Reference-Dependent Preferences in First Price Auctions

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Abstract: In this paper I develop a Prospect theory based model to explain bidding in first-price auctions. As suggested in the literature, bidding occurs in these auctions in an inherently ambiguous environment due to lack of information about bidders' risk attitudes and bidding strategies. I show that bidding in first-price auctions can be rationalized as a combination of reactions to underlying ambiguity and anticipated loss aversion. Using data from experimental auctions, I provide evidence that in induced value auctions against human bidders this approach works well. In auctions with prior experience and /or against risk-neutral Nash bidders where ambiguity effects could be altogether irrelevant, anticipated loss aversion by itself can explain aggressive bidding. This is a novel result in the literature. Using data from experiments I find that ambiguity effects become negligible in auctions with prior experience (with loss aversion) against (i) experienced human bidders and (ii) Nash computer bidders. The estimates for loss aversion are similar in auctions against human bidders.

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1. Introduction

Auctions have become extremely popular for transferring goods and services. Their use can be traced back to 500 B.C. in ancient Babylon. Since Vickrey (1961)[1] economists have tried to explore bidding and auction outcomes under various experimental settings.[2] In induced value first-price auctions, subjects bid in excess of the risk-neutral-Nash predictions in classroom conditions ("Overbidding" Anomaly: Cox et al 1982, 1988, 1996; Harrison 1989). Although risk aversion can explain such aggressive behavior, skepticism surrounding risk aversion as the sole explanation has prompted scholars to explore other behavioral alternatives[3] (Salo and Weber 1995, Goeree et al. 2002, Dorsey and Razzolini 2003, Morgan, Steiglitz and Reis 2003, Kagel 1995, Filiz-Ozbay and Ozbay 2007). In this paper I propose a different alternative which combines elements of Prospect theory: loss aversion and nonlinear probability weighting.

In first-price sealed bid auctions, the probability of winning for a given bid depends on the joint distribution of induced values, risk attitudes, and the unknown strategies of rival bidders. Thus, missing information about other bidders' induced values, risk posture, and/or bidding strategies exposes bidders to submit bids in an inherently "ambiguous"[4] environment. Ambiguity effects as captured in Ellsberg paradox (1961) have been observed in market experiments (Camerer and Kunreuther 1989, Sarin and Weber 1993)[5] and could influence bidding in auctions as well (Salo and Weber 1995, Chen et al 2007)[6]. In auctions against human bidders, prior bidding experience could make it easier to derive missing information thereby reducing the level of ambiguity. Moreover, additional controls for missing information have been applied which present even smaller levels of ambiguity in these auctions. For example, when bidding against risk-neutral Nash computer bidders, there is no uncertainty about bidders' risk attitudes and bidding strategies. Therefore, ambiguity effects should become smaller in these

[1] Vickrey (1961) provides the theoretical foundations of various auction mechanisms.
[2] There is a rich variation of classroom and field experiments that employ various types of subjects and auctioned objects.
[3] Some other behavioral explanations include-nonlinear probability weighting (ambiguity aversion), spiteful preferences, regret aversion, etc.
[4] Thus, ambiguity reflects a scenario where missing probabilistic information must be derived.
[5] In Sarin and Weber (1993) the market prices for ambiguous assets were consistently below the corresponding prices for equivalent unambiguous assets. An asset is a two-stage lottery with risk (well-defined probabilities) and ambiguity (probabilities not well-defined). This effect was stronger when these assets were traded simultaneously. However there is weaker evidence that ambiguity affects insurance markets in Camerer and Kunreuther (1989).
[6] Ambiguity (unlike risk) better characterizes decision making in many real-world situations. E.g., the success rate of new drugs, insurance against previously unknown environmental hazards, terrorist activities, outcomes of R&D and success of new products in consumer goods markets (see references in Chen et al. 2007).

auction environments. While efforts have been made to explore the effect of ambiguity on bidding in first-price auctions (Chen et al. 2007) some other behavioral explanations can't explain overbidding in auctions against Nash computer bidders'.[7] In this paper, I exploit the difference between bidding against human bidders versus computer bidders to demonstrate the existence of ambiguity effects as well as another determinant of behavior: loss aversion.

I base the analysis in this paper on a model of loss aversion with endogenous reference points similar to Koszegi and Rabin (2006). This is different from an approach with an exogenous fixed reference point in which winning the auction is interpreted as a "gain" while losing leaves the initial wealth unaffected. I argue that the reference point may get influenced by expected gains and therefore auction outcomes could be interpreted as "gains" or "losses." It is plausible that a bidder who draws a high value and expects to win the auction interprets "not winning" as a "loss" and likewise that a bidder with low induced value interprets winning the auction as a "gain." This has been observed in other contexts. For example, loss aversion has been observed in trading of various commodities – from chocolate bars to coffee mugs, coins, or sportscards – for money or other goods (Knetsch 1989; Tversky and Kahneman 1991; Kahneman, Knetsch, and Thaler 1990; Benartzi and Thaler 1995, List 2003). I show that loss aversion by itself (irrespective of other behavioral explanations) can explain aggressive bidding in first-price auctions and captures an important behavioral influence on bidding. Thus, my approach provides a justification for aggressive bidding in auctions where ambiguity effects could be minimal or altogether absent. Other behavioral explanations-spiteful preferences, nonlinear probability weighting, anticipated regret aversion, disappointment aversion - could explain aggressive bidding loss aversion. Unlike a regret-based explanation (Ozbay and Filiz-Ozbay 2007), my approach does not rely on ex-post information to explain aggressive bidding; spiteful preferences (Morgan, Steiglitz and Reis 2003) can't explain why human bidders bid aggressively against computer bidders. And finally, when the auction winner earns only the monetary profit as in classroom experiments,[8] my approach is equivalent to the disappointment aversion model as in Gul (1991).[9]

[7] Spiteful preferences or ambiguity aversion cannot explain why humans bid aggressively against computers whose bidding strategies are known, and therefore the objective probability of winning the auction conditional on bid can be derived fairly easily or conveyed to the human bidder.
[8] This is different in *field* where auction object is exchanged for a monetary price (bid). In Lange and Ratan (2009) we discuss the differences that could arise between the auctions conducted in *induced value (classroom)* settings and *field* in the context of the model, offered here.

Two prominent approaches to address ambiguity attitudes in the literature are the maximin expected utility (MMEU model) (Gilboa and Schmeidler 1989) and Choquet expected utility (CEU) model (Gilboa 1987, Schmeidler 1989). I take the CEU approach, which allows subjective distortion of objective probability measures to capture attitudes towards ambiguity, exactly as in Salo and Weber (1995) and Goeree et al (2002). This is consistent with Prospect theory, which allows nonlinear probability weighting and loss aversion. I propose a model of endogenous expectations, similar to Koszegi and Rabin (2006), to accommodate reference-dependent preferences and attitudes towards ambiguity.

Theoretically, as special cases of my approach, either *non-linear probability weighting* or *loss aversion* can explain observed bidding outcomes. I show that when bidders are loss-averse and fully anticipate potential losses, overbidding is justified even without nonlinear probability weighting. Thus, I suggest loss aversion as an alternative explanation for aggressive bidding in auctions. When I rely on nonlinear probability weighting alone, my approach is behaviorally equivalent to previous explanations that explain overbidding in terms of risk aversion or ambiguity aversion (Salo and Weber 1995; Goeree et al 2002).

Using data from experimental auctions, I provide evidence that the general approach that combines loss aversion and nonlinear probability weighting provides a good fit for observed bids. This approach is capable of addressing the differences in ambiguity across auction environments and explains aggressive bidding in auctions with prior experience (with loss aversion) against (i) experienced human bidders and (ii) risk-neutral Nash-computer bidders. In these auctions, drawing probabilistic inferences (conditional on bids) is relatively easier, and ambiguity effects could be irrelevant,[10] and therefore smaller deviations between subjective and objective probabilities are expected.[11]

I estimate the behavioral parameters in my models using experimental data (Cox et al 1982, Harrison 1989) and test the hypothesis for probability weighting under less ambiguous

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[9] Since I allow nonlinear probability weighting, my approach differs from Gul's approach; in the special case of linear probability weighting, the two approaches are similar. This equivalence breaks down in field auctions where the auction object is exchanged for the bid. The implications of a model based on loss aversion for various auction settings are further explored in Lange and Ratan (2009).

[10] In an experiment reported in Dorsey and Razzolini (2003), the probability of winning conditional on bids is conveyed to the subjects.

[11] The evidence on ambiguity attitudes suggests that ambiguity aversion is more prevalent. In addition to the experiments that are replications of the Ellsberg paradox (Fox and Tversky 1998), Sarin and Weber (1993) find that the price of ambiguous two-stage lotteries is lower than equivalent unambiguous lotteries obtained through double-market auctions.

circumstances. I provide evidence that in auctions against human bidders aggressive bidding can be rationalized as a combination of "loss aversion" and "ambiguity aversion"; the estimates for loss aversion in auctions with human bidders (irrespective of prior experience) are similar, whereas probability weighting becomes less convex in auctions that present successively reduced levels of ambiguity. This results in smaller deviations between subjective and objective probabilities. When loss aversion is allowed, this yields an almost linear probability weighting in auctions with prior experience against (i) experienced human and (ii) risk-neutral Nash bidders.

In the following sections I motivate the general Prospect theory model for bidding in auctions (sections 2 and 3). I apply the model to auctions with risk-neutral Nash bidders (section 4), and analyze the experimental data in sections 5 and 6. Finally, I discuss my results and conclude (sections 7 and 8).

2. Prospect Theory: Reference-Dependence and Nonlinear Probability Weighting

In this section I describe the behavioral assumptions in my approach to address bidding in classroom first-price auctions. In classroom auctions, induced values are induced and profits are paid in monetary units. Thus, consumption occurs in a single dimension.[12] Following Koszegi and Rabin (2006), an individual's utility $u(c \mid r)$ depends both on her consumption $c \in \mathbb{R}$ and her reference level $r \in \mathbb{R}$. The "direct" consumption utility $v(c)$ is obtained when realized consumption is the same as the reference level, i.e., $v(c) = u(c \mid c)$, and the individual utility when her consumption differs from her reference is defined as

$$u(c \mid r) = v(c) - k_l \max[0, v(r) - v(c)] \qquad (1)$$

with $0 \leq k_l$. k_l is the scalar gradient which captures the sensation of "loss" when less favorable outcomes are realized.[13]

Ex ante, both reference levels and consumption could be stochastic. Following Köszegi and Rabin (2006), the reference level is a probability measure G over $\mathbb{R}$ and consumption is drawn

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[12] Unlike classroom auctions where induced values are induced in money and profits are paid in monetary units, in real auctions the object is awarded to the winner in return for money. In Lange and Ratan (2009), we discuss the implications arising from this difference when loss aversion associated with the object and money may differ.

[13] I normalize psychological "gains" to zero.

according to the probability measure H over $\mathbb{R}$. Then, the individual's overall expected utility over risky outcomes is given by

$$U(H \mid G) = \int\int u(c \mid r) dG(r) dH(c) \qquad (2)$$

In an equilibrium (for a first-price auction) captured by a strictly increasing symmetric bidding function, the bid determines the probability of winning and the consequent profits for a bidder. Since no further action is possible after placing the bid, the bid not only defines the probability of consumption outcomes (H) but also defines the probability of reference outcomes (G). Thus, for a bidder with rational expectations $H = G$, and the reference point G is endogenously determined.[14]

The other important feature of prospect theory is nonlinear probability weighting (Kahneman and Tversky 1979). As discussed earlier, auction environments could vary in terms of underlying ambiguity. Two prominent approaches to address ambiguity attitudes in the literature are maximin expected utility (MMEU) (Gilboa and Schmeidler 1989) and Choquet expected utility (CEU) (Gilboa 1987, Schmeidler 1989). In the MMEU model, decision makers have a set of priors over outcomes and choose the actions that maximize the minimum expected utility over the set of priors. In the CEU model, decision makers' beliefs are represented by a set of non-additive probability measure (capacities).[15] I take the CEU approach, which allows subjective distortion of objective probability measures to capture attitudes towards ambiguity.[16] Ambiguity effects should become smaller in auctions with prior bidding experience and/or against risk-neutral Nash bidders, thereby producing smaller distortions of objective probabilities. I therefore assume that each bidder distorts the objective probability measure P through the following probability weighting function as in Salo and Weber (1995) and Goeree et al (2002):

[14] Alternative reference-dependent models with endogenous definition of reference points are given by Sugden (2003) and Munro and Sugden (2003) who assume the reference to be given by the *current endowment* which might adjust over the time. One other fixed reference could be the weighted expected value of the prospect, which is also determined endogenously in one-shot games (Kahnemann and Tversky 1979).

[15] Some recent contributions aim at characterizing ambiguity without restricting attention to specific decision models, or functional-form considerations. E.g. Klibanoff, Marinacci and Mukerji (2005).

[16] Thus, I assume that probability distortions arise entirely as a response to ambiguity. This approach is similar to Salo and Weber (1995) and Goeree et al (2002).

