and h_0, all dynamic, competitive equilibria are characterized by sequences of $\{k_t^P, h_t^P\}$ that satisfy the two equilibrium paths as expressed above.

3.3 Political Economy of Two-Armed Intergenerational Transfers

While in the previous two subsections we deal with only one fiscal instrument at a time, this subsection deals with the situation when both forward and backward intergenerational goods are present, that is, when the economy is two-armed. To the best of our knowledge, this is the first attempt towards studying simultaneous political decisions on two intergenerational tax instruments in a general equilibrium setup. In this setting, government has two public programs and the budget balance conditions require $e_t = \theta_t w_t h_t$ and $b_t = \tau_t w_t h_t$ where θ_t and τ_t are the education and pension tax rate respectively imposed on the wage income of the middle-aged workers. Given the tax rates, an agent maximizes (1) subject to $c_t^m = (1 - \theta_t - \tau_t) w_t h_t - s_t$ and $c_{t+1}^o = R_{t+1} s_t + b_{t+1}$. Following the same argument as in subsection 3.1, we here exclude the private investment in education from agent's choice set. Solving the generation$-t$ agent's optimization problem, we obtain

$$s_t^X = \frac{(1 - \theta_t - \tau_t)\delta w_t h_t - b_{t+1}/R_{t+1}}{1 + \delta}. \tag{13}$$

We will now consider the welfare effects of social security tax and education tax on the middle-aged workers and the old retirees. It can be verified that for the old, (given τ_t) the welfare effect of education tax, i.e., $\partial V_t^o / \partial \theta_t = 0$, and (given θ_t) the welfare effect of social security tax, i.e., $\partial V_t^o / \partial \tau_t = u'(c_t^o) w_t h_t$, remain same as in the previous cases. Since neither of the two public policies can modify the current states of the economy, they can have only the direct effect of taxation on the old. Further, it can be verified that (given θ_t) the welfare effect of social security tax on the middle-aged is the same as in the case (see equation 11) in

[14] A point to note here is that our analysis can generate the results in Gonzalez-Eiras and Niepelt (2008) as a special case. Gonzalez-Eiras and Niepelt (2008) uses a two-period overlapping generations model that features a positive population growth n_t in order to study the pension tax. If we impose $\beta = 0$ to shut out the education investment in our model, the equilibrium pension tax is $\tau^P = [\omega - \alpha(1 + \alpha\delta)/(1 - \alpha)]/(\omega + 1 + \alpha\delta)$, which is exactly equal to the result of Gonzalez-Eiras and Niepelt (2008) with $n_t = 1$.

which only social security policy is present; however, (given τ_t) the welfare effect of education tax changes to

$$\frac{\partial V_t^m}{\partial \theta_t} = \mathcal{J} + \underbrace{\delta u'\left(c_{t+1}^o\right) \frac{\partial\left(\tau_{t+1} w_{t+1} h_{t+1}\right)}{\partial \theta_t}}_{\mathcal{K}} \tag{14}$$

where J denotes the right hand side of (8). Further, when a pension tax accompanies an education tax, the term K represents an extra welfare effect of education tax on the middle-aged where

$$\frac{\partial\left(\tau_{t+1} w_{t+1} h_{t+1}\right)}{\partial \theta_t} = \tau_{t+1} w_{t+1} \frac{\partial h_{t+1}}{\partial \theta_t} + \tau_{t+1} h_{t+1}\left(\frac{\partial w_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \theta_t} + \frac{\partial w_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \theta_t}\right) + w_{t+1} h_{t+1} \frac{\partial \tau_{t+1}}{\partial \theta_t}.$$

As we can see K captures the welfare effect of education tax that the agents pay in the middle age on the social security benefit that they will receive in the future when old. As discussed previously, the general equilibrium effects of public education are $\partial k_{t+1}/\partial \theta_t < 0$ and $\partial h_{t+1}/\partial \theta_t > 0$, along with $\partial w_{t+1}/\partial k_{t+1} > 0$ and $\partial w_{t+1}/\partial h_{t+1} < 0$. Given the sign of these effects along with the fact that $\partial \tau_{t+1}/\partial \theta_t = 0$ (as will be shown below), the sign of K becomes indeterminate. Note that in the economy where only education policy is in existence and is politically determined, the middle aged agents support the education tax because they can gain from the investment in public education through parental altruism as well as from the positive general equilibrium effect of education tax on the interest rate. Our analysis confirms that in the presence of pension, the effect of education tax is not necessarily positive. The result explains that introducing PAYG may not increase the benefits from public education monotonically. This is completely in contrast to the existing results that appear in Pogue and Sgontz (1977), Becker and Murphy (1988) and Rangel (2003). They argue that since the enhanced human capital of the next generation can increase the future social security benefit, the presence of social security would increase the incentive of the middle-aged in investing in the public education. However, this results somewhat supports the findings by Soares (2006) and very recently by Kaganovich and Zilcha (2012). This remarkable result is due to the general equilibrium effects and to the best of our knowledge, is the first in nature.

