A theory of sharecropping: the role of price behavior and imperfect competition

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Abstract

This paper proposes a theory of sharecropping on the basis of price behavior in agriculture and imperfectly competitive nature of rural product markets. We consider a contractual setting between one landlord and one tenant with seasonal variation of price, where the tenant receives a low price for his output while the landlord can sell his output at a higher price by incurring a cost of storage. We consider two different classes of contracts: (i) tenancy contracts and (ii) crop-buying contracts. It is shown that sharecropping is the optimal form within tenancy contracts and it also dominates crop-buying contracts provided the price variation is not too large. Then we consider interlinked contracts that have both tenancy and crop-buying elements and show that there are multiple optimal interlinked contracts. Finally, proposing an equilibrium refinement that incorporates imperfect competition in the rural product market, it is shown that the unique contract that is robust to this refinement results in sharecropping.

Keywords: Sharecropping, price variation, imperfect competition, tenancy contracts, crop-buying contracts, interlinkage, the ε-agent

JEL Classification: D02, D23, J43, O12, O17, Q15

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1 Introduction

Over the years, sharecropping has remained a widely prevalent, and perhaps the most controversial, tenurial system in agriculture. While writings on this institution can be traced back earlier, modern economic theories of sharecropping are centered around its criticism of Alfred Marshall (1920). The essence of the Marshallian critique is that sharecropping is an inefficient system. Under a share contract, the tenant-cultivator pays the landlord a stipulated proportion of the output. This leads to suboptimal application of inputs: even though there is gain in surplus from employing additional inputs, it does not pay the tenant to do so since he keeps only a fraction of the marginal product. In contrast, the tenant has the incentive to maximize the surplus under a fixed rental contract where he keeps the entire output and pays only a fixed rent to the landlord. The landlord, who usually has the bargaining power, can then extract the entire additional surplus by appropriately determining the rent. Thus, apart from being inefficient, sharecropping is also apparently suboptimal for the landlord. The wide prevalence of this institution has therefore remained a puzzle and several theories have been put forward to explain its existence. In particular, it has been argued that sharecropping can be explained by the trade-off between risk-sharing and incentive provision (Stiglitz, 1974; Newbery, 1977; Newbery and Stiglitz, 1979), informational asymmetry (Hallagan, 1978; Allen, 1982; Muthoo, 1998), moral hazard (Reid, 1976; Eswaran and Kotwal, 1985; Laffont and Matoussi, 1995; Ghatak and Pandey, 2000) or limited liability (Shetty, 1988; Basu, 1992; Sengupta, 1997; Ray and Singh, 2001).[1]

The present paper is motivated by an aspect of agriculture that has not received much attention in the theoretical literature of sharecropping. Given that the core of the contention here is sharing of the agricultural product between the contracting parties, a natural question is: does the price behavior in agriculture influence the resulting tenancy contracts? This question is usually sidestepped in the existing literature of sharecropping as it is always implicitly assumed that price is competitively determined in agriculture and the contracting parties take the same price as given. While price in agriculture is often regarded to be competitive, it is also well-known that it does exhibit variation—seasonal, spatial or both. The seasonal variation has a broad pattern: the price is the lowest right after the harvest, then it rises and finally reaches its peak just before the next harvest.[2] In less-developed agrarian economies, a landlord can take advantage of price variations by 'hoarding' (i.e., storing the output for a few months and sell it when the price is high) or transporting the produce to a location that offers a better price (e.g., from the village to the town market). A tenant-farmer, on the other hand, has to sell the output at low price immediately after the harvest due to various reasons such as not having enough buffer wealth to pay for

[1]See also Cheung (1969), Bardhan and Srinivasan (1971), Bardhan (1984), Binswanger and Rosenzweig (1984), Hayami and Otsuka (1993) and recent papers of Ray (1999) and Roy and Serfes (2001). The literature of sharecropping is enormous and we do not attempt to summarize it here. We refer to Singh (1989) for a comprehensive survey.

[2]For example, the Summary Report (2000: 8) of Bangladesh Agricultural Research Council states: "The overall findings of the market survey regarding the prices of rice over the twelve months indicate that there had been seasonal variation of prices of rice and other foodgrains. The average retail price of coarse rice in the selected three regions reached its peak (Tk.16.05/kg) in Chaitra (mid-March to mid-April) and went down to its minimum (Tk.11.12/kg) in Jaiyastha (mid-May to mid-June). This means pre-harvesting price of rice was the highest and the immediate post-harvesting price was the lowest with a 44.3 percent difference from the minimum to maximum prices."

essential commodities for immediate consumption, urgency for clearing his debts or the lack of necessary storage and transportation facilities. Generally speaking, one can say that a landlord has better access to the market and as a result the price that he receives for the produce is higher than the price received by the tenant. We argue that that this innate difference of the two parties can explain sharecropping even in the absence of factors such as risk aversion or informational asymmetry. The underlying intuition is simple. A fixed rental contract leaves the entire output with the tenant. Since the tenant receives a low price for the output, the revenue and consequently the rent to the landlord is low. The landlord may prefer a share contract because it enables him to take advantage of price variation by allowing him to keep a proportion of the output.

We formalize the intuition above in a landlord-tenant model with seasonal variation of price, where the tenant receives a low price for his output while the landlord can sell his output at a higher price by incurring a cost of storage, and show the superiority of sharecropping over fixed rental contracts. We also consider another type of contracts that seem to arise naturally in this setting. These are "crop-buying" contracts, where the landlord specifies a price at which he buys the entire output from the tenant. We show that as long as the price variation is not too large, sharecropping dominates crop-buying contracts. A crop-buying contract is high-incentive in nature, enabling the landlord to have a higher output, but the downside is he pays the tenant a high unit price to get this output. Under a share contract, the landlord keeps a share of relatively low output, but his unit profit margin is higher since he has to make no payment for this share. If the price variation is not too high, the gain from higher volume of output is outweighed by the loss from lower unit profit and the landlord prefers sharecropping over crop-buying contracts.

After analyzing tenancy and crop-buying contracts separately, we subsequently consider more general contracts where the landlord specifies the shares for both parties, a rental transfer and a price at which he offers to buy the tenant's share of output. These are *interlinked* contracts that enable the landlord to interact with the tenant in two markets: land (through share and rent) and product (through his offer of price).[3] We show that the landlord has multiple optimal interlinked contracts. The intuition behind the multiplicity is simple. The tenant's incentive is determined by (i) his share and (ii) the price he receives for his share, so the optimal level of incentive can be sustained by multiple combinations of these two variables. To resolve this multiplicity, we appeal to the nature of the rural product markets and propose an equilibrium refinement that takes into consideration the fact that although the landlord has monopoly power over the land he owns, this is not necessarily the case in the product market, where he could face competition from other entities (e.g., traders, intermediaries) who might be interested in trading with the tenant. In fact, a rural product market closely resembles what one might call a situation of *imperfect competition*, in the line suggested by Stiglitz (1989: 25):

> "There is competition; inequality of wealth itself does not imply that landlords can exercise their power unbridled. On the other hand, markets in which there are a

[3]The theoretical literature on interlinkage has mainly focused on credit contracts, considering (i) land-credit linkage (e.g., Bhaduri, 1973; Braverman & Stiglitz, 1982; Mitra, 1982; Basu, 1983; Bardhan, 1984; Gangopadhyay & Sengupta, 1986; Ray & Sengupta, 1989; Banerji, 1995; Basu et al., 2000) and (ii) product-credit linkage (e.g., Gangopadhyay & Sengupta, 1987; Bell & Srinivasan, 1989). See also Chapter 14 of Basu (1998) and Chapter 9 of Bardhan and Udry (1999).

large number of participants...need not be highly competitive...transaction costs and, in particular, information costs imply that some markets are far better described by models of imperfect competition than perfect competition."

The refinement criterion we propose incorporates imperfect competition in the following way. Suppose there is a small but positive probability that a third agent (who can also sell the output at a higher price by incurring a storage cost) emerges in the end of production to compete with the landlord as a potential buyer for the tenant's share of output. Then the question is, out of the multiple contracts obtained before, which ones will the landlord choose when he anticipates such a possibility? We show that the unique contract that is robust to this refinement criterion is a sharecropping contract. To see the intuition, observe that incentive provision to the tenant demands that a relatively high share for the landlord has to be compensated by a relatively high price at which the landlord offers to buy the tenant's share. The possibility of a third agent as another potential buyer enables the landlord to have a high share of output for himself without incurring the loss of buying the tenant's output at high price. We show that competition in the product market generates a Pareto improving subset of share contracts out of the multiple contracts obtained before. It is optimal for the landlord to choose that specific contract in this subset where his own share is maximum. The upshot is that the unique robust contract results in sharecropping where the tenant's share is high enough to ensure that the third agent trades with the tenant and just breaks even.

While the specific aim of this paper is to provide a theoretical analysis of sharecropping, the paper relates itself to some of the more general themes of development economics. Rural economies of poor countries are subject to volatilities of different kinds such as in weather, prices and wages that severely effect the people living there [see, e.g., Bliss and Stern (1982), Rosenzweig and Binswanger (1993), Rosenzweig and Wolpin (1993), Jayachandran (2006)]. It is also important to note that the effect of these volatilities are different across agents. In their study of Indian villages for 1975-84, Rosenzweig and Binswanger (1993) find evidence that facing possible income volatilities, wealthier households engaged in significantly more risky production activities and on the average obtained a much higher return than poorer households. Studying the effect of productivity shocks on agricultural workers using wage data from India for 1956-87, Jayachandran (2006) finds support for her theoretical prediction that such shocks cause higher wage fluctuations for poor workers that make them worse off, but in contrast, rich landowners are better off since negative productivity shocks are compensated by lower wages. Thus, our basic premise that landlords can take advantage of price fluctuations while the tenant-farmers cannot, is part of a much broader phenomenon of agrarian economies that shows that rich and poor agents respond differently to volatilities.

Our theoretical conclusion that tenancy contracts could be endogenous to the nature of price fluctuations is consistent with the well observed aspect of rural economies that institutions and contractual forms often emerge to cope with the volatilities mentioned above. Specifically, various formal and informal rural insurance systems in this regard have been extensively studied in a large literature [see, e.g., Platteau and Abraham (1987), Udry (1990), Townsend (1994), Ravallion and Chaudhuri (1997), Munshi and Rosenzweig (2007)]. It should be mentioned that interlinked contracts in our model play the role of implicitly providing insurance to the tenant-farmer. When the landlord specifies a price to buy the tenant's share of output, the tenant is insured against two contingencies: (i) if the immediate post-harvest price is even lower than expected, he is assured of a higher price from the landlord and (ii) the already standing offer from the landlord improves the tenant's position as a seller vis-à-vis another potential buyer (e.g., a third agent of the kind described before). The landlord needs to provide such insurance to make sure that the tenant's incentive stays at its optimal level. If there are other entities (e.g., the government or a big outside firm that does not have a stake at small village-level competition) that can reliably assure the tenant of a high price, the price differential between the two contracting parties will be reduced and the resulting tenancy contracts will also evolve. This is similar in spirit to the conclusion of Jayachandran (2006) who finds evidence that access to financial services such as banks reduces wage fluctuations for agricultural workers.

The paper is organized as follows. In Section 2 we present a few case studies to provide support for our premise that landlords store output to take advantage of price variation. We present the model and derive the optimal contracts in Section 3. The model with interlinked contracts is studied in Section 4. We conclude in Section 5. Some proofs are relegated to the Appendix.

2 Empirical evidence

The basic premise of our proposed theory is that landlords store the agricultural output in order to take advantage of price fluctuations. A key question is whether we observe landlords storing output. We provide some evidence on this from four studies to motivate our theoretical analysis.

The first evidence is taken from Myers (1984) who studies four villages in north China for the period 1890-1949. The village Ssu pei ch'ai, located at Luan-che'eng county, was one of the villages covered in this study. Two main tenurial systems of this village were *shao-chung-ti* (a form of share tenancy) and *pao-chung-ti* (a form of fixed rent). Cotton was the main marketed crop and the large market located in the county seat of Luan-che'eng was the major outlet for landlords and traders. The immediate post-harvest market there is described as follows (ibid: 79):

> "On the supply side, absentee landlords also sold cotton to the market, but their percentage of total supply marketed was very small. They naturally preferred to sell long after the harvest when cotton prices resumed their rise...Cotton prices were high during the winter months and low during the summer period...landlords retained their cotton and sold during the early spring..."

The source of the second evidence is Baker (1984) who studies three sub-regional economies of the south Indian region of Tamilnad from 1880s to early 1950s. Landlords having customary rights in land were called *mirasidars* in this region. The mirasidars usually leased their lands using a specific form of sharecropping called *waram*. The description of the paddy market there makes it clear that not only did landlords store the produce, but also their crop-sharing decision was influenced by such marketing activities:

> "...[T]here was a distinct pattern to the annual marketing cycle...The first stage came immediately after the main harvest in the months from January to April. This was the time when cultivators had to pay their government revenue and service their debts.

> Many cultivators, particularly the smaller ones, were obliged to unload their produce immediately. Perhaps half of the entire crop was sold at this point and naturally enough the prices were low...Substantial mirasidars...would procure stocks of rice in order to store against an expected price rise. They accumulated stocks through crop-shares they received from waram tenants; the mirasidars who were really interested in the market would have provided the seed and the cattle for the waram tenant in order that they might take away a very substantial crop-share (p. 239)...in the final stage of the marketing year...mirasidars...would release stocks on the eve of the next harvest when prices reached their peak. (p. 241)"

The next evidence is from Bolivia. In the pre-land reform Bolivia during 1920-50, different forms of land tenurial systems such as sharecropping and *colonato* (a kind of labor-rent system) existed [see, e.g., Mendelberg (1965: 46), Jackson (1994: 162-163), Assies (2006: 580)]. In his study of pre-reform agriculture markets of the north highlands of Bolivia, Clark (1968) finds that most landlords there were absentees, who lived in the city of La Paz that was also the major marketing center of the highlands area. It is clearly documented that landlords engaged in storing and marketing of the produce in a fairly organized manner:

> "At the time of the harvest the landlord visited the firm to make sure that he received the agricultural produce that was due him (p.157)...In the last seven to ten years before 1952 many landlords began to use their own or rented trucks to bring produce to La Paz...Once in La Paz agricultural produce was stored and subsequently sold in the store or *aljería* owned by the landlord...The person who worked in the store was called an *aljiri*...The specific obligations of an aljiri were to go and tell the retailers in the city markets who had done business with the landlord previously of the arrival of products from the farm...If the buyer was interested the aljiri would call the landlord...to come and make a sale...These sales were usually made in large quantities to established retailers in the La Paz markets...when sales were difficult to make in large quantities at a good price, the landlord would sell directly to consumers in small quantities (p. 158)."

The last evidence is taken from Sharma (1997) whose study is based on fieldworks of a village in the Indian state of Uttar Pradesh, conducted in the early nineties. Sharecropping was the dominant form of tenurial system in this village. It is reported that the rich landlords there stored output to take advantage of price variation (ibid: 270-271):

> "Two of the rich peasant households in the village each own a large diesel-operated machine for wheat-threshing and winnowing and rice-shelling which enables them...to process and bag much of their grains in the village (eliminating the middlemen and the cost of transport to the mills), and to sell it directly to grain merchants in Aligarh and Delhi for a much higher return. The imposing brick-made *godown* (grain-storage barn) in the centre of the village...not only acts as a storage bin, but also allows the rich land-owners periodically to withhold grain from the market until prices improve."

The studies above show that price variation and the concomitant selling behavior of landlords is an aspect that is commonly observed in agriculture. Given that, it is plausible that it may play a role in determining tenurial institutions. It can be also noted that such price variations can be seasonal as well as spatial in nature. Although our theoretical analysis will be presented in terms of seasonal variation of price, it will also apply for spatial variations of the kind mentioned in some of these studies.

3 The model

Consider a small village consisting of one landlord and many potential tenants. The landlord owns a piece of land that can grow only one crop. The landlord leases out his land to a tenant to carry out production.

• *The Production Process:* There is only one input of production: labor (ℓ). In the land leased out by the landlord, the production function is $f(\ell)$, where $f(0) = 0$. We assume that f is twice continuously differentiable with $f'(\ell) > 0$ and $f''(\ell) < 0$ for $\ell > 0$, i.e., f is strictly increasing and strictly concave. Moreover, $\lim_{\ell \downarrow 0} f'(\ell) = \infty$ and $\lim_{\ell \to \infty} f'(\ell) = 0$. The cost of ℓ units of labor is $w(\ell)$, where $w(0) = 0$. It is assumed that w is twice continuously differentiable, strictly increasing and convex, i.e., $w'(\ell) > 0$ and $w''(\ell) \geq 0$ for $\ell > 0$.

• *Price Behavior:* The market price of the product exhibits seasonal variation which is modeled as follows. There are two seasons 1 and 2. Season 1 can be viewed as the immediate post-harvest period when the price is $p_1 > 0$. Season 2 corresponds to a future period sometime after season 1 (but before the next harvest), when the price is $p_2 > p_1$. We assume that these prices are determined by economy-wide demand-supply conditions. The price is low in season 1 due to large aggregate supply immediately after the harvest. In season 2 price rises due to a fall in the aggregate supply. We normalize $p_1 = 1$ and denote $p_2 \equiv p > 1$.

The landlord can store any output q in season 1 and sell it later in season 2 at price $p > 1$ by incurring a storage cost $c(q)$ that is strictly increasing and strictly convex.[4] The tenant, on the other hand, sells any output at his disposal in season 1 at price 1. There are two main reasons behind this difference in the selling behavior of the two parties: (i) the landlord has storage facilities that the tenant lacks and (ii) unlike the landlord, the tenant does not have enough buffer wealth, so he has to sell his output in season 1 to pay for essential commodities for immediate consumption. The passage cited from Baker (1984: 239) in the last section provides empirical support to this. See also Myers (1984: 79-80).

We assume that the output held by any agent of the village is very small compared to the aggregate supply. So in any season, an agent of the village can sell his output at the existing market price of that season without affecting the price. This assumption seems reasonable for season 1 as the aggregate supply immediately after the harvest is large. Regarding season 2, it can be seen from the empirical evidence given in the last section that landlords who seek to take advantage of price fluctuations usually sell their produce in large town markets (e.g., markets in the county seat of Luan-che'eng or in cities like La Paz or Aligarh). It is assumed that although the aggregate supply falls in season 2, still it is very large in a town market and a landlord, being a small player in such a market, does not affect the price.

• *The Set of Contracts:* The landlord can lease out his land to the tenant through two different classes of contracts: (1) *tenancy contracts* and (2) *crop-buying contracts.* For both classes, we restrict to linear contracts.

[4]See Section 3.2.1 for a detailed specification of the storage cost.

A tenancy contract is a pair (α, β), where $\alpha \in [0, 1]$ is the share of the output of the tenant and $\beta \in \mathrm{R}$ is the fixed lump-sum cash transfer from the tenant to the landlord. If the tenant works under the contract (α, β) and produces output Q: (i) he keeps αQ and leaves the rest $(1 - \alpha)Q$ with the landlord and (ii) makes the lump-sum transfer β to the landlord. We say that (α, β) is a *share* contract if the landlord and the tenant share the output, i.e., if $0 < \alpha < 1$. If $0 < \alpha < 1$ and $\beta = 0$, we have a *pure share* contract. If $\alpha = 1$ and $\beta > 0$, the resulting contract is a *fixed rental* contract, where the tenant keeps the entire output and pays the fixed rent β to the landlord.

A crop-buying contract is a number $\gamma > 0$ where γ is the price at which the landlord offers to buy the output produced by the tenant. If the tenant works under the crop-buying contract γ and produces Q, he obtains γQ if he sells the output to the landlord. Since the tenant can sell the output in season 1 at price 1, he will not sell it to the landlord if $\gamma < 1$. On the other hand, since the landlord obtains the price p in season 2, he makes a loss if he buys from the tenant at a price $\gamma > p$. Therefore we can restrict $\gamma \in [1, p]$.

• *The Strategic Interaction:* The strategic interaction between the landlord and the tenant is modeled as a game G in extensive form that has the following stags. In the first stage, the landlord either offers a tenancy contract (α, β) or a crop-buying contract γ to the tenant. In the second stage, the tenant either rejects the contract in which case the game terminates with both parties get their reservation payoffs, or he accepts in which case the game moves to the third stage where the tenant chooses the amount of labor for carrying out production and output is realized. In the fourth stage, the tenant pays the landlord in accordance with the contract. If the tenant works under a tenancy contract (α, β) and the output is Q: (i) he keeps αQ which he sells in season 1 at price 1 and leaves the rest $(1 - \alpha)Q$ with the landlord and (ii) makes the lump-sum cash transfer β to the landlord. If the tenant works under a crop-buying contract γ and the output is Q, he sells it to the landlord at price γ. In the fifth stage, the landlord, who can store the output at his disposal by incurring a storage cost, decides on his storing strategy (i.e. how much to store for selling in season 2 and how much to sell in season 1). Finally payoffs are realized and the game terminates. The solution concept is the notion of Subgame Perfect Equilibrium (SPE).