$\omega(P) = P^{\beta}$ where $\beta > 0$ [17] (3)

Under this assumption H and G in (2) could be nonlinearly weighted measures of probability as defined in (3).[18] Thus an individual solves the following program:

$\max U(H \mid H)$ (4)

This specification is however slightly different from the general setting discussed by Köszegi and Rabin (2006). In their approach, action takes place *after* a reference distribution has been formed. Given a reference distribution G, the individual therefore chooses $H(G)$ to maximize $U(H \mid G)$. In equilibrium, rational expectations then require that the consumption distribution is chosen such that it is consistent with the formulation of the reference point, i.e. $H(G) = G$. In sealed-bid auction equilibrium, given the beliefs of bidders' bidding strategies, a bid uniquely determines the probability of various auction outcomes for each bidder. This allows the formulation of a probability distribution over consumption and reference outcomes simultaneously. A rational bidder applies the same weighting to the objective probability measure associated with reference and consumption levels. This allows a complete specification of overall expected utility for a bidder who fully anticipates ensuing losses as defined in (4).

3. The First-Price Auction Environment

In this section I discuss the bidding problem in a first-price auction for a bidder with behavioral characteristics as described in the previous section.

I consider n bidders $i = 1,...,n$. I assume symmetric behavioral preferences, i.e. that bidders share the same characteristics for loss aversion and probability weighting; this is common knowledge. In my framework, unique identification of risk preferences and nonlinear probability weighting may not be possible. Therefore, bidders are assumed to be risk-neutral in the numeraire

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[17] β governs the elevation of the probability weighting function with respect to the 45-degree line. The 45-degree line describes linear probability weighting. $\beta < (>) 1$ implies overweighting (underweighting) of probability. This functional specification is a special case of the probability weighting function described in Prelec (1998): $\omega(P) = \exp(-\beta(-\log P)^{\alpha})$, in which $\alpha = 1$; thus, my approach is less general. Moreover, in previous attempts to fit the more general form for bidding in first-price auctions, I found that $\alpha \rightarrow 1$. Later I discuss other evidence in the literature that supports this functional form for uncertain circumstances where probabilities are derived and not known exclusively.

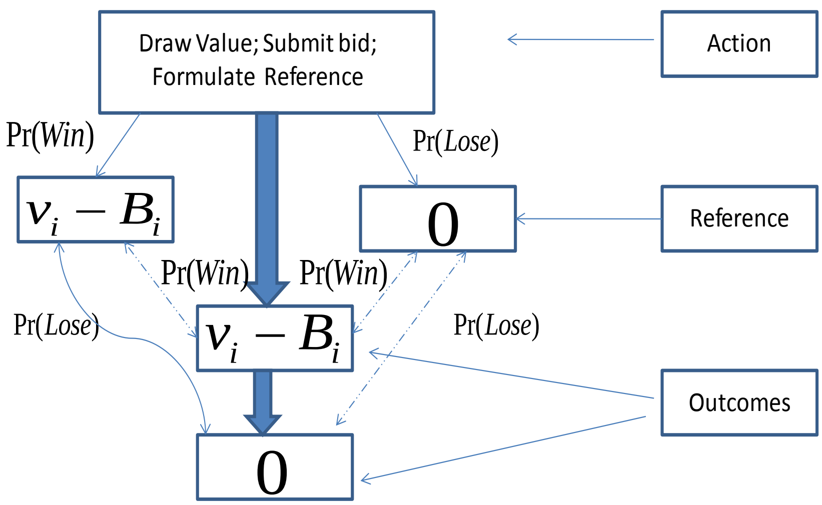
[18] Later, I show how the auction outcomes are weighted in my model.

consumption, i.e. $v_i(c) = c$. In the classroom auction, v_i is directly induced in monetary units. Each bidder draws her induced value v_i from a probability distribution defined by the distribution function F defined over $[\underline{v}, \bar{v}]$ ($\bar{v} \geq \underline{v} \geq 0$); each bidder knows his induced value, and knows that other bidders' induced values are also drawn independently from distribution F.[19]

The bidding problem for a typical bidder in a classroom first-price auction is described in figure 1. In equilibrium for symmetric bidders, which can be depicted through a strictly increasing bid function $B_j(v_j) = B(v_j)$ where $j \neq i$ for all other bidders, a bid B_i for bidder i defines her objective probability of winning the auction. This is weighted nonlinearly by the bidder. Thus, a bidder can formulate an endogenous reference lottery for each feasible bid that captures his expectations (beliefs) of various auction outcomes. The auction outcome follows. Ex-ante, losing the auction could be interpreted as loss and weighted with respect to the endogenous reference formulated at the time of bidding.

Note that a bidder's reference is defined by her beliefs about the relevant outcomes held between the time she formulates her bid and shortly before the auction outcome is observed. The degenerate utility in a first -price sealed bid auction that captures the gain-loss utility as described in (1) takes the following values:

Figure 1: Bidding Problem in a First-Price Auction



[19] In classroom auctions, overbidding beyond induced value entails negative payoff and is always suboptimal. However, in Harrison (1989) this restriction is not imposed explicitly.

$$u_{PT}(c\,|\,r)=\begin{cases} v_i - B_i & when\ \ c=r=v_i - B_i \\ v_i - B_i & when\ \ c=v_i - B_i, r=0 \\ -k_l(v_i - B_i) & when\ \ c=0, r=v_i - B_i \\ 0 & when\ \ c=r=0 \end{cases}$$

The overall expected utility for a bidder with preferences as given in section 2 (based on conditions (1)-(4)) is given by:

$$\pi_{PT}(v_i,B_i)=\omega(f(B_i))(v_i-B_i)-k_l\omega(f(B_i))(1-\omega(f(B_i)))(v_i-B_i) \qquad (5)$$

where $f(B_i)$ and $\omega(f(B_i))$ are the objective and weighted probability of winning for a given bid. The first probability term captures direct consumption utility and the second captures the psychological losses when the bidder unexpectedly loses the auction.[20] Note that bidding yields nonnegative payoff for moderate levels of loss aversion; for high levels of loss aversion bidding $B_i = v_i$ maximizes overall payoff.[21] Also note that weighted expected value is also determined endogenously for an equilibrium bid and could be used as a fixed reference to evaluate the reference-dependent utility of various outcomes (Kahnemann and Tversky 1979). This is equivalent to the lottery (Koszegi-Rabin) approach as discussed in the previous section and yields the same overall expected utility as in (5). [22]As mentioned before, with linear probability weighting and induced value (classroom) settings where auction winner earns the monetary profit, my approach is equivalent to the disappointment aversion model as in Gul (1991).[23]

It should be noted that (5) implies that a non-negative expected utility gain $\pi_{PT}(v_i,B_i)$ from participating in the auction can only result if $1>k_l(1-\omega(f(B_i)))$. That is, auction yields positive utility only for agents with $\omega(f(B_i))>1-1/k_l$. If $k_l<1$, this condition holds for all agents. If

[20] Note when there is no loss aversion, $k_l = 0$, this becomes a probability weighted model alone. In addition, when there is no probability weighting, this becomes a risk-neutral Nash model.

[21] The maximum payoff in this case is zero.

[22] In the fixed reference approach, only the auction outcome of not winning yields psychological loss with respect to the reference of the expected value for a given bid.

[23] In Gul's model, disappointment could arise from paying a higher than expected price and/or losing the profit (based on higher price) due to losing the lottery. In a first-price auction, the price paid equals the bid in case of winning; so the only source of disappointment arises from not realizing the expected profit (certainty equivalent- $f(B_i)(v_i-B_i)$) when the auction is lost which occurs with probability $(1-f(B_i))$. The last term therefore fully captures the disappointment as discussed in Gul (1991).

$k_l \geq 1$, the condition implies only agents with a sufficiently large probability to win derive positive payoff from placing positive bids.

I restrict attention to symmetric monotonically increasing equilibria in pure strategies. In equilibrium, the chances of player i to win are given by $H^{n-1}(v_i)$. With the above argument, auction yields positive utility only if $\omega(H^{n-1}(v_i)) > 1 - 1/k_l$. Given $\omega(.)$, the threshold value $\hat{v}(k_l)$ beyond which positive utility is realized is defined by

$$\omega(H^{n-1}(\hat{v}(k_l))) = \max[0, 1 - 1/k_l] \qquad \text{(5a)}$$

Note that $\hat{v}(k_l) = \underline{v}$ if $k_l \leq 1$. Agents with $v_j \in [\hat{v}(k_l), \overline{v}]$ shall place positive equilibrium bids that yield positive overall payoff. Maximizing (5) with respect to B_i yields a strictly increasing (optimal) bid function.

Proposition 1: **(-First Price Auction against Human bidders-)** *The unique monotonically increasing symmetric Bayesian Nash equilibrium (BNE) bid function for loss-averse bidders who weigh probabilities nonlinearly is*

$$B(v_i)_{PT} = \begin{cases} \dfrac{\int_{\hat{v}(k_l)}^{v_i} x[1 - k_l(1 - 2\omega(F^{n-1}(x))] d\omega(F^{n-1}(x))}{\omega(F^{n-1}(v_i))[1 - k_l(1 - \omega(F^{n-1}(v_i)))]} & \text{for} \quad v_i \geq \hat{v}(k_l) \\ v_i & \text{for} \quad v_i < \hat{v}(k_l) \end{cases}$$

Proof: See Appendix.

It is clear from the above that (i) $\hat{v}(k_l)$ varies with k_l and β and (ii) for agents with $v_i \geq \hat{v}(k_l)$ the equilibrium bid depends on k_l and β.[24] Thus, for $v_i \geq \hat{v}(k_l)$ we can explore the marginal effects of changes in β and k_l on equilibrium bids.

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[24] For $v_i < \hat{v}(k_l)$ equilibrium bid $B(v_i) = v_i$ does not depend on k_l and β.

Proposition 2 (i) ***(-Effect of loss aversion-)*** *Greater loss aversion yields aggressive bidding, i.e.,* $\frac{\partial B_{PT}}{\partial k_l} > 0$ **(ii)** ***(-Effect of probability weighting-)*** *Greater* β *(more convex probability weighting) yields more aggressive bidding, i.e. ,* $\frac{\partial B_{PT}}{\partial \beta} \geq 0$ *except when* $0.9951 < k_l < 1$ *and bidders with very small induced values such that* $Z(y_i) = 2k_l^2 y_i^2 + 3k_l(1-k_l)y_i + (1-k_l)^2 + k_l(1-k_l)y_i \ln y_i < 0$ *(where* $y_i = F(v_i)^{\beta(n-1)}$ *) who bid less aggressively i.e.* $\frac{\partial B_{PT}}{\partial \beta} < 0$.

Proof: See Appendix.

Intuitively, the tradeoff that determines the optimal bid for loss-averse bidders differs from the tradeoff without loss aversion. Loss-averse bidders are willing to pay a higher price to avoid the "loss" from not realizing the profits upon winning. This induces more aggressive bidding for any monotonic probability weighting. Thus, anticipated loss aversion by itself explains overbidding with respect to risk-neutral Nash equilibrium.[25] For example, if the ambiguity confronting the bidder is smaller, (such that ambiguity effects could be smaller or altogether irrelevant[26]) then anticipated loss aversion would suffice to rationalize aggressive bidding.

Before I explore the effect of probability weighting on equilibrium bidding it is noteworthy that bidders could avoid losses in the following ways: (a) if the value draw is not high enough then bid upto their value to avoid losses, (b) and if the value draw is high enough they could either bid (i) more aggressively or (ii) less aggressively, in response to more convex probability weighting. In the latter scenario, when the value draw is high enough less aggressive bidding could happen because bid also affects the expectation of auction outcomes simultaneously. Higher β means lower elevation of the probability weighting curve and causes more aggressive bidding which suggests ambiguity aversion (or bidder pessimism) in most circumstances except the following: when $0.9951 < k_l < 1$ some bidders with very small induced

[25] In addition to other behavioral influences that would suggest aggressive bidding with respect to the RNNE bid.
[26] For example, in auctions with experienced bidders and/or against risk-neutral Nash bidding strategies, deriving missing information regarding the probability of winning for a bid could be easier. Such auctions therefore present smaller levels of ambiguity for a bidder.

values could bid less aggressively. [27] Therefore, as a special case of Proposition 1, one can justify aggressive bidding entirely as a response to underlying ambiguity with nonlinear probability weighting (without loss aversion $k_l = 0$). Aggressive bidding with respect to the RNNE would then suggest that "ambiguity aversion" or "bidder pessimism" causes underweighting the probability of winning for given bids (Salo and Weber 1995, Goeree. et al 2002).

Proposition 3: G*reater competition (more bidders) yields more aggressive bidding, i.e.,* $\frac{\partial B_{PT}(v_i)}{\partial n} \geq 0$ *except when* $0.9951 < k_l < 1$ *and bidders with very small induced values such that* $Z(y_i) = 2k_l^2 y_i^2 + 3k_l(1-k_l)y_i + (1-k_l)^2 + k_l(1-k_l)y_i \ln y_i < 0$ *(where* $y_i = F(v_i)^{\beta(n-1)}$ *) who bid less aggressively i.e.* $\frac{\partial B_{PT}(v_i)}{\partial n} < 0$.

Proof: See Appendix.