In equilibrium, the objective function for the political candidates becomes $\mathcal{W}\left(\mathcal{S}^t; \theta_t, \tau_t\right) = \omega V_t^o\left(\mathcal{S}^t; \theta_t, \tau_t\right) + V_t^m\left(\mathcal{S}^t; \theta_t, \tau_t\right)$. By substituting agent's optimal savings (13), government's balanced budgets for the two programs, factor prices and equilibrium market clearing condition into $\mathcal{W}\left(\mathcal{S}^t; \theta_t, \tau_t\right)$, and then dropping the irrelevant terms, the political objective function $\mathcal{W}\left(\mathcal{S}^t; \theta_t, \tau_t\right)$ reduces to

$$\mathcal{W}\left(\mathcal{S}^t; \theta_t, \tau_t\right) \simeq \omega \ln\left[\alpha + \tau_t(1-\alpha)\right] + (1+\alpha\delta)\ln\left(1-\theta_t-\tau_t\right) + \beta\delta(1-\alpha+\phi)\ln\left(\theta_t\right).$$

The optimal tax rates are simultaneously determined in a repeated probabilistic voting. We

solve for the two tax rates by solving the two first order conditions $\partial \mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right)/\partial\theta_t = 0$ and $\partial \mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right)/\partial\tau_t = 0$ simultaneously. We term this equilibrium as Simultaneous Nash Markov Perfect Equilibrium (SNMPE). As before, we make the conjecture that the future equilibrium policy τ_{t+1} is independent of current political choice of θ_t and τ_t and will verify that it is indeed the case. Solving the SNMPE for this probabilistic-voting problem gives us the following lemma.

Lemma 4 *In a political economy with public education and PAYG pension determined in a probabilistic-voting setting, there exists a unique set of instruments* $\left(\theta^X,\tau^X\right)$ *under SNMPE where* $\tau^X = \tau^P$ *and*

$$\theta^X = \frac{\beta\delta\left(1-\alpha+\phi\right)/\left(1-\alpha\right)}{\omega+\Omega} \in (0,1) \ \ \forall t.$$

Under logarithmic utility, the equilibrium social security tax is independent of the states of the economy, verifying the conjecture. Under this repeated stage game, every subgame thus has an unique Nash equilibrium. This in fact confirms that this is the unique Nash equilibrium for the entire game. Thus the short run taxes that have been solved simultaneously at each period t, are valid not only for each t but also valid for the long run. Using these two equilibrium tax rates, the equilibrium market clearing condition and the budget constraints, the two-dimensional first-order dynamical system of the economy can be defined by $k_{t+1}^X = \alpha\delta\left(1+\alpha\delta\right)Ak_t^{\alpha}h_t^{1-\alpha}/\left[\omega+\alpha\delta\left(\omega+\Omega\right)\right]$ and $h_{t+1}^X = B\left[A\beta\delta\left(1-\alpha+\phi\right)/\left(\omega+\Omega\right)\right]^{\beta}k_t^{\alpha\beta}h_t^{1-\alpha\beta}$. Given k_0 and h_0, all dynamic, competitive equilibria are characterized by sequences of $\left\{k_t^X,h_t^X\right\}$ that satisfy the above two equilibrium paths. Finally we complete the whole section by presenting the steady state equilibrium for the above three political economies.

Lemma 5 *In each of the three political economies presented above, there exists a unique steady state with balanced growth where the human and physical capital grow at a same constant rate.*

4 Welfare Implications of Political Intergenerational Transfers

As two of the largest public transfer programs in most economies, public education and social security evoke tremendous research interest in the academia and have been intensively studied in the literature. Both public education and social security are commonly viewed as a mechanism of intergenerational transfers, but as mentioned in the introduction, there is a wide diversity of studies on these two transfers but most of them are in isolation. We present a unified framework for studying intergenerational transfer in a general equilibrium political economy model. The results could be misleading if we ignore the existence of the other policy due to the possible interaction between the two intergenerational transfers in a political economy.

This unified framework allows us to compare[15] the results of different treatments of the two intergenerational transfers that have been partially conducted in previous studies. Furthermore, within our framework, we can investigate the difference in the role played by one of the two intergenerational transfers when the other transfer running in the opposite direction is either absent or present.

4.1 Public Education

As is the standard practice, we first examine the role of public education in a political economy in the absence of PAYG social security program. In all that follows, we particularly concentrate on the welfare effects of the intergenerational transfer policy on the accumulation of both human and physical capital in the short run as well as their long-run growth. Comparison of laissez-faire equilibrium with the equilibrium of the Political economy of public education leads to the following proposition as given below:

Proposition 1 *Under MPE, the introduction of public education in a laissez-faire economy by the process of probabilistic voting*

1) at each t, given S^t, increases the human capital accumulation and reduces physical capital accumulation. Further it

2) increases the long-run growth rate when α is small.

The above proposition claims that while on one hand, investment in education is higher when education is publicly provided (compared to when it is not, as in a laissez faire economy), an economy with public education generates lower amount of physical capital in the short run. The first part of the proposition reconfirms the celebrated result of Glomm and Ravikumar (1992) that investment in education is higher when it is publicly provided. However, our results are generated through a probabilistic voting mechanism where voting power is distributed intergenerationally. Since implementation of a system of public education has no impact on the current old retirees when pension is absent, evidently it is the middle aged that decide to increase education investment in the political process. As a voter, when deciding on education investment, besides the altruism effect, the middle-aged agent also takes into account the positive general equilibrium effect of education investment which she is unaware of as a consumer. As a consequence, the education investment under the political process is higher than the private education investment. This increase in education investment in turn crowds out savings, therefore reducing the physical capital investment. If we shut out the general equilibrium effect of education investment in the voting process, i.e., C and D in equation (8), the public education investment would be exactly equal to the private education investment. Although the

[15] Whenever a comparison is made in our analysis, we naturally assume that the state variables at period t are same for all the cases. Changes appear from $t+1$ and the crucial variables are represented with the superscripts then.

public education generates countervailing effects on the accumulation of physical capital and human capital, if the production function is human capital intensive, then the human capital enhancement effect dominates so that public education ensures higher growth in the long run.

Now we will investigate the role of public education in the presence of pay-as-you-go social security. Following the same line of comparison as above we show that the presence of PAYG social security does not change the qualitative welfare effects of public education. That is introducing public education generates the same results as in Proposition 1 even when the PAYG social security has already been politically determined in the economy. This is stated below as Proposition (2).