3.1 The tenant's problem

We consider the tenant's problem under two different classes of contracts.

• *Tenancy contracts:* Under a tenancy contract (α, β), the payoff of the tenant has two components: (i) the profit that he obtains from his share α of the produced output and (ii) the lump-sum transfer β that he has to make to the landlord. If the tenant chooses labor input ℓ, the output is $f(\ell)$. When the tenant's share of output is α, he keeps $\alpha f(\ell)$ which he sells in season 1 at price 1, thus earning the revenue $\alpha f(\ell)$. As the cost of ℓ units of labor is $w(\ell)$, the tenant's profit is $\alpha f(\ell) - w(\ell)$. So his payoff under the contract (α, β) when he employs ℓ units of labor is

$$\alpha f(\ell) - w(\ell) - \beta \tag{1}$$

and β being a constant, his problem reduces to choosing ℓ to maximize $\alpha f(\ell) - w(\ell)$.

• *Crop-buying contract:* Under a crop-buying contract γ, the tenant sells the output to the landlord at price γ. If the tenant chooses labor input ℓ, the output is $f(\ell)$. When the tenant sells this output at price γ, he obtains the revenue $\gamma f(\ell)$. The cost of ℓ units of labor is $w(\ell)$. Hence the tenant's payoff under the contract γ when he employs ℓ units of labor is

$$\gamma f(\ell) - w(\ell) \tag{2}$$

and his problem is to choose ℓ to maximize $\gamma f(\ell) - w(\ell)$.

For $x \geq 0$, let us define

$$\phi^x(\ell) := x f(\ell) - w(\ell). \tag{3}$$

Then by (1) and (2), it follows that: (i) under the tenancy contract (α, β), the tenant's problem is to maximize $\phi^\alpha(\ell)$ and (ii) under the crop-buying contract γ, his problem is to maximize $\phi^\gamma(\ell)$. So it will be useful to solve the problem of maximizing $\phi^x(\ell)$ for any $x \geq 0$.

Since $f'' < 0$ and $w'' \geq 0$, by (3), $\phi^x(\ell)$ is strictly concave in ℓ for $x > 0$. For $x \geq 0$, let $\ell(x)$ be the unique maximizer of $\phi^x(\ell)$. Clearly $\ell(0) = 0$. For $x > 0$, $\ell(x)$ is obtained from the first-order condition $x f'(\ell) = w'(\ell)$. Hence

$$\ell(0) = 0 \text{ and } x f'(\ell(x)) = w'(\ell(x)) \text{ for } x > 0. \tag{4}$$

Now define the composite functions $F, \Phi : \mathrm{R}_+ \to \mathrm{R}_+$ as

$$F(x) := f(\ell(x)) \text{ and } \Phi(x) := \phi^x(\ell(x)) = xF(x) - w(\ell(x)). \tag{5}$$

The following lemma, which characterizes the solution to the tenant's problem under different classes of contracts, follows from (1)-(5) and by the envelope theorem.

Lemma 1 (i) *Under the tenancy contract (α, β), the tenant chooses labor input $\ell(\alpha)$, the output produced is $F(\alpha)$ and the tenant obtains the payoff $\Phi(\alpha) - \beta$.*

(ii) *Under the crop-buying contract γ, the tenant chooses labor input $\ell(\gamma)$, the output produced is $F(\gamma)$ and the tenant obtains the payoff $\Phi(\gamma)$.*

(iii) $\ell(0) = 0$, $F(0) = 0$ *and* $\Phi(0) = 0$.

(iv) $\ell'(x) > 0$, $F'(x) > 0$ *and* $\Phi'(x) = F(x) > 0$ *for* $x > 0$.

Having characterized the solution of the tenant's problem under any contract offered by the landlord, we are in a position to solve the landlord's problem of determining his optimal contracts. Before solving that problem, we qualify two more aspects of our model. First, we impose more structure to the model by making an additional assumption and second, we specify the reservation payoff of the tenant in terms of the function $\Phi(.)$ and provide the economic interpretation behind this specification.

3.1.1 Assumption: Concavity of $F(x)$

Consider the function $F(x) = f(\ell(x))$. As $F'(x) = f'(\ell(x))\ell'(x)$, we have

$$F''(x) = f''(\ell(x))[\ell'(x)]^2 + f'(\ell(x))\ell''(x).$$

Since $f'' < 0$, the first term of the expression above is positive, but the sign of the second term is ambiguous. We make the following additional assumption, which is a sufficient condition to ensure that the landlord's problem will have a unique solution.

Assumption A1: The functions $f(\ell)$ and $w(\ell)$ are such that $F(x)$ is concave, i.e., $F''(x) \leq 0$.

Assumption A1 holds for $f(\ell) = \ell^a$ and $w(\ell) = k\ell^b$ for $k > 0$, $a < 1 \leq b$ and $a/b \leq 1/2$.

3.1.2 Reservation payoff of the tenant

In specifying the reservation payoff of the tenant, we posit the following situation which is plausible in a less developed village economy. We assume that a potential tenant is a small or marginal farmer in the village who has limited employment opportunities outside. Moreover such a farmer lacks the necessary storage facilities, which prevents him from taking advantage of the seasonal variation of price. If he does not have a contract with the landlord, his only viable alternative is to cultivate his own land, which is smaller and possibly of inferior quality than the land leased out by the landlord and his alternative payoff is the profit from this land when the output is sold at price 1 (the price of season 1). This profit, being a good approximation of the opportunity cost of a potential tenant, is assumed to be the reservation payoff of the tenant.

To formalize the situation described above, observe that if a farmer cultivates the land leased out by the landlord without any contractual obligation and sells the output at price 1, under his optimal choice of labor, the profit that he obtains is $\Phi(1)$ (take $\alpha = 1$, $\beta = 0$ in Lemma 1). So a small farmer, who cultivates a smaller and possibly inferior quality land and sells his output at price 1, obtains $\underline{\Phi} < \Phi(1)$. We consider this profit $\underline{\Phi}$ $[0 < \underline{\Phi} < \Phi(1)]$ to be the reservation payoff of the tenant. Since $\Phi(0) = 0$ and $\Phi(.)$ is strictly increasing (Lemma 1), there is a constant $\underline{\alpha} \in (0, 1)$ such that $\underline{\Phi} = \Phi(\underline{\alpha})$. For the rest of the paper we assume that the reservation payoff of the tenant is $\Phi(\underline{\alpha})$ for a small positive fraction $\underline{\alpha}$.

3.2 The landlord's problem

Observe that under any contract the landlord potentially has some output $Q > 0$ at his disposal. To solve the landlord's problem, first we determine his optimal storing strategy for any $Q > 0$ and then obtain his payoff under any contract by using the revenue from his optimal storing strategy.

3.2.1 Storage cost and optimal storing strategy

Let $c(x)$ denote the landlord's cost of storing output x. We assume that $c(0) = 0$, $c(x)$ is twice continuously differentiable, strictly increasing and strictly convex, i.e., $c'(x) > 0$ and $c''(x) > 0$ for $x > 0$. We also assume that $c'(0) = 0$ and $\lim_{x\to\infty} c'(x) = \infty$. Under these assumptions, for any $p > 1$, $\exists\ 0 < \overline{Q}_p < \infty$ such that

$$p - 1 \gtreqqless c'(x) \Leftrightarrow x \lesseqqgtr \overline{Q}_p. \tag{6}$$

Moreover

$$\overline{Q}_p \text{ is strictly increasing, } \overline{Q}_1 = 0 \text{ and } \lim_{p\to\infty} \overline{Q}_p = \infty. \tag{7}$$

As the price of the output is 1 in season 1 and $p > 1$ in season 2, the marginal revenue of the landlord from storing across seasons is $p - 1$, so by (6), the marginal cost of storing exceeds the marginal revenue $p - 1$ beyond $\overline{Q}_p$. Therefore storing is worthwhile for the landlord as long as the amount stored does not exceed $\overline{Q}_p$.

Now suppose the landlord has output $Q \in (0, \infty)$ at his disposal (Q will depend on the contract offered by the landlord and the output produced by the tenant). For any Q, a typical storing strategy for the landlord is $\langle Q - x, x\rangle$ for $x \in [0, Q]$, where (i) x is the amount he stores in season 1 and sells in season 2 and (ii) $Q-x$ is the amount he sells in season 1. Since the price in season 2 is p and the storage cost is $c(x)$, the net revenue of the landlord from storing x is $R_2(x) = px - c(x)$. For the remaining output $Q-x$, he obtains price 1 and there is no cost of storing, so his revenue is $R_1(Q-x) = Q-x$. Hence the landlord's revenue under the storing strategy $\langle Q-x, x\rangle$ is $\psi^p_Q(x) = R_2(x) + R_1(Q-x) = Q + (p-1)x - c(x)$. Denoting

$$\zeta^p(x) = (p-1)x - c(x), \tag{8}$$

we have

$$\psi^p_Q(x) = Q + \zeta^p(x). \tag{9}$$

By (8) and (9), for any Q, the problem of finding an optimal storing strategy reduces to choosing $x \in [0, Q]$ to maximize $\zeta^p(x)$. The following lemma characterizes the optimal storing strategy of the landlord for any Q.

Lemma 2 (i) *For $x \geq 0$, $\partial\zeta^p(x)/\partial x \gtreqless 0 \Leftrightarrow x \lesseqgtr \overline{Q}_p$.*

(ii) *Let $Q \in (0,\infty)$. The unique maximum of $\psi^p_Q(x)$ over $x \in [0, Q]$ is attained at $x = Q$ if $Q < \overline{Q}_p$ and at $x = \overline{Q}_p$ if $Q \geq \overline{Q}_p$. Consequently when the landlord has output Q at his disposal, his optimal storing strategy is $\langle 0, Q\rangle$ if $Q < \overline{Q}_p$ and $\langle Q - \overline{Q}_p, \overline{Q}_p\rangle$ if $Q \geq \overline{Q}_p$.*

(iii) *The revenue of the landlord under his optimal storing strategy when he has output Q is*

$$\Psi^p(Q) = \begin{cases} Q + \zeta^p(Q) = pQ - c(Q) & \text{if } Q < \overline{Q}_p, \\ Q + \zeta^p(\overline{Q}_p) = Q + (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) & \text{if } Q \geq \overline{Q}_p. \end{cases} \tag{10}$$

(iv) $\Psi^p(0) = 0$, *$\Psi^p(Q)$ is strictly increasing in both Q and p, and $\lim_{p\to\infty}\Psi^p(Q) = \infty$.*

(v) *For any $p > 1$, $\Psi^p(Q)$ is concave in Q. If $Q_1, Q_2 \geq 0$ is such that (a) at least one of Q_1, Q_2 is positive and (b) at least one of them is less than $\overline{Q}_p$, then*

$$\Psi^p\left(\lambda Q_1 + (1-\lambda)Q_2\right) > \lambda\Psi^p(Q_1) + (1-\lambda)\Psi^p(Q_2) \text{ for any } \lambda \in (0,1).$$

(vi) *Let $p > 1$. For any $Q_1, Q_2 > 0$, $\Psi^p(Q_1 + Q_2) < \Psi^p(Q_1) + \Psi^p(Q_2)$.*

Proof. Parts (i)-(iii) follow by (6), (8) and (9). Part (iv) follows by (7) and (10).

For (v)-(vi), note that $\Psi^p(Q)$ is continuous, but it has a kink at $Q = \overline{Q}_p$. The details of the proof are left in the Appendix. ■

The optimal storing strategy given by the lemma above is fairly intuitive. Since the marginal revenue from storing $p-1$ falls below the marginal cost of storing once the output exceeds $\overline{Q}_p$, it does not pay the landlord to store any output beyond $\overline{Q}_p$. Also observe that the revenue $\Psi^p(Q)$ under the optimal storing strategy exhibits decreasing returns to scale. This result will be useful later in our analysis.

3.2.2 Tenancy contracts

Consider a tenancy contract (α, β) where $\alpha \in [0,1]$ and $\beta \in$ R. By Lemma 1, the tenant's optimal choice of labor under this contract is $\ell(\alpha)$ that yields the output $F(\alpha)$. As the landlord's share is $(1-\alpha)$, the output at his disposal is $(1-\alpha)F(\alpha)$. Define $H : [0,1] \to R_+$ as

$$H(\alpha) := (1-\alpha)F(\alpha). \tag{11}$$

Taking $Q = H(\alpha)$ in (10) of Lemma 2, the revenue of the landlord under his optimal storing strategy is $\Psi^p(H(\alpha))$. As the landlord also obtains the fixed rent β, his payoff is

$$\Pi^p(\alpha, \beta) = \Psi^p(H(\alpha)) + \beta. \tag{12}$$

By Lemma 1, when the tenant acts optimally under the contract (α, β), he obtains $\Phi(\alpha) - \beta$. Since the reservation payoff of the tenant is $\Phi(\underline{\alpha})$, the tenant will accept the contract (α, β) only if

$$\Phi(\alpha) - \beta \geq \Phi(\underline{\alpha}). \tag{13}$$

Under the class of tenancy contracts, the landlord's problem is to choose (α, β) to maximize (12) subject to (13). For any α, the optimal β for the landlord is

$$\beta^{\underline{\alpha}}(\alpha) = \Phi(\alpha) - \Phi(\underline{\alpha}) \tag{14}$$

that binds the tenant's participation constraint (13). So it is sufficient to consider contracts $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ for $\alpha \in [0, 1]$. Under $(\alpha, \beta^{\underline{\alpha}}(\alpha))$, the payoff of the landlord is

$$\Pi^{p,\underline{\alpha}}(\alpha) = \Psi^p(H(\alpha)) + \Phi(\alpha) - \Phi(\underline{\alpha}). \tag{15}$$

Observe that for any α, the total surplus (sum of payoffs of the landlord and the tenant) is $\Psi^p(H(\alpha)) + \Phi(\alpha)$.

Let us first consider the situation when there is no price variation across seasons (i.e. $p = 1$). Then the landlord's optimal storing strategy is to sell $H(\alpha)$ in season 1 at price 1 to obtain the revenue $\Psi^1(H(\alpha)) = H(\alpha)$ and then the total surplus is

$$s(\alpha) = H(\alpha) + \Phi(\alpha). \tag{16}$$

The following lemma, which characterizes the basic properties of the functions $H(\alpha)$ and $s(\alpha)$, will be useful for solving the landlord's problem. The proof is standard and hence omitted. Assumption A1 [concavity of $F(.)$] is used to prove the strict concavity of $H(.)$.

Lemma 3 (i) $H(0) = H(1) = 0$.
(ii) $H'(\alpha) = (1 - \alpha)F'(\alpha) - F(\alpha)$ *and* $H(\alpha)$ *is strictly concave for* $\alpha \in [0, 1]$.
(iii) *There is a constant* $\widetilde{\alpha} \in (0, 1)$ *such that* $H'(\alpha) \gtreqless 0 \Leftrightarrow \alpha \lesseqgtr \widetilde{\alpha}$.
(iv) $s'(\alpha) = (1 - \alpha)F'(\alpha)$, $s(\alpha)$ *is strictly increasing for* $\alpha \in [0, 1]$ *and* $s(1) > s(\alpha)$ *for* $\alpha \in [0, 1)$.

Before solving the landlord's problem under general tenancy contracts, we recap the Marshallian inefficiency argument against sharecropping and show how the Marshallian critique loses some of its force in the presence of price variation (Proposition 1). This proposition formalizes the basic intuition of this paper and forms the basis of the remaining results, where properties of different contracts are derived in more detail.

3.2.3 The Marshallian inefficiency argument

To see the Marshallian inefficiency argument against sharecropping, we begin with *fixed rental* contracts. A fixed rental contract is of the form $(1, \beta)$, where the tenant keeps the entire output ($\alpha = 1$) and pays only a fixed rent $\beta > 0$ to the landlord. Under the contract $(1, \beta)$, the landlord has no output at his disposal [$H(1) = 0$] so his payoff is simply the fixed rent β. Taking $\alpha = 1$ in (14), the optimal fixed rental contract for the landlord is $(1, \beta^{\underline{\alpha}}(1))$ where

$$\beta^{\underline{\alpha}}(1) = \Phi(1) - \Phi(\underline{\alpha}). \tag{17}$$

Now let $0 < \alpha < 1$ and consider the *share contract* $(\alpha, \beta^{\underline{\alpha}}(\alpha))$. Since $\Phi(.)$ is monotonic, by (14) and (17), compared to the fixed rental contract $(1, \beta^{\underline{\alpha}}(1))$, the share contract $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ entails a *rental loss* for the landlord. This loss is

$$\beta^{\underline{\alpha}}(1) - \beta^{\underline{\alpha}}(\alpha) = \Phi(1) - \Phi(\alpha) > 0. \tag{18}$$

However, under the latter contract, the landlord has output $H(\alpha) > 0$ at his disposal. Therefore the share contract $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ can be superior to the fixed rental contract $(1, \beta^{\underline{\alpha}}(1))$ for the landlord only if his revenue from output $H(\alpha)$ can recover the rental loss in (18). Define $A : (0, 1) \to R_+$ as

$$A(\alpha) = [\Phi(1) - \Phi(\alpha)]/H(\alpha). \tag{19}$$

The function $A(\alpha)$ presents the rental loss *per unit* of output when the landlord switches from the fixed rental contract $(1, \beta^{\underline{\alpha}}(1))$ to the share contract $(\alpha, \beta^{\underline{\alpha}}(\alpha))$.

When there is no price variation across seasons (i.e. $p = 1$), a share contract always results in a lower total surplus $s(\alpha) < s(1)$ [Lemma 3(iv)]. This is the essence of the Marshallian inefficiency argument against sharecropping. This argument can be put in *per unit* terms as follows. When $p = 1$, the landlord receives the same unit price for the output as the tenant. So if he switches from the fixed rental contract $(1, \beta^{\underline{\alpha}}(1))$ to the share contract $(\alpha, \beta^{\underline{\alpha}}(\alpha))$, it is never possible for him to recover the rental loss through his revenue from $H(\alpha)$. In the presence of price variation ($p > 1$), however, the Marshallian inefficiency argument does not have its unequivocal force. If the landlord can receive a higher price for the output $H(\alpha)$, it may enable him to recover the rental loss. Proposition 1 makes this point precise, where we show that for any $p > 1$, there is a share contract $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ such that the landlord's per unit profit (net of storage cost) from output $H(\alpha)$ is higher than the per unit rental loss $A(\alpha)$ in (19).

Proposition 1 (i) $A(\alpha) > 1$ *for all* $\alpha \in (0, 1)$ *and* $lim_{\alpha \uparrow 1} A(\alpha) = 1$.

(ii) (*Marshallian Inefficiency of Share Contracts*) *When* $p = 1$, *the unique optimal tenancy contract for the landlord is the fixed rental contract* $(1, \beta^{\underline{\alpha}}(1))$.

(iii) *For any* $p > 1$, *there is a share contract that yields higher payoff to the landlord compared to the fixed rental contract* $(1, \beta^{\underline{\alpha}}(1))$.

Proof. (i) Consider the function $s(\alpha) = H(\alpha) + \Phi(\alpha)$ (the total surplus when $p = 1$) given in (16). Since $H(1) = 0$, we have $s(1) = \Phi(1)$. By Lemma 3(iv), $s(1) = \Phi(1) > s(\alpha) = H(\alpha) + \Phi(\alpha)$ for any $\alpha \in (0, 1)$. As $H(\alpha) > 0$ for $\alpha \in (0, 1)$, we have $A(\alpha) = [\Phi(1) - \Phi(\alpha)]/H(\alpha) > 1$ for $\alpha \in (0, 1)$.

As $\lim_{\alpha \uparrow 1}[\Phi(1) - \Phi(\alpha)] = 0$ and $\lim_{\alpha \uparrow 1} H(\alpha) = H(1) = 0$, by L'Hospital's rule, $\lim_{\alpha \uparrow 1} A(\alpha) = \lim_{\alpha \uparrow 1}[-\Phi'(\alpha)/H'(\alpha)]$. Since $\Phi'(\alpha) = F(\alpha)$ (Lemma 1) and $H'(\alpha) = (1 - \alpha)F'(\alpha) - F(\alpha)$ (Lemma 3), we have $\lim_{\alpha \uparrow 1} A(\alpha) = \lim_{\alpha \uparrow 1} [F(\alpha)/\{F(\alpha) - (1 - \alpha)F'(\alpha)\}] = 1$.

(ii) It is sufficient to consider contracts $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ for $\alpha \in [0, 1]$ where $\beta^{\underline{\alpha}}(\alpha)$ is given by (14). Clearly $\alpha = 0$ is not optimal. For $0 < \alpha < 1$, the landlord incurs a per unit rental loss $A(\alpha)$. When $p = 1$, the per unit profit of the landlord from output $H(\alpha)$ is 1. The proof is complete by noting that $A(\alpha) > 1$ [by part (i)].