The marginal response to greater competition is similar to the effect of probability weighting; as before, when value draw is high enough, bidders could avoid losses more or less aggressively, in response to more competition; this happens because their bid affects their expectation of auction outcomes simultaneously. The effect of greater competition is analogous to more convex probability weighting and causes aggressive bidding in most circumstances except the following: when $0.9951 < k_l < 1$ some bidders with very small induced values could bid less aggressively. Thus, despite behavioral preferences, in most circumstances bidders respond to greater competition along conventional lines by bidding more aggressively.[28]

In the following sections, I provide evidence that my approach that allows loss aversion performs quite well in induced value auctions, but identifying suitable reference points[29]presents a major

[27] For any bidder, more convex probability weighting, affects overall payoffs by affecting the weighted probability of winning, direct expected payoff and anticipated losses; for most bidders the net effect of more convex probability is such that it yields more aggressive bidding; however when $0.9951 < k_l < 1$ for some bidders with low induced values the net effect could yields less aggressive bidding.

[28] Except when $0.9951 < k_l < 1$ some bidders with very small induced values the net effect of greater competition yields less aggressive bidding, as in the case of probability weighting before.

[29] How people develop reference points could be contextual and plausible reference points could differ under different circumstances.

challenge in applying Prospect theory based approaches to other contexts, e.g., in common value auctions.

As discussed earlier, the general model is capable of addressing the differences in ambiguity across auction environments. Intuitively, ambiguity effects should become smaller in auctions where bidders have prior bidding experience against (i) experienced human bidders and (ii) risk-neutral Nash bidders, thereby producing smaller deviations between weighted and objective probabilities. I shall explore this hypothesis in the following section. It should be noted, however, that bidding against Nash risk-neutral bidders is not a special case of the equilibrium bid as discussed so far. Instead, it merely represents the best response of the player. In the following I derive the best response bid under given behavioral assumptions in these auctions.

4. Auctions against Nash (risk-neutral) bidders

In this section I address bidding in induced value auctions against Nash risk-neutral computer bidders. In these auctions bidders are informed ex-ante that other bidders always bid a certain fraction of their induced values.[30] The auction environment is the same except that bidders face Nash risk-neutral computer bidders. There is no uncertainty in these auctions about risk attitudes and equilibrium strategies that rival bidders employ. Thus, the ambiguity confronting the bidder becomes smaller in these auctions. Some other behavioral explanations for overbidding (considered in isolation) do not apply in these auctions. E.g., it is unlikely that humans will show spite against computer bidders; thus, spiteful preferences cannot explain aggressive bidding in these auctions. Similarly, risk aversion does not yield estimates of risk attitudes similar to those observed in auctions against human bidders.[31] Although combining risk aversion with spite could explain overbidding against risk-neutral Nash computerized bidders, such a model by itself is not capable of addressing the changes in ambiguity on bidding behavior.[32] The Prospect theoretic framework is capable of addressing changes in ambiguity on bidding behavior.

[30] In some variants of these experiments (Dorsey and Razzolini, 2003), probability of winning, conditional upon bids was also shown to bidders.

[31] This is obvious by looking at the estimates of the probability-weighted model (no loss aversion) in auctions against risk- neutral Nash bidders (Table 5). Variations in probability weighting would therefore suggest variation in risk attitudes.

[32] Among other explanations, ambiguity aversion and risk aversion could also rationalize bidding outcomes in these auctions. However, uniquely identifying risk and ambiguity attitudes could be extremely difficult when they are modeled together.

Each bidder relies only on her induced characteristics, as described in the preference structure defined in (1)-(4).[33] Consistent with the experimental setup, I assume that induced values are drawn from a uniform distribution over the support $[0,1]$. Since it is known that bidders' bids are Nash (risk-neutral) best responses,[34] the bidder need not take into account the strategic consequences of his bids.

This yields the following overall expected utility for the bidder who maximizes expected payoffs:

$$\max_{\underline{v}\le B_i\le\overline{v}} \quad \pi_{PT}(v_i,B_i)=\left[\omega((\theta B_i)^{n-1})-k_l\omega((\theta B_i)^{n-1})(1-\omega((\theta B_i)^{n-1})\right](v_i-B_i) \tag{7}$$

where $\theta = n/(n-1)$, $(\theta B_i)^{n-1}$, and $\omega((\theta B_i)^{n-1})$ are the objective and weighted probability of winning conditional on bid. The first probability term captures direct consumption utility and the second captures the psychological losses when the bidder loses but had expected to win the auction. Given the risk-neutral-Nash opponent bidders, agents can ensure winning by placing a bid- $(n-1)\overline{v}/n$.

As before, (7) implies that a non-negative expected utility gain $\pi_{PT}(v_i,B_i)$ from participating in the auction can only result if $1>k_l(1-\omega(f(B_i)))$. That is, auction yields positive utility only for agents with $\omega(f(B_i))>1-1/k_l$. If $k_l<1$, this condition holds for all agents. If $k_l\ge 1$, the condition implies only agents with a sufficiently large probability of winning shall derive positive payoff from the auction by placing positive bids.

I restrict attention to symmetric monotonically increasing equilibria. In equilibrium, the chances of player i to win, are given by $(\theta B_i)^{n-1}=H^{n-1}(v_i)$. With the above argument, auction yields positive utility only if $\omega(H^{n-1}(v_i))>1-1/k_l$. Given $\omega(.)$, the threshold value $\hat{v}(k_l)$ beyond which positive utility is realized is defined by

$$\omega(H^{n-1}(\hat{v}))=\max[0,1-1/k_l] \tag{7a}$$

Note that $\hat{v}(k_l)=\underline{v}$ if $k_l\le 1$. For agents with $v_i\le\hat{v}(k_l)$ bidding their induced value ensures maximizes overall payoff. Agents with $v_j\in[\hat{v}(k_l),\overline{v}]$ shall place positive equilibrium bids that

[33] We don't need to assume symmetric behavioral characteristics to derive the optimal bid response.
[34] For example, in a first-price auction with 4 bidders, computers always bid three-quarters of their induced value.

yield positive overall payoff. Maximizing (7) with respect to B_i yields a strictly increasing (optimal) bid response function.[35]

Proposition 4: **(-First-price auction against Nash bidders-)** *The unique optimal bid for loss-averse bidders who weigh probabilities nonlinearly (against Nash risk-neutral bidders) is captured through the following monotonic relationship:*

$$v_i = \begin{cases} \min\left\{ B_i + \frac{B_i}{(n-1)\beta}\left(\frac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right), \frac{n\text{-}1}{n}\bar{v} \right\} & \text{for} \quad v_i \geq \hat{v}(k_l) \\ B_i & \text{for} \quad v_i < \hat{v}(k_l) \end{cases}$$

Proof: See Appendix.

It is clear from the above that (i) $\hat{v}(k_l)$ varies with k_l and β and (ii) for agents with $v_i \geq \hat{v}(k_l)$ the equilibrium bid depends on k_l and β. For $v_i \geq \tilde{v}$, the optimal bid attains a corner solution i.e. $B_i + \frac{B_i}{(n-1)\beta}\left(\frac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right) = \frac{n\text{-}1}{n}\bar{v}$. This suggests that beyond the threshold induced value $\tilde{v}$ it is optimal for bidders to bid $(n-1)\bar{v}/n$ that ensures winning the auction. If a bidder chooses a bid below $(n-1)\bar{v}/n$ and anticipates losses, then her bid is adjusted against loss aversion. For agents with $v_i < \hat{v}(k_l)$ equilibrium bid $B(v_i) = v_i$ does not depend on k_l and β. As a special case, when bidders aren't loss-averse and do not weigh probabilities nonlinearly, this yields a best response in a Nash equilibrium. This allows characterizing the effect of loss aversion on bidding.

Proposition 5: **(i) (-Effect of Loss Aversion-)** *In auctions with induced values (against Nash risk-neutral bidders), for* $\hat{v}(k_l) \leq v_i < \tilde{v}$ *loss aversion induces more aggressive bidding, i.e.* $\frac{\partial B_{PT}}{\partial k_l} > 0$ **(ii) (-Effect of probability weighting-)** *Greater* β *(more convex probability*

[35] For all plausible parameters β and k_l the payoff function has a unique interior or corner optimum.

weighting) yields more aggressive bidding i.e. $\frac{\partial B_{PT}}{\partial \beta} \geq 0$ *except when* $k^* < k_l < 1$ *and bidders such that* $[1-k_l(1-2\omega(\theta B_i))] < \frac{n\beta(k_l-1)\ln(\theta B_i)\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, *who bid less aggressively i.e.* $\frac{\partial B_{PT}}{\partial \beta} < 0$.

Proof: See appendix.

This suggests that loss aversion has no effect on bidding when bidders either have very high or low induced values. Beyond a certain threshold induced value $\tilde{v}$ it is optimal to bid $(n-1)\bar{v}/n$ and ensure winning the auction against risk-neutral Nash computer bidders.[36] Bidders with very low induced values, avoid losses by bidding their upto their value. However, for most bidders with intermediate range of induced values, anticipated loss aversion justifies aggressive bidding, with or without nonlinear probability weighting. Since the role of probability weighting is limited in these auctions, loss aversion by itself provides a sufficient justification for aggressive bidding, as evident in auction outcomes obtained thru classroom experimentation.

While discussing the effect of probability weighting on bidding, it is important to understand that the effect of probability weighting in such auctions could be limited. Nevertheless, just like in auction against human bidders, bidders could avoid losses in the following ways: (a) if the value draw is not high enough then bid upto their value to avoid losses, (b) and if the value draw is high enough they could either bid (i) more aggressively or (ii) less aggressively, in response to more convex probability weighting; this happens because bid affects the expectation of auction outcomes simultaneously. Higher β means lower elevation of the probability weighting curve and in most circumstances causes more aggressive bidding which suggests ambiguity aversion (or bidder pessimism); except when $k^* < k_l < 1$ and for v_i such that $[1-k_l(1-2\omega(\theta B_i))] < \frac{n\beta(k_l-1)\ln(\theta B_i)\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, more convex probability weighting causes less aggressive bidding.

[36] Note that, $(n-1)\bar{v}/n$ is the highest possible bid in a risk-neutral Nash model.

Proposition 6: (-Effect of greater competition-) *For most human bidders (in most circumstances) greater competition yields more aggressive bidding i.e.* $\frac{\partial B_{PT}}{\partial n} > 0$ *except when* $\hat{k} < k_l < 1$ *and bidders such that* $[1 - k_l(1 - 2\omega(\theta B_i))] < \frac{(n-1)[\beta(1-\theta B_i) - B_i \ln(\theta B_i)]\omega(\theta B_i)k_l}{[1 - k_l(1 - \omega(\theta B_i))]}$, *who bid less aggressively i.e.* $\frac{\partial B_{PT}}{\partial n} < 0$.

Proof: See Appendix

The marginal response to greater competition (more bidders) is similar to the marginal effect of probability weighting; as before, bidders could more or less aggressively, in response to greater competition; this happens because their bid also affects their expectation of auction outcomes simultaneously. The effect of greater competition is analogous to more convex probability weighting and in most circumstances causes more aggressive bidding; for $\hat{k} < k_l < 1$ and for v_i such that $[1 - k_l(1 - 2\omega(\theta B_i))] < \frac{(n-1)[\beta(1-\theta B_i) - B_i \ln(\theta B_i)]\omega(\theta B_i)k_l}{[1 - k_l(1 - \omega(\theta B_i))]}$, bidders bid less aggressively in response to greater competition. Thus, despite behavioral preferences, in most circumstances bidders respond to greater competition along conventional lines by bidding more aggressively.

In the following section I fit the general model with probability weighting and loss aversion and its restricted versions which take into account loss aversion and nonlinear probability weighting in isolation to explain bidding using data from auctions with (i) human bidders and (ii) risk-neutral Nash computer bidders. Note that the equilibrium bidding behavior as specified in Propositions 1 and 3 differs across these auctions.

5. Empirical Analysis

Data

I use data from induced value first-price auctions reported in Cox et al. (1982) and Harrison (1989). Cox et al. (1982) reports 210 auctions with different number of bidders, totaling 1170 bids in first-price auctions.[37] A description of the data in Cox et al. (1982) is provided in Table 1.

[Table 1 here]

The experiments in Cox et al. (1982) employed undergraduate students enrolled in introductory economics classes at the University of Arizona and Indiana University and were conducted over a number of years in the 1980s. The results based on this data have formed a benchmark for investigation of bidding outcomes in first-price auctions experiments (see Harrison 1989, Salo and Weber 1995, Goeree et al. 2002). The first-price auctions were conducted in sessions along with Dutch and second-price auctions for single (hypothetical) objects. All sessions consisted of 30 sequential auctions (e.g., 10 Dutch, 10 first-price, and 10 Dutch). These auctions had the following features: Identifying variables include auction series, type of auction, observed bid/price, number of bidders, period, subject, and the support of the uniform distribution from which induced values were drawn and induced. Bidders were paid $3.00 for participation and a series of 30 auctions had an expected profit of $12. Thus, the total expected earnings were about $15 per subject. A session lasted for about one hour. Induced values (in discrete multiples of 10 cents) were induced from uniform distributions with support over 0 and an upper limit that varied across different sets of auctions (see Table 2 for description). The number of bidders and the support from which induced values were drawn (with replacement) were varied such that expected gains were similar across auctions. Overbidding beyond induced values was not allowed, the object was awarded to the highest bidder at his bid, and the winning bid was displayed after the auction was concluded. The winner's identity and bid were not conveyed to the other bidders.[38] The summary statistics of the data reported in Cox et al (1982) is provided in table 2.