Proposition 2 *The results of Proposition 1 hold in the presence of pay-as-you-go social security.*

In addition to exploring the welfare analysis of an intergenerational transfer policy in different scenarios, in this paper we are also interested in how the introduction of an intergenerational transfer policy affects the other transfer policy that is already present in the economy. The next proposition follows directly from Lemma 4.

Proposition 3 *When PAYG social security is already present in the economy, introducing politically determined public education has no effect on the political equilibrium tax rate of PAYG social security.*

When PAYG is already present, bringing in education tax does not affect the welfare effect of social security tax on the present old, but would influence the middle aged via direct taxation effect as well as through the general equilibrium effects. For the middle-aged, it is a replacement of private education, e_t^P, by the public education investment, $\theta_t^X w_t h_t$, and therefore they are forced to pay more for the education of the next generation. This has a direct consequences through the change in the level of education of their descendents and not only that, the individual decision (of savings) and equilibrium market prices also change under such a replacement. However Proposition 3 surprisingly shows that the welfare effects of social security on the middle aged remains unchanged in spite of these changes.

This result is striking and has important implications and is also in line with what we observe in the economy. First it shows that when there is a change in the amount of public fund for education, there is rarely a change in the tax rate of PAYG social security. The stronger implication of the result is that the reduction in social security tax rate can easily be ruled out in a voting equilibrium. People should not expect that government would increase the public education investment by sacrificing the benefit of PAYG social security for the current old. A notable example is that when U.S. government had to face a tight budget during the recent financial crisis, it had shut down many elementary and middle schools and cut down funding

support for public universities, but it never thought of reducing the benefit of PAYG social security.

Second, it has been argued by many authors that by suitably extending the forwarding arm, backward arm pension can be balancing and stable. An implied view is that by increasing the level of education so that the future income is expanded, the burden of social security tax may be reduced. However, we have been able to show that, although the public education program provides more funding for education investment in the next generation than parental investment, if PAYG social pension is politically determined, the pension tax rate would be unchanged by the political decision on public education. Thus our result has a very clear implication. Gonzalez-Eiras and Niepelt (2008) as well as the social security administration have projected that the social security tax rate should increase it in the future in response to demographic change. We show that even after adopting the remedial measure of augmenting the level of education, there might be immense difficulty in bringing about any change in tax rate in a democratic setup.

In sum, in this subsection we have shown that although the quantity of the welfare effects may differ, the qualitative welfare effects of public education on capital accumulation and long-run growth remains the same irrespective of whether the PAYG social security is present or not. Further, it turns out that where the tax rate is concerned, social security program would not be affected by the political decision on public education even when the two instruments are generationally well linked.

4.2 PAYG Social Security

Just as in the previous subsection, we first consider the role of PAYG social security in a political economy that stays away from public education. To do that, we will examine the welfare effects of PAYG social security by comparing the laissez-faire economy with a political economy that is accompanied by private education and chooses PAYG social security through the voting process. After that, following the same exercise, we will study the role of PAYG social security in an economy where public funding in education that has been decided through a political process is present. Finally, we discuss how PAYG social security affects the political decision on public education.

We start with focusing on the role of PAYG social security in the case of private education and present our first proposition below.

Proposition 4 *Under MPE, there exist two threshold weights*

$$\widehat{\omega} = \frac{\alpha\Omega}{1-\alpha} \text{ and } \widetilde{\omega} = \frac{\delta(1-\alpha)\Omega}{1+\alpha\delta+\beta\delta\phi}$$

such that introducing PAYG social security by probabilistic voting in a laissez-faire economy, at each t, given S^t,

1) increases the physical capital accumulation if and only if $\omega \leq \widehat{\omega}$.

2) and also given $\widetilde{\omega} \leq \widehat{\omega}$ or $\widetilde{\omega} \geq \widehat{\omega}$, increases the human capital accumulation if and only if $\omega \in [\widetilde{\omega}, \widehat{\omega}]$ or $\omega \in [\widehat{\omega}, \widetilde{\omega}]$, where $\widehat{\omega} \gtreqless \widetilde{\omega}$ is equivalent to $\alpha(1 + \beta\delta\phi + 2\delta) \gtreqless \delta$.

Note that the political equilibrium of social security tax τ^P is increasing in the political power of the old retirees ω. In addition, it can easily be verified that $\widehat{\omega}$ is the value at which τ^P equals to zero. Hence the equilibrium social security tax is positive if and only if $\omega \geq \widehat{\omega}$, and consequently, as is standard in the literature, positive (negative) social security tax decreases (increases) total savings as well as physical capital investment of the economy, from which the first statement of Proposition 4 follows directly.

Proposition 4 also demonstrates that, depending on the relative political weight of the old retirees, introducing the PAYG social security program could increase or decrease human capital investment, and the relationship between the political weight and the welfare effect of PAYG social security on human capital investment is non-monotonic. The result is intuitive if we look into the expression of human capital investment in the political economy of PAYG social security, (9). From (9), we can know that the political weight of the old retirees affects the human capital investment via the following ways. 1) social security crowd-out effect: as the political weight of the old retirees rises, the current social security tax would also rise and thus directly crowd out education investment; 2) social security benefit effect: the political weight of the old retirees, on the other hand, would affect the future social security benefit b_{t+1}/R_{t+1}, which in equilibrium equals $(1-\alpha)\tau^P k_{t+1}/\alpha$. Mathematically we have

$$\frac{\partial e_t^P}{\partial \omega} = \frac{\beta\delta\phi}{1+\delta+\beta\delta\phi}\left[\underbrace{-w_t h_t \frac{\partial \tau^P}{\partial \omega}}_{\mathcal{L}} + \underbrace{\frac{(1-\alpha)\,k_{t+1}^P}{\alpha}\frac{\partial \tau^P}{\partial \omega}}_{\mathcal{M}} + \underbrace{\frac{(1-\alpha)\,\tau^P}{\alpha}\frac{\partial k_{t+1}^P}{\partial \omega}}_{N}\right].$$