(iii) Let $p > 1$. Consider $0 < \alpha < 1$. Under the share contract $(\alpha, \beta^{\underline{\alpha}}(\alpha))$, the landlord gets output $H(\alpha) > 0$. If he stores $H(\alpha)$ in season 1 and sells it in season 2 at price $p > 1$ (it is a *feasible* storing strategy), his profit net of storage cost is $pH(\alpha) - c(H(\alpha))$. So his per unit profit is

$$\overline{A}^p(\alpha) = p - c(H(\alpha))/H(\alpha) > p - c'(H(\alpha)),$$

the last inequality following by the strict convexity of $c(.)$. Since $c'(0) = 0$ and $H(1) = 0$, we have

$$\lim_{\alpha \uparrow 1} \overline{A}^p(\alpha) \geq \lim_{\alpha \uparrow 1} [p - c'(H(\alpha))] = p > 1 = \lim_{\alpha \uparrow 1} A(\alpha).$$

So it is possible to choose $\alpha \in (0, 1)$ such that the per unit profit $\overline{A}^p(\alpha)$ is more than the per unit rental loss $A(\alpha)$. This completes the proof. ■

Remarks. The proposition above shows the optimality of share contracts under two implicit assumptions: (i) the share contract has a rental component with it and (ii) the landlord has full bargaining power over the tenant and he uses the rent to drive down the tenant's payoff to its reservation level. Now we discuss the plausibility of (i) and (ii) and see the extent to which the result above is sensitive to these assumptions.

1. *Bargaining power*: A poor rural economy is usually labor-surplus in nature. As the number of small or marginal farmers (who are potential tenants) in a village is large compared to the number of landlords, in this context it is arguably natural to assume that a landlord holds relatively large bargaining power over a tenant. We assume full bargaining power for analytical convenience and our conclusions will continue to hold qualitatively under situations where the landlord has less than full bargaining power, as long as it is not too small. This is illustrated below with regard to Proposition 1.

When the tenant's share is α, his profit is $\Phi(\alpha)$. Recall that the tenant's reservation payoff is $\Phi(\underline{\alpha})$ for a small $\underline{\alpha} \in (0, 1)$. Consider the following alternative bargaining arrangement:

- The tenant rejects any contract that has $\alpha < \underline{\alpha}$.
- For $\alpha \in [\underline{\alpha}, 1]$, the additional profit of the tenant over his reservation payoff $\beta^{\underline{\alpha}}(\alpha) = \Phi(\alpha) - \Phi(\underline{\alpha})$ is divided as follows: the landlord gets $b\beta^{\underline{\alpha}}(\alpha)$ and the tenant gets $(1-b)\beta^{\underline{\alpha}}(\alpha)$ where $b \in (0, 1)$ is a constant that presents the bargaining power of the landlord.

Under this arrangement it is sufficient to consider contracts $(\alpha, b\beta^{\underline{\alpha}}(\alpha))$ for $\alpha \in [\underline{\alpha}, 1]$. Then the optimal fixed rental contract for the landlord is $(1, b\beta^{\underline{\alpha}}(1))$ that yields the rent $b\beta^{\underline{\alpha}}(1)$. If he switches from this contract to a share contract $(\alpha, b\beta^{\underline{\alpha}}(\alpha))$, his rent becomes $b\beta^{\underline{\alpha}}(\alpha)$. So the rental loss is $b[\Phi(1) - \Phi(\alpha)]$, yielding a per unit rental loss $b[\Phi(1) - \Phi(\alpha)]/H(\alpha) = bA(\alpha)$. By Proposition 1(i), $\lim_{\alpha \uparrow 1} bA(\alpha) = b < 1$ and the conclusion of Proposition 1(iii) continues to hold.

2. *Share contracts without rental component*: Now consider *pure* share contracts $(\alpha, 0)$ that have no rental component. When the landlord switches from the fixed rental contract $(1, \beta^{\underline{\alpha}}(1))$ to a share contract $(\alpha, 0)$, his rental loss is $\beta^{\underline{\alpha}}(1) = \Phi(1) - \Phi(\underline{\alpha})$ and the unit rental loss is $[\Phi(1) - \Phi(\underline{\alpha})]/H(\alpha)$. Since there is a unique maximizer $\widetilde{\alpha} \in (0, 1)$ of $H(\alpha)$ (Lemma 3), the unit rental loss is at least $[\Phi(1) - \Phi(\underline{\alpha})]/H(\widetilde{\alpha})$ which is more than 1 for small values of $\underline{\alpha}$. As the landlord can obtain at most p for each unit he sells, the unit profit can recover the rental loss only when p is relatively large (see Proposition 2). Thus our basic intuition that price variation leads to share contracts still holds, but expectedly a weaker result is obtained when the contract forms are restricted.

Do share contracts observed in practice include side payments? The evidence is mixed. For example, Forster (1957: 236) finds that *mètayage* (sharecropping) contracts of 18th century Toulose in France included substantial side payments:

> "The sharecropper had to pay for the use of the farm animals as well as for the use of the land. In 1728 François Caseneuve, a tenant of Astre de Blagnac, was obligated under his contract for half the harvest, twenty four pairs of fowl, all the cartage necessary to carry the farm produce to the Toulose market, and a *prélèvement* (supplementary rent) of eighty *setiers* of wheat."

Evidence of various implicit and explicit side payments can be found in sharecropping contracts from mid-19th century south India (Reddy, 1996: 80-81) and early 20th century South Carolina, United States (Taylor, 1943: 125-128) as well. However, sometimes it is also the case, such as in 19th century north China (Myers, 1970: 93) and Bolivia in the 1920s (Jackson, 1994: 163), where the landlords are primarily interested in collecting the share of the harvest and there is no significant side payment. It is plausible that including a side payment in the share contract may involve some transaction cost. As pure share contracts are of some independent interest, we analyze them in the next section.

3.2.4 Pure share contracts

Under *pure share* contracts the contracting parties share the output ($0 < \alpha < 1$) without any rental transfer ($\beta = 0$). Taking $\beta = 0$ in (12), the landlord's payoff under the pure share contract $(\alpha, 0)$ is

$$\Pi_S^p(\alpha) = \Psi^p(H(\alpha)). \tag{20}$$

Taking $\beta = 0$ in (13), the tenant's participation constraint is $\Phi(\alpha) \geq \Phi(\underline{\alpha})$. By the monotonicity of $\Phi(.)$, this constraint reduces to $\alpha \geq \underline{\alpha}$. So under pure share contracts, the landlord's problem is to choose $\alpha \geq \underline{\alpha}$ to maximize (20).

Under the contract $(\alpha, 0)$, the landlord has output $H(\alpha)$ at his disposal. If there is no storage cost, he can sell $H(\alpha)$ in season 2 at price p to obtain the $pH(\alpha)$. As $\widetilde{\alpha} \in (0,1)$ is the unique maximizer of $H(\alpha)$ (Lemma 3), in the absence of any storage cost, $(\widetilde{\alpha}, 0)$ is the optimal *unconstrained* pure share contract and it will be the optimal pure share contact if $\widetilde{\alpha} \geq \underline{\alpha}$. Henceforth we shall assume that the tenant's reservation payoff $\Phi(\underline{\alpha})$ is small enough so that the following holds.

Assumption A2: $0 < \underline{\alpha} < \widetilde{\alpha}$.

A2 is not a crucial assumption, but it helps to simplify our analysis as it renders the tenant's participation constraint to be non-binding. Moreover A2 is consistent with the interpretation that the reservation payoff of the tenant is sufficiently small as a potential tenant is a small or marginal farmer with limited employment opportunities outside the village.

Proposition 2 *Consider the set of all pure share contracts $\mathbb{S} = \{(\alpha, 0) | \alpha \in (0,1)\}$. For any $p \geq 1$, the landlord has a unique optimal pure share contract $(\widetilde{\alpha}, 0)$ that does not depend on p or the storage cost $c(.)$. The optimal contract has the following properties, where $\widetilde{p} > 1$ is a constant.*

(i) *The tenant obtains $\Phi(\widetilde{\alpha}) > \Phi(\underline{\alpha})$.*

(ii) *The output produced is* $F(\widetilde{\alpha})$. *The output at the landlord's disposal is* $H(\widetilde{\alpha})$ *which exceeds* $\overline{Q}_p$ *if and only if* $p < \widetilde{p}$.

(iii) *The landlord obtains*

$$\Pi_S^p(\widetilde{\alpha}) = \Psi^p(H(\widetilde{\alpha})) = \begin{cases} H(\widetilde{\alpha}) + (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) & \text{if } p \in [1, \widetilde{p}], \\ pH(\widetilde{\alpha}) - c\left(H(\widetilde{\alpha})\right) & \text{if } p > \widetilde{p}. \end{cases}$$

(iv) $\Pi_S^p(\widetilde{\alpha})$ *is strictly increasing in* p. *Specifically* $\mathrm{d}\Pi_S^p(\widetilde{\alpha})/\mathrm{d}p$ *equals* $\overline{Q}_p$ *if* $p \in [1, \widetilde{p}]$ *and* $H(\widetilde{\alpha})$ *if* $p > \widetilde{p}$. *Moreover* $lim_{p\to\infty}\Pi_S^p(\widetilde{\alpha}) = \infty$.

(v) *For any* $0 < \underline{\alpha} < \widetilde{\alpha}$, $\exists\ p_R^S(\underline{\alpha}) > 1$, *which is strictly decreasing in* $\underline{\alpha}$, *such that*

- *for the landlord, the optimal pure share contract* $(\widetilde{\alpha}, 0)$ *is superior to the optimal fixed rental contract* $(1, \beta^{\underline{\alpha}}(1))$ *if and only if* $p > p_R^S(\underline{\alpha})$,
- *if* $p > p_R^S(\underline{\alpha})$, *then both the landlord and the tenant prefer* $(\widetilde{\alpha}, 0)$ *over* $(1, \beta^{\underline{\alpha}}(1))$.

Proof. By (20) the landlord's problem under pure share contracts is to choose $\alpha \in (0,1)$ to maximize $\Psi^p(H(\alpha))$ subject to $\alpha \geq \underline{\alpha}$. Since $\Psi^p(.)$ is monotonic (Lemma 2) and $\widetilde{\alpha}$ is the unique maximizer of $H(\alpha)$, $\widetilde{\alpha}$ is the unique maximizer of $\Psi^p(H(\alpha))$. Since $\widetilde{\alpha} > \underline{\alpha}$ (Assumption A2), $(\widetilde{\alpha}, 0)$ is the unique optimal pure share contract. Part (i) is direct. Now we prove (ii)-(v).

(ii) Since $\overline{Q}_p$ is monotonic, $H(\widetilde{\alpha}) > 0 = \overline{Q}_1$ and $\lim_{p\to\infty} \overline{Q}_p = \infty$, $\exists\ \widetilde{p} > 1$ such that $\overline{Q}_p \lesseqgtr H(\widetilde{\alpha}) \Leftrightarrow p \lesseqgtr \widetilde{p}$.

(iii) Follows directly by taking $Q = H(\widetilde{\alpha})$ at (10) of Lemma 2.

(iv) Noting that $p - 1 = c'(\overline{Q}_p)$, (iv) follows directly from (iii).

(v) The landlord obtains $\beta^{\underline{\alpha}}(1) = \Phi(1) - \Phi(\underline{\alpha})$ under $(1, \beta^{\underline{\alpha}}(1))$. Let $\Delta^{\underline{\alpha}}(p) := \Pi_S^p(\widetilde{\alpha}) - \beta^{\underline{\alpha}}(1)$. For the landlord $(\widetilde{\alpha}, 0)$ is superior to $(1, \beta^{\underline{\alpha}}(1))$ if and only if $\Delta^{\underline{\alpha}}(p) > 0$. The monotonicity and limiting properties of $\Pi_S^p(\widetilde{\alpha})$ yield (a) $\Delta^{\underline{\alpha}}(p)$ is strictly increasing in p and (b) $\lim_{p\to\infty} \Delta^{\underline{\alpha}}(p) = \infty$. Since $(1, \beta^{\underline{\alpha}}(1))$ is the unique optimal tenancy contract for $p = 1$ (Proposition 1), $\Delta^{\underline{\alpha}}(1) < 0$. Hence $\exists\ p_R^S(\underline{\alpha}) > 1$ such that $\Delta^{\underline{\alpha}}(p) \lesseqgtr 0 \Leftrightarrow p \lesseqgtr p_R^S(\underline{\alpha})$, proving the first statement of (v). The last statement is direct. ∎

Under pure share contracts, the landlord's only consideration is to keep the maximum possible output at his disposal so that his revenue from price variation is maximum. For any α, he has output $H(\alpha)$ and he chooses $\alpha = \widetilde{\alpha}$ that maximizes $H(\alpha)$. Compared to the optimal fixed rental contract, the pure share contract $(\widetilde{\alpha}, 0)$ entails a rental loss $\Phi(1) - \Phi(\widetilde{\alpha})$ which can be recovered from his revenue from $H(\widetilde{\alpha})$ provided the price p he receives is high enough. Finally observe that it is possible that both the landlord and the tenant prefer the pure share contract over the fixed rental contract, which shows that share contracts can be optimal under alternative bargaining arrangements as well.

3.2.5 Tenancy contracts that include both share and rent

Now we consider general tenancy contracts (α, β) where $\alpha \in [0,1]$ and $\beta \in \mathrm{R}$. We know that it is sufficient to consider contracts $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ where $\beta^{\underline{\alpha}}(\alpha) = \Phi(\alpha) - \Phi(\underline{\alpha})$. By (15), the payoff of the landlord under $(\alpha, \beta^{\underline{\alpha}}(\alpha))$ is

$$\Pi_{S+R}^{p,\underline{\alpha}}(\alpha) = \Psi^p(H(\alpha)) + \Phi(\alpha) - \Phi(\underline{\alpha}). \tag{21}$$

Taking $Q = H(\alpha)$ in (10) of Lemma 2 and noting that $s(\alpha) = H(\alpha) + \Phi(\alpha)$, we can write

$$\Pi^{p,\underline{\alpha}}_{S+R}(\alpha) = \begin{cases} s(\alpha) + (p-1)H(\alpha) - c(H(\alpha)) - \Phi(\underline{\alpha}) \text{ if } H(\alpha) < \overline{Q}_p, \\ s(\alpha) + (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) - \Phi(\underline{\alpha}) \text{ if } H(\alpha) \geq \overline{Q}_p. \end{cases} \tag{22}$$

To see the interpretation of the payoff above, let us consider the case when there is no price variation ($p = 1$). Then the surplus generated under share α is $s(\alpha)$, the tenant is paid his reservation payoff $\Phi(\underline{\alpha})$ and the landlord obtains $s(\alpha) - \Phi(\underline{\alpha})$. For $p > 1$, the additional terms in (22) represent the additional surplus that the landlord obtains from his output $H(\alpha)$ due to price variation. If $H(\alpha) < \overline{Q}_p$, he obtains an additional unit revenue of $p - 1$ while his cost of storage is $c(H(\alpha))$ yielding the additional surplus $(p-1)H(\alpha) - c(H(\alpha))$. If $H(\alpha) \geq \overline{Q}_p$, however, he obtains an additional unit revenue of $p - 1$ only for output $\overline{Q}_p$. The remaining output $H(\alpha) - \overline{Q}_p$ is sold at price 1, so it yields no additional surplus. Hence the additional surplus is $(p-1)\overline{Q}_p - c\left(\overline{Q}_p\right)$.

Observe that (i) $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ is continuous at all α and (ii) it is twice continuously differentiable at all α except when $H(\alpha) = \overline{Q}_p$. The following lemma will be useful to determine the optimal tenancy contracts.

Lemma 4 *Let $p > 1$ and $\widetilde{\alpha} \in (0,1)$ be the unique maximizer of $H(\alpha)$.*

(i) $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ *is strictly increasing for* $\alpha \in [0, \widetilde{\alpha})$.

(ii) *If* $H(\alpha) > \overline{Q}_p$, *then* $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ *is strictly increasing in* α.

(iii) *If* $H(\alpha) < \overline{Q}_p$, *then* $\Pi'^{p}_{S+R}(\alpha) = (1-\alpha)F'(\alpha) + [p - 1 - c'(H(\alpha))]H'(\alpha)$ *and* $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ *is strictly concave in* α.

(iv) *Suppose* $H(\widetilde{\alpha}) > \overline{Q}_p$. *Then* $\exists\, \overline{\alpha}_p \in (\widetilde{\alpha}, 1)$ *such that for* $\alpha \in [\widetilde{\alpha}, 1]$, $H(\alpha) \gtreqqless \overline{Q}_p \Leftrightarrow \alpha \lesseqqgtr \overline{\alpha}_p$. *Consequently* $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ *is strictly increasing for* $\alpha \in [\widetilde{\alpha}, \overline{\alpha}_p)$.

(v) $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ *is strictly decreasing at* $\alpha = 1$.

Proof. See the Appendix. ■

Now we characterize the optimal tenancy contracts.

Proposition 3 *Consider the set $\mathbb{T} = \{(\alpha, \beta) | \alpha \in [0,1], \beta \in \mathrm{R}\}$ of all tenancy contracts. For any $p > 1$, the landlord has a unique optimal tenancy contract $(\alpha^*_p, \beta^{\underline{\alpha}}(\alpha^*_p))$. It is a share contract $(0 < \alpha^*_p < 1)$ where $\beta^{\underline{\alpha}}(\alpha^*_p) = \Phi(\alpha^*_p) - \Phi(\underline{\alpha})$ that binds the tenant's participation constraint. The optimal contract has the following properties.*

(i) *The output produced is $F(\alpha^*_p)$. The output at the landlord's disposal is $H(\alpha^*_p) < \overline{Q}_p$.*

(ii) *The landlord obtains $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha^*_p) = pH(\alpha^*_p) - c(H(\alpha^*_p)) + \Phi(\alpha^*_p) - \Phi(\underline{\alpha})$.*

(iii) $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha^*_p)$ *is strictly increasing in p, $\partial \Pi^{p,\underline{\alpha}}_{S+R}(\alpha^*_p)/\partial p = H(\alpha^*_p)$ and $lim_{p\to\infty}\, \Pi^{p,\underline{\alpha}}_{S+R}(\alpha^*_p) = \infty$.*

(iv) $\alpha^*_p > \widetilde{\alpha}$ *for all $p > 1$, i.e., the tenant's share is higher than the share he gets under the optimal pure share contract.*

(v) α^*_p *is strictly decreasing, i.e., as price variation increases, the contract prescribes lower share to the tenant and higher share to the landlord.*

(vi) $lim_{p\to\infty}\alpha_p^* = \widetilde{\alpha}$ (*as price variation increases indefinitely, the tenant's share converges to his share under the optimal pure share contract*) *and* $lim_{p\downarrow 1}\alpha_p^* = 1$ (*as price variation diminishes, the tenant's share converges to* 1).

Proof. Let M^p_{S+R} be the set of all maximizers of $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ over $\alpha \in [0,1]$. By Lemma 4(i), $M^p_{S+R} \subseteq [\widetilde{\alpha}, 1]$. If $H(\widetilde{\alpha}) > \overline{Q}_p$, then by Lemma 4(iv), $M^p_{S+R} \subseteq [\overline{\alpha}_p, 1] \subset [\widetilde{\alpha}, 1]$ where $\overline{\alpha}_p \in (\widetilde{\alpha}, 1)$ is such that $H(\overline{\alpha}_p) = \overline{Q}_p$. Let us define

$$\widehat{\alpha}_p = \begin{cases} \overline{\alpha}_p \text{ if } \ H(\widetilde{\alpha}) > \overline{Q}_p = H(\overline{\alpha}_p), \\ \widetilde{\alpha} \text{ if } \ H(\widetilde{\alpha}) \leq \overline{Q}_p. \end{cases} \tag{23}$$

Using (23), by Lemma 4 [(i) & (iv)], $M^p_{S+R} \subseteq [\widehat{\alpha}_p, 1]$. Let $\alpha \in (\widehat{\alpha}_p, 1]$. Then by (23), $H(\alpha) < \overline{Q}_p$ and Lemma 4(iii) yields $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ is strictly concave in α and

$$\Pi'^{p}_{S+R}(\alpha) = [p - 1 - c'(H(\alpha))]H'(\alpha) + (1-\alpha)F'(\alpha) \text{ for } \alpha \in (\widehat{\alpha}_p, 1]. \tag{24}$$

Observe by (23) that (a) if $H(\widetilde{\alpha}) > \overline{Q}_p$, then $H(\widehat{\alpha}_p) = H(\overline{\alpha}_p) = \overline{Q}_p$ and $p - 1 - c'(H(\widehat{\alpha}_p)) = p - 1 - c'(\overline{Q}_p) = 0$ and (b) if $H(\widetilde{\alpha}) \leq \overline{Q}_p$, then $H'(\widehat{\alpha}_p) = H'(\widetilde{\alpha}) = 0$. Thus, in either case the first term of (24) is zero at $\alpha = \widehat{\alpha}_p$ and the right derivative of $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ at $\alpha = \widehat{\alpha}_p$ is

$$\overline{\Pi}'^{p}_{S+R}(\widehat{\alpha}_p) = \lim_{\alpha\downarrow\widehat{\alpha}_p} \Pi'^{p}_{S+R}(\alpha) = (1-\widehat{\alpha}_p)F'(\widehat{\alpha}_p) > 0.$$

Hence $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ is strictly increasing in the neighborhood of $\alpha = \widehat{\alpha}_p$. Since it is strictly decreasing at $\alpha = 1$ [Lemma 4(v)], $\exists$ a unique $\alpha_p^* \in (\widehat{\alpha}_p, 1)$ such that $\Pi'^{p}_{S+R}(\alpha_p^*) = 0$ and α_p^* is the unique maximizer of $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$, proving that the optimal tenancy contract for the landlord is a share contract. Now we prove properties (i)-(vi).