[Table 2 here]

[37] I ignore auctions with 3 bidders in these experiments. The results for these auctions are considered anomalous, and breakdown of non-cooperative bidding is suspected (Cox et al. 1982)

[38] This is quite unlike in real first-price auctions where such information can be public. The non-availability of ex-post information that becomes the basis of "regret" therefore weakens anticipated "regret" as an explanation for overbidding in these auctions (Ozbay and Filiz-Ozbay 2007). Note that my explanation is invariant to ex-post information structure in these auctions.

The series of auctions where bidders have prior experience of bidding in first-price auctions have a suffix "x" in the name (see Table 1). Thus, for auctions with 4 and 5 bidders, we can explore the effect of "experience" on behavioral parameters.

I also use data from Harrison (1989) in addition to Cox et al. (1982). Six experimental sessions were conducted using the design indicated in Table 3. The general procedures follow those introduced by Cox, Smith and Walker (1985b) and Cox et al. (1988), and are broadly similar to Cox et al.(1982). All subjects were economics undergraduates at the University of Western Ontario and received $3 just for showing up at the experimental session. The expected profit for a session of 20 auctions was roughly $10. Therefore, total expected earnings were $13 for each subject. All experimental sessions had 4 bidders whose induced values were drawn from a uniform distribution with lower and upper valuations of $0.01 (or 1 point) and $10.00 (or 1000 points). A description of the data reported in Harrison (1989) is provided in Table 3.

[Table 3 here]

I restrict my analysis to auctions with dollar payoff and compare the auctions with auctions involving inexperienced human rival in the following treatments: (i) subject experience and (ii) use of computer-simulated "Nash risk-neutral bidders." Subjects have a similar level of experience in series 1, 2, and 3, respectively. In auctions against risk-neutral Nash bidders, a computer entered risk-neutral Nash equilibrium bids for the 3 bidders that each human bidder faces in an auction. Subjects were informed ex-ante that the computer would bid 75% of the valuation that it drew for each of the 3 simulated bidders. The auctions in Harrison (1989) are different from the auctions in Cox et al. (1982) in the following ways: Bidding beyond induced value is allowed in Harrison. Bidders (human or simulated) were assigned randomly in each period. This controls for the use of multi-period strategies that can be employed when this randomization procedure is not in use. Valuations vary across agents in a given replication and across periods. Each replication in a given period also employs the same N valuations, since replications occur simultaneously in a given experiment. The summary statistics of the auctions in Harrison (1989) is provided in Table 4.

[Table 4 Here]

Pooling of data

1. Induced value distributions were varied across auctions with varying numbers of bidders in Cox et al. (1982) such that expected gains from participation in auctions were roughly

similar. In my framework this design may not have the desired effect. Also, auctions with different numbers of bidders may present unique levels of ambiguity. Therefore, I do not pool the data from all the auctions together.

2. In Cox et al. (1982) there are two series of auctions, “fdf” and “dfd” each composed of 10 consecutive auctions of a type. For example, “fdf” represents 10 first-price, 10 dutch, and 10 first-price auctions, and “dfd” represents 10 dutch, 10 first-price, and 10 dutch auctions. I pool data from 20 first-price auctions from the series “fdf” and 10 first-price auctions from the series “dfd”.

   Similarly, data from 20 sequential first-price auctions are pooled together from Harrison (1989). As observed earlier, randomization procedures adopted in Harrison (1989) control for the use of multi-period strategies that can be employed when this randomization procedure is not in use. (1989).

An overview of bidding behavior

An overview of bidding across auctions in Cox et al. (1982) and Harrison (1989) (in tables 2 and 4) reveals the following: (a) in auctions with 4 bidders, the number of bids above the risk-neutral Nash (henceforth overbids) ranges between 81-91% in Harrison (1989), as compared to 77.5-82.5% in Cox et al. (1982); (b) and in auctions with 5 or more bidders in Cox et al. (1982), the number of overbids ranges between 66-86%. For all auctions (a) the amount by which bids exceed the risk-neutral Nash bids (overbid[39]) in Harrison (1989) is also higher (around 22%) than in Cox et al (1982) (around 16%) and (b) the percentage absolute deviation[40] around RNNE is also higher in Harrison (19-24%) than in Cox et al. (1982) (12-20%).

In Cox et al.(1982)(a) in the second set of auctions with 6 bidders (series b), the number of overbids is substantially lower (66.7%) than in any other auctions; the average percentage overbid is also the lowest among all auctions, whereas the average percentage bid below the risk neutral Nash (henceforth underbid) is similar to other auctions; (b) in the other set of auctions with 6 bidders (series a) the number of overbids is 78.3%, which is similar to other auctions, but the average percentage underbid is around 23%, which is somewhat high; (c) in both series of auctions with 6 bidders, 4 out of 10 bidders bid below RNNE in 50% of the auctions; and (d) in

[39] Overbid=(bid-RNNE)/RNNE; Underbid=(RNNE-bid)/RNNE.
[40] Absolute deviation=abs(bid-RNNE)/RNNE.

auctions with 9 bidders, low valuation bidders tended to bid close to zero, which yields an unusually high average underbid of around 27% below RNNE; 4 out of 10 bidders bid below RNNE 50% of the time. Clearly, observed bids reflect differences in auction procedures, payoffs, and bidder characteristics.

In Harrison (1989) prior experience seems to affect bidding in against human bidders and against Risk-neutral Nash bidders. The number of bids above RNNE declines from 91% in auctions with inexperienced bidders to 89% in auctions with experienced bidders. This further declines to 81% in auctions with experienced bidders who face Nash bidders (see Table 4). The average percentage overbid above the RNNE declines from 23% to 21% in auctions against human bidders. This declines further to 18% in auctions with experienced bidders against Nash bidders. The average percentage absolute deviation around RNNE declines from 24% to 21% in auctions against human bidders. This further declines to 19% in auctions with experienced bidders against Nash bidders.

Such effects are not obvious in auctions in Cox et al. (1982). In auctions with 4 bidders, number of overbids increase from 77.5% with inexperienced bidders to 82.5% with experienced bidders. However, average overbid (underbid) declines from 16.3% (34.2%) to 15.5% (20.9%). This yields a decline in average absolute deviation around RNNE from 20% to 16.3%. Thus, prior experience lowers absolute deviation around RNNE. However, an opposite effect is observed in auctions with 5 bidders. Although the number of bids with prior experience above RNNE declines from 86.7% to 80%, the average percentage overbid declines from 14.2% to 13.8%; the average percentage underbid however increases from 17.6 to 20.5%. The average percentage absolute deviation around RNNE increases from 14.6% to 15.1%. Clearly, the effect of experience in auctions with 5 bidders, in terms of average percentage absolute deviations around RNNE, is different from that observed in other auctions.

Omitted Observations

In Cox et al. (1982), I estimate the parameters for different levels of competition without pooling the data. In auctions with 9 bidders, bidders with low induced values tend to bid close to zero, clearly suggesting that cognitive costs of bidding exceed potential gains from optimal bidding. All bids that suggest more than 20% absolute deviation around RNNE (most of these bids are underbids close to zero) are therefore ignored for estimation purposes. I ignore bids that exceed induced values in Harrison (1989). In auctions against risk-neutral Nash bidders, only those bids that do not exceed the highest possible bid of 750 have been considered. Thus, the number of bids considered for estimation purposes are less than the number of bids reported in Harrison (1989). Outliers have been removed throughout.

Estimation Procedure

I use nonlinear least squares estimation to identify the parameters for the bidding function in a symmetric Bayesian Nash equilibrium.[41] This estimation has been done for the general model (outlined in Proposition 1) and the restricted versions of the general model which allow loss aversion and nonlinear probability weighting in isolation. I have used MATLAB to implement a "Trust-region reflective Newton" search for the best-fitting parameters[42].

Estimates

The combined results for all the auctions are listed in Table 5; the table lists estimated behavioral parameters for auctions with varying levels of experience, number of bidders, and nature of opponent bidders (humans or risk-neutral Nash bidders). The estimates for auctions against risk-neutral Nash bidders are reported in the last set of rows in Table 5.

[Table 5 here]

i. Probability weighting and loss aversion in the general model

The estimates of β are greater than 1 (and significantly different from zero in most cases[43]) in auctions against human bidders in both Cox et al. (1982) and Harrison (1989). Except for the auctions with 6 bidders in Cox et al. (1982), the estimates of β are greater than 1.[44] This yields convex probability weighting and therefore suggests "ambiguity aversion" along the lines of Salo

[41] If the errors between the predicted and observed bids are assumed independent identical normal random variables i.e. $\varepsilon_i \sim NID(0,\sigma^2)$, then maximum likelihood and nonlinear least squares estimation are equivalent. ML estimates are consistent, asymptotically efficient and asymptotically normal; however, if this does not hold nonlinear least squares though not efficient remain consistent and asymptotically normal.

[42] The programming code underlying all the ensuing results is available upon request.

[43] Based on t-ratio.

[44] In auctions with 6 bidders (series B), the number of overbids is substantially lower (66.7%) than for any other auctions; the average overbid is also the lowest among all auctions, whereas the average underbid is similar to other auctions. In the other set of auctions with 6 bidders (series a) the number of overbids is 78.3%, which is similar to other auctions, but the average underbid is around 23%, which is somewhat high. In both series of auctions with 6 bidders, 4 out of 10 bidders bid below RNNE in 50% of the auctions. These auctions are therefore unusual and the estimates of β which suggest overweighting (concave probability weighting), are somewhat out of order.

and Weber (1995) and Goeree et al. (2002). In Harrison (1989), the estimates of β successively decline from 1.51 in auctions with inexperienced human bidders to 1.16 in auctions against human bidders and prior experience; this further declines to 1.01 in auctions against risk-neutral Nash bidders and prior experience. Note that a model based on risk-aversion alone cannot explain these changes.[45]

The estimates of k_l are approximately close to 1 and significantly different from zero in most auctions against human bidders in Cox et al. (1982) and Harrison (1989). Except for auctions against risk-neutral Nash bidders in Harrison (1989), where the estimate of k_l is smaller but not significantly different from zero, the estimates are approximately close to 1, which supports loss aversion based on my model.

ii. Probability weighting without loss aversion

Although the estimates of β are greater than 1 and significantly different from zero in all auctions against human bidders for β in both Cox et al. (1982) and Harrison (1989), their magnitude is much larger. This yields more convex probability weighting and suggests larger deviations between the objective and weighted probabilities of auction outcomes.[46] The estimates for auctions with 6 bidders in Cox et al. (1982) are much lower than the estimates for all other auctions. In Harrison (1989) the estimates of β decline from 3.02 in auctions with inexperienced bidders to 2.32 in auctions with human bidders and prior bidding experience; this further declines to 1.70 in auctions against risk-neutral Nash bidders and prior experience. As before, a model based on risk-aversion alone cannot explain these changes.

iii. Loss aversion without probability weighting

The estimates of k_l for most auctions in Cox et al (1982), except for auctions with 6 bidders (series B), are approximately close to 1 and significantly different from zero. The estimates of k_l in auctions in Harrison (1989) are 1.00, 1.01, and 0.91 respectively and significantly different

[45] Another aspect of the estimates for β relate to the deviation from 1 in the expected utility based models. In most auctions, when the estimates are greater than 1 in more than 50% cases (more than half of the auctions) they significantly improve the explanatory power of the model based on sum of squared errors and F-test.

[46] Also note than when loss aversion was considered the estimates for probability weighting were quite similar to each other which is not true when loss aversion is ignored.

from zero. Thus, even when probability weighting is ignored, based on the estimates obtained for auctions in Harrison (1989), these estimates become smaller in auctions with smaller ambiguity levels (with human bidders and prior experience or Nash bidders).

The estimates of the gradient associated with losses k_l are approximately close to 1 in models where loss aversion is allowed except for auctions against risk-neutral-Nash bidders in Harrison (1989) where the estimate is smaller than 1 and significantly different from zero.

The implied ratio of loss-gain utility is therefore close to 2. Tversky and Kahneman (1991)[47] suggest a ratio of 2:1 for the "gains" and "losses" based on acceptable lottery gambles.[48] The estimates I obtain suggest that the ratio of "gain-loss" utility is qualitatively similar to that observed in Tversky and Kahneman (1991) and reported elsewhere (Ho, Lim and Camerer, 2006).[49] Note that my model with linear probability weighting and $k_l = 1$ is equivalent to a model with risk-aversion with Arrow-Pratt coefficient of 0.5. This similarity is supported by the estimates obtained for β and k_l, in auctions with least ambiguous circumstances. However, unlike the model based on risk-aversion (constant relative risk-aversion or CRRAM) alone, the general prospect theory model, can address changes in ambiguity levels; the estimates for probability weighting obtained across these auctions, is consistent with how individuals should respond to changes in underlying circumstances.

In the following section, I state the results based on differences in estimates for β and k_l obtained in auctions with prior bidding experience and/or against Nash risk-neutral bidders; in section 7, I further discuss the implications of my results in the context of related literature.

6. The effect of bidding experience and type of opponent bidders

Ambiguity aversion has attracted attention because individuals are typically not aware of precise probabilities in the real world. In auctions, the probability of winning for a given bid depends on bidders' bidding strategies, which is not readily known in most induced value auctions. Clearly,

[47] "…these findings suggest that a loss aversion coefficient of about two may explain both risky and riskless choices involving monetary outcomes and consumption goods" (Tversky and Kahneman, 1991, p.1053)

[48] As mentioned earlier, not winning the auction does not result in monetary losses; thus a ratio of losses to gains would be $(1+k_l)/1$.