According to the expression of k_{t+1}^P, the physical capital investment is monotonically decreasing in the weight ω, which implies $L < 0$, $M > 0$ and $N < 0$ (when $\tau^P > 0$) or $N > 0$ (when $\tau^P < 0$). The aggregate welfare effect of ω on the education investment is thus non-monotonic. As shown in the proof for Proposition 4, e_t^P actually is concave in ω. When ω is sufficiently small so that social security tax is negative, $N > 0$ and M is large due to enhanced physical capital investment k_{t+1}^P. In this case, the welfare effect of ω on education investment is dominated by the social security benefit effect, and education investment is increasing in the political weight of the old. The opposite is true when ω is sufficiently large so that the social security crowd-out effect dominates. On the other hand, the human capital investment in the laissez-faire economy is independent of the political weight. Hence introducing the PAYG social security program could increase the human capital investment if and only if the old have intermediate-level

political power.

Note that according to Proposition 4, if $\widetilde{\omega} \leq \widehat{\omega}$, there could exists a range of ω, i.e., $\omega \in [\widetilde{\omega}, \widehat{\omega}]$, in which bringing the PAYG social security into an economy that education investment source is private could increase both physical capital investment and human capital investment. In this case, the PAYG social security tax is negative, but it undoubtedly enhances the long run growth rate. However, if the political weight is beyond that range, $\omega \notin [\widetilde{\omega}, \widehat{\omega}]$ or $\omega \notin [\widehat{\omega}, \widetilde{\omega}]$, the social security discourages both physical capital investment and human capital investment, thereby hurting the long run growth. Another interesting result that one needs to note is that, if $\widehat{\omega} \leq \widetilde{\omega}$ and $\omega \in [\widehat{\omega}, \widetilde{\omega}]$, the politically determined PAYG social security program could increase the parental investment in education even when the social security tax is positive. In this case, the social security has countervailing effect on both physical and human capital investment.

Now we focus on the economy where education is publicly provided and compare the situation in which PAYG social security is present with the one where it is not. Interestingly, when the source of education is public, although the welfare effect of PAYG social security on physical capital investment is the same as stated in the above proposition, its welfare effect on human capital investment differs radically.

Proposition 5 *Consider a political economy where the source of education is public. Introducing PAYG social security that is politically determined through a voting process would increase both physical and human capital investment if and only if* $\omega < \widehat{\omega}$.

Since the politically decided social security tax rate is same irrespective of whether education is privately or publicly funded, the welfare effect of social security on physical capital investment is same as before and thus straightforward.

We have learned from Proposition 4 that when expenditure on education is a private decision as in a laissez-faire economy, introducing politically determined PAYG social security increases the education investment if and only if the political power of the old is at intermediate level. However, here when education investment is determined by a voting process, introducing politically determined PAYG social pension would increase the education investment if and only if the political power of the old is so small that the social security tax is negative. Before the PAYG social security is injected into the political economy of public education, the old have no role in the political decision on public education. Thus the education tax is independent of the political weight ω. When the PAYG social security program is introduced to the economy, the public education tax rate becomes monotonically decreasing in the political weight of the old. High political weigh of the old yields high social security tax rate which in turn leads to a lower public education investment. As mentioned before, $\widehat{\omega}$ is the value at which τ^P equals to zero. Hence introducing politically determined PAYG social pension would increase human capital investment if and only if $\omega < \widehat{\omega}$ so that social security tax is negative.

Since now the welfare effect of PAYG social security on the physical and human capital goes in the same direction, the effect of PAYG social pension on the long run growth rate is straightforward. That is, introducing politically determined PAYG social pension into a political economy with public education would increase the long run growth rate if and only if $\omega < \widehat{\omega}$, which is totally different from the previous case where public education is absent. If we further restrict ω to be above $\widehat{\omega}$, that is if, $\omega \geq \widehat{\omega}$ so that the PAYG social security is positive, then PAYG social security always hurts the long-run growth rate.

We finally check how the introduction of a new policy would change the political decision of the other policy that is already in existence in the economy. The subsection 4.1 had shown that when PAYG social pension is already present in the economy, introducing politically determined public education into the economy results in no change in the PAYG social security tax rate. Interestingly, here, we see a different picture.

Proposition 6 *When public education is already present in the economy, introducing politically determined PAYG social security would increase the equilibrium tax rate of public education if and only if the political power of the old is small enough, i.e.,* $\omega \leq \widehat{\omega}$.

The reason behind this phenomenon is that, introducing PAYG at present would compete the government revenue against the public education program that is already present. When ω is in the range of $\omega \geq \widehat{\omega}$ so that indeed the pension tax rate is positive, bringing PAYG social security to an economy where public education is present always reduces the public education investment as well as the tax rate. Earlier studies, like Pogue and Sgontz (1977) and Rangel (2003), argue that the presence of PAYG social security would increase the middle aged' support for public education, because they will be rewarded with larger social security benefit when they are old owning to the enhanced human capital of the next generation. However, Soares (2006) and Kaganovich and Zilcha (2012) have shown that this is not necessarily true in a general equilibrium model, because the positive effect of increased education investment on future social security benefit due to the increased tax base would be cancelled out by the negative general equilibrium effect of education investment on factor prices. Their result is attained in our study even if we allow PAYG social security program to be endogenously determined in the model.