(i) As $H(\alpha) < \overline{Q}_p$ for $\alpha \in (\widehat{\alpha}_p, 1]$ and $\alpha_p^* \in (\widehat{\alpha}_p, 1)$, (i) follows.

(ii) Using (i), (ii) follows by the first expression of (22).

(iii) The first part follows by the envelope theorem. Since $\lim_{p\to\infty} \Pi^p_S(\widetilde{\alpha}) = \infty$ (Proposition 2) and $\Pi^p_{S+R}(\alpha_p^*) \geq \Pi^p_S(\widetilde{\alpha})$, the limiting property follows.

(iv) Since $\alpha_p^* \in (\widehat{\alpha}_p, 1)$ and $\widehat{\alpha}_p \geq \widetilde{\alpha}$ [by (23)], (iv) follows.

(v) Let $1 < p_1 < p_2$. Since $\Pi'^{p}_{S+R}(\alpha_p^*) = 0$ for any $p > 1$, by (24), $\Pi'^{p_2}_{S+R}(\alpha_{p_1}^*) = (p_2 - p_1)H'(\alpha_{p_1}^*)$. Since $H'(\alpha) < 0$ for $\alpha > \widetilde{\alpha}$ (Lemma 3) and $\alpha_{p_1}^* > \widetilde{\alpha}$, we have $\Pi'^{p_2}_{S+R}(\alpha_{p_1}^*) < 0 = \Pi'^{p_2}_{S+R}(\alpha_{p_2}^*)$. The strict concavity of $\Pi^{p,\underline{\alpha}}_{S+R}(\alpha)$ then yields $\alpha_{p_2}^* < \alpha_{p_1}^*$.

(vi) Since $\lim_{p\to\infty} \overline{Q}_p = \infty$, for large values of p, $H(\alpha) \leq H(\widetilde{\alpha}) < \overline{Q}_p$ for all $\alpha \in [\widetilde{\alpha}, 1]$. Now consider any small $\delta > 0$ and let $\alpha \in [\widetilde{\alpha} + \delta, 1]$. Since $H'(\alpha) < 0$ for $\alpha > \widetilde{\alpha}$ and $c'(.)$, $H'(.)$ and $F'(.)$ are all bounded, from (24) it follows that $\exists$ $P(\delta) > 1$ such that for any $p > P(\delta)$, $\Pi'^{p}_{S+R}(\alpha) < 0$ for all $\alpha \in [\widetilde{\alpha} + \delta, 1]$. Hence $\alpha_p^* \in (\widetilde{\alpha}, \widetilde{\alpha} + \delta)$ for $p > P(\delta)$, proving that $\lim_{p\to\infty} \alpha_p^* = \widetilde{\alpha}$.

When $p = 1$, the optimal tenancy contract is the fixed rental contract $(1, \beta^{\underline{\alpha}}(1))$, i.e., $\alpha_1^* = 1$ (Proposition 1). Hence $\lim_{p\downarrow 1} \alpha_p^* = 1$. ■

Proposition 3 qualifies the result of Proposition 1 by showing that under price variation, the unique optimal tenancy contract is a sharecropping contract. One interesting result is that the tenant's share under the optimal pure share contract $\widetilde{\alpha}$ forms a lower bound of his share from the optimal tenancy contract. To see the intuition, observe from (21) that the landlord's payoff has two components: (i) $\Psi^p(H(\alpha))$ (landlord's revenue from output $H(\alpha)$ under his optimal storing strategy) and (ii) $\Phi(\alpha) - \Phi(\underline{\alpha})$ (the rent). The rent is increasing in α. If $\alpha < \widetilde{\alpha}$, $H(\alpha)$ is also increasing, resulting both components of the payoff to move in the same direction. The landlord is then better off raising the tenant's share until it reaches $\widetilde{\alpha}$. When $\alpha \geq \widetilde{\alpha}$, there is a trade-off: $H(\alpha)$ then starts falling, so a higher rent can be obtained only at the cost of a lower revenue $\Psi^p(H(\alpha))$. This trade-off is settled by the extent of price variation. As p increases, the revenue $\Psi^p(H(\alpha))$ has a relatively higher weight in the landlord's payoff and he chooses a relatively small value of α that raises $H(\alpha)$. In the two extremes, the tenancy contract converges to two "pure" contractual forms: towards a pure share contract for large values of p and a fixed rental contract when p is close to 1.

3.2.6 Crop-buying contracts

Now we consider crop-buying contracts. Such a contract given by a number $\gamma \in [1, p]$, where γ is the unit price at which the landlord offers to buy the entire produced output from the tenant. By Lemma 1, under the contract γ, the tenant's optimal choice of labor is $\ell(\gamma)$, the output is $F(\gamma)$ and the tenant's payoff is $\Phi(\gamma)$. So the tenant's participation constraint is $\Phi(\gamma) \geq \Phi(\underline{\alpha})$. Since $\underline{\alpha} \in (0, 1)$, by the monotonicity of $\Phi(.)$ (Lemma 1), the participation constraint of the tenant does not bind for any $\gamma \geq 1$. Under the crop-buying contract γ, the output at the landlord's disposal is $F(\gamma)$. Taking $Q = F(\gamma)$ in Lemma 2, the landlord's revenue from this output under his optimal storing strategy is $\Psi^p(F(\gamma))$. Since the landlord has to pay the tenant the price γ for each unit, his payoff is

$$\Pi_B^p(\gamma) = \Psi^p(F(\gamma)) - \gamma F(\gamma). \tag{25}$$

The landlord's problem is to choose $\gamma \in [1, p]$ to maximize $\Pi_B^p(\gamma)$. Taking $Q = F(\gamma)$ in (10) of Lemma 2, by (25) we have

$$\Pi_B^p(\gamma) = \begin{cases} (p - \gamma)F(\gamma) - c(F(\gamma)) \text{ if } F(\gamma) < \overline{Q}_p \\ (1 - \gamma)F(\gamma) + (p - 1)\overline{Q}_p - c\left(\overline{Q}_p\right) \text{ if } F(\gamma) \geq \overline{Q}_p \end{cases} \tag{26}$$

Note that $\Pi_B^p(\gamma)$ is continuous at all γ and it is twice continuously differentiable at all γ except when $F(\gamma) = \overline{Q}_p$. The following lemma will be useful to determine the optimal crop buying contracts.

Lemma 5 (i) *If $F(\gamma) > \overline{Q}_p$, then $\Pi_B^p(\gamma)$ is strictly decreasing in γ.*
(ii) *If $F(1) \geq \overline{Q}_p$, then the unique maximizer of $\Pi_B^p(\gamma)$ over $\gamma \in [1, p]$ is $\gamma_p^* = 1$.*
(iii) *If $F(1) < \overline{Q}_p < F(p)$, then $\exists\ \overline{\gamma}_p \in (1, p)$ such that for $\gamma \in [1, p]$, $F(\gamma) \lesseqqgtr \overline{Q}_p \Leftrightarrow \gamma \lesseqqgtr \overline{\gamma}_p$. Consequently $\Pi_B^p(\gamma)$ is strictly decreasing for $\gamma \in (\overline{\gamma}_p, p]$.*

Proof. (i) If $F(\gamma) > \overline{Q}_p$, then by (26), $\Pi_B^p(\gamma) = (1 - \gamma)F(\gamma) +$ a constant. Hence $\Pi_B'^p(\gamma) = (1 - \gamma)F'(\gamma) - F(\gamma) < 0$ [since $F(.) > 0$, $F'(.) > 0$ and $\gamma \geq 1$] which proves the result.

(ii) If $F(1) \geq \overline{Q}_p$, then by the monotonicity of $F(.)$, $F(\gamma) > \overline{Q}_p$ for all $\gamma \in (1, p]$ and (ii) follows by (i).

(iii) The first part of (iii) follows by the monotonicity of $F(.)$. The second part follows by (i). ■

Now we characterize the optimal crop-buying contracts.

Proposition 4 *Consider the set of all crop-buying contracts $\mathbb{B} = \{\gamma | \gamma \in [1,p]\}$. For any $p \geq 1$, the landlord has a unique optimal crop-buying contract γ_p^*. The optimal contract has the following properties, where $\overline{p} \equiv 1 + c'(F(1))$ and $\overline{\overline{p}} \equiv \overline{p} + F(1)/F'(1)$.*

(i) $\gamma_p^* = 1$ *if* $p \in [1, \overline{\overline{p}}]$ *and* $\gamma_p^* > 1$ *if* $p > \overline{\overline{p}}$.

(ii) *The tenant obtains* $\Phi(1)$ *if* $p \in [1, \overline{\overline{p}}]$ *and* $\Phi(\gamma_p^*) > \Phi(1)$ *if* $p > \overline{\overline{p}}$.

(iii) *The landlord has the entire output* $F(\gamma_p^*)$ *at his disposal. It exceeds* $\overline{Q}_p$ *if and only if* $p < \overline{p}$.

(iv) *The landlord obtains*

$$\Pi_B^p(\gamma_p^*) = \begin{cases} (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) & \text{if } p \in [1, \overline{p}], \\ (p-1)F(1) - c\left(F(1)\right) & \text{if } p \in (\overline{p}, \overline{\overline{p}}], \\ (p-\gamma_p^*)F(\gamma_p^*) - c\left(F(\gamma_p^*)\right) & \text{if } p > \overline{\overline{p}}. \end{cases}$$

(v) $\Pi_B^p(\gamma_p^*)$ *is strictly increasing in* p. *Specifically* $\mathrm{d}\Pi_B^p(\gamma_p^*)/\mathrm{d}p$ *equals* $\overline{Q}_p$ *if* $p \in [1, \overline{p}]$, $F(1)$ *if* $p \in (\overline{p}, \overline{\overline{p}}]$ *and* $F(\gamma_p^*)$ *if* $p > \overline{\overline{p}}$. *Moreover* $lim_{p\to\infty}\Pi_B^p(\gamma_p^*) = \infty$.

(vi) γ_p^* *is strictly increasing for* $p > \overline{\overline{p}}$ *and* $\gamma_p^* \to \infty$ *as* $p \to \infty$.

Proof. The proof depends on whether $F(1)$ exceeds $\overline{Q}_p$ or not. Recall by (6) that $F(1) \lesseqqgtr \overline{Q}_p \Leftrightarrow p - 1 \gtreqqless c'(F(1))$. As $\overline{p} \equiv 1 + c'(F(1))$, $F(1) \lesseqqgtr \overline{Q}_p \Leftrightarrow p \gtreqqless \overline{p}$.

Case 1. $p \in [1, \overline{p}]$. Then $F(1) \geq \overline{Q}_p$ and by Lemma 5(ii), the unique maximizer of $\Pi_B^p(\gamma)$ is $\gamma_p^* = 1$.

Case 2. $p > \overline{p}$. Then $F(1) < \overline{Q}_p$. Let M_B^p be the set of all maximizers of $\Pi_B^p(\gamma)$ over $\gamma \in [1,p]$. If $F(1) < \overline{Q}_p < F(p)$ then by Lemma 5(iii), $M_B^p \subseteq [1, \overline{\gamma}_p]$ where $F(\overline{\gamma}_p) = \overline{Q}_p$. Let us define

$$\widehat{\gamma}_p = \begin{cases} \overline{\gamma}_p \text{ if } F(1) < \overline{Q}_p = F(\overline{\gamma}_p) < F(p), \\ p \text{ if } F(p) \leq \overline{Q}_p. \end{cases} \tag{27}$$

By (27) and Lemma 5(iii), $M_B^p \subseteq [1, \widehat{\gamma}_p]$. Consider $\gamma \in [1, \widehat{\gamma}_p)$. Then $F(\gamma) < \overline{Q}_p$ and $\Pi_B^p(\gamma)$ is given by the first expression of (26). Denoting $g(\gamma) := \gamma + c'(F(\gamma))$, we have

$$\Pi_B'^p(\gamma) = [p - g(\gamma)]F'(\gamma) - F(\gamma) \text{ for } \gamma \in [1, \widehat{\gamma}_p). \tag{28}$$

By (28), $\Pi_B'^p(\gamma) < 0$ if $p - g(\gamma) \leq 0$ [since both $F'(.)$ and $F(.)$ are positive]. To determine the sign of $p - g(\gamma)$, observe that $g(\gamma)$ is strictly increasing [$g'(\gamma) = 1 + c''(F(\gamma))F'(\gamma) > 0$ since $c''(.) > 0$ and $F'(.) > 0$]. Since $c'(\overline{Q}_p) = p - 1$, by (27) we have

$$g(\widehat{\gamma}_p) = \begin{cases} \overline{\gamma}_p + c'(F(\overline{\gamma}_p)) = \overline{\gamma}_p + p - 1 \text{ if } F(1) < \overline{Q}_p = F(\overline{\gamma}_p) < F(p), \\ p + c'(F(p)) \text{ if } F(p) \leq \overline{Q}_p. \end{cases}$$

Hence $g(\widehat{\gamma}_p) > p$. As $g(1) = 1 + c'(F(1)) \equiv \overline{p} < p$, $\exists\ \widetilde{\gamma}_p \in (1, \widehat{\gamma}_p)$ such that $p - g(\gamma) \gtreqqless 0 \Leftrightarrow \gamma \lesseqqgtr \widetilde{\gamma}_p$. So by (28), $\Pi_B'^p(\gamma) < 0$ for $\gamma \in [\widetilde{\gamma}_p, \widehat{\gamma}_p)$, implying that $M_B^p \subseteq [1, \widetilde{\gamma}_p]$. Note from (28) that

$$\Pi_B''^p(\gamma) = [p - g(\gamma)]F''(\gamma) - g'(\gamma)F'(\gamma) - F'(\gamma).$$

Since $p - g(\gamma) \geq 0$ for $\gamma \in [1, \widetilde{\gamma}_p]$, $F''(.) \leq 0$ (Assumption A1), $g'(.) > 0$ and $F'(.) > 0$, it follows that $\Pi_B^p(\gamma)$ is strictly concave for $\gamma \in [1, \widetilde{\gamma}_p]$. As $\Pi_B'^p(\widetilde{\gamma}_p) < 0$, $\Pi_B^p(\gamma)$ has a unique maximizer $\gamma_p^* \in [1, \widetilde{\gamma}_p)$. Whether $\gamma_p^* > 1$ or $\gamma_p^* = 1$ depends on $\Pi_B'^p(1)$. As $g(1) = \overline{p}$, by (28), $\Pi_B'^p(1) = [p - \overline{p}]F'(1) - F(1)$. Denoting $\overline{\overline{p}} \equiv \overline{p} + F(1)/F'(1)$, $\Pi_B'^p(1) \gtreqless 0 \Leftrightarrow p \gtreqless \overline{\overline{p}}$. So we conclude that (a) if $p \in (\overline{p}, \overline{\overline{p}}]$, then $\gamma_p^* = 1$ and (b) if $p > \overline{\overline{p}}$, then $\gamma_p^* \in (1, \widetilde{\gamma}_p)$ satisfying $\Pi_B'^p(\gamma_p^*) = 0$.

Combining (a) and (b) with Case 1, it follows that the landlord has a unique crop-buying contract γ_p^* where $\gamma_p^* = 1$ if $p \in [1, \overline{p}]$ and $\gamma_p^* > 1$ if $p > \overline{p}$. This proves (i). Part (ii) is direct from (i). Now we prove properties (iii)-(v).

(iii) Follows by noting that (a) $\gamma_p^* = 1$ and $F(1) \geq \overline{Q}_p$ for $p \in [1, \overline{p}]$, (b) $\gamma_p^* = 1$ and $F(1) < \overline{Q}_p$ for $p \in (\overline{p}, \overline{\overline{p}}]$, and (c) $F(\gamma_p^*) < \overline{Q}_p$ for $p > \overline{\overline{p}}$.

(iv) Follows by (iii) and (26).

(v) The first part follows from (iv) by the envelope theorem and noting that $p - 1 = c'(\overline{Q}_p)$. To prove the limiting property, let $p > \overline{p}$. Then $F(1) < \overline{Q}_p$ and by (26), $\Pi_B^p(1) = (p-1)F(1) - c(F(1))$, so $\lim_{p\to\infty} \Pi_B^p(1) = \infty$. Since $\Pi_B^p(\gamma_p^*) \geq \Pi_B^p(1)$, the result follows.

(vi) Let $\overline{\overline{p}} < p_1 < p_2$. Then $\widetilde{\gamma}_{p_1} < \widetilde{\gamma}_{p_2}$ [since $g(.)$ is monotonic]. As $\gamma_p^* \in (1, \widetilde{\gamma}_p)$ for $p > \overline{\overline{p}}$, both $\gamma_{p_1}^*, \gamma_{p_2}^* \in (1, \widetilde{\gamma}_{p_2})$. Since $\Pi_B'^p(\gamma_p^*) = 0$ for $p > \overline{p}$, by (28), $\Pi_B'^{p_2}(\gamma_{p_1}^*) = (p_2 - p_1)F'(\gamma_{p_1}^*) > 0$. The strict concavity of $\Pi_B^{p_2}(\gamma)$ over $\gamma \in [1, \widetilde{\gamma}_{p_2}]$ yields $\gamma_{p_2}^* > \gamma_{p_1}^*$ proving the monotonicity of γ_p^* for $p > \overline{\overline{p}}$.

Consider any $K > 1$. For sufficiently large values of p, $K < \widehat{\gamma}_p$. Then for $\gamma \in [1, K]$, $\Pi_B'^p(\gamma)$ is given by (28). As $g(.)$, $F'(.)$, and $F(.)$ are all bounded for $\gamma \in [1, K]$, by (28) $\Pi_B'^p(\gamma) > 0$ for sufficiently large values of p. So for any $K > 1$, $\gamma_p^* > K$ for all sufficiently large values of p proving that $\lim_{p\to\infty} \gamma_p^* = \infty$. ■

Crop-buying contracts are high-incentive in nature. Since the tenant sells the entire output to the landlord at a high price, the output produced is high. For the landlord, the marginal cost of storing goes up with higher volumes of output, so if p is small and he has a large amount, his optimal storing strategy involves disposing off substantial part of the output in season 1 at low price 1. Setting a high γ in the crop-buying contract is not worthwhile for the landlord for small values of p, because for a substantial part of the output he makes a net loss (pays price $\gamma > 1$ but sells at price 1). This is the reason why for small values of p, the landlord buys the output at the minimum possible price 1. As p goes up, it becomes more profitable for the landlord to have a higher output at his disposal. As a result, he raises γ to create higher incentive for the tenant.

3.3 Comparison of different contracts

Now we are in a position to compare different contracts. To simplify notations, the optimal contract for the landlord within each class is denoted as follows: $R \equiv (1, \beta^{\underline{\alpha}}(1))$ (fixed rental contract), $S \equiv (\widetilde{\alpha}, 0)$ (pure share contract), $S + R \equiv (\alpha_p^*, \beta^{\underline{\alpha}}(\alpha_p^*))$ (tenancy contract) and $B \equiv \gamma_p^*$ (crop-buying contract). Also we use $\succ$ to stand for the preference of the landlord between two contracts (i.e. $R \succ S$ means the landlord prefers R to S etc.).

Proposition 5 (1) *The output produced is $F(\widetilde{\alpha})$ under S, $F(\alpha_p^*) > F(\widetilde{\alpha})$ under $S + R$, $F(1) > F(\alpha_p^*)$ under R and $F(\gamma_p^*) \geq F(1)$ under B.*

(2) *The tenant obtains his reservation payoff* $\Phi(\underline{\alpha})$ *under* R *and* $S+R$*; he obtains* $\Phi(\widetilde{\alpha}) > \Phi(\underline{\alpha})$ *under* S *and* $\Phi(\gamma_p^*) \geq \Phi(1) > \Phi(\widetilde{\alpha})$ *under* B.

(3) *There are numbers* $p_{S+R}^B(\underline{\alpha})$, $p_R^B(\underline{\alpha})$, $p_R^S(\underline{\alpha}) > 1$, *all strictly decreasing in* $\underline{\alpha}$, *and a constant* $p_S^B > 1$ *such that for the landlord*:

- (i) $R \succ S$ *if and only if* $p \leq p_R^S(\underline{\alpha})$.
- (ii) $S+R \succ B$ *if and only if* $p \leq p_{S+R}^B(\underline{\alpha})$.
- (iii) $R \succ B$ *if and only if* $p \leq p_R^B(\underline{\alpha})$.
- (iv) $S \succ B$ *if and only if* $p \leq p_S^B$.

(4) $Min\{p_S^B, p_R^S(\underline{\alpha})\} \leq p_R^B(\underline{\alpha}) \leq Max\{p_S^B, p_R^S(\underline{\alpha})\}$.