[49] The estimated coefficient for loss aversion makes my model equivalent to a model with risk-aversion coefficient of 0.5 without nonlinear probability weighting; the generality due to nonlinear probability weighting adds to the explanatory power of my model over a model with risk-aversion alone.

deriving probabilities in these auctions is a complicated task, and therefore ambiguity could affect bidding as in other market experiments (Sarin and Weber 1993, Salo and Weber 1995). As people become familiar and gain experience of bidding, deriving probabilities of various outcomes could become easier.[50] In my model, this could result in smaller deviations between subjective and objective probabilities under less ambiguous circumstances. The data for auctions where bidders have prior bidding experience and/or face risk-neutral Nash bidders present an opportunity to explore these effects. Since these induced value auctions are similar, besides variations in experience level and the nature of opponent bidders, as a preliminary hypothesis one could argue that changes in the underlying circumstances (ambiguity) are not likely to affect the degree of loss aversion (the gradient for loss aversion).[51],[52] In this section I discuss the experimental evidence which supports my hypothesis and suggests minimal role for nonlinear probability weighting in auctions characterized by less ambiguous circumstances.

Based on my discussion above, I propose the following hypothesis.

Hypothesis: (a) *The deviations between weighted and objective probabilities become smaller as auctions environments become less ambiguous, i.e,*

$$\beta_{inexperienced}^{humanrivals} > \beta_{experienced}^{humanrivals} > \beta_{experienced}^{RNNrivals}$$

whereas ***(b)*** *the coefficient of loss-gain utility* k_l *is similar across auction environments.*

Since my hypothesis pertains to both loss aversion and probability weighting, I shall focus only on the results from the general model to explore the effect of prior bidding experience against experienced human and risk-neutral Nash bidders[53].

I first test the following hypothesis for (gradient of) loss aversion using a generalized likelihood ratio test:

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[50] Such expertise is likely to develop faster in other contexts, e.g., in games of chance.

[51] Loss aversion may vary across commodities (Horowitz and McConnell 2002, Koszegi and Rabin 2006) and could potentially depend on availability of substitutes and trading intentions (Kahneman, Knetsch and Thaler 1990; List 2003).

[52] The assumption in Kahnemann and Tversky (1979), which suggests that probability weighting and loss aversion are independent, is too simplistic. There is some literature that suggests that probability weighting and loss aversion could be related. Intuitively it is plausible that loss aversion could become smaller in less ambiguous circumstances (Chambers and Melkonyan 2008, Plott and Zeiler 2005).

[53] Going by the sum of squared residuals (SSE) alone, the restricted versions of the general model do not throw unambiguous evidence in favor of one approach over the other. As observed earlier, the similarity of estimates suffer, when either of these influences on behavior is ignored.

$$H_0 : k_l^{i,g} = k_l^{j,h}, \quad H_1 : Not\ H_0$$

where *i*,*j*=level of experience and *g*,*h*=nature of bidders. Then I test the following hypothesis for probability weighting:

$$H_0 : \beta_i^g = \beta_j^h; \quad H_1 : Not\ H_0$$

If the first test does not reject the null hypothesis, I test the following hypothesis for probability weighting under the assumption that loss aversion remains the same for robustness:

$$H_0 : \beta_i^g = \beta_j^h \mid k_{li}^g = k_{lj}^h; \quad H_1 : Not\ H_0$$

The likelihood ratio has a χ_r^2 distribution where r is the number of restrictions imposed in the null hypothesis. On the basis of these tests (see Table 6), I obtain the following result. (figures 1-5 in appendix for bidding functions and probability weighting functions, which are based on the estimates listed in Table 5, supplement the results below).

Result 1.A: (-Less convex probability weighting due to experience-) ***Prior bidding experience reduces the nonlinearity of probability weighting in auctions (i) against experienced human bidders and (ii) against risk-neutral Nash bidders. This yields smaller deviations between subjective and objective probabilities of equilibrium auction outcomes.***

This result addresses the effect of prior experience on bidding in auctions which present successively smaller levels of ambiguity as opponents change from (i) experienced human bidders to (ii) risk-neutral Nash bidders.

First, I shall address the former auctions. The estimates for β are smaller in these auctions with 4 bidders and prior bidding experience (compared to auctions with bidders without experience) in Harrison (1989) and Cox et al. (1982). This decline is significant at the 1% level for auctions in Harrison (1989) and not significant for auctions with 4 bidders in Cox et al. (1982) (see Table 6). In auctions with 5 bidders, the increase in the estimate of β for experienced bidders in Cox et al. (1982) contradicts my hypothesis but is not significant. If prior experience is expected to reduce deviations with respect to the risk-neutral Nash bid then the deviations obtained in auctions with 5 bidders belies the expectation, which parallels the movements obtained for β .

Next, in auctions against risk neutral Nash bidders (Harrison 1989), bidders have prior bidding experience as well. Thus, of all auctions under consideration, bidding in these auctions occurs in least ambiguous circumstances. In these auctions, the decline in the estimate of β as compared to auctions without prior bidding experience is significant. This supports my primary hypothesis about the effect of ambiguity on bidding in these auctions.

Result 1.B: (-Less convex probability weighting due to fixed opponents' strategies-) ***In auctions with prior bidding experience against risk-neutral Nash bidders, fixing the opponents' bidding strategies reduces the nonlinearity of probability weighting (with and without loss aversion). This yields smaller deviations between subjective and objective probabilities of equilibrium auction outcomes.***

While the previous result compares the estimates for β with prior bidding experience, the auctions against risk-neutral Nash rivals differ from the auctions with human opponent bidders (with same experience levels) since the opponents bidding strategies are fixed. The focus of previous attempts (Salo and Weber 1995) to explain aggressive bidding relates to the ambiguous circumstances arising due to uncertain behavior of opponent bidders. The extra control in bidding against risk-neutral Nash bidders allows us to examine the implications for β using my approach. As before, in auctions against risk-neutral Nash bidders (Harrison 1989), the decline in the estimate of β as compared to auctions against human bidders, is significant.

Thus, so far, as we move from auctions with inexperienced bidders to auctions with experienced bidders and risk-neutral Nash opponent bidders, the estimates of β display significant downward movement with successively smaller levels of ambiguity. It is therefore appropriate to reflect on the role of ambiguity attitudes in auctions with least ambiguous circumstances, based on the estimates obtained for behavioral parameters.

Result 1.C: (-Linear probability weighting in least ambiguous circumstances-) ***In auctions, with prior bidding experience, against risk-neutral Nash bidders, by allowing loss aversion, an almost linear probability weighting function is obtained.***

Without loss aversion, although nonlinearity of probability weighting declines with successively smaller levels of ambiguity, the deviations between subjective and objective probabilities remain. However, with loss aversion, I obtain almost linear probability weighting which suggests that aggressive bidding can be rationalized by loss aversion alone without invoking ambiguity effects.

I shall now turn to the estimates for loss aversion observed in various auctions.

Result 2.A: (-No effect on loss aversion due to experience-) ***Prior bidding experience has no effect on loss aversion in auctions against experienced human bidders.***

Result 2.B: (-Loss aversion declines in least ambiguous circumstances-) ***The degree of loss aversion obtained in auctions with prior bidding experience against risk-neutral Nash bidders is smaller than that obtained in auctions with human opponent bidders.***

The estimates for k_l are almost identical in all the auctions except in auctions against risk-neutral Nash bidders, where the estimated gradient for losses k_l is smaller. This decline is significant when compared to the estimates obtained in auctions with human bidders in Harrison (1989). This allows a reflection of the possible shortcomings of my approach. In more general field settings, the degree of loss aversion may vary across commodities (Horowitz and McConnell 2002, Koszegi and Rabin 2006). It may be affected by the availability of market substitutes (Horowitz and McConnell 2002) or trading intentions (List 2003, 2004; Kahnemann, Knetsch and Thaler 1990). The difference in loss aversion obtained in induced values settings (where the above do not apply) possibly suggests that behavioral influences, other than probability weighting and loss aversion, coexist. For example, if bidders display spite against human bidders and not against Nash bidders (computers), the differences in loss aversion as obtained are expected.[54]

7. Further discussion of the empirical findings

In this section I discuss the significance of my findings in the context of the experimental literature on auctions as well as the experimental literature in general. I compare my findings to previous literature that explores probability weighting and loss aversion in experiments.

[54] The changes in estimates for β and k_l (when considered in isolation) are also similar to the change in estimates obtained in the general model.

Several studies on decision under risk show the tendency of subjects to overweight small objective probabilities and underweight medium and large objective probabilities (Tversky and Kahneman 1992, Camerer and Ho 1994, Fox and Tversky 1998, Gonzalez and Wu 1999). This pattern yields an inverted S-shaped probability weighting function as in Kahneman and Tversky (1979).[55]In the real world actual probabilities may not be known precisely. Recent evidence (Barron and Erev 2003; Hertwig et al. 2004; Barron and Ursino 2007) suggests that the inverted S-shaped curve does not capture decision making under uncertainty where probabilities are typically derived through repeated sampling (experience)[56].This literature suggests that individuals underweight small probabilities under uncertainty, which is different from what they do under risky circumstances as reflected in the inverted S-shaped probability weighting (Prelec 1998, Wu and Gonzalez 1999).[57] In an auction equilibrium, winning is a rare event for bidders with low induced values. Thus, the estimated convex probability weighting in my models (with or without loss aversion) is consistent with this literature. As discussed earlier, this is also consistent with Salo and Weber (1995) and Goeree et al (2002) who suggest "ambiguity aversion" in auctions.[58]

The literature suggests loss aversion in various settings and provides experimental evidence for choices over trade of mugs, pens, candy bars, subscription for electric services, job attributes, sportscards, etc. (Knetsch 1989, Tversky and Kahneman 1991, Kahneman, Knetsch, and Thaler 1990, Benartzi and Thaler 1995, List 2003). The estimate for the ratio of the slopes of the value function in two domains, for small and moderate gains and losses of money, is about 2:1 (Tversky and Kahneman 1991). In a slightly different context, Kahneman, Knetsch, and Thaler (1990) investigate loss aversion in a purely deterministic environment. In an experiment, half of

[55] This function typically intersects the linear probability weighting function somewhere between 0.3 and 0.4.

[56] In these experiments subjects were asked to choose among two options; for example, when asked to choose between a sure $3 , and $4 with probability 0.8, and $0 with probability 0.2. In one treatment the probabilities are specified clearly (descriptive) and in the other the probabilities are derived by random sampling of the options (experience-based learning). The proportion of subjects who choose the risky ($4 with probability 0.8) option is significantly higher in the treatment with uncertainty (experience-induced learning).

[57] In experiments, underweighting of rare events could occur due to sampling errors. For example, people are likely to draw rare events less often than objective probability implies, especially if their samples are small. Barron and Ursino (2007) find that underweighting of rare events as observed in one-shot decisions is robust to removal of unrepresentative samples. This suggests that underweighting of rare events in experience-based decisions occurs due to overweighting of most recent outcomes.

[58] In Chen et al. (2007), ambiguity attitudes could get confounded with the pessimistic reasoning that applies to symmetric bidders. For example, when a rival's induced value distribution is unknown , a bidder with low valuation might assume that the rival also makes a similar assumption about his values (symmetry). This could produce lower bids in equilibrium. Thus the experimental design in Chen et al. (2007) does not separates "ambiguity attitudes" from such ex ante pessimistic reasoning.

a group of Cornell students are given a Cornell insignia coffee mug, while the other half are not. When mug owners are given an opportunity to trade and nonowners are given an opportunity to buy, Kahneman, Knetsch, and Thaler (1990) found that the reservation prices for the two groups were significantly different. Specifically, the ratio of the median of the reservation price of the sellers to the buyers is roughly 2.5:1. My findings are broadly consistent with this literature (Tversky and Kahneman, 1991; Ho, Lim and Camerer, 2006).[59]

It is however important to emphasize that doubts have been raised in the literature about the robustness of loss aversion as a description of individual preferences. List (2003, 2004) provides evidence using field experiments that loss aversion attenuates with previous trading experience. Plott and Zeiler (2005) suggest that an endowment effect arises due to subject misconceptions (ambiguity) about experimental tasks. They suggest that when all known controls for subject misconceptions are employed the WTA-WTP disparity is not observed.[60] The lessons from this literature suggest the following possibilities: (i) ambiguity affects loss aversion; (ii) trading intentions could affect choices such that loss aversion disappears and (iii) market experience, which could affect both ambiguity and/or trading intentions and thereby loss aversion. My results that are obtained within the context of induced value classroom experiments add to this literature and provide support along the lines of List (2003, 2004) which suggest that loss aversion could become smaller in the field. However, unlike List (2003, 2004), my results do not suggest that loss aversion will disappear completely. This might be due to the complexity of the auction environment. If cognitive capital that attenuates loss aversion develops slowly, then such learning is likely to be slower in auctions than in other simpler choice/trading environments as in List (2003, 2004). My results also suggest that ambiguity could affect loss aversion since the estimates for loss aversion are slightly smaller in auctions against risk-neutral Nash bidders. However, in *field* auctions, even if ambiguity effects can be ruled out, trading intentions could still influence loss aversion. [61]

[59] Note however that because loss aversion is modeled slightly differently in my approach, this equivalence is not obvious. If $u(x) = x^{\alpha}$ *for* $x > 0; -\lambda(-x^{\beta})$ *for* $x \leq 0$. Therefore, $k_l = \lambda - 1$. Clearly, these estimates suggest $k_l > 0$. My approach rules out very high levels of loss aversion so bidding remains acceptable.