Since public education has been existent in the history for several hundred years, and the PAYG social security came into the picture only in the twentieth century, the result of Proposition 6 has a very important implication for the real economies. It suggests that the public education investment would be generally hurt if an economy decides to implement PAYG social security through a voting process. On the other hand, in term of recent discussion on reforming the current PAYG social security program, our result implies that dismantling or reducing the size of the current PAYG social security program could help increase the public education investment and therefore benefit the long run growth rate.

4.3 Fully Fiscal

In previous subsections, we investigated the role of public education or PAYG social security by sequentially introducing the two intergenerational transfer policies into an economy one by one. In this subsection, we deal with the extreme case where PAYG social pension and public education are simultaneously brought to a laissez-faire economy by a voting process, and examine their aggregate welfare effects on the economy. Comparison of laissez-faire equilibrium with the political equilibrium of the economy with public education and PAYG social security simultaneously determined gives

Proposition 7 *In a political economy under MPE, simultaneous determination of subsidy on education and social security through the political process of voting would*

1) increase the capital investment if and only if $\omega \leq \overline{\omega}_1$*, and*

2) increase the education investment if and only if $\omega \leq \overline{\omega}_2$*, where* $\overline{\omega}_2 > \widehat{\omega} > \overline{\omega}_1$*.*

Proposition 7 shows that, if an economy decides to simultaneously build up public education program and PAYG social security program by a political process, there exists three regimes of the welfare states: a) when $\omega \in [0, \overline{\omega}_1]$, the investment of both physical and human capital increase; b) when $\omega \in [\overline{\omega}_1, \overline{\omega}_2]$, where physical capital investment decreases while the human capital investment still increases; c) when $\omega \in [\overline{\omega}_2, \infty)$, the investment of both physical and human capital decrease. If we restrict ω in the range of $\omega \geq \widehat{\omega}$ so that the PAYG social security is positive, the simultaneous determination of public education and PAYG social security would always reduce the physical capital investment, but would increase the human capital investment if and only if the political weight of the old is relatively small so that the size of PAYG social security program is thin. This result is intuitive: we have shown in Proposition 1 and 2 that as public education is introduced to an economy, it always increases the human capital investment no matter whether the PAYG social security is present or not, but here due to the presence of PAYG social security, which, as shown in Proposition 6, would discourage the public education investment. The human capital investment thus can be enhanced (compared to the laissez-faire private human capital investment) if and only if the PAYG social security is small so that its effect on public education investment is limited.

In Boldrin and Montes (2005), prescribing simultaneous establishment of pubic education and PAYG social security is justified by their role in restoring the complete market solution when credit market is imperfect. In our political economy, if we check the welfare effects of the two intergenerational transfer policies on the long-run growth, whether simultaneous establishment of these two-armed intergenerational transfer are justifiable depends on the distribution of the political power. If the political weight of the old is relatively low so that the PAYG social security tax is negative, i.e., $\omega \leq \overline{\omega}_1$, building up public education and PAYG social security simultaneously in a political economy would increase investment in both the human and physical

capital, thereby increasing the long-run growth rate. However, if we consider the more realistic case that the political weight of the old is relatively large so that the PAYG social security tax is positive, then simultaneous subsidy on education and social security would definitely hurt the long-run growth of the economy when $\omega \geq \overline{\omega}_2$, and may or may not hurt the long-run growth of the economy when $\omega \in [\widehat{\omega}, \overline{\omega}_2]$. In the latter case, human capital investment is enhanced, but the physical capital investment falls. As discussed above, in such a case, only when the production function is human capital intensive, i.e., α is small, then the human capital enhancement effect dominates so that the long-run growth rate can be enhanced. In sum, in our political model, the rationale for simultaneous establishment of standard pattern of intergenerational transfers (the PAYG social security is positive) is pretty limited.

5 Conclusion

Education (a FIG) and pension (a BIG) are the two biggest sources of spending by any welfare state. Though the sources of funding for education and pension in an economy are two widely discussed topics, a complete and comprehensive study of these intergenerational goods in a unified framework was missing from the literature. This paper tries to fill in this gap through a political economy approach. More particularly, the paper relies on probabilistic voting which can also be seen as a benevolent planner's problem with particular weights being assigned to different generations. The solving technique involves the idea of a MPE and for some results we employ the notion of SNMPE. We consider an economy where altruism is observed and middle-aged and old agents vote to choose their own tax rates. Because of the age restriction, young cannot participate in voting. The intergenerational instruments we consider are education subsidy and pension. We introduce these instruments in a sequential manner in a laissez-faire economy. We also study the case where both these instruments exist at the same time in the economy. The laissez-faire economy here is one in which no intergenerational instruments are present. Thus young agents depend fully on parental investment for their education. We assume that the agents care about the (general equilibrium) effect of the present policies on the future factor prices and other future crucial variables that are materialized in their life-time.

Given this framework, our study reveals many important results which make our understanding of the effects of these instruments clear. We show that when public education is introduced in an economy through a political process of voting, it always reduces the accumulation of physical capital but increases the accumulation of human capital. However, it has no effect on the PAYG social security tax rate. This is in contrast to the popular wisdom that changing the public funds available for education may reduce the tax rate on PAYG social security. On the other hand, we have been able to show that introducing PAYG social security that has been politically determined, most definitely reduces physical capital accumulation.

But human capital will be reduced with certainty if only if the public education is already present in the economy. Otherwise, human capital actually increases. Further, we demonstrate that the general equilibrium effects are crucial in sustaining the social security program, and explain why the presence of social security may not provide sufficient incentive for public investment in education. Finally, when we introduce both public education and positive PAYG in a laissez-faire economy, we find that the presence of both the instruments always reduces the physical capital investment but increases human capital if the relative political weight to the old is small and therefore the PAYG program is thin. If the question arises whether simultaneous arrangement of these two-armed intergenerational transfers are justifiable for an increase in the long run growth, the answer in our model would depend on the distribution of political power and thus making the result of Boldrin and Montes (2005) study "conditional" in our political economy setup.