(5) *If the landlord is restricted to only fixed rental, pure share and crop buying contracts, then the following hold.*

- (i) *Suppose* $p_S^B \leq p_R^S(\underline{\alpha})$. *The optimal contract is* R *if* $p \leq p_R^B(\underline{\alpha})$ *and it is* B *if* $p > p_R^B(\underline{\alpha})$.
- (ii) *Suppose* $p_S^B > p_R^S(\underline{\alpha})$. *The optimal contract is* R *if* $p \leq p_R^S(\underline{\alpha})$, *it is* S *if* $p_R^S(\underline{\alpha}) < p < p_S^B$ *and it is* B *if* $p \geq p_S^B$.

Proof. (1)-(2) follow directly from Propositions 1-4. Part (3)(i) has been proved in Proposition 2.

(3)(ii) Let $\Delta_{S+R}^{\underline{\alpha}}(p) := \Pi_B^p(\gamma_p^*) - \Pi_{S+R}^{p,\underline{\alpha}}(\alpha_p^*)$. We prove the first part by showing that $\Delta_{S+R}^{\underline{\alpha}}(p)$ is (a) negative when $p=1$, (b) strictly increasing in p and (c) positive for large values of p. By Prop. 4(iv), $\Pi_B^1(\gamma^*(1)) = \Pi_B^1(1) = 0$, proving (a). Since $H(\alpha_p^*) < \overline{Q}_p$ and $H(\alpha_p^*) < F(1) \leq F(\gamma_p^*)$, by Props. 3(iii) and 4(iii), $\partial\Delta_{S+R}^{\underline{\alpha}}(p)/\partial p > 0$ for all $p \geq 1$ which proves (b). To prove (c), observe that $\Pi_B^p(\gamma_p^*) \geq \Pi_B^p(1)$. As $F(1) < \overline{Q}_p$ for large values of p (since $\lim_{p\to\infty} \overline{Q}_p = \infty$), by (26), $\Pi_B^p(\gamma_p^*) \geq \Pi_B^p(1) = (p-1)F(1) - c(F(1))$. As $\Pi_{S+R}^{p,\underline{\alpha}}(\alpha_p^*) \leq pH(\alpha_p^*) + \Phi(\alpha_p^*)$ [by Prop. 2(ii)], for large values of p

$$\Delta_{S+R}^{\underline{\alpha}}(p) \geq p[F(1) - H(\alpha_p^*)] - \Phi(\alpha_p^*) - F(1) - c(F(1)).$$

Since $\lim_{p\to\infty} \alpha_p^* = \widetilde{\alpha}$ and $H(\widetilde{\alpha}) < F(1)$, it follows that $\lim_{p\to\infty} \Delta_{S+R}^{\underline{\alpha}}(p) = \infty$ which proves (c).

(3)(iii) Let $\Delta_R^{\underline{\alpha}}(p) := \Pi_B^p(\gamma_p^*) - \beta^{\underline{\alpha}}(1)$. By the monotonicity and limiting properties of $\Pi_B^p(\gamma_p^*)$ (Proposition 4), $\Delta_R^{\underline{\alpha}}(p)$ is strictly increasing in p and $\lim_{p\to\infty} \Delta_R^{\underline{\alpha}}(p) = \infty$. The proof is complete by noting that $\Delta_R^{\underline{\alpha}}(1) = -\beta^{\underline{\alpha}}(1) < 0$ [since $\Pi_B^1(\gamma^*(1)) = \Pi_B^1(1) = 0$].

(3)(iv) Let $\Delta_S(p) := \Pi_B^p(\gamma_p^*) - \Pi_S^p(\widetilde{\alpha})$. Consider the constants $\widetilde{p}$ (Proposition 2) and $\overline{p}$ (Proposition 4). As $\overline{Q}(\widetilde{p}) = H(\widetilde{\alpha}) < F(1) = \overline{Q}(\overline{p})$, by the monotonicity of $\overline{Q}(.)$ we have $\widetilde{p} < \overline{p}$. The result is proved by showing that $\Delta_S(p)$ is (a) a negative constant for $p \in [1, \widetilde{p}]$, (b) strictly increasing for $p > \widetilde{p}$ and (c) positive for large values of p. For $p \in [1, \widetilde{p}]$, by Props. 2(ii) and 4(iv), $\Delta_S(p) = -H(\widetilde{\alpha})$ which proves (a). Since $H(\widetilde{\alpha}) < \overline{Q}(\overline{p})$ for $p > \widetilde{p}$ and $F(\gamma_p^*) \geq F(1) > H(\widetilde{\alpha})$, by Props. 2(iii) and 4(iii), $\mathrm{d}\Delta_S(p)/\mathrm{d}p > 0$ for $p > \widetilde{p}$, proving (b). Since $\Delta_S(p) \geq \Delta_{S+R}^{\underline{\alpha}}(p)$ and $\lim_{p\to\infty} \Delta_{S+R}(p) = \infty$, we have $\lim_{p\to\infty} \Delta_S(p) = \infty$ which proves (c).

For the proof of (4)-(5), we ignore non-generic values of p where the landlord is indifferent between multiple contracts.

(4) By contradiction, if the result is not true, then either (a) $p_R^B(\underline{\alpha}) < \min\{p_S^B, p_R^S(\underline{\alpha})\}$ or (b) $p_R^B(\underline{\alpha}) > \max\{p_S^B, p_R^S(\underline{\alpha})\}$. If (a) holds, then $\exists\ p$ such that $p_R^B(\underline{\alpha}) < p < \min\{p_S^B, p_R^S(\underline{\alpha})\}$. By (3)(i), (iii) and (iv), for such a p, we have $B \succ R$, $R \succ S$ and $S \succ B$, a contradiction. If (b) holds, then $\exists\ p$ such that $\max\{p_S^B, p_R^S(\underline{\alpha})\} < p < p_R^B(\underline{\alpha})$. By (3)(i), (iii) and (iv), for such a p, we have $B \succ S$, $S \succ R$ and $R \succ B$, again a contradiction.

(5)(i) Suppose $p_S^B \leq p_R^S(\underline{\alpha})$. Then by (4), $p_S^B \leq p_R^B(\underline{\alpha}) \leq p_R^S(\underline{\alpha})$. If $p < p_R^B(\underline{\alpha}) \leq p_R^S(\underline{\alpha})$, then $R \succ B$ and $R \succ S$, proving that R is optimal. If $p > p_R^B(\underline{\alpha}) \geq p_S^B$, then $B \succ R$ and $B \succ S$ which proves that B is optimal.

(5)(ii) Suppose $p_S^B > p_R^S(\underline{\alpha})$. Then by (4), $p_R^S(\underline{\alpha}) < p_R^B(\underline{\alpha}) < p_S^B$. If $p < p_R^S(\underline{\alpha}) < p_R^B(\underline{\alpha})$, then $R \succ S$ and $R \succ B$ proving that R is optimal. If $p_R^S(\underline{\alpha}) < p < p_S^B$, then $S \succ R$ and $S \succ B$ which proves that S is optimal. If $p > p_S^B > p_R^B(\underline{\alpha})$, then $B \succ S$ and $B \succ R$ proving that B is optimal. This completes the proof. ■

Proposition 5 shows that the landlord prefers crop-buying contracts over share contracts only if the price variation is relatively large. A crop-buying contract results in higher output, but the landlord has to pay the tenant a high unit price for this output that lowers the unit profit margin. Under a share contract, the landlord keeps a share of relatively low output [bounded by $F(1)$], but he does not have to pay for his share. A higher volume of output with a lower unit profit margin is worthwhile only if p is large, which explains the result.

Pure share contracts dominate fixed rentals for relatively large values of p, while they dominate crop-buying contracts when p is relatively small. The last part of Proposition 5 shows that depending on the specifics of the model, it is possible to have a range of intermediate values of p where pure share contracts can be superior to both of these other forms. We provide an example to show that the set of parameter values where the pure share contract is optimal is not vacuous.

An Example: Let $f(\ell) = \ell^{1/3}$ (production function), $w(\ell) = \ell$ (cost of labor) and $c(q) = q^2/2$ (storage cost). Since $p - 1 = c'(\overline{Q}_p)$, for this example, $\overline{Q}_p = p - 1$.

For this example, $\ell(x) = x\sqrt{x}/3\sqrt{3}$, $F(x) = \sqrt{x/3}$, $\Phi(x) = 2x\sqrt{x}/3\sqrt{3}$ and $\widetilde{\alpha} = 1/3$. It can be shown that $[1.7, 2] \subset (\overline{p}, \overline{\overline{p}})$ ($\overline{p}$ and $\overline{\overline{p}}$ are constants defined in Prop 4). If $p \in [1.7, 2]$ then (i) $H(\widetilde{\alpha}) < \overline{Q}_p$ and $\Pi_S^p(\widetilde{\alpha}) = 2p/9 - 2/81$, (ii) $\Pi_B^p(\gamma_p^*) = (p-1)/\sqrt{3} - 1/6$ and (iii) $\beta^{\underline{\alpha}}(1) = 2[1 - \underline{\alpha}\sqrt{\underline{\alpha}}]/3\sqrt{3}$. If $p \in [1.7, 2]$ and $\underline{\alpha} \in [1/5, 1/3]$ then for the landlord the pure share contract $(1/3, 0)$ dominates both fixed rental and crop-buying contracts.

Take $p = 1.8$, $\underline{\alpha} = 1/4$. Then $\Pi_S^p(\widetilde{\alpha}) \approx 0.375$, $\Pi_B^p(\gamma_p^*) \approx 0.295$ and $\beta^{\underline{\alpha}}(1) \approx 0.336$.

3.4 Some implications of the results

Our results show that as long as the gain from price variation is not sufficiently large, tenancy contracts will be active. This is fairly consistent with what is observed in rural economies where purely trade-based contracts are not frequently used. The landlord might also prefer a share contract if he foresees the possibility of other competing buyers who may trade with the tenant—this rationale for sharecropping is developed later in the paper.

One issue of share contracts that has received much attention in the literature is that crop-sharing patterns do not show much variation within a region. For example, Rudra and Bardhan (1983: 38) find that most villages in their survey had one or two sharing patterns. In this regard, Stiglitz (1989: 22) points out:

> "...[T]he range of contract forms seems far more restricted than theory would suggest: most contracts have, for instance, shares of one-half, one-third, or two-thirds. Although there have been several attempts to explain this uniformity, none has gained general acceptance."

Further, there is a lack of explanation of "...why minute changes in the determining factors do not bring about minute changes in the share proportion." (Rudra, 1992: 288). Our results provide a somewhat partial explanation of the uniformity of share contracts. Observe from the last part of Proposition 5 that when general tenancy contracts are not allowed, there could be an interval of values of p where the pure share contract $(\widetilde{\alpha}, 0)$ is optimal for the landlord. As the share $\widetilde{\alpha}$ does not depend on p, for relatively small changes in p within an interval, the share contract will continue to stay optimal and the sharing pattern will also remain the same. In other words, "minute changes" in price variation will not change the share. However, this only explains that the specific share contract offered by a single landlord may be relatively stable over time. As the share $\widetilde{\alpha}$ depends on the agent-specific aspects (production function, labor cost), our results cannot explain uniformity of contract forms if these aspects vary a lot within a region.

Observe that the price thresholds obtained in Proposition 5 depend on production and storage cost functions of the landlord. If two landlords are relatively asymmetric, it is possible that they might prefer to offer different types of contracts. For example, for the same p, a landlord with a low storage cost may choose sharecropping while a landlord with a higher cost of storage may offer a fixed rental contract. Thus, our results suggest that tenancy contracts in a region may vary across agents depending on agent-specific characteristics. This is similar in spirit to the conclusion of the screening models of sharecropping [e.g., Newbery and Stiglitz (1979), Hallagan (1982), Allen (1982), Muthoo (1998)] that argue that tenants of different skills may be offered types of contracts. Like the screening models, our theory may also provide an explanation of the coexistence of different forms of tenancy contracts in a given region [see, e.g., Myers (1984: 227-229), Rudra (1992: 293)].

Apart from the agent-specific variation of contracts discussed above, there is another kind of variation that is also observed. It is crop-specific. For example, in his study of West Godavari district of the state of Andhra Pradesh in India, Rao (1971: 584-585) finds that:

> "...[W]ithin the same district, share-lease and cash-lease arrangements coexist, the latter being negligible in the rice zone and predominant in the tobacco zone...Also, the rice crop, for which the share-lease system is extensive, is a major marketed or cash-crop of the region, so that the share-lease system cannot readily be explained in terms of the subsistence nature of the crop."

Since foodgrains like rice are more likely to exhibit seasonal fluctuations of price compared to non-food crops like tobacco, our theory may provide an explanation of the kind of crop-specific variation described above. However, there could be other compelling reasons behind this variation. For example, farmers of two different crops may be systematically different from each other in terms of wealth, skill and other characteristics.

To conclude, it should be said that although our model is consistent with some of the stylized facts of tenancy contracts, further empirical work is needed to identify the extent to which price variation plays a role in specific contexts.

4 Interlinked contracts

So far we have separately considered tenancy and crop-buying contracts. Now we consider general contracts that combine both. A contract offered by the landlord is now a triplet (α, β, γ), where $\alpha \in [0,1]$ is the tenant's share of output, $\beta \in \mathrm{R}$ is the rental transfer from the tenant to the landlord and $\gamma \in [1, p]$ is the unit price at which the landlord offers to buy the tenant's output. Such a contract is an *interlinked contract*, as it enables the landlord to interact with the tenant in two markets: the land market (through share α and rent β) and the product market (through price γ).

The strategic interaction is modeled as an extensive form game G_1 that has the following stages. In the first stage, the landlord offers a contract (α, β, γ) to the tenant. In the second stage, the tenant can reject the contract, in which case the game terminates with both parties getting their reservation payoffs, or he can accept, in which case the game moves on to the third stage where the tenant decides on the amount of labor for carrying out production and output is realized. If the output is Q: (i) the tenant keeps αQ and leaves the rest $(1-\alpha)Q$ with the landlord, (ii) makes the rental transfer β to the landlord and (iii) sells his share of output αQ to the landlord at price γ. The solution concept is Subgame Perfect Equilibrium (SPE). We continue to maintain the assumptions of Section 3.

4.1 The tenant's problem

Under the contract (α, β, γ), the tenant's payoff has two components: (i) the profit from his share α of the output that he sells at price γ and (ii) the rental transfer β. If the tenant chooses labor input ℓ, the output is $f(\ell)$ and he obtains the revenue $\gamma\alpha f(\ell)$ by selling his share $\alpha f(\ell)$ to the landlord at price γ. As the cost of ℓ units of labor is $w(\ell)$, the profit of the tenant from his share is $\gamma\alpha f(\ell) - w(\ell)$ and his payoff is $\gamma\alpha f(\ell) - w(\ell) - \beta$. Defining $\theta := \gamma\alpha$, this payoff is

$$\theta f(\ell) - w(\ell) - \beta$$

and β being a constant, the tenant's problem is to choose ℓ to maximize

$$\phi^{\theta}(\ell) = \theta f(\ell) - w(\ell).$$

Note that θ is the *effective unit price* of the output for the tenant when he works under the contract (α, β, γ). As $\alpha \in [0,1]$ and $\gamma \in [1, p]$, we have $\theta \in [0, p]$. The following lemma, which characterizes the tenant's optimal choice, follows from (3), (4) and (5) by taking $x = \theta$.

Lemma 6 *Consider a contract (α, β, γ) and let $\theta = \gamma\alpha$.*

(i) *Under this contract, the tenant's optimal labor input is $\ell(\theta)$ where $\ell(0) = 0$ and $\theta f'(\ell(\theta)) = w'(\ell(\theta))$ for $\theta > 0$.*

(ii) *The output produced is $F(\theta)$ and the tenant obtains the payoff $\Phi(\theta) - \beta$ where $F(\theta) = f(\ell(\theta))$ and $\Phi(\theta) = \phi^{\theta}(\ell(\theta)) = \theta F(\theta) - w(\ell(\theta))$.*

(iii) *$F(0) = \Phi(0) = 0$, both $F(\theta)$ and $\Phi(\theta)$ are strictly increasing with $\Phi'(\theta) = F(\theta)$ for $\theta > 0$.*

4.2 The landlord's problem

By Lemma 6, when the tenant acts optimally under the contract (α, β, γ), the output produced is $F(\theta)$ where $\theta = \gamma\alpha$. The payoff of the landlord has the following components.

(a) The landlord has his share of output $(1-\alpha)F(\theta)$. Moreover the tenant sells his share $\alpha F(\theta)$ to the landlord. So the landlord has the total output $F(\theta)$ at his disposal. Taking $Q = F(\theta)$ in (10) of Lemma 2, his revenue from this output under his optimal storing strategy is $\Psi^p(F(\theta))$.

(b) The tenant sells his share of output $\alpha F(\theta)$ to the landlord at price γ. So the landlord pays $\gamma\alpha F(\theta) = \theta F(\theta)$ to the tenant.

(c) The landlord obtains the fixed rent β from the tenant.

By (a)-(c), the payoff of the landlord is

$$\Pi^p(\alpha, \beta, \gamma) = \Pi^p(\theta, \beta) = \Psi^p\left(F(\theta)\right) - \theta F(\theta) + \beta.$$

By Lemma 6, the tenant's payoff under his optimal labor input is $\Phi(\theta) - \beta$. As the tenant's reservation payoff is $\Phi(\underline{\alpha})$, his participation constraint is $\Phi(\theta) - \beta \geq \Phi(\underline{\alpha})$. For any θ, the optimal β for the landlord is the one that binds this constraint:

$$\beta^{\underline{\alpha}}(\theta) = \Phi(\theta) - \Phi(\underline{\alpha}).$$

So it is sufficient to consider contracts $(\alpha, \beta^{\underline{\alpha}}(\theta), \gamma)$ where $\alpha \in [0,1]$, $\gamma \in [1,p]$ and $\theta = \gamma\alpha \in [0,p]$. Under such a contract, the landlord's payoff is a function of θ, given by

$$\Pi^{p,\underline{\alpha}}(\theta) = \Psi^p\left(F(\theta)\right) - \theta F(\theta) + \Phi(\theta) - \Phi(\underline{\alpha}). \tag{29}$$

Therefore the landlord's problem reduces to choosing $\theta \in [0,p]$ to maximize $\Pi^{p,\underline{\alpha}}(\theta)$.

Note that the total surplus generated by θ is $\Psi^p\left(F(\theta)\right) - \theta F(\theta) + \Phi(\theta)$. By (29), the landlord's problem is to choose θ to maximize this surplus (leaving the tenant with his reservation payoff). It will be useful to begin with the case when there is no price variation (i.e. $p = 1$). Then the landlord's revenue from output $F(\theta)$ is $\Psi^1(F(\theta)) = F(\theta)$ and the total surplus is

$$s(\theta) = (1-\theta)F(\theta) + \Phi(\theta). \tag{30}$$

As $\Phi'(\theta) = F(\theta)$, we have $s'(\theta) = (1-\theta)F'(\theta)$ and the unique maximizer of $s(\theta)$ is $\theta = 1$. For $p = 1$, the unique contract that supports $\theta = 1$ has $\alpha = 1$, $\gamma = 1$ and $\beta = \beta^{\underline{\alpha}}(1) = \Phi(1) - \Phi(\underline{\alpha})$. Expectedly, this is a fixed rental contract.

Now let $p > 1$. Taking $Q = F(\theta)$ in (10) and by (29) and (30), the payoff of the landlord is

$$\Pi^{p,\underline{\alpha}}(\theta) = \begin{cases} s(\theta) + (p-1)F(\theta) - c\left(F(\theta)\right) - \Phi(\underline{\alpha}) \text{ if } F(\theta) < \overline{Q}_p \\ s(\theta) + (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) - \Phi(\underline{\alpha}) \text{ if } F(\theta) \geq \overline{Q}_p \end{cases} \tag{31}$$

We have seen that for $p = 1$, the total surplus is $s(\theta)$ and the landlord obtains $s(\theta) - \Phi(\underline{\alpha})$. The additional terms in (31) represent the additional surplus that the landlord obtains from his output $F(\theta)$ due to price variation . These expressions of (31) are similar to (22) [$H(\alpha)$

being replaced by $F(\theta)$]. Note that $\Pi^{p,\underline{\alpha}}(\theta)$ is continuous at all θ and it is twice continuously differentiable at all θ except when $F(\theta) = \overline{Q}_p$.

Lemma 7 (i) *If* $F(\theta) < \overline{Q}_p$, *then* $\Pi'^p(\theta) = [p - \theta - c'(F(\theta))]F'(\theta)$ *and* $\Pi'^p(\theta) \gtreqless 0 \Leftrightarrow p \gtreqless \theta + c'(F(\theta))$.
(ii) *If* $F(\theta) > \overline{Q}_p$, *then* $\Pi'^p(\theta) = s'(\theta) = (1-\theta)F'(\theta)$.
(iii) $\Pi^{p,\underline{\alpha}}(\theta)$ *is strictly increasing for* $\theta \in [0,1)$.