[60] Although, recent research seems to raise doubts about the claims in Plott and Zeiler (2005) (see Isoni, Loomes and Robert Sugden ,2009)

[61] This is further explored in Ratan (2009).

8. Conclusions

In this chapter, I provide a model of bidding in first-price auctions that combines loss aversion and nonlinear probability weighting. This approach applies to a wider domain of auction environments which differ in terms of levels of ambiguity. In auctions against human bidders, missing information about bidders' risk postures and bidding strategies present greater levels of uncertainty (ambiguity) in comparison to bidding against risk-neutral Nash (computer) bidders. The analysis of experimental auction data suggests that aggressive bidding against inexperienced human bidders can be rationalized by anticipated loss aversion and ambiguity effects. Interestingly, my approach suggests that ambiguity effects become less relevant as levels of ambiguity decline with prior experience in auctions against (i) experienced human bidders and (ii) risk-neutral Nash bidders. When loss aversion is taken into account, the best-fitting parameters in auctions with smaller levels of ambiguity yield almost linear probability weighting.

However, other behavioral explanations that induce aggressive bidding in these auctions may coexist with the influences that are prominent in my approach. For example, theoretically, risk aversion could be combined with spiteful preferences and/or nonlinear probability weighting (ambiguity aversion) to create a bidding response that is observationally equivalent to my approach. However, using this approach, in auctions against risk-neutral Nash bidders where ambiguity effects and spitefulness could be altogether irrelevant, the obtained level of aggregate risk aversion is still very high.[62] This brings out the advantages of my approach over other approaches: it provides a reasonable account of aggregate bidding behavior, and addresses the role of ambiguity very well. The declining role of ambiguity effects in auctions that present successively smaller levels of ambiguity is consistent with the smaller levels of nonlinear probability weighting obtained using my approach. This enhances the performance criteria for other behavioral approaches that can be applied in auction environments. Further research is required to disentangle the effects of various behavioral influences in auctions to attain this objective.

[62] For example, using constant-risk-aversion approach and linear probability weighting (similar to that in obtained using my approach), the Arrow-Pratt measure for auctions in Harrison (1989) with prior experience in auctions against (a) human bidders and (b) risk-neutral Nash bidders would vary between 0.42-0.52.

More investigation of the indirect effects of ambiguity on loss aversion could possibly help refine the Prospect theory based accounts of behavior under risk and/or uncertainty. However, attaining these objectives within the complexity of auction environments could be difficult.

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Appendix A

Proof for Proposition 1

For $v_i \geq \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \geq 0$ maximizing (5), agent i chooses B_i according to

$$\omega' f'(B_i)(v_i - B_i) - \omega f^i - \omega' f'(B_i)[1-2\omega(f^i)k_l](v_i - B_i) + \omega(f^i)(1-\omega(f^i))k_l = 0 \qquad \text{(A.1)}$$

Here $f^i = f(B_i) = F^{n-1}(B^{-1}(B_i))$ and therefore $f'(B_i) = (F^{n-1})'(B^{-1}(B_i))(B^{-1})'(B_i)$. In equilibrium, we have $B^{-1}(B_i) = v_i$, $(B^{-1})'(B_i) = 1/B'(v_i)$, and $f^i = F^{n-1}(v_i)$. Rearranging (A.1) gives

$$\omega'(F^{n-1})'(v_i)v_i[1-(1-2\omega(F^{n-1}(v_i)))k_l] = [\omega(F^{n-1}(v_i))(1-k_l(1-\omega(F^{n-1}(v_i))))B(v_i)]' \qquad \text{(A.2)}$$

Integrating yields

$$B(v_i)_{PT} = \frac{\int_{\hat{v}(k_l)}^{v_i} x[1-k_l(1-2\omega(F^{n-1}(x))]d\omega(F^{n-1}(x))}{\omega(F^{n-1}(v_i))[1-k_l(1-\omega(F^{n-1}(v_i)))]}$$

(A.3)

as the unique candidate for a symmetric monotonic bidding equilibrium. Monotonicity of $B(v_i)_{PT}$ can easily be established by differentiating (A.3) and using the following:

for $v_i \geq \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \geq 0 \Rightarrow [1-k_l(1-2f(B(v_i)))] \geq 0$.

It remains to show the second order condition for the maximization problem. Using the envelope theorem and (A.1), this is equivalent to $\partial^2 \pi_{PT}(B(v_i), v_i)/\partial B \partial v_i \geq 0$ which holds true since

$\partial^2 \pi_{PT}(B(v_i), v_i)/\partial B \partial v_i = \omega' f'(B(v_i))[1-k_l(1-2f(B(v_i)))] \geq 0$ since $[1-k_l(1-f(B(v_i)))] \geq 0$

Applying L'hospital's rule to (A.3) yields the bid for lowest induced value.

For $v_i < \hat{v}(k_l), B(v_i) = v_i$ maximizes payoff (yields zero payoff).

■

Proof for Proposition 2 (i)

Note first that by definition of $\hat{v}(k_l)$, $\dfrac{\partial \hat{v}(k_l)}{\partial k_l} > 0$; for $v_i \geq \hat{v}(k_l)$ rewrite bid function (A.3) as

$$B(v_i)_{PT} = v_i - \frac{\int_{\underline{v}}^{v_i} \omega(F^{n-1}(x))[1-k_l(1-\omega(F^{n-1}(x))]dx}{\omega(F^{n-1}(v_i))[1-k_l(1-\omega(F^{n-1}(v_i)))]} \qquad (2.1)$$

From above

$$den^2 \frac{\partial B_{PT}}{\partial k_l} = -[den \int_{\underline{v}}^{v_i} -\omega(x)(1-\omega(x))dx + \omega(v_i)(1-\omega(v_i)) \int_{\underline{v}}^{v_i} (\omega(x) - k_l \omega(x)(1-\omega(x)))dx]$$

where $den = \omega(v_i) - \omega(v_i)(1-\omega(v_i)k_l; \omega(v_i) = \omega(F(v_i)^{n-1}), \omega(x) = \omega(F(x)^{n-1})$. Upon expansion and cancellation this reduces to $den^2 \dfrac{\partial B_{PT}}{\partial k_l} = \int_{\underline{v}}^{v_i}(1-\omega(x))dx - \int_{\underline{v}}^{v_i}(1-\omega(v_i))dx$. For all $x < v_i$ and monotonic probability weighting $\omega(v_i) > \omega(x)$. Thus, $\dfrac{\partial B_{PT}}{\partial k_l} > 0$.

Proof for Proposition 2 (ii)

Note first that by definition of $\hat{v}(k_l)$, $\dfrac{\partial \hat{v}(k_l)}{\partial \beta} > 0$; for $v_i \geq \hat{v}(k_l)$ I show that (i) $\dfrac{\partial}{\partial \beta} B(v_i) \geq 0$ for $k_l \geq 1$ (ii) and $\dfrac{\partial}{\partial \beta} B(v_i) \geq 0$ is guaranteed for $0 \leq k_l \leq 0.995066$

Let $P(x) = F(x)^{n-1}$ and $P(v) = F(v)^{n-1}$ and drop subscript i for simplicity. From (2.1)

$$\frac{\partial}{\partial \beta} B(v_i) \geq 0 \Leftrightarrow \frac{\frac{\partial}{\partial \beta}[\omega(P(v)) \ 1-k_l(1-\omega(P(v))) \ ]}{\omega(P(v)) \ 1-k_l(1-\omega(P(v)))} \geq \frac{\int_{\underline{v}}^{v_i} \frac{\partial}{\partial \beta}\left[\omega(P(x)) \ 1-k_l(1-\omega(P(x))) \ \right]dx}{\int_{\underline{v}}^{v_i} \omega(P(x)) \ 1-k_l(1-\omega(P(x))) \ dx}$$

(3.1)

Now

$$\frac{\partial}{\partial \beta}\left[\omega(P(v)) \ 1-k_l(1-\omega(P(v))) \ \right] = \ 1-k_l(1-2\omega(P(v))) \ \frac{\partial \omega(P(v))}{\partial \beta}$$

where

$$\frac{\partial\omega(P(v))}{\partial\beta} = \frac{\partial(P(v)^{\beta})}{\partial\beta} = \omega(P(v))\ln P(v)$$

Since $\ln P(v) \le 0$ (3.1) is equivalent to

$$\frac{\int_{\underline{v}}^{v_i}\left[\omega(P(x))\ 1-k_l(1-\omega(P(x)))\ \right]dx}{\omega(P(v))\ 1-k_l(1-\omega(P(v)))} \le \frac{\int_{\underline{v}}^{v_i}\ln P(x)\omega(P(x))\ 1-k_l(1-2\omega(P(x)))\ dx}{\ln P(x)\omega(P(v))\ 1-k_l(1-2\omega(P(v)))}$$

To show this, it is sufficient to show that for

$x \le v$

$$\frac{\omega(P(x))\ 1-k_l(1-\omega(P(x)))}{\omega(P(v))\ 1-k_l(1-\omega(P(v)))} \le \frac{\ln P(x)\omega(P(x))\ 1-k_l(1-2\omega(P(x)))}{\ln P(v)\omega(P(v))\ 1-k_l(1-2\omega(P(v)))}$$

$$\Leftrightarrow \frac{1-k_l(1-\omega(P(x)))}{1-k_l(1-\omega(P(v)))} \le \frac{\ln P(x)\ 1-k_l(1-2\omega(P(x)))}{\ln P(v)\ 1-k_l(1-2\omega(P(v)))} \tag{3.1a}$$

which is equivalent to

$$\frac{\ln P(x)\ 1-k_l(1-2\omega(P(x)))}{1-k_l(1-\omega(P(x)))} = \frac{1}{\beta}\frac{\ln P(x)^{\beta}\ 1-k_l(1-2\omega(P(x)))}{1-k_l(1-\omega(P(x)))}$$

$$= \frac{1}{\beta}\frac{\ln\omega(P(x)\ 1-k_l(1-2\omega(P(x)))}{1-k_l(1-\omega(P(x)))}$$

being increasing in x ; Or equivalently

$T(y) = \dfrac{\ln y\ 1-k_l(1-2y)}{1-k_l(1-y)}$ being increasing in y when $0 \le y \le 1$;

i.e. $\dfrac{\partial T}{\partial y} = \dfrac{1}{y}[1-k_l(1-y)][1-k_l(1-2y)] + k_l(1-k_l)\log y \ge 0$ (3.2)

Case 1: For $k_l = 1$, $\dfrac{\partial T}{\partial y} \ge 0 \Rightarrow \dfrac{\partial B}{\partial\beta} \ge 0$ for $0 \le y \le 1$.

For $k_l > 1$, $\hat{v}(k_l) > \underline{v}$. (i) Since $\frac{\partial \hat{v}}{\partial \beta} > 0$ more bidders bid $B = v$ (bid more aggressively in response to greater ambiguity). (ii) For $v_i \geq \hat{v}(k_l)$,

$[1 - k_l(1 - f(B(v_i)))] \geq 0 \Rightarrow [1 - k_l(1 - 2f(B(v_i)))] \geq 0$

$\frac{\partial T}{\partial y} \geq 0 \Rightarrow \frac{\partial B}{\partial \beta} \geq 0$ for $0 \leq y \leq 1$.

Case 2: For $k_l < 1$, all $v_i \geq \hat{v}(k_l)$ when $k_l \to 0$ or $k_l \to 1$, $\frac{\partial T}{\partial y} \geq 0 \Rightarrow \frac{\partial B}{\partial \beta} \geq 0$

Let $Z(y) = 2k_l^2 y^2 + 3k_l(1-k_l)y + (1-k_l)^2 + k_l(1-k_l)y \ln y$ (3.3)

Then we need to show $Z(y) \geq 0$ for $0 \leq y \leq 1$

$Z(y)$ is strictly convex with a strict minimum attained at y^* such that $\left.\frac{\partial}{\partial y} Z(y)\right|_{y=y^*} = 0$ (3.4)

i.e. $\frac{4k_l y^*}{1-k_l} + \ln y^* = -4$ (3.5)

The function [$\frac{4k_l y}{1-k_l} + \ln y$] is a strictly monotonically increasing continuous function of y which increases from $-\infty$ at $y = 0$ to $\frac{4k_l}{1-k_l}$ at $y = 1$. Hence there exists a unique y^* at which (3.4) holds. Using (3.3) it can be shown that $Z(y^*) = (1-k_l)^2 - 2k_l^2 y^{*2} - k_l(1-k_l)y^*$; rearranging (3.5) yields

$k_l = \frac{4 + \ln y^*}{4 + \ln y^* - 4y^*}$ (3.5)

Again using (3.4) it can be shown that $Z(y^*) = (1-k_l)\left[1 - k_l(1 - \frac{y^* \ln y^*}{2} - y^*)\right]$. Thus

$Z(y^*) \geq 0$ iff $k_l[1 - \frac{y^* \ln y^*}{2} - y^*] \leq 1$

i.e. $\frac{4 + \ln y^*}{4 + \ln y^* - 4y^*}[1 - \frac{y^* \ln y^*}{2} - y^*] \leq 1$. Suppose $\ln y = -4$ then $y = e^{-4}$; and since

$\frac{4k_l y}{1-k_l} + \ln y \geq \ln y \Rightarrow y^* \leq e^{-4} \Rightarrow 4 + \ln y^* - 4y^* \leq 0$.