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6 Appendix A

Proof of Lemma 1. First, using the expressions for the paths of capital accumulation, we have

$$\frac{k_{t+1}^L}{h_{t+1}^L} = \frac{[\delta A(1-\alpha)]^{1-\beta}}{B(1+\delta+\beta\delta\phi)^{1-\beta}(\beta\phi)^{\beta}}\left(\frac{k_t^L}{h_t^L}\right)^{\alpha-\alpha\beta}.$$

Then by denoting $\overline{k}_{t+1}^L \equiv k_{t+1}^L/h_{t+1}^L$, we can observe that $\overline{k}_{t+1}^L$ is concave in $\overline{k}_t^L$ with $\lim_{\overline{k}_t^L\to 0}\overline{k}_{t+1}^L = 0$ and $\lim_{\overline{k}_t^L\to 0} d\overline{k}_{t+1}^L/d\overline{k}_t^L = \infty$. Hence there exists a unique non-trivial steady state value of $\overline{k}^L$ which is given by

$$\overline{k}^L = \left\{\frac{[\delta A(1-\alpha)]^{1-\beta}}{B(1+\delta+\beta\delta\phi)^{1-\beta}(\beta\phi)^{\beta}}\right\}^{\frac{1}{1-\alpha(1-\beta)}}.$$

Further, at the steady state, the growth rate of the human capital is given by

$$\frac{h_{t+1}^L}{h_t^L} = B\left[\frac{\beta\delta\phi A(1-\alpha)}{1+\delta+\beta\delta\phi}\right]^{\beta}\left(\overline{k}^L\right)^{\alpha\beta} = \left\{\frac{B^{1-\alpha}(\beta\phi)^{\beta(1-\alpha)}[\delta A(1-\alpha)]^{\beta}}{(1+\delta+\beta\delta\phi)^{\beta}}\right\}^{\frac{1}{1-\alpha+\alpha\beta}},$$

which is a constant. It is straightforward to verify that this is also the steady state growth rate for physical capital. Hence the proof. ■

Proof of Lemma 2. Note that θ_t^G is the solution to the first order condition, $\partial\mathcal{W}(\mathcal{S}^t;\theta_t)/\partial\theta_t = 0$. It is to be noted that $\theta_t^G = \theta^G\ \forall t$. Further, we can verify $\lim_{\theta_t\to 0}\mathcal{W}(\mathcal{S}^t;\theta_t) = \lim_{\theta_t\to 1}\mathcal{W}(\mathcal{S}^t;\theta_t) = -\infty$, and $\partial^2\mathcal{W}(\mathcal{S}^t;\theta_t)/\partial\theta_t^2 < 0$ so that the second order sufficient condition is satisfied. Hence the proof. ■

Proof of Lemma 3. It is easy to check that τ_t^P is the solution to $\partial\mathcal{W}(\mathcal{S}^t;\tau_t)/\partial\tau_t = 0$. It can also be verified that $\partial^2\mathcal{W}(\mathcal{S}^t;\tau_t)/\partial\tau_t^2 < 0$ with $\lim_{\tau_t\to 0}\mathcal{W}(\mathcal{S}^t;\tau_t) = \omega\ln\alpha$ and $\lim_{\tau_t\to 1}\mathcal{W}(\mathcal{S};\tau_t) = -\infty$. Hence the proof. ■

Proof of Lemma 4. Since education subsidy and pension tax are simultaneously chosen under a Nash setting, one tax rate is determined assuming the other tax rate is given at the optimum level. Thus, we find out the optimal tax rates $\left(\theta_t^X, \tau_t^X\right)$ by simultaneously solving the two first order conditions as follows:

$$\begin{aligned}
\frac{\partial\mathcal{W}(\mathcal{S}^t;\theta_t,\tau_t)}{\partial\tau_t} &= \frac{\omega(1-\alpha)}{\alpha+\tau_t^X(1-\alpha)} - \frac{1+\alpha\delta}{1-\tau_t^X-\theta_t^X} = 0\\
\frac{\partial\mathcal{W}(\mathcal{S}^t;\theta_t,\tau_t)}{\partial\theta_t} &= \frac{\beta\delta(1-\alpha+\phi)}{\theta_t^X} - \frac{1+\alpha\delta}{1-\tau_t^X-\theta_t^X} = 0.
\end{aligned}$$

It can also be verified that the Hessian of $\mathcal{W}(\mathcal{S}^t;\theta_t,\tau_t)$,

$$\begin{pmatrix} -\frac{\omega(1-\alpha)^2}{\left[\alpha+\tau_t^X(1-\alpha)\right]^2} - \frac{1+\alpha\delta}{\left(1-\theta_t^X-\tau_t^X\right)^2} & -\frac{1+\alpha\delta}{\left(1-\theta_t^X-\tau_t^X\right)^2} \\ -\frac{1+\alpha\delta}{\left(1-\theta_t^X-\tau_t^X\right)^2} & -\frac{1+\alpha\delta}{\left(1-\theta_t^X-\tau_t^X\right)^2} - \frac{\beta\delta(1-\alpha+\phi)}{\left(\theta_t^X\right)^2} \end{pmatrix},$$

is negative definite so that the second order sufficient condition is satisfied. Hence the proof. ■