Proof. (i) If $F(\theta) < \overline{Q}_p$, (31) yields $\Pi'^p(\theta) = s'(\theta) + [p - 1 - c'(F(\theta))]F'(\theta)$. Since $s'(\theta) = (1-\theta)F'(\theta)$, the first part follows. The second part follows by noting that $F'(\theta) > 0$.

(ii) Follows directly from (31).

(iii) We shall prove (iii) for $F(\theta) \neq \overline{Q}_p$. By continuity, the result will also hold for $F(\theta) = \overline{Q}_p$. First let $\theta \in [0,1)$ be such that $F(\theta) < \overline{Q}_p$. Then $p - \theta > p - 1 > c'(F(\theta))$ [by (6)] and by (i), $\Pi'^p(\theta) > 0$. Next let $\theta \in [0,1)$ be such that $F(\theta) > \overline{Q}_p$. Then by (ii), $\Pi'^p(\theta) = (1-\theta)F'(\theta) > 0$ for $\theta \in [0,1)$ and the proof is complete. ■

Now we characterize the optimal interlinked contracts.

Proposition 6 *Consider the set* $\mathbb{I} = \{(\alpha, \beta, \gamma) | \alpha \in [0,1], \beta \in \mathrm{R}, \gamma \in [1,p]\}$ *of all interlinked contracts. For any* $p > 1$, *the landlord has multiple optimal interlinked contracts. The optimal contracts have the following properties, where* $\overline{p} \equiv 1 + c'(F(1))$.

(i) *For any* $p > 1$, $\exists$ *a unique* $\theta_p^* \in [1,p)$ *such that the set of all optimal interlinked contracts is*

$$\mathbb{I}_p^* = \{(\alpha, \beta^{\underline{\alpha}}(\theta_p^*), \gamma) |\, \alpha \in [\theta_p^*/p, 1], \gamma \in [\theta_p^*, p], \gamma\alpha = \theta_p^*\}$$

where $\beta^{\underline{\alpha}}(\theta_p^*) = \Phi(\theta_p^*) - \Phi(\underline{\alpha})$ *that binds the tenant's participation constraint.*

(ii) $\theta_p^* = 1$ *if* $p \in [1, \overline{p}]$ *and* $\theta_p^* > 1$ *if* $p > \overline{p}$. *For* $p > \overline{p}$, θ_p^* *satisfies* $p = \theta_p^* + c'(F(\theta_p^*))$.

(iii) *The landlord has the entire output* $F(\theta_p^*)$ *at his disposal. Specifically,* $F(\theta_p^*) = F(1) \geq \overline{Q}_p$ *if* $p \in [1, \overline{p}]$ *and* $F(\theta_p^*) < \overline{Q}_p$ *if* $p > \overline{p}$.

(iv) *The landlord's net revenue from the product market* (*revenue under his optimal storing strategy net of the payment he makes for the tenant's share*), *given by* $\Psi^p(F(\theta_p^*)) - \theta_p^* F(\theta_p^*)$, *is positive.*

(v) *The landlord's total payoff is* $\Pi^{p,\underline{\alpha}}(\theta_p^*) = \Psi^p(F(\theta_p^*)) - \theta_p^* F(\theta_p^*) + \beta^{\underline{\alpha}}(\theta_p^*)$. *Specifically*

$$\Pi^{p,\underline{\alpha}}(\theta_p^*) = \begin{cases} (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) + \Phi(1) - \Phi(\underline{\alpha}) & \text{if } p \in [1, \overline{p}], \\ pF(\theta_p^*) - c(F(\theta_p^*)) - \theta_p^* F(\theta_p^*) + \Phi(\theta_p^*) - \Phi(\underline{\alpha}) & \text{if } p > \overline{p}. \end{cases}$$

(vi) $\Pi^{p,\underline{\alpha}}(\theta_p^*)$ *is strictly increasing in p. Specifically* $\partial \Pi^{p,\underline{\alpha}}(\theta_p^*)/\partial p$ *equals* $\overline{Q}_p$ *if* $p \in [1, \overline{p}]$ *and* $F(\theta_p^*)$ *if* $p > \overline{p}$. *Moreover* $lim_{p\to\infty} \Pi^{p,\underline{\alpha}}(\theta_p^*) = \infty$.

(vii) θ_p^* *is strictly increasing for* $p > \overline{p}$ *and* $\theta_p^* \to \infty$ *as* $p \to \infty$.

Proof. We have shown that under the set $\mathbb{I}$, the landlord's problem reduces to choosing $\theta \in [0,p]$ to maximize $\Pi^{p,\underline{\alpha}}(\theta)$ given in (31). By Lemma 7(iii), it is sufficient to consider $\theta \in [1,p]$. The proof depends on whether $F(1)$ exceeds $\overline{Q}_p$ or not. Note by (6) that $F(1) \lesseqgtr \overline{Q}_p \Leftrightarrow p \gtreqless 1 + c'(F(1)) \equiv \overline{p}$.

Case 1. $p \in [1, \overline{p}]$. Then $F(1) \geq \overline{Q}_p$. As $F(.)$ is monotonic, $F(\theta) > \overline{Q}_p$ for all $\theta \in (1, p]$. Then by Lemma 7(ii), $\Pi'^p(\theta) = (1-\theta)F'(\theta) < 0$ for $\theta \in (1, p]$. So the unique maximizer of $\Pi^{p,\underline{\alpha}}(\theta)$ is $\theta_p^* = 1$.

Case 2. $p > \overline{p}$. Then $F(1) < \overline{Q}_p$. If $F(1) < \overline{Q}_p < F(p)$ then by the monotonicity of $F(.)$, $\exists$ $\overline{\theta}_p \in (1, p)$ such that for $\theta \in [1, p]$, $F(\theta) \lesseqqgtr \overline{Q}_p \Leftrightarrow \theta \lesseqqgtr \overline{\theta}_p$. Let us define

$$\widehat{\theta}_p = \begin{cases} \overline{\theta}_p \text{ if } \ F(1) < \overline{Q}_p = F(\overline{\theta}_p) < F(p), \\ p \text{ if } \ F(p) \leq \overline{Q}_p. \end{cases} \tag{32}$$

If $\theta \in (\widehat{\theta}_p, p]$, then by (32), $F(\theta) > \overline{Q}_p$ and Lemma 7(ii) yields $\Pi'^p(\theta) = (1-\theta)F'(\theta) < 0$ (since $\theta > \widehat{\theta}_p > 1$), so it is sufficient to consider $\theta \in [1, \widehat{\theta}_p]$. If $\theta \in [1, \widehat{\theta}_p)$, then $F(\theta) < \overline{Q}_p$ and by Lemma 7(i),

$$\Pi'^p(\theta) \lesseqqgtr 0 \Leftrightarrow p \lesseqqgtr \theta + c'(F(\theta)). \tag{33}$$

Let $g(\theta) := \theta + c'(F(\theta))$. Since $c''(.) > 0$ and $F'(.) > 0$, $g(\theta)$ is strictly increasing. Note by (32) that

$$p - g(\widehat{\theta}_p) = \begin{cases} p - \overline{\theta}_p - c'(\overline{Q}_p) = 1 - \overline{\theta}_p \text{ if } \ F(1) < \overline{Q}_p < F(p), \\ -c'(F(p)) \text{ if } \ F(p) \leq \overline{Q}_p. \end{cases}$$

Hence $p - g(\widehat{\theta}_p) < 0$. As $p - g(1) = p - 1 - c'(F(1)) = p - \overline{p} > 0$, by the monotonicity of $g(.)$ $\exists$ $\theta_p^* \in (1, \widehat{\theta}_p)$ such that for $\theta \in [1, \widehat{\theta}_p)$, $p \gtreqqless g(\theta) \Leftrightarrow \theta \lesseqqgtr \theta_p^*$. Then by (33), the unique maximizer of $\Pi^{p,\underline{\alpha}}(\theta)$ over $\theta \in [0, p]$ is θ_p^*. The multiplicity of optimal contracts follows from the fact that $\theta = \theta_p^*$ can be sustained by multiple combinations of α and γ. Now we prove properties (i)-(iv).

Parts (i)-(iii) follows directly from Cases 1 and 2 of the proof above.

(iv) First let $p \in [1, \overline{p}]$. Then $\theta_p^* = 1$ and the landlord's net revenue from the product market is $\Psi^p(F(1)) - F(1)$. As $F(1) \geq \overline{Q}_p$ for this case, by (10), $\Psi^p(F(1)) = F(1) + (p-1)\overline{Q}_p - c\left(\overline{Q}_p\right) > F(1)$. Now let $p > \overline{p}$. Then $F(\theta_p^*) < \overline{Q}_p$ and by (10), $\Psi^p(F(\theta_p^*)) = pF(\theta_p^*) - c(F(\theta_p^*))$, so his net revenue is $(p - \theta_p^*)F(\theta_p^*) - c(F(\theta_p^*)) = c'(F(\theta_p^*))F(\theta_p^*) - c(F(\theta_p^*))$ [by (ii)]. Since $c(q)$ is strictly convex, at any $q > 0$, $c'(q) > c(q)/q$ which proves that the net revenue is positive.

(v) Follows from (10) by using (ii) and (iii).

(vi) The monotonicity for $p \in [1, \overline{p}]$ follows from (v) by noting that $p - 1 = c'(\overline{Q}_p)$. For $p > \overline{p}$, it follows by the envelope theorem. Since $\Pi^{p,\underline{\alpha}}(\theta_p^*) \geq \Pi_B^p(\gamma_p^*)$ and $\lim_{p \to \infty} \Pi_B^p(\gamma_p^*) = \infty$ (Proposition 4), the limiting property follows.

(vii) Note by (ii) that for $p > \overline{p}$, $p = g(\theta_p^*)$ where $g(\theta) = \theta + c'(F(\theta))$. Since $g(\theta)$ is strictly increasing, θ_p^* is strictly increasing for $p > \overline{p}$. To prove the limiting property of θ_p^*, let $K > 0$ and $p > \max\{g(K), \overline{p}\}$. Then $g(\theta) \leq g(K) < p = g(\theta_p^*)$ for any $\theta \in [0, K]$. So for any $K > 0$, $\theta_p^* > K$ for all sufficiently large values of p which proves that $\lim_{p \to \infty} \theta_p^* = \infty$. ■

The reason behind the multiplicity of optimal contracts is clear. As the tenant's incentive depends on his effective unit price $\theta = \gamma\alpha$, the optimal level of incentive can be sustained by multiple combinations of γ and α. Observe from (31) that for any θ, the landlord's payoff has two components: (i) $s(\theta)$ (the total surplus when $p = 1$) and (ii) the additional surplus due to price variation. First consider $s(\theta)$. It rises for $\theta < 1$, reaches its maximum at $\theta = 1$ and falls for $\theta > 1$. Now consider the additional surplus due to price variation. It is increasing for small volumes of output and becomes a constant beyond a threshold $\overline{Q}_p$ (which is increasing in p). As higher θ results in higher output, the additional surplus is weakly monotonic in θ. So for $\theta < 1$, both components of the landlord's payoff rise with θ and the landlord is better off increasing θ until it reaches 1. For $\theta > 1$, $s(\theta)$ starts falling, so raising θ beyond this point could be worthwhile only if it leads to substantial additional surplus. This is not the case for small values of p. Due to increasing marginal costs of storing, it does not pay the landlord to store a large volume when p is small, so he sells most of the output in season 1 at price 1. As a result, it is optimal to set $\theta = 1$ for relatively small values of p. Once p reaches a threshold level ($p > \overline{p}$), additional surplus is generated from larger volumes of output and the landlord has incentive to raise θ above 1. But how far should he raise θ? Observe that the landlord's marginal cost of storage at output $F(\theta)$ is $c'(F(\theta))$. Since he also pays the tenant $\gamma\alpha F(\theta) = \theta F(\theta)$, his *effective marginal cost* becomes $\theta + c'(F(\theta))$. The optimal level of θ is determined by simply equating the marginal revenue p with the effective marginal cost.

As the optimal θ in Proposition 6 is at least 1, if an optimal contract has $\alpha \in (0, 1)$, then $\gamma > 1$. So in this model a share contract is necessarily accompanied by interlinkage (the landlord buying the tenant's output at a price γ that is higher than price 1 of season 1). This contract can be viewed alternatively as follows: if the output is Q, the landlord effectively provides a subsidy of $(\gamma - 1)\alpha Q$ to the tenant. Thus, an interlinked transaction in our model can be interpreted as a cost-sharing arrangement. Under this broader interpretation, our theory has some empirical support as share contracts in practice often involve cost-sharing [see, e.g., Bardhan and Rudra (1978: 99-100), Rudra (1992: 293-294), Reddy (1996: 52-53), Sharma and Drèze (1996: 8)]. So far as empirical support for general interlinked contracts is concerned, one problem is the lack of sufficient empirical work on this, as recently pointed out by Bardhan (2005: 88):

> "There is now quite a bit of theoretical literature...on interlinked contracts in a poor agrarian economy, but there is even now very little empirical work on the subject. Our dataset [Bardhan & Rudra (1978)] is one of the earliest and still the largest that exists on such interlinked contracts."

Evidence of tenancy-credit linkage can be found in Bardhan and Rudra (1978: 99). Jodha (1984) finds that under a "fairly broad" definition of interlinked operations, between 6 to 21 percent of tenancy transactions of his survey had some form of interlinkage (ibid: 110). The following is a specific evidence from Akola district in the state of Maharashtra in India (ibid: 111):

> "In the Akola villages, the few interlinked transactions concerned primarily land lease, credit and marketing. One of the reasons for this pattern was the public intervention in the form of the monopoly purchase of cotton by the Cotton Marketing Federation in Maharashtra...Small farmers with a limited holding capacity sometimes had to use large farmers as informal intermediaries to do their cotton marketing, a practice that led to interlinked tenancy credit and market transactions."

Although the tenancy-marketing interlinkage above arose out of special circumstances, nevertheless the nature of the transaction is very similar to the one considered in this paper where the tenant, lacking storage facility, sells his output to the landlord.

4.3 Motivation for a refinement criterion: imperfect competition in rural product markets

Going back to Proposition 6, now the question is, how to resolve the multiplicity of optimal contracts obtained there? So far we have implicitly assumed that the landlord is a monopolist in the land market and a monopsonist in the product market. While the landlord can exercise monopoly power over the land he owns, empirical evidence suggests that this is not necessarily the case in the rural product market, which closely resembles what one might call a situation of *imperfect competition* [see, e.g., Baker (1984: 239), Rudra (1992: 53-54), Haymai et al. (1999: 81-83)]. We resolve the multiplicity by proposing an equilibrium refinement that takes into consideration the fact that the landlord might face potential competition in the product market. The following description of a rural product market, taken from Rudra (1992: 53-54), will be helpful in motivating our analysis:

> "Our investigations in more than 200 villages in West Bengal and Bihar indicate the following ranking among different categories of traders in terms of prices paid by them as purchasers of grains.
>
> 1. village retail shops.
> 2. big farmers acting as traders.
> 3. village wholesalers.
> 4. travelling traders (or itinerant merchants) and other village level traders.
> 5. *hats* (that is, non-permanent markets centres functioning on a number of days per month or per week), market wholesalers, and rice mills.
>
> The lowest prices are paid by the village retail shops and the highest prices are paid by the rice mills, market wholesalers, and *hats*."

Now we posit a situation that is plausible in a poor agrarian economy and broadly reflective of the essential findings of Rudra (1992). Consider a small farmer who works as a tenant for a landlord. Out of the marketing channels given above, the price is the highest at *hats* (category 5) that serves as the dominant outlet for the landlord. Due to transportation and other costs, the tenant does not have access to this channel. If no intermediate channel is available, the tenant has to sell his product in village retail shops (category 1) that pay the lowest price. As a buyer of the tenant's output, the landlord belongs to category 2 above and pays a price no lower than that paid by the retail shops. It is evident that in trading with the tenant, the landlord does not enjoy monopoly power and he might face competition from other agents (belonging to categories 3 and 4). Suppose such an agent appears with some small but positive probability. Then the question is, out of all contracts obtained in Proposition 6, what are the ones that the landlord will choose once he anticipates such a possibility? It will be shown that there is a unique contract that satisfies this refinement criterion and it results in a sharecropping contract.

4.4 The perturbed game $G_1(\varepsilon)$

The possibility of competition in the product market is modeled by the perturbed game $G_1(\varepsilon)$ that has the following stages. In the first stage, the landlord offers a contract (α, β, γ)

to the tenant. In the second stage, the tenant either rejects the contract, in which case the game terminates with both parties getting their reservation payoffs, or he accepts, in which case the game moves to the third stage where the tenant carries out production and output is realized. If the output is Q: (i) the tenant keeps αQ and leaves the rest $(1-\alpha)Q$ with the landlord and (ii) makes the rental transfer β to the landlord. At the end of this stage: (a) with probability $\varepsilon \in (0,1)$, a third agent, who we call *the ε-agent*, emerges and (b) with probability $1-\varepsilon$, he does not emerge.

If the ε-agent does not emerge, then $G_1(\varepsilon)$ proceeds like the unperturbed game G_1: the tenant sells his share αQ to the landlord at price γ, the landlord has output Q at his disposal that he sells using his optimal storing strategy, payoffs are realized and the game terminates.

If the ε-agent emerges, then in the fourth stage he decides whether to buy the tenant's output or not. Accordingly he offers a trading contract to the tenant. The tenant then decides whether to sell his output to the landlord or the ε-agent and trade takes place between one of the following buyer-seller pairs: (landlord—tenant) or (ε-agent—tenant). In the fifth stage the ε-agent decides whether to buy the landlord's output or not and he offers a trading contract to the landlord. The landlord then decides whether to keep his output or sell it to the ε-agent and trade takes place potentially between the ε-agent and the landlord.[5] In the last stage, any party that has positive output at his disposal from the previous stage of trading (the landlord or the ε-agent or both), sells it using his optimal storing strategy. Finally payoffs are obtained and the game terminates. The solution concept is the notion of Subgame Perfect Bayesian Equilibrium (SPBE).

Definition: Let $p > 1$. Consider the set of all optimal interlinked contracts $\mathbb{I}_p^*$ of the unperturbed game G_1. We say that a contract $(\alpha, \beta, \gamma) \in \mathbb{I}_p^*$ is *robust to the emergence of the ε-agent* if there is a sequence $\{(\alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon))\}$ such that: (i) for $\varepsilon \in (0,1)$, $(\alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon))$ is the contract offered by the landlord in an SPBE of $G_1(\varepsilon)$ and (ii) $(\alpha(\varepsilon), \beta(\varepsilon), \gamma(\varepsilon)) \to (\alpha, \beta, \gamma)$ as $\varepsilon \to 0+$.

4.5 The problem of the ε-agent

4.5.1 Storage cost and optimal storing strategy of the ε-agent

Like the landlord, the ε-agent can store his output in season 1 and sell it in season 2. We assume that the ε-agent has the same storage cost as the landlord (see Section 3.2.1). Due to symmetry of storage costs, the ε-agent has the same optimal storing strategy as the landlord (see Lemma 2) and his revenue under his optimal storing strategy when he has output Q at his disposal is $\Psi^p(Q)$, given by (10) of Lemma 2.

Remarks. The ε-agent represents a small trader in the village. Given that, it is more realistic to assume that he has a higher storage cost compared to the landlord. We assume symmetric storage costs for analytic convenience. Later we argue that (see page 37) that our qualitative conclusions will not be altered if the ε-agent has a higher storage cost than the landlord, as long as it is not too high. If the storage cost of the ε-agent is too high, then obviously he does not pose any serious competition to the landlord.

[5] This trading sequence ensures that the number of transactions is minimum and trade takes place at most once between two parties.

4.5.2 Trading contracts of the ε-agent

As in the rest of the paper, we restrict to linear contracts. We assume that if the ε-agent trades with a party (tenant/landlord), he has to buy the entire output that is at that party's disposal. Then there is no loss of generality in restricting to trading contracts where the ε-agent offers to buy the output in return for a fixed lump-sum payment. The set of trading contracts is

$$\mathbb{V} = \{(\lambda_T, \mu_T, \lambda_L, \mu_L) | \lambda_T \in \{0,1\}, \lambda_L \in \{0,1\}, \mu_T \in \mathrm{R}_+, \mu_L \in \mathrm{R}_+\}.$$

The variables λ_T and λ_L are indicator variables: $\lambda_T = 0$ means the ε-agent does not offer to trade with the tenant; $\lambda_T = 1$ means he does. Similarly λ_L represents whether or not he offers to trade with the landlord. The variables μ_T and μ_L are the lump-sum payments at which the ε-agent offers to buy the outputs of the tenant and the landlord. It is clear that μ_i comes into play only if $\lambda_i = 1$.

Bargaining power. The ε-agent, being a small trader, has an economic position in the village that is somewhere between the landlord and the tenant. So it can be expected that the ε-agent has a relatively large bargaining power over the tenant, while the landlord has a relatively large bargaining power over the ε-agent.