Hence $Z(y^*) \geq 0$ iff $(4+\ln y^*)(2-y^*\ln y^*-2y^*) \geq 2(4+\ln y^*-4y^*)$ i.e. , $y^* \geq e^{-6}$ (3.6)

From (3.5) k_l increases as y^* decreases. Thus $y^* \geq e^{-6}$ iff $k_l \leq \dfrac{4+\ln e^{-6}}{4+\ln e^{-6}-4e^{-6}} = 0.995066$.

Thus, when $k_l \leq \dfrac{4+\ln e^{-6}}{4+\ln e^{-6}-4e^{-6}} = 0.995066$ $\dfrac{\partial T}{\partial y} \geq 0 \Rightarrow \dfrac{\partial B}{\partial \beta} \geq 0$ is guaranteed.

From (3.6) when $k_l > \dfrac{4+\ln e^{-6}}{4+\ln e^{-6}-4e^{-6}}$, if there exists $y^* = F(v)^{\beta(n-1)} < e^{-6} = 0.0025$; then for very small induced values such that $Z(y) < 0$ as specified in (3.3), $\dfrac{\partial T}{\partial y} < 0 \Rightarrow \dfrac{\partial B}{\partial \beta} < 0$.

Proof for Proposition 3

Note first that by definition of $\hat{v}(k_l)$, $\dfrac{\partial \hat{v}(k_l)}{\partial n} > 0$; for $v_i > \hat{v}(k_l)$ I show that (i) $\dfrac{\partial}{\partial n} B(v_i) \geq 0$ for $k_l \geq 1$ (ii) and $\dfrac{\partial}{\partial n} B(v_i) \geq 0$ is guaranteed for $0 \leq k_l \leq 0.995066$.

Let $P(x) = F(x)^{n-1}$ and $P(v) = F(v)^{n-1}$ and drop subscript i for simplicity

$$\frac{\partial}{\partial n} B(v_i) \geq 0 \Leftrightarrow \frac{\frac{\partial}{\partial n}[\omega(P(v))\ 1-k_l(1-\omega(P(v)))\ ]}{\omega(P(v))\ 1-k_l(1-\omega(P(v)))} \geq \frac{\int_{\hat{v}}^{v_i} \frac{\partial}{\partial n}\left[\omega(P(x))\ 1-k_l(1-\omega(P(x)))\ \right] dx}{\int_{\hat{v}}^{v_i} \omega(P(x))\ 1-k_l(1-\omega(P(x)))\ dx} \tag{4.1}$$

Also

$$\frac{\partial}{\partial n}\left[\omega(P(v))\ 1-k_l(1-\omega(P(v)))\ \right] = \ 1-k_l(1-2\omega(P(v)))\ \frac{\partial \omega(P(v))}{\partial n}$$

Where

$$\frac{\partial \omega(P(v))}{\partial n} = \frac{\partial \omega(F(v)^{\beta(n-1)})}{\partial n} = \beta \ln F(v) F(v)^{\beta(n-1)} = \beta \ln F(v) \omega(P(v)) < 0$$

Since $\ln P(v) \leq 0$ (4.1) is equivalent to

$$\frac{\int_{\hat{v}}^{v_i}\left[\omega(P(x))\ 1-k_l(1-\omega(P(x)))\ \right] dx}{\omega(P(v))\ 1-k_l(1-\omega(P(v)))} \leq \frac{\int_{\hat{v}}^{v_i} \ln F(x) \omega(P(x))\ 1-k_l(1-2\omega(P(x)))\ dx}{\ln F(x) \omega(P(v))\ 1-k_l(1-2\omega(P(v)))}$$

Upon multiplying both sides by β we get the same inequality (3.1a). Thus, the rest of the proof is the same as outlined above for proposition 2(ii) for various range of value for k_l. The same conclusions follow.

■

Proof for Proposition 4

I shall first characterize the interior solution underlying the first-order condition for the objective function, assuming a monotonic bid-value relationship exists. Then show that (i) the best-response bid-value relationship is strictly increasing for $\hat{v}(k_l) \le v_i \le \tilde{v}$ and (ii) the expected payoff is local and global maximum at the optimal bid.

(i) $$\max_{\underline{v} \le B_i \le \bar{v}} \pi_{PT}(v_i, B_i) = \left[\omega((\theta B_i)^{n-1}) - k_l \omega((\theta B_i)^{n-1})(1-\omega((\theta B_i)^{n-1}))\right](v_i - B_i) \quad (2.1)$$

For $v_i \ge \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \ge 0$ and $v_i \le \tilde{v}$,

$$\frac{\partial \pi_{PT}}{\partial B_i} = 0 \Rightarrow v_i = B_i + \frac{\omega((\theta B_i)^{n-1})\left[1-(1-\omega((\theta B_i)^{n-1}))k_l\right]}{\frac{\partial \omega((\theta B_i)^{n-1})}{\partial B_i}\left[1-(1-2\omega((\theta B_i)^{n-1}))k_l\right]} \quad (2.2)$$

This defines a unique bid for each value.

For $v_i < \hat{v}(k_l), B(v_i) = v_i$ maximizes payoff (yields zero payoff).

For $v_i > \tilde{v}$ the following holds: $B(v_i) = \left(\frac{n\text{-}1}{n}\right)\bar{v}$

(ii) For $\beta > 0,\ v_i \ge \hat{v}(k_l), [1-k_l(1-f(B(v_i)))] \ge 0$ and $v_i \le \tilde{v}$ using (2.2) we obtained the following above: $v_i = B_i + \frac{B_i}{(n-1)\beta}\left(\frac{1-k_l+k_l(\theta B_i)^{\beta(n-1)}}{1-k_l+2k_l(\theta B_i)^{\beta(n-1)}}\right)$

$\frac{\partial v}{\partial B_i} = 1 + \frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) + \frac{Z}{(n-1)\beta}$ where $Z = \left(\frac{1-k_l+k_l(\theta B_i)^{\beta(n-1)}}{1-k_l+2k_l(\theta B_i)^{\beta(n-1)}}\right) \in [0,1]$; and

$\frac{\partial Z}{\partial B_i} = \frac{-(1-k_l)k_l\theta^{\beta(n-1)}B_i^{\beta(n-1)-1}}{(1-k_l+2k_l(\theta B_i)^{\beta(n-1)})^2} \Rightarrow \frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) = \frac{B_i}{(n-1)\beta}\left(\frac{-(1-k_l)k_l\theta^{\beta(n-1)}B_i^{\beta(n-1)-1}}{(1-k_l+2k_l(\theta B_i)^{\beta(n-1)})^2}\right)$ For

$0 < k_l < 1$, $-1 < \frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) < 0 \Rightarrow \frac{\partial v}{\partial B_i} > 0$; For $k_l \geq 1$, the numerator and denominator are such that $\frac{B_i}{(n-1)\beta}\left(\frac{\partial Z}{\partial B_i}\right) > 0 \Rightarrow \frac{\partial v}{\partial B_i} > 0$.

Thus the bid-value relationship is strictly increasing for $\hat{v}(k_l) \leq v_i \leq \tilde{v}$.

(iii) For $v_i \geq \hat{v}(k_l),[1-k_l(1-f(B(v_i)))] \geq 0$ and $v_i \leq \tilde{v}$ at the optimal bid $\frac{\partial \pi_{PT}}{\partial B_i} = 0$. To show $\frac{\partial^2 \pi_{PT}}{\partial B_i^2} \leq 0$. Differentiate the first order condition $\frac{\partial \pi_{PT}}{\partial B_i} = 0$ with respect to v_i yields $\frac{\partial^2 \pi_{PT}}{\partial B_i^2}\frac{\partial B_i}{\partial v_i} + \frac{\partial^2 \pi_{PT}}{\partial B_i v_i} = 0$. Then we need to show that $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial v_i} \geq 0$ for the proof to work since $\frac{\partial B}{\partial v_i} > 0$. Differentiating (2.1) with respect to v_i yields

$$\frac{\partial^2 \pi_{PT}}{\partial B_i \partial v_i} = \frac{\partial \omega((\theta B_i)^{n-1})}{\partial B_i}[1-k_l(1-2\omega(f(B_i)))]$$

. Since $\frac{\partial \omega((\theta B_i)^{n-1})}{\partial B_i} > 0$ and $[1-k_l(1-\omega(f(B_i))] \geq 0 \Rightarrow [1-k_l(1-2\omega(f(B_i))] \geq 0$; therefore $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial v_i} \geq 0$. Thus, the first order condition describes a global optimum. Bidding $\frac{n-1}{n}\bar{v}$ ensures that the auction is won.

Therefore for $v_i \geq \tilde{v}$, the global optimum is given by $B = \frac{n-1}{n}\bar{v}$

■

Proof for Proposition 5

For $\hat{v}(k_l) \leq v_i \leq \tilde{v}$ we need to show that $\frac{\partial B_i}{\partial k_l} > 0$. Differentiate $\frac{\partial \pi_{PT}}{\partial B_i} = 0$ with respect to k_l yields $\frac{\partial^2 \pi_{PT}}{\partial B_i^2}\frac{\partial B_i}{\partial k_l} + \frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} = 0$. Then we need to show that $\frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} \geq 0$ for the proof to work since it has been shown (above for proposition 4) that $\frac{\partial^2 \pi_{PT}}{\partial B_i^2} \leq 0$. Differentiating (2.1) with respect to k_l yields

$$\frac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l} = \frac{-B_i}{(n-1)\beta}\left(\frac{(1-k_l+2k_lY)(Y-1)-(1-k_l+k_lY)(2Y-1)}{(1-k_l+2k_lY)^2}\right) = \frac{-B_i}{(n-1)\beta}\left(\frac{Y-2Y}{(1-k_l+2k_lY)^2}\right) \geq 0$$

where $Y=(\theta B_i)^{\beta(n-1)}>0$; thus from above, $\dfrac{\partial^2 \pi_{PT}}{\partial B_i \partial k_l}>0 \Rightarrow \dfrac{\partial B_i}{\partial k_l}>0$

▪

Proof: for Proposition 5(ii)

If $v_i - B_i - \dfrac{B_i}{(n-1)\beta}\left(\dfrac{1\text{-}k_l+k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l+2k_l(\theta B_i)^{(n-1)\beta}}\right)=0$ is equivalent to $F(\beta, B(\beta))=0$ where subscript i is dropped for simplicity. By implicit function theorem, if $\dfrac{\partial F}{\partial B}\neq 0$, then $\dfrac{\partial B}{\partial \beta}=\dfrac{\partial F}{\partial \beta}/\dfrac{\partial F}{\partial B}$

$$\frac{\partial B}{\partial \beta} = \frac{\dfrac{B\Gamma(.)}{\beta^2(n-1)} - \dfrac{A(.)X_2(.)(k_l-1)}{D(.)^2}}{1+\dfrac{\Gamma(.)}{\beta(n-1)}+\dfrac{A(.)X_1(.)(k_l-1)}{D(.)^2}}$$ where

$$\Gamma(.)=\left(\frac{1\text{-}k_l+k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l+2k_l(\theta B_i)^{(n-1)\beta}}\right),\ A(.)=\frac{B_i}{(n-1)\beta},\ X_2(.)=k_l\omega(.)n\ln(\theta B)<0,\ X_1(.)=k_l\beta n\frac{(\theta B)^{\beta(n-1)}}{(\theta B)}$$

and $D(.)=1\text{-}k_l+2k_l(\theta B_i)^{(n-1)\beta}$

It is relatively straightforward to show that for $v>\hat{v}(k_l),\ \Gamma(.)>0,\ D(.)>0$

$$1+\frac{\Gamma(.)}{\beta(n-1)}+\frac{A(.)X_1(.)(k_l-1)}{D(.)^2}>0$$
$$\Leftrightarrow \beta(n-1)D(.)^2+\Gamma(.)D(.)^2 > A(.)X_1(.)(k_l-1)\beta(n-1)$$

Since $\Gamma(.)D(.)^2>0$ the above holds if $\beta(n-1)D(.)^2>(n-1)\beta\omega(.)(1-k_l)k_l$. Note that this holds when $k_l\geq 1$. When $k_l<1$, the above is equivalent to

$$D(.)^2>\omega(.)(1-k_l)\Leftrightarrow[(1-k_l)+4\omega(.)k_l]+4\omega(.)^2k_l^2/(1-k_l)>\omega(.)$$
$$\Leftrightarrow (1-k_l)_l>\omega(.)(1-4k)$$

which holds for all $k_l<1$. Therefore $1+\dfrac{\Gamma(.)}{\beta(n-1)}+\dfrac{A(.)X_1(.)(k_l-1)}{D(.)^2}>0$.

Thus $\frac{\partial F}{\partial B} > (<) 0$ iff $\frac{B\Gamma(.)}{\beta^2(n-1)} - \frac{A(.)X_2(.)(k_l-1)}{D(.)^2} > (<) 0$

i.e. $\frac{B}{\beta(n-1)}\left[\frac{\Gamma(.)}{\beta} - \frac{X_2(.)(k_l-1)}{D(.)^2}\right] > (<) 0 \Leftrightarrow \Gamma(.)D(.)^2 > (<) n\beta\ln(\theta B)(k_l-1)[\omega(\theta B)k_l]$

when $k_l \geq 1$, the LHS exceeds the RHS since $\ln(\theta B) \leq 0$. Therefore $\frac{\partial B_{PT}}{\partial \beta} \geq 0$.