Proof of Lemma 5. This proof follows the same approach of proof of Lemma 1. First we can show that for each political economy, there exists a unique non-trivial steady state value of $\overline{k}_t^i \equiv k_t^i/h_t^i$, $i = \{L, P, X\}$, which are respectively given by

$$\begin{aligned}
\overline{k}^G &= \left\{ \frac{\delta(1-\theta^G)(1-\alpha)A}{B(1+\delta)\left[\theta^G(1-\alpha)A\right]^\beta} \right\}^{\frac{1}{1-\alpha+\alpha\beta}}, \\
\overline{k}^P &= \left\{ \frac{\alpha\delta A(1-\alpha)(1-\tau^P)/\left\{\beta\delta\phi A\left(1-\tau^P\right)\left[\alpha+(1-\alpha)\tau^P\right]\right\}^\beta}{B\left[\alpha(1+\delta+\beta\delta\phi)+(1-\alpha)(1+\beta\delta\phi)\tau^P\right]^{1-\beta}} \right\}^{\frac{1}{1-\alpha+\alpha\beta}}, \\
\overline{k}^X &= \left\{ \frac{A\alpha\delta(1+\alpha\delta)}{B\left[\omega+\alpha\delta(\omega+\Omega)\right]\left[A\beta\delta(1-\alpha+\phi)/(\omega+\Omega)\right]^\beta} \right\}^{\frac{1}{1-\alpha+\alpha\beta}}.
\end{aligned}$$

Further, we can show that for each political economy, at the steady state, both the human capital and physical capital grow at a constant rate and are respectively given by

$$\begin{aligned}
\frac{h_{t+1}^G}{h_t^G} &= B\left[\theta^G A(1-\alpha)\right]^\beta \left(\overline{k}^G\right)^{\alpha\beta}, \\
\frac{h_{t+1}^P}{h_t^P} &= B\left\{ \frac{\beta\delta\phi A\left(1-\tau^P\right)\left[\alpha+(1-\alpha)\tau^P\right]}{\alpha(1+\delta+\beta\delta\phi)/(1-\alpha)+(1+\beta\delta\phi)\tau^P} \right\}^\beta \left(\overline{k}^P\right)^{\alpha\beta}, \\
\frac{h_{t+1}^X}{h_t^X} &= B\left(A\beta\delta\frac{1-\alpha+\phi}{\omega+\Omega}\right)^\beta \left(\overline{k}^X\right)^{\alpha\beta}.
\end{aligned}$$

Hence the proof. ■

Proof of Proposition 1. The proof of Part 1 relies on the comparison between $\left(e_t^L, k_{t+1}^L\right)$ and $\left(e_t^G, k_{t+1}^G\right)$ element-wise. To prove $e_t^G > e_t^L$, we need to show that $\theta^G > \beta\delta\phi/(1+\delta+\beta\delta\phi)$ and for $k_{t+1}^G < k_{t+1}^L$, we have to show that $(1+\delta)/(1+\delta+\beta\delta\phi) > (1-\theta^G)$. Using the equilibrium value of θ^G, it is straight forward to verify that both the above conditions are equivalent to the condition $\delta\beta(1-\alpha)(1+\delta+\phi\delta)/\Omega > 0$ and which always holds given our specifications.

For the second part, note that the long-run growth rate in the political economy with public education is higher than that in the laissez-faire economy if and only if $\left(h_{t+1}^G/h_t^G\right)/\left(h_{t+1}^L/h_t^L\right) > 1$ holds at the steady state. Since $\left(h_{t+1}^G/h_t^G\right)/\left(h_{t+1}^L/h_t^L\right)$ is continuous in α, to prove part 2, it is enough to show that $\left(h_{t+1}^G/h_t^G\right)/\left(h_{t+1}^L/h_t^L\right) > 1$ holds when $\alpha = 0$. Given $\alpha = 0$, some algebra yields $\left(h_{t+1}^G/h_t^G\right)/\left(h_{t+1}^L/h_t^L\right) = \left[\theta^G(1+\delta+\beta\delta\phi)/(\beta\delta\phi)\right]^\beta$, which is larger than one by directly following the proof of part 1 presented above. Hence the proof. ■

Proof of Proposition 2. Using the equilibrium factor prices, government budget and the

expression for k_{t+1}^P, some tedious algebra yields the equilibrium investment in education

$$e_t^P = \frac{\omega\beta\delta\phi\Omega}{(\omega+\Omega)\left[\alpha\delta\Omega+\omega\left(1+\alpha\delta+\beta\delta\phi\right)\right]}Ak_t^\alpha h_t^{1-\alpha}.$$

In addition, we have $e_t^X = \theta_t^X w_t h_t = \beta\delta\left(1-\alpha+\phi\right)Ak_t^\alpha h_t^{1-\alpha}/\left(\omega+\Omega\right)$, from which we can get

$$e_t^X - e_t^P = \frac{\beta\delta Ak_t^\alpha h_t^{1-\alpha}}{\omega+\Omega}\frac{\alpha\delta\Omega\left(1-\alpha+\phi\right)+\omega\left(1-\alpha\right)\left(1+\alpha\delta\right)}{\alpha\delta\Omega+\omega\left(1+\alpha\delta+\beta\delta\phi\right)} > 0.$$

Further, it is straight forward that

$$\frac{k_{t+1}^P}{k_{t+1}^X} = \frac{\omega+\alpha\delta\Omega/\left(1+\alpha\delta\right)}{\omega\left(1+\alpha\delta+\beta\delta\phi\right)/\Omega+\alpha\delta}.$$

Since it can be verified that $\left(1+\alpha\delta+\beta\delta\phi\right)/\Omega < 1$ and $\Omega/\left(1+\alpha\delta\right) > 1$, obviously we have $k_{t+1}^P/k_{t+1}^X > 1$. Hence the proof. ■

Proof of Proposition 4. Firstly it can be shown that

$$\frac{k_{t+1}^L}{k_{t+1}^P} = \frac{1-\alpha}{\alpha\left(1+\delta+\beta\delta\phi\right)}\left[\alpha\delta+\frac{\omega\left(1+\alpha\delta+\beta\delta\phi\right)}{\Omega}\right].$$

and there exists an unique threshold value $\widehat{\omega} = \alpha\Omega/\left(1-\alpha\right)$ such that $k_{t+1}^L \geq k_{t+1}^P$ if and only if $\omega \geq \widehat{\omega}$. Hence the proof.