We assume that the ε-agent has full bargaining power over the tenant (so that in equilibrium, the tenant will be made just indifferent between the offers of the landlord and the ε-agent). As before, the full bargaining power assumption is done for analytic convenience and our qualitative conclusions will not be altered for small variations in full bargaining power (see page 37).

Regarding the relative bargaining powers between the landlord and the ε-agent, we make a very weak assumption. It is only assumed that the landlord has a very slight edge in bargaining which is modeled as follows: there is a small but positive constant $\underline{\tau} > 0$ such that the landlord trades with the ε-agent only if the difference between the landlord's payoff from trading and not trading is at least $\underline{\tau}$. It is shown below that this slight edge in bargaining is enough to ensure that the ε-agent does not trade with the landlord.

Recall that due to symmetry of storage costs, for both the landlord and the ε-agent, the revenue from output Q under the optimal storing is $\Psi^p(Q)$, given by (10).

Lemma 8 *Consider the game $G_1(\varepsilon)$ and suppose the landlord has offered to buy the tenant's output at price $\gamma \in [1, p]$. The following hold in any SPBE of $G_1(\varepsilon)$.*

(i) *If the tenant has output Q_T and the ε-agent trades with the tenant, then $\mu_T = \gamma Q_T$.*

(ii) *If the landlord has output Q_L and the ε-agent trades with the landlord, then $\mu_L = \Psi^p(Q_L) + \underline{\tau}$.*

(iii) $\lambda_L = 0$, *i.e., the ε-agent does not offer to trade with the landlord.*

Proof. (i) As the landlord has offered price γ, the tenant will trade with the ε-agent only if $\mu_T \geq \gamma Q_T$. Given the full bargaining power of the ε-agent, he will choose $\mu_T = \gamma Q_T$.

(ii) The landlord's revenue from output Q_L under the optimal storing strategy is $\Psi^p(Q_L)$. Given the landlord's bargaining edge with the ε-agent, he will sell his output to the ε-agent only if $\mu_L \geq \Psi^p(Q_L) + \underline{\tau}$ and in equilibrium, the ε-agent will choose $\mu_L = \Psi^p(Q_L) + \underline{\tau}$.

(iii) W.l.o.g., consider only the trading contracts that are accepted. Under such a contract $(\lambda_T, \mu_T, \lambda_L, \mu_L) \in \mathbb{V}$, the ε-agent obtains output $\lambda_T Q_T + \lambda_L Q_L$, pays $\lambda_T \mu_T + \lambda_L \mu_L$ and earns revenue $\Psi^p(\lambda_T Q_T + \lambda_L Q_L)$ by using the optimal storing strategy. So his payoff is

$\Psi^p(\lambda_T Q_T + \lambda_L Q_L) - \lambda_T \mu_T - \lambda_L \mu_L$. Using the equilibrium values of μ_T and μ_L from (i) and (ii), this payoff would be

$$\pi^p(\lambda_T, \lambda_L) = \Psi^p(\lambda_T Q_T + \lambda_L Q_L) - \lambda_T \gamma Q_T - \lambda_L[\Psi^p(Q_L) + \underline{\tau}].$$

Hence we have

$$\pi^p(\lambda_T, 0) - \pi^p(\lambda_T, 1) = \begin{cases} \underline{\tau} \text{ if } \lambda_T = 0 \\ \Psi^p(Q_T) + \Psi^p(Q_L) - \Psi^p(Q_T + Q_L) + \underline{\tau} \text{ if } \lambda_T = 1 \end{cases}$$

As $\Psi^p(Q_T)+\Psi^p(Q_L)-\Psi^p(Q_T+Q_L) > 0$ for $Q_L, Q_T > 0$ [Lemma 2(vi)], $\pi^p(\lambda_T, 0) > \pi^p(\lambda_T, 1)$. So, in equilibrium, $\lambda_L = 0$, i.e., the ε-agent does not offer to trade with the landlord. ■

The result above is driven mainly by the decreasing returns to scale of the revenue function $\Psi^p(Q)$ [Lemma 2(vi)]. Let $Q_T, Q_L > 0$. The total *trading surplus* does not depend on transfers from one party to another, so it is simply the sum of revenues of the ε-agent and the landlord from their optimal storing strategies. If the ε-agent trades only with the tenant, then the trading surplus is $\Psi^p(Q_T) + \Psi^p(Q_L)$. However, if the ε-agent trades with both parties, then the surplus is $\Psi^p(Q_T + Q_L) < \Psi^p(Q_T) + \Psi^p(Q_L)$. The surplus goes down due to decreasing returns (as $Q_T + Q_L$ is being stored in only the facility of the ε-agent instead of being divided with the facility of the landlord). As the landlord and the tenant have to be paid their opportunity costs, a lower trading surplus results in lower revenue for the ε-agent. Now, if the ε-agent trades *only* with the landlord, his revenue $\Psi^p(Q_L)$ equals the opportunity cost of the landlord and even a slight edge in the landlord's bargaining power (given by $\underline{\tau} > 0$) induces the ε-agent to not trade with the landlord.

4.6 Stage game of $G_1(\varepsilon)$ following the landlord's contract offer

Now consider the stage game of $G_1(\varepsilon)$ that follows the landlord's contract (α, β, γ). Following this contract, the tenant sells his output at price γ regardless of who the buyer is [Lemma 8(i)]. So his problem stays the same as in the unperturbed game G_1 and it depends only on $\theta = \gamma\alpha$. The output produced is $F(\theta)$ and the tenant obtains $\Phi(\theta) - \beta$ (Lemma 6). As before, the landlord sets the rent $\beta^{\underline{\alpha}}(\theta) = \Phi(\theta) - \Phi(\underline{\alpha})$ that binds the tenant's participation constraint. So for the landlord, it is sufficient to consider contracts $(\alpha, \beta^{\underline{\alpha}}(\theta), \gamma)$. As the ε-agent does not trade with the landlord [Lemma 8(iii)], his trading options are (i) to trade with the tenant and (ii) not to trade at all.

Lemma 9 *Let $p > 1$. Suppose the landlord offers the contract $(\alpha, \beta^{\underline{\alpha}}(\theta), \gamma)$ to the tenant, where $\theta = \gamma\alpha \in (0, p]$, $\alpha \in [\theta/p, 1]$ and $\gamma \in [\theta, p]$. In any SPBE of $G_1(\varepsilon)$ after this offer the following hold.*

(i) *The output produced is $F(\theta)$ and after paying the landlord's share and rent, the output at the tenant's disposal is $\alpha F(\theta)$.*

(ii) *If the ε-agent trades with the tenant, then he pays the tenant $\theta F(\theta)$ and obtains*

$$\pi_A^p(\theta, \alpha) = \Psi^p(\alpha F(\theta)) - \theta F(\theta).$$

The ε-agent trades with the tenant if and only if $\pi_A^p(\theta, \alpha) \geq 0$.

(iii) *$\pi_A^p(\theta, \alpha)$ is strictly increasing in α, $\pi_A^p(\theta, 1) = \Psi^p(F(\theta)) - \theta F(\theta)$ and $\pi^p(\theta, \theta/p) < 0$.*

(iv) *Trading between the ε-agent and the tenant depends on θ and α as follows.*

(a) *If* $\pi^p_A(\theta,1) < 0$, *then for any* $\alpha \in [\theta/p, 1]$, *the* ε*-agent does not trade with the tenant.*

(b) *If* $\pi^p_A(\theta,1) = 0$, *then the* ε*-agent trades with the tenant if and only if* $\alpha = 1$.

(c) *If* $\pi^p_A(\theta,1) > 0$, *then* $\exists\ \alpha^p_A(\theta) \in (\theta/p, 1)$ *such that for* $\alpha \in [\theta/p, 1]$, $\pi^p_A(\theta,\alpha) \lesseqqgtr 0 \Leftrightarrow \alpha \lesseqqgtr \alpha^p_A(\theta)$. *Consequently the* ε*-agent trades with the tenant if and only if* $\alpha \geq \alpha^p_A(\theta)$.

Proof. Part (i) is direct. To prove (ii), take $Q_T = \alpha F(\theta)$ in Lemma 8(i). As the tenant obtains $\gamma\alpha F(\theta) = \theta F(\theta)$ if he sells his output to the landlord, in equilibrium, the ε-agent pays the tenant $\theta F(\theta)$. If the ε-agent buys the tenant's output $\alpha F(\theta)$, then by his optimal storing strategy he obtains the revenue $\Psi^p(\alpha F(\theta))$ yielding the payoff $\pi^p_A(\theta,\alpha) = \Psi^p(\alpha F(\theta)) - \theta F(\theta)$.

(iii) As $F(\theta) > 0$ for $\theta > 0$, the first part follows by the monotonicity of $\Psi^p(.)$ (Lemma 2). The second part is direct. To prove the third part, note that $\Psi^p(Q) < pQ$ for any $Q > 0$ due to positive cost of storage. Taking $(\theta/p)F(\theta) = Q$, we have $\theta F(\theta) = pQ$ and $\pi^p(\theta, \theta/p) = \Psi^p(Q) - pQ < 0$.

Part (iv) follows directly by part (iii). ■

The result above is fairly intuitive. To buy the tenant's output $\alpha F(\theta)$, the ε-agent pays him $\gamma\alpha F(\theta) = \theta F(\theta)$. So for fixed θ, the ε-agent's revenue goes up as the tenant's share α increases. It is maximum when $\alpha = 1$, given by $\pi^p_A(1,\theta) = \Psi^p(F(\theta)) - \theta F(\theta)$ which is also the landlord's net revenue from the product market in the unperturbed game G_1 (the revenues are equal due to symmetry of storage costs). Now, if $\pi^p_A(1,\theta) < 0$, then regardless of α, it is not worthwhile for the ε-agent to trade with the tenant. However, if $\pi^p_A(1,\theta) > 0$, then trading is worthwhile for relatively large values of α. We know that $\pi^p_A(1,\theta) > 0$ under the optimal θ of the unperturbed game G_1 [Prop. 6(iv), page 27], so for $\alpha \geq \alpha^p_A(\theta)$, trade between the ε-agent and the tenant is feasible. In what follows it will be shown that in the perturbed game $G_1(\varepsilon)$, it is optimal for landlord to choose $\alpha = \alpha^p_A(\theta)$ that ensures that in trading with the tenant, the ε-agent just breaks even.

4.7 The problem of the landlord

Now we are in a position to solve the landlord's problem. As we know, for the landlord it is sufficient to consider contracts $(\alpha, \beta^{\underline{\alpha}}(\theta), \gamma)$ for $\theta = \gamma\alpha \in [0,p]$, $\alpha \in [\theta/p, 1]$ and $\gamma \in [\theta, p]$.[6] We know that under this contract the output produced is $F(\theta)$. To determine the expected payoff of the landlord in the perturbed game $G_1(\varepsilon)$, we observe the following, where $\Psi^p(Q)$ [given by (10)] is the landlord's revenue under his optimal storing strategy when he has output Q.

(a) Regardless of the emergence or trading nature of the ε-agent, the landlord obtains the rental transfer $\beta^{\underline{\alpha}}(\theta) = \Phi(\theta) - \Phi(\underline{\alpha})$ from the tenant.

(b) If the ε-agent does not trade with the tenant, the landlord has the total output $F(\theta)$ (his own share $1-\alpha$ plus the tenant's share α) at his disposal. The revenue from this output under his optimal storing strategy is $\Psi^p(F(\theta))$. Moreover the landlord has to pay $\gamma\alpha F(\theta) = \theta F(\theta)$ to the tenant, so his net revenue from the product market is $\Psi^p(F(\theta)) - \theta F(\theta)$.

[6] As $\gamma \leq p$, we have $\theta = \gamma\alpha \leq \alpha p$ implying $\alpha \geq \theta/p$. As $\alpha \leq 1$, we have $\theta = \gamma\alpha \leq \gamma$, so $\gamma \geq \theta$.

(c) If the ε-agent trades with the tenant, then (i) the landlord does not have to make any payment for the tenant's share of output and (ii) he has only his share $(1-\alpha)F(\theta)$. So his revenue from the product market is simply the revenue from output $(1-\alpha)F(\theta)$ under his optimal storing strategy, which is $\Psi^p((1-\alpha)F(\theta))$.

By Lemma 9(iv), whether the ε-agent trades with the tenant or not depends on θ and α. Let $\lambda_T^p(\theta,\alpha)$ be the indicator variable that equals 1 if the ε-agent trades with the tenant and 0 otherwise. Since the ε-agent emerges with probability ε, by (a)-(c), the expected payoff of the landlord is

$$\Pi_\varepsilon^{p,\underline{\alpha}}(\theta,\alpha)=\beta^{\underline{\alpha}}(\theta)+(1-\varepsilon)\underbrace{[\Psi^p(F(\theta))-\theta F(\theta)]}_{\text{revenue in (b)}}$$

$$+\varepsilon\,[\lambda_T^p(\theta,\alpha)\underbrace{\Psi^p((1-\alpha)F(\theta))}_{\text{revenue in (c)}}+(1-\lambda_T^p(\theta,\alpha))\underbrace{\{\Psi^p(F(\theta))-\theta F(\theta)\}}_{\text{revenue in (b)}}]\,. \tag{34}$$

When the ε-agent does not trade with the tenant, the landlord's payoff is the same as in the unperturbed game G_1. It depends only on θ, given by

$$\Pi^{p,\underline{\alpha}}(\theta)=\Psi^p(F(\theta))-\theta F(\theta)+\beta^{\underline{\alpha}}(\theta).$$

When the ε-agent trades with the tenant, the payoff changes from $\Pi^{p,\underline{\alpha}}(\theta)$ (the change could be positive, negative or zero). This change is the difference between the revenues of (c) and (b):

$$\Omega^p(\theta,\alpha)=\Psi^p((1-\alpha)F(\theta))-[\Psi^p(F(\theta))-\theta F(\theta)]\,. \tag{35}$$

By (34) and (35), the landlord's expected payoff in $G_1(\varepsilon)$ is

$$\Pi_\varepsilon^{p,\underline{\alpha}}(\theta,\alpha)=\Pi^{p,\underline{\alpha}}(\theta)+\varepsilon\lambda_T^p(\theta,\alpha)\Omega^p(\theta,\alpha). \tag{36}$$

Let us define

$$\mathbb{J}^p=\{(\theta,\alpha)|\alpha\in[\theta/p,1],\theta\in[0,p]\}.$$

The landlord's problem in the first stage of $G_1(\varepsilon)$ is to choose $(\theta,\alpha)\in\mathbb{J}^p$ to maximize $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta,\alpha)$. As the functions in (36) are bounded for $(\theta,\alpha)\in\mathbb{J}^p$, the maximization problem has a solution, i.e., the game $G_1(\varepsilon)$ has a Subgame Perfect Bayesian Equilibrium (SPBE) for any $\varepsilon\in(0,1)$.

4.7.1 SPBE of $G_1(\varepsilon)$

Note that whether the ε-agent trades with the tenant $[\lambda_T^p(\theta,\alpha)=1]$ or not $[\lambda_T^p(\theta,\alpha)=0]$ depends on whether $\pi_A^p(\theta,1)=\Psi^p(F(\theta))-\theta F(\theta)$ is positive or not [Lemma 9(iv)]. It will be useful for our analysis to partition $[0,p]=\mathbb{E}^p\cup\overline{\mathbb{E}}^p$ where

$$\mathbb{E}^p=\{\theta\in[0,p]\,|\,\Psi^p(F(\theta))-\theta F(\theta)>0\}\text{ and }\overline{\mathbb{E}}^p=[0,p]/\mathbb{E}^p.$$

Also recall by Proposition 6 that for $p>1$, θ_p^* is the unique maximizer of $\Pi^{p,\underline{\alpha}}(\theta)$ over $\theta\in[0,p]$.

Lemma 10 *The following hold for any $p>1$ and $\varepsilon\in(0,1)$.*
(i) *Let $\theta\in\overline{\mathbb{E}}^p$. Then $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta,\alpha)=\Pi^{p,\underline{\alpha}}(\theta)$ for any $\alpha\in[\theta/p,1]$.*

(ii) *Let* $\theta \in \mathbb{E}^p$. *Then* $\exists\ \alpha_A^p(\theta) \in (\theta/p, 1)$, *satisfying* $\pi_A^p(\alpha_A^p(\theta), \theta) = 0$ (*i.e., at* $\alpha = \alpha_A^p(\theta)$, *the* ε*-agent just breaks even in his trade with the tenant*), *such that*

$$\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha) = \begin{cases} \Pi^{p,\underline{\alpha}}(\theta) \text{ if } \alpha \in [\theta/p, \alpha_A^p(\theta)), \\ \Pi^{p,\underline{\alpha}}(\theta) + \varepsilon \Omega^p(\theta, \alpha) \text{ if } \alpha \in [\alpha_A^p(\theta), 1]. \end{cases} \tag{37}$$

(a) $\Omega^p(\theta, \alpha)$ *is strictly decreasing in* α *and* $\exists\ \alpha_L^p(\theta) \in (\alpha_A^p(\theta), 1)$ *such that for* $\alpha \in [\alpha_A^p(\theta), 1]$, $\Omega^p(\theta, \alpha) \gtreqqless 0 \Leftrightarrow \alpha \lesseqqgtr \alpha_L^p(\theta)$.

(b) (*Pareto improving region*) *If* $\alpha \in [\alpha_A^p(\theta), \alpha_L^p(\theta)]$, *then* $\pi_A^p(\theta, \alpha) \geq 0$ (*the* ε*-agent obtains a non-negative payoff by trading with the tenant*) *and* $\Omega^p(\theta, \alpha) \geq 0$ (*change in the landlord's payoff due to trading between the* ε*-agent and the tenant is non-negative*).

(c) $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha_A^p(\theta)) > \Pi^{p,\underline{\alpha}}(\theta)$ *and the unique maximum of* $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha)$ *over* $\alpha \in [\theta/p, 1]$ *is attained at* $\alpha = \alpha_A^p(\theta)$.

(iii) $\theta_p^* \in \mathbb{E}^p$. *Consequently, if* $\theta \in \overline{\mathbb{E}}^p$, *then* $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta_p^*, \alpha_A^p(\theta_p^*)) > \Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha)$ *for any* $\alpha \in [\theta/p, 1]$.

(iv) *Let* (θ_M, α_M) *be a maximizer of* $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha)$ *over* $(\theta, \alpha) \in \mathbb{J}^p$. *Then* $\theta_M \in \mathbb{E}^p$ *and* $\alpha_M = \alpha_A^p(\theta_M)$.

(v) *Let* $\{(\theta_p(\varepsilon), \alpha_A^p(\theta_p(\varepsilon)))\}$ *be a sequence such that for* $\varepsilon \in (0, 1)$, $(\theta_p(\varepsilon), \alpha_A^p(\theta_p(\varepsilon))$ *is a maximizer of* $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha)$ *over* $(\theta, \alpha) \in \mathbb{J}^p$. *Then* $lim_{\varepsilon \downarrow 0}((\theta_p(\varepsilon), \alpha_A^p(\theta_p(\varepsilon)) = (\theta_p^*, \alpha_A^p(\theta_p^*))$.

Proof. (i) Let $\theta \in \overline{\mathbb{E}}^p$, i.e., $\Psi^p(F(\theta)) - \theta F(\theta) \leq 0$. If $\Psi^p(F(\theta)) - \theta F(\theta) < 0$, then $\lambda_T^p(\theta, \alpha) = 0$ for all $\alpha \in [\theta/p, 1]$ [Lemma 9(iv)] and (36) yields the result. If $\Psi^p(F(\theta)) - \theta F(\theta) = 0$, then $\lambda_T^p(\theta, \alpha) = 0$ for $\alpha \in [\theta/p, 1)$ [Lemma 9(iv)] and for $\alpha = 1$, $\Omega^p(\theta, 1) = 0$ [by (35)], so by (36), the result follows.

(ii) Let $\theta \in \mathbb{E}^p$, i.e., $\Psi^p(F(\theta)) - \theta F(\theta) > 0$. Then by Lemma 9(iv) $\exists\ \alpha_A^p(\theta) \in (\theta/p, 1)$ such that $\lambda_T^p(\theta, \alpha) = 0$ if $\alpha \in [\theta/p, \alpha_A^p(\theta))$ and $\lambda_T^p(\theta, \alpha) = 1$ if $\alpha \in [\alpha_A^p(\theta), 1]$. Then (37) follows by (36).

(ii)(a) As $\Psi^p(.)$ is monotonic, by (35), $\Omega^p(\theta, \alpha)$ is strictly decreasing in α. Since $\Psi^p(0) = 0$, (35) yields $\Omega^p(\theta, 1) < 0$ for $\theta \in \mathbb{E}^p$. We complete the proof by showing that $\Omega^p(\theta, \alpha_A^p(\theta)) > 0$. As $\pi_A^p(\theta, \alpha_A^p(\theta)) = \Psi^p(\alpha_A^p(\theta) F(\theta)) - \theta F(\theta) = 0$ [Lemma 9(iv)], by (35) we have

$$\Omega^p(\theta, \alpha_A^p(\theta)) = \Psi^p\left((1 - \alpha_A^p(\theta))\, F(\theta)\right) + \Psi^p\left(\alpha_A^p(\theta) F(\theta)\right) - \Psi^p(F(\theta))$$

which is positive since $\Psi^p(Q_1) + \Psi^p(Q_2) > \Psi^p(Q_1 + Q_2)$ for any $Q_1, Q_2 > 0$ [Lemma 2(vi)].