Next we compare the levels of investment in education. It is straight forward to check that

$$\frac{e_t^L}{e_t^P} = \frac{1-\alpha}{1+\delta+\beta\delta\phi}\left[1+2\alpha\delta+\beta\delta\phi+\frac{\omega^2\left(1+\alpha\delta+\beta\delta\phi\right)+\alpha\delta\Omega^2}{\omega\Omega}\right],$$

from which we know the comparison of e_t^L with e_t^P depends on the value of ω. Taking derivative of e_t^L/e_t^P with respect to ω yields

$$\frac{\partial\left(e_t^L/e_t^P\right)}{\partial\omega} = \frac{1-\alpha}{1+\delta+\beta\delta\phi}\left(\frac{1+\alpha\delta+\beta\delta\phi}{\Omega}-\frac{\alpha\delta\Omega}{\omega^2}\right).$$

Evidently e_t^L/e_t^P is convex in ω for all t, along with $e_t^L/e_t^P \to \infty$ as $\omega \to 0$ or $\omega \to \infty$. Hence either $e_t^L/e_t^P = 1$ has two roots or $e_t^L/e_t^P > 1$ for all ω. It can be checked that $\widehat{\omega}$ is one of the solutions to $e_t^L/e_t^P = 1$. Therefore there must exist another root of $e_t^L/e_t^P = 1$. Denote $\widetilde{\omega}$ as the other root for $e_t^L/e_t^P = 1$. By solving $e_t^L/e_t^P = 1$, we have

$$\widetilde{\omega} = \frac{\delta\left(1-\alpha\right)\Omega}{1+\alpha\delta+\beta\delta\phi}.$$

Further, it can be checked that

$$\left.\frac{\partial\left(e_t^L/e_t^P\right)}{\partial\omega}\right|_{\omega=\widehat{\omega}}=\frac{\alpha+\alpha\beta\delta\phi+2\alpha\delta-\delta}{\alpha}.$$

Thus $\alpha+\alpha\beta\delta\phi+2\alpha\delta-\delta\gtrless 0\Leftrightarrow\widehat{\omega}\gtrless\widetilde{\omega}$ and since e_t^L/e_t^P is convex in ω, we must have $e_t^L<e_t^P$ for $\omega\in[\widetilde{\omega},\widehat{\omega}]$ or $[\widehat{\omega},\widetilde{\omega}]$ depending on which root is bigger, and clearly $e_t^L>e_t^P\ \forall\ \omega\notin[\widetilde{\omega},\widehat{\omega}]$ or $[\widehat{\omega},\widetilde{\omega}]$. Hence the proof. ■

Proof of Proposition 5. First we have

$$\begin{aligned}\frac{k_{t+1}^G}{k_{t+1}^X} &= \frac{1-\alpha}{\alpha\left(1+\delta\right)}\left[\alpha\delta+\frac{\omega\left(1+\alpha\delta\right)}{\Omega}\right]\\ \frac{e_t^G}{e_t^X} &= \left(1+\frac{\omega}{\Omega}\right)\left(1-\alpha\right).\end{aligned}$$

Then it can be easily checked that $k_{t+1}^G\geq k_{t+1}^X$ and $e_t^G\geq e_t^X$ if and only if $\omega\geqslant\widehat{\omega}$. Hence the proof. ■

Proof of Proposition 7. It can be verified that

$$\frac{k_{t+1}^L}{k_{t+1}^X}=\frac{1-\alpha}{\alpha\left(1+\delta+\beta\delta\phi\right)}\left(\omega+\frac{\alpha\delta\Omega}{1+\alpha\delta}\right)$$

which is monotonically increasing with respect to the parameter ω. Hence there exists a threshold value

$$\overline{\omega}_1=\alpha\frac{\left(1+\alpha\delta+\beta\delta\phi\right)\left(1+\alpha\delta\right)-\delta\beta\delta\left(1-\alpha+\phi\right)\left(1-\alpha\right)}{\left(1-\alpha\right)\left(1+\alpha\delta\right)}$$

such that $k_{t+1}^L\leq k_{t+1}^X$ if and only if $\omega\leq\overline{\omega}_1$.

Similarly, we have

$$\frac{e_t^L}{e_t^X}=\frac{\phi\left(1-\alpha\right)}{1+\delta+\beta\delta\phi}\left(\beta\delta+\frac{1+\alpha\delta+\omega}{1-\alpha+\phi}\right).$$

Evidently there exists a threshold value of $\overline{\omega}_2$, particularly,

$$\overline{\omega}_2=\frac{\left(1+\delta+\alpha\phi\beta\delta\right)\left(1-\alpha+\phi\right)-\phi\left(1-\alpha\right)\left(1+\alpha\delta\right)}{\phi\left(1-\alpha\right)}$$

such that $e_t^L\leq e_t^X$ if and only if $\omega\leq\overline{\omega}_2$. With some tedious algebra, it can be shown that $\overline{\omega}_2>\overline{\omega}_1$ is equivalent to

$$1+\delta+\delta\phi+\alpha\phi\beta\delta+\frac{\alpha\phi\beta\delta^2\left(1-\alpha+\phi\right)}{1+\alpha\delta}>0$$

and which is obviously true. Hence the proof. ■