(ii)(b) Follows by Lemma 9(iv) and part (ii)(a).

(ii)(c) As $\Omega^p(\theta, \alpha_A^p(\theta)) > 0$ [by (a)], (37) yields $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha_A^p(\theta)) > \Pi^{p,\underline{\alpha}}(\theta)$. Since $\Omega^p(\theta, \alpha)$ is strictly decreasing in α, by (37), $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha_A^p(\theta)) > \Pi_\varepsilon^{p,\underline{\alpha}}(\theta, \alpha)$ for $\alpha \in (\alpha_A^p(\theta), 1]$ which proves (c).

(iii) That $\theta_p^* \in \mathbb{E}^p$ is direct by Prop. 6(iv). Then by (ii), $\Pi_\varepsilon^{p,\underline{\alpha}}(\theta_p^*, \alpha_A^p(\theta_p^*)) > \Pi^{p,\underline{\alpha}}(\theta_p^*)$. Since θ_p^* is the unique maximizer of $\Pi^{p,\underline{\alpha}}(\theta)$, for any $\theta \in \overline{\mathbb{E}}^p$, $\Pi^{p,\underline{\alpha}}(\theta_p^*) > \Pi^{p,\underline{\alpha}}(\theta)$ and the result follows by (i).

(iv) By (iii), $\theta_M \in \mathbb{E}^p$ and then by (ii), $\alpha_M = \alpha_A^p(\theta_M)$.

(v) Since $\theta_p^* \in \mathbb{E}^p$, $\exists$ a small $\delta_p > 0$ such that $\mathbb{N}^{\delta_p}(\theta_p^*) \equiv [\theta_p^* - \delta_p, \theta_p^* + \delta_p] \subseteq \mathbb{E}^p$. As θ_p^* is the unique maximizer of $\Pi^{p,\underline{\alpha}}(\theta)$, by (36), $\exists\ \overline{\varepsilon}_p \in (0, 1)$ such that: if $(\theta_p(\varepsilon), \alpha_A^p(\theta_p(\varepsilon))$ is a maximizer of $\Pi_{\varepsilon}^{p,\underline{\alpha}}(\theta,\alpha)$, then $\theta_p(\varepsilon) \in \mathbb{N}^{\delta_p}(\theta_p^*)$ for all $\varepsilon \in (0,\bar{\varepsilon}_p)$. Hence for $\varepsilon \in (0,\bar{\varepsilon}_p)$, the maximization problem reduces to choosing $\theta \in \mathbb{N}^{\delta_p}(\theta_p^*)$ to maximize $\Pi_{\varepsilon}^{p,\underline{\alpha}}(\theta,\alpha_A^p(\theta))$. Since $\alpha_A^p(\theta)$ is the solution of $\Psi^p(\alpha F(\theta)) - \theta F(\theta) = 0$, it is continuous in θ and so is $\Pi_{\varepsilon}^{p,\underline{\alpha}}(\theta,\alpha_A^p(\theta))$. When $\varepsilon = 0$, $\Pi_0^{p,\underline{\alpha}}(\theta,\alpha) = \Pi^{p,\underline{\alpha}}(\theta)$ and θ_p^* is its unique maximizer. Hence $\lim_{\varepsilon\downarrow 0}\theta_p(\varepsilon) = \theta_p^*$ and consequently $\lim_{\varepsilon\downarrow 0}\alpha_A^p(\theta_p(\varepsilon)) = \alpha_A^p(\theta_p^*)$. This completes the proof. ■

As the ε-agent trades with the tenant for $\alpha > \alpha_A^p(\theta)$, the landlord is left with his own share $(1-\alpha)F(\theta)$. His revenue falls with α, so the drawback of a fixed rental contract is immediate. At $\alpha = 1$, his revenue from the product market drops from $\Psi^p(F(\theta)) - \theta F(\theta) > 0$ to zero $[\Omega^p(\theta,1) < 0]$. The function $\Omega^p(\theta,\alpha)$ continues to stay negative until α falls to $\alpha_L^p(\theta)$. The set $[\alpha_A^p(\theta), \alpha_L^p(\theta)]$ of share contracts presents the *Pareto improving* (PI) region: trading between the ε-agent and the tenant improves the payoffs of both the landlord and the ε-agent, keeping the tenant indifferent. However, the gains of the landlord and the ε-agent move in opposite directions: each prefers to have a high output at his disposal, so the landlord prefers α to be low while the ε-agent prefers it to be high. The landlord, having the advantage of choosing the contract, sets $\alpha = \alpha_A^p(\theta)$, which is the lowest possible α in the PI region, and the ε-agent just breaks even.

Why is there a non-empty PI region, i.e., why it is the case that $\alpha_A^p(\theta) < \alpha_L^p(\theta)$? When there is trade between the ε-agent and the tenant, the tenant's payoff does not change, the landlord's change in payoff is $\Omega^p(\theta,\alpha) = \Psi^p((1-\alpha)F(\theta)) - [\Psi^p(F(\theta)) - \theta F(\theta)]$ and the ε-agent obtains $\pi_A^p(\theta,\alpha) = \Psi^p(\alpha F(\theta)) - \theta F(\theta)$. The total trading surplus is

$$\Omega^p(\theta,\alpha) + \pi_A^p(\theta,\alpha) = \Psi^p(\alpha F(\theta)) + \Psi^p((1-\alpha)F(\theta)) - \Psi^p(F(\theta)).$$

Due to decreasing returns to scale property of the revenue function $\Psi^p(Q)$ [Lemma 2(vi)], the total trading surplus is positive. Hence if the surplus of one party is zero, the other party must have a positive surplus. As $\pi_A^p(\theta,\alpha) = 0$ at $\alpha = \alpha_A^p(\theta)$, $\Omega^p(\theta,\alpha_A^p(\theta))$ must be positive. As $\Omega^p(\theta,\alpha)$ is decreasing in α and $\Omega^p(\theta,\alpha_L^p(\theta)) = 0$, it follows that $\alpha_A^p(\theta) < \alpha_L^p(\theta)$, resulting in a non-empty PI region.

1. *Variation in bargaining power*: Observe that since the PI region is a non-empty interval, our qualitative conclusions will not be altered for small variations in the relative bargaining powers of the parties. For example, if the tenant has a larger bargaining power, the ε-agent has to pay him more to buy his output. So the break-even point of the ε-agent will go up above $\alpha_A^p(\theta)$. It will still be optimal for the landlord to choose a share contract, but the specific share will change.

2. *Asymmetry of storage costs*: The PI region will not disappear if the ε-agent has a higher storage cost, provided it is not too high. For higher costs, his payoff falls to $\widetilde{\pi}_A^p(\theta,\alpha) = \widetilde{\Psi}^p(\alpha F(\theta)) - \theta F(\theta)$ where $\widetilde{\Psi}^p(\alpha F(\theta)) < \Psi^p(\alpha F(\theta))$, so his break-even point goes up to $\widetilde{\alpha}_A^p(\theta) > \alpha_A^p(\theta)$. As long as the storage cost difference is not too high, $\widetilde{\alpha}_A^p(\theta)$ will stay below $\alpha_L^p(\theta)$ and there will still be a non-empty PI region. Now suppose the ε-agent's storage cost is lower. The result that he does not trade with the landlord (Lemma 8) will continue to hold provided his cost is not too low. His break-even point will fall below $\alpha_A^p(\theta)$ for lower costs and the PI region will expand. To sum up, our results are robust to relatively small asymmetries in storage costs.

Now we present the result on equilibrium refinement.

Proposition 7 (1) *Let $p > 1$. For any $\varepsilon \in (0,1)$ the perturbed game $G_1(\varepsilon)$ has an SPBE. Let (α,β,γ) be the contract offered by the landlord in an SPBE of $G_1(\varepsilon)$ and let $\theta = \gamma\alpha$.*

Then the following hold.

(i) $\alpha = \alpha_A^p(\theta)$ *and* $\beta = \Phi(\theta) - \Phi(\underline{\alpha})$.

(ii) *The tenant obtains his reservation payoff* $\Phi(\underline{\alpha})$*. If the* ε*-agent emerges, he buys the tenant's share of output* $\alpha_A^p(\theta)F(\theta)$ *by paying the tenant* $\theta F(\theta)$*. If the* ε*-agent does not emerge, the landlord buys the tenant's share by making the same payment.*

(iii) *By trading with the tenant the* ε*-agent just breaks even. He obtains* $\pi_A^p(\theta, \alpha_A^p(\theta)) = 0$.

(iv) *The landlord obtains the expected payoff* $\Pi^{p,\underline{\delta}}(\theta) + \varepsilon\Omega^p(\theta, \alpha_A^p(\theta)) > \Pi^{p,\underline{\delta}}(\theta)$.

(2) *Let* $p > 1$*. Consider the set* $\mathbb{I}_p^*$ *of all optimal interlinked contracts for the landlord in the unperturbed game* G_1*. There is a unique* $(\alpha, \beta, \gamma) \in \mathbb{I}_p^*$ *that is robust to the emergence of the* ε*-agent. The robust contract has the following properties where* $\overline{p} \equiv 1 + c'(F(1))$.

(i) *It prescribes positive shares of output for both the tenant and the landlord. Specifically,* $\alpha = \alpha_A^p(\theta_p^*)$, $\beta = \Phi(\theta_p^*) - \Phi(\underline{\alpha})$ *and* $\gamma = \theta_p^*/\alpha_A^p(\theta_p^*)$.

(ii) $\alpha_A^p(\theta_p^*)$ *is strictly decreasing for* $p \in (1, \overline{p})$ *and* $lim_{p\downarrow 1}\alpha_A^p(\theta_p^*) = 1$.

Proof. (1) and 2(i) follow by Lemma 10. For 2(ii), note that $\pi_A^p(\theta, \alpha_A^p(\theta)) = \Psi^p(\alpha_A^p(\theta)F(\theta)) - \theta F(\theta) = 0$. By Proposition 6, for $p \in (1, \overline{p})$, $\theta_p^* = 1$, so $\alpha_A^p(\theta_p^*) = \alpha_A^p(1)$ and $\Psi^p(\alpha_A^p(1)F(1)) = F(1)$. Since for any $\alpha > 0$, $\Psi^p(\alpha F(1))$ is strictly increasing in p, it follows that $\alpha_A^p(1)$ is strictly decreasing for $p \in (1, \overline{p})$. For $p = 1$, the unique contract has $\alpha = 1$ and the limiting property is immediate. ■

Proposition 7 shows that the unique contract that is robust to the emergence of the ε-agent results in a share contract. Competition in the product market generates a subset of Pareto improving share contracts out of the multiple optimal contracts of the unperturbed game G_1. It is optimal for the landlord to choose that specific contract in this subset where his own share is maximum. As a result, the ε-agent just breaks even in trading with the tenant.

Recall from Proposition 6 that for $p \in (1, \overline{p})$, $\theta_p^* = 1$ and the output stays fixed at $F(1)$. For this reason, as p falls, the tenant's share has to be increased to make sure that the ε-agent trades with the tenant. Now consider $p > \overline{p}$. For $\alpha > 0$, the ε-agent's payoff when he trades with the tenant is $\pi_A^p(\theta_p^*, \alpha) = \Psi^p(\alpha F(\theta_p^*)) - \theta_p^* F(\theta_p^*)$. For $p > \overline{p}$, the output $F(\theta_p^*)$ increases due to increasing θ_p^*, so both components of the ε-agent's payoff increase in p. Due to this reason, $\alpha_A^p(\theta_p^*)$ may or may not be monotonic $p > \overline{p}$. More structure in the model is needed to have a clear conclusion on how $\alpha_A^p(\theta_p^*)$ behaves for $p > \overline{p}$.

5 Concluding remarks

In this paper we have proposed a theory of sharecropping on the basis of price behavior in agriculture and imperfectly competitive nature of rural product markets. Considering a contractual setting where the landlord can take advantage of seasonal variation of price but the tenant-farmer cannot, first we have shown the optimality of sharecropping in the class of tenancy contracts. We have also shown that share contracts dominate crop-buying contracts provided the price variation is not too large. Then considering interlinked contracts that have both tenancy and crop-buying elements, it has been shown that there are multiple optimal contracts. Finally proposing an equilibrium refinement that incorporates imperfect competition in the rural product market, we have shown that the unique contract that is robust to this refinement results in a share contract. In our model, the price differential between the contracting parties is the main driving force behind sharecropping. It is further reinforced by the emergence of a small trader who seeks to gain from arbitrage. When the price differential goes down, this rationale for sharecropping will gradually disappear. For example, if there are entities (e.g., the government or an outside firm that has no stake at small village-level competition) that can credibly assure the tenant of a high price, then fixed rental contracts would be gradually more preferable for the landlord.

In proposing a theory of tenancy contracts based on price fluctuations, this paper relates itself to two general themes of development economics: (i) volatilities of different kinds have important effects on rural economies of poor countries and (ii) institutions and contractual forms can often be endogenous to these volatilities. Our model is consistent with some of the stylized facts of tenancy contracts (e.g., agent-specific and crop-specific variation of contractual forms, incidence of cost-sharing with share contracts). However, further empirical work is necessary to see the extent to which price variation plays a role in explaining these facts in specific contexts.

One important question regarding sharecropping is whether it results in lower productivity compared to fixed rental contracts. There is a large empirical literature that addresses this question [see, e.g., Rao (1971), Bell (1977), Chattopadhyay (1979), Bliss and Stern (1982), Shaban (1987)], but the evidence is mixed. Our theoretical results show that if the landlord is restricted to only tenancy contracts, then the output from sharecropping is low (Prop 5), but this is no longer the case under more general contracts (Prop 6). This is intuitive. If the contract forms allow the landlord to extract more surplus, he will have incentive to raise productivity. Thus, sharecropping in itself might not necessarily lead to low productivity.

It is well recognized that there cannot be a single explanation of the sharecropping institution. As Singh (1989: 34) points out:

> "Sharecropping has existed in various times and places in various forms. It has disappeared over time and reappeared. Sometimes the output share equals the cost share; sometimes it does not. Sometimes the tenant's share is one-half; sometimes it is not. Sometimes productivity is higher on sharecropped land than on other types of tenancy or with self-cultivation; sometimes it is not. Sometimes sharecroppers are poor; sometimes they are prosperous. Sometimes sharecroppers produce risky cash crops; sometimes they produce for subsistence. I do not think a single theory can capture all of these aspects of sharecropping"

In this spirit, it can be said that this paper complements the existing theories of the literature.

Appendix

Proof of Lemma 2(v)-(vi) To prove (v), note that (a) $\Psi^p(Q)$ is strictly concave for $Q \in [0, \overline{Q}_p]$ [since $c(.)$ is strictly convex] and (b) $\Psi^p(Q)$ is linear for $Q \geq \overline{Q}_p$.

To complete the proof, let $Q_1 \in [0, \overline{Q}_p)$, $Q_2 > \overline{Q}_p$ and $Q_3 = \lambda Q_1 + (1-\lambda)Q_2$ for $\lambda \in (0,1)$. In what follows, we show that

$$\Psi^p(Q_3) > \lambda \Psi^p(Q_1) + (1-\lambda)\Psi^p(Q_2). \tag{38}$$

First let $Q_3 \geq \overline{Q}_p$. Then by (10), $\Psi^p(Q_3) = \Psi^p(Q_2) - (Q_2 - Q_3) = \Psi^p(Q_2) - \lambda(Q_2 - Q_1)$. Using this fact and (10), we have $\Psi^p(Q_3) - [\lambda\Psi^p(Q_1) + (1-\lambda)\Psi^p(Q_2)] = \lambda[\zeta^p(\overline{Q}_p) - \zeta^p(Q_1)] > 0$ which proves (38).

To prove (38) for $Q_3 < \overline{Q}_p$, note that since $Q_1 < \overline{Q}_p < Q_2$, $\exists\ \overline{\lambda} \in (0,1)$ such that $\overline{Q}_p = \overline{\lambda} Q_1 + (1-\overline{\lambda})Q_2$. By the last paragraph, we know that (38) holds for $\overline{Q}_p$, hence

$$\Psi^p(\overline{Q}_p) > \overline{\lambda}\Psi^p(Q_1) + (1-\overline{\lambda})\Psi^p(Q_2). \tag{39}$$

Now let $Q_3 < \overline{Q}_p$. Then $\exists\ \underline{\lambda} \in (0,1)$ such that $Q_3 = \underline{\lambda} Q_1 + (1-\underline{\lambda})\overline{Q}_p$. Since $\Psi^p(Q)$ is strictly concave over $Q \in [0, \overline{Q}_p]$, we have

$$\Psi^p(Q) > \underline{\lambda}\Psi^p(Q_1) + (1-\underline{\lambda})\Psi^p(\overline{Q}_p). \tag{40}$$

As $Q_3 = \lambda Q_1 + (1-\lambda)Q_2 = \underline{\lambda} Q_1 + (1-\underline{\lambda})\overline{Q}_p$ and $\overline{Q}_p = \overline{\lambda} Q_1 + (1-\overline{\lambda})Q_2$, we have $\lambda = \underline{\lambda} + (1-\underline{\lambda})\overline{\lambda}$. Using this fact and combining (39) and (40), (38) follows.

(vi) First let Q_1, Q_2 be such that both are at least $\overline{Q}_p$. Then clearly $Q_1 + Q_2$ is more than $\overline{Q}_p$ and by (10), $\Psi^p(Q_1) + \Psi^p(Q_2) - \Psi^p(Q_1 + Q_2) = \zeta^p(\overline{Q}_p) > 0$.

Now let $Q_1, Q_2 > 0$ be such that at least one is less than $\overline{Q}_p$. If $Q_1 = Q_2$, then $Q_1 = (1/2)(Q_1+Q_2) + (1/2)0$. Since $\Psi^p(0) = 0$, by the last statement of (v), $\Psi^p(Q_1) > (1/2)\Psi^p(Q_1 + Q_2)$ proving that $\Psi^p(Q_1 + Q_2) < \Psi^p(Q_1) + \Psi^p(Q_2)$.

Finally let $Q_1 \neq Q_2$ and w.l.o.g., suppose $Q_1 < Q_2$ and $Q_1 < \overline{Q}_p$. Denote $\lambda \equiv Q_1/Q_2 \in (0,1)$. Then (a) $Q_1 = (1-\lambda)0 + \lambda Q_2$ and (b) $Q_2 = \lambda Q_1 + (1-\lambda)(Q_1 + Q_2)$. Hence by (v), we have

$$\Psi^p(Q_1) > \lambda\Psi^p(Q_2) \text{ and } \Psi^p(Q_2) > \lambda\Psi^p(Q_1) + (1-\lambda)\Psi^p(Q_1 + Q_2)$$

where the inequalities are strict due to the last statement of (v). Adding the inequalities above yields $\Psi^p(Q_1) + \Psi^p(Q_2) > \Psi^p(Q_1 + Q_2)$. ■

Proof of Lemma 4 (i) Since $\Psi^p(.)$ and $\Phi(.)$ are both monotonic and $H(\alpha)$ is strictly increasing, the result follows by (21).

(ii) If $H(\alpha) > \overline{Q}_p$, then (22) yields $\Pi^{p,\alpha}_{S+R}(\alpha) = s(\alpha) +$ a constant. Since $s(\alpha)$ is strictly increasing for $\alpha \in [0,1]$ (Lemma 3), (ii) follows.

(iii) Let $H(\alpha) < \overline{Q}_p$. As $s'(\alpha) = (1-\alpha)F'(\alpha)$ (Lemma 3), (22) yields the first part, and

$$\Pi''^{p}_{S+R}(\alpha) = [(1-\alpha)F''(\alpha) - F'(\alpha)] - c''(H(\alpha))[H'(\alpha)]^2 + [p - 1 - c'(H(\alpha))]H''(\alpha).$$

As $F''(.) \leq 0$ (Assumption A1) and $F'(.) > 0$, the first term of the expression above is negative. Since $c''(.) < 0$, the second term is negative. Since (a) $H''(.) < 0$ (Lemma 3) and (b) $p - 1 > c'(H(\alpha))$ [by (6), since $H(\alpha) < \overline{Q}_p$], the last term is also negative, proving that $\Pi''^{p}_{S+R}(\alpha) < 0$.

(iv) Let $H(\widetilde{\alpha}) > \overline{Q}_p$. The first statement follows by noting that $H(\alpha)$ is strictly decreasing for $\alpha \in (\widetilde{\alpha}, 1]$ and $H(1) = 0 < \overline{Q}_p$ (Lemma 3). The last statement follows by (ii).

(v) Since $H(1) = 0 < \overline{Q}_p$, (iii) applies for $\alpha = 1$. As $H'(1) < 0$ (Lemma 3) and $c'(0) = 0$, taking $\alpha = 1$ in (iii) yields $\Pi'^{p}_{S+R}(1) = (p-1)H'(1) < 0$ which proves (iv). ■

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