When $k_l < 1$, as $B_i \to 0$ or $B_i \to \theta$, the LHS exceeds the RHS ; given the bids and values are monotonically increasing therefore for the extreme induced values $\frac{\partial B_{PT}}{\partial \beta} \geq 0$; for some $k^* < k_l < 1$ for v_i it follows from above, if $[1-k_l(1-2\omega(\theta B_i))] < \frac{n\beta(k_l-1)\ln(\theta B_i)\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$,

then $\frac{\partial B_{PT}}{\partial \beta} < 0$.

Proof: for Proposition 6

As before, $v_i - B_i - \frac{B_i}{(n-1)\beta}\left(\frac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right) = 0$ is equivalent to $F(n, B(n)) = 0$ where subscript i is dropped for simplicity. By implicit function theorem, if $\frac{\partial F}{\partial B} \neq 0$, then

$$\frac{\partial B}{\partial n} = \frac{\partial F}{\partial n} / \frac{\partial F}{\partial B} \quad \text{i.e. } \frac{\partial B}{\partial n} = \frac{\frac{B\beta\Gamma(.)}{\beta^2(n-1)^2} - \frac{A(.)X_2(.)(k_l-1)}{D(.)^2}}{1 + \frac{\Gamma(.)}{\beta(n-1)} + \frac{A(.)X_1(.)(k_l-1)}{D(.)^2}} \quad \text{where}$$

$\Gamma(.) = \left(\frac{1\text{-}k_l + k_l(\theta B_i)^{(n-1)\beta}}{1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}}\right)$, $A(.) = \frac{B_i}{(n-1)\beta}$, $X_2(.) = k_l[\omega(.)(B\ln(\theta B) + \beta\frac{B(n-1)-n}{n})] < 0$,

$X_1(.) = k_l\beta n\frac{(\theta B)^{\beta(n-1)}}{(\theta B)}$ and $D(.) = 1\text{-}k_l + 2k_l(\theta B_i)^{(n-1)\beta}$

It is relatively straightforward to show (as shown before in the proof for Prop.5(ii)) that for $v > \hat{v}(k_l)$ and for all k_l,

$1 + \frac{\Gamma(.)}{\beta(n-1)} + \frac{A(.)X_1(.)(k_l-1)}{D(.)^2} > 0$.

Thus $\frac{\partial F}{\partial B} > (<) 0$ iff $\frac{B\beta\Gamma(.)}{\beta^2(n-1)^2} - \frac{A(.)X_2(.)(k_l-1)}{D(.)^2} > (<) 0$

i.e. $\frac{B}{\beta(n-1)}\left[\frac{\Gamma(.)}{n-1} - \frac{X_2(.)(k_l-1)}{D(.)^2}\right] > (<) 0$ (6.1)

which can be shown to be equivalent to

$$(1+\frac{k_l\omega(.)}{1-k_l})(1-k_l+2k_l\omega(.)) > (<) -(n-1)k_l\omega(.)[B\ln(\theta B)+\beta(B(1-1/n)-1)]$$

When $k_l \geq 1$, the LHS exceeds the RHS in equation (6.1) since $X_2 < 0$. Therefore $\frac{\partial B_{PT}}{\partial n} > 0$.

When $k_l < 1$, as $B_i \to 0$ or $B_i \to \theta$, the LHS exceeds the RHS ; given the bids and values are monotonically increasing therefore for the extreme induced values $\frac{\partial B_{PT}}{\partial n} > 0$; for some $\hat{k} < k_l < 1$ for v_i it follows from above that if

$[1-k_l(1-2\omega(\theta B_i))] < \frac{(n-1)[\beta(1-\theta B_i)-B_i\ln(\theta B_i)]\omega(\theta B_i)k_l}{[1-k_l(1-\omega(\theta B_i))]}$, then $\frac{\partial B_{PT}}{\partial n} < 0$.

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Appendix B: Tables

Table 1: Auctions in Cox, Roberson and Smith (1982)				
	n=4 (No. of Auctions)	n=5 (No. of Auctions)	n=6 (No. of Auctions)	n=9 (No. of Auctions)
Inexperienced Bidders	fdf8 (20) dfd8 (10)	fdf9 (20) dfd9 (10)	fdf2(20) , fdf4 (20) dfd2 (10), dfd2 (10)	fdf5 (20) dfd5 (10)
Experienced Bidders	fdf8x (20) dfd8x (10)	fdf9x (20) dfd9x (10)		

(1) "n" denotes the number of bidders in a first-price auction

Table 2: Descriptive Statistics for Auctions in Cox, Roberson and Smith (1982)

Observations	No. of Bidders (Experience)		Highest Value	Value	Bid	No of Overbids (%)	Average Overbid (%)	Average Underbid (%)	Average Deviation (%)
120	4 (Inexperienced)	Mean (Std)	8.1 (-)	4.0 (2.3)	3.4 (2.1)	77.5 (-)	16.3 (-)	34.2 (-)	20.0 (-)
120	4 (Experienced)	Mean (Std)	8.1 (-)	4.5 (2.3)	3.8 (2.0)	82.5 (-)	15.5 (-)	20.9 (-)	16.3 (-)
150	5 (Inexperienced)	Mean (Std)	12.1 (-)	6.5 (3.4)	5.8 (3.1)	86.7 (-)	14.2 (-)	17.6 (-)	14.6 (-)
150	5 (Experienced)	Mean (Std)	12.1 (-)	5.6 (3.5)	5.1 (3.2)	80.0 (-)	13.8 (-)	20.5 (-)	15.1 (-)
180	6-seriesA (Inexperienced)	Mean (Std)	16.9 (-)	8.6 (4.9)	7.7 (4.5)	78.3 (-)	12.2 (-)	22.9 (-)	14.3 (-)
180	6-series B (Inexperienced)	Mean (Std)	16.9 (-)	8.8 (5.0)	7.6 (4.5)	66.7 (-)	9.5 (-)	21.0 (-)	13.1 (-)
270	9 (Inexperienced)	Mean (Std)	36.1 (-)	19.2 (10.0)	17.9 (10.0)	77.4 (-)	7.4 (-)	26.8 (-)	11.8 (-)

Note: (i) Overbid % defined with respect to RNNE i.e. no. of bids above the RNNE (ii) Overbid is 100*(bid-RNNE)/RNNE for each bid above RNNE (iii) Underbid is 100*(RNNE-bid)/RNNE for each bid below RNNE (iv) Deviation is 100*|(bid-RNNE)|/RNNE

Table 3: First-Price Auctions in Harrison (1989)					
Common Design Features: $N = 4$, $\underline{v} = \$0.01$ or 1 Point, $\bar{v} = \$10.00$ or 1000 Points, 20 Periods					
Experiment	Level of Experience	Payoff in Dollars or Lottery Points	Simulated Nash Opponent?	Number of Replications per period?	Total Number of Human Bids?
1	Inexperienced	Dollars	No	4	320
1P	Inexperienced	Points	No	4	320
2	Experienced	Dollars	No	5	400
2P	Experienced	Points	No	4	320
3	Experienced	Dollars	Yes	14	280
3P	Experienced	Points	Yes	16	320

Table 4: Descriptive Statistics for Auctions in Harrison (1989)									
Observations	No. of Bidders (Experience) Rivals		Highest Value	Value	Bid	No. of Overbids (%)	Average Overbid (%)	Average Underbid (%)	Average Deviation (%)
320	4 (Inexperienced) Human	Mean (Std)	10 (-)	5.09 (2.64)	4.56 (2.40)	91 (-)	23 (-)	26 (-)	24 (-)
400	4 (Experienced) Human	Mean (Std)	10 (-)	5.09 (2.64)	4.42 (2.31)	89 (-)	21 (-)	25 (-)	21 (-)
280	4 (Experienced) Nash	Mean (Std)	10 (-)	4.65 (2.26)	3.85 (1.98)	81 (-)	18 (-)	27 (-)	19 (-)
Note: (i) Overbid % defined with respect to RNNE i.e. no. of bids above the RNNE (ii) Overbid is 100*(bid-RNNE)/RNNE for each bid above RNNE (iii) Underbid is 100*(RNNE-bid)/RNNE for each bid below RNNE (iv) Deviation is 100*|(bid-RNNE)|/RNNE									

Table 5: Prospect Theory Models of Bidding

Cox, Roberson and Smith(1982)

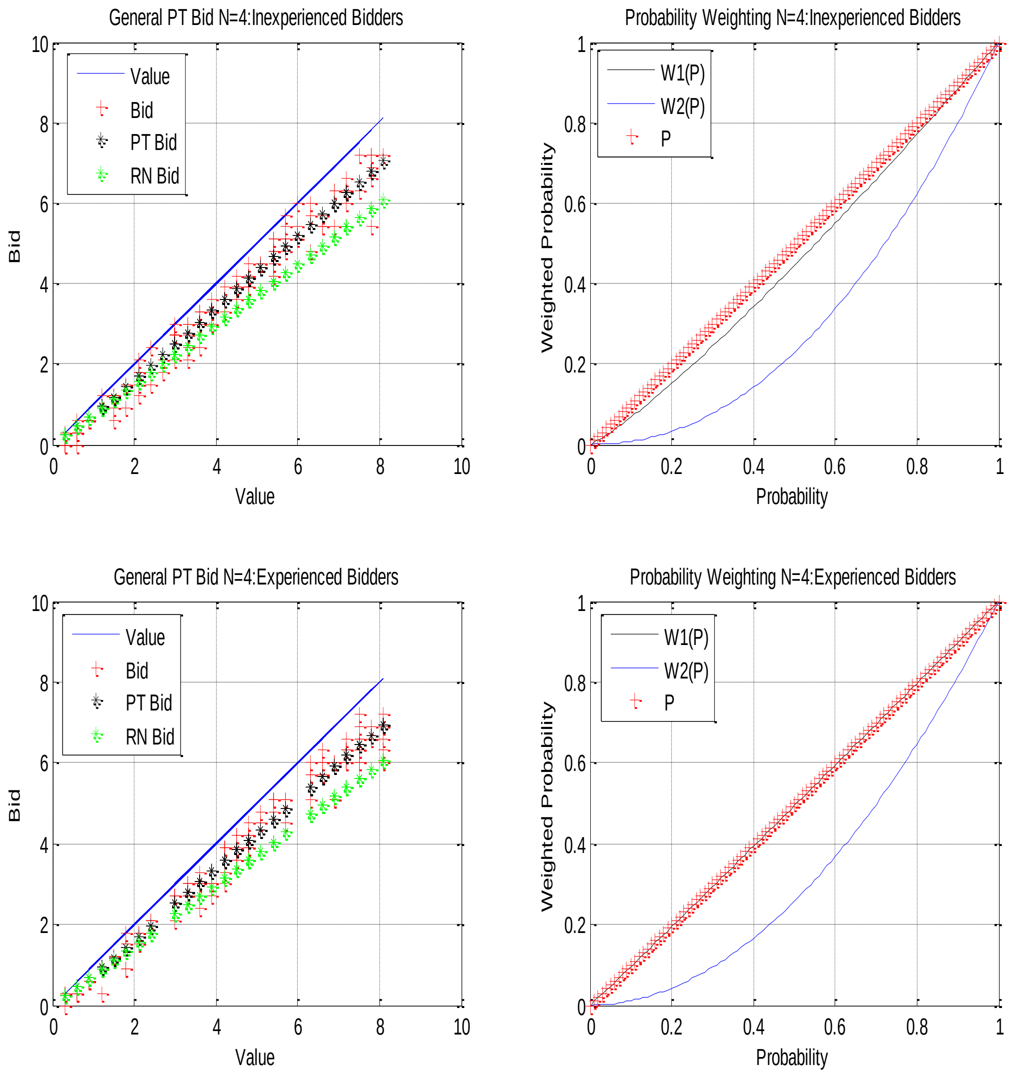
No. of Bidders (Experience) (Rivals)	No. of Observations (Periods × Bidders-outliers)	Model	$\hat{\beta}(S.E.)$	$\hat{k}_l(S.E.)$	Residual Sum of Squares (SSE)
4 (Inexperienced) Human	115	General	1.17(0.78)	0.98(0.16)**	12.28
		PW	2.13(0.76)**	-	13.26
		RD	-	0.99(0.06)**	12.85
		RNN	-	-	46.15
4 (Experienced) Human	118	General	1.02(0.48)*	0.99(0.11)**	12.43
		PW	1.96(0.59)**	-	12.81
		RD	-	0.99(0.07)**	12.44
		RNN	-	-	46.59
5 (Inexperienced) Human	146	General	1.17(0.50)**	1.00(0.02)**	26.62
		PW	2.26(0.74)*	-	27.22
		RD	-	1.00(0.004)**	28.10
		RNN	-	-	107.21
5 (Experienced) Human	146	General	1.20(0.52)**	1.00(0.01)**	24.40
		PW	2.31(0.82)**	-	25.50
		RD	-	1.00(0.003)**	26.09
		RNN	-	-	94.64
6-series A (Inexperienced) Human	175	General	0.92(0.48)	1.00(0.003)**	142.55
		PW	1.89(0.83)**	-	142.76
		RD	-	1.00(0.002)**	143.63
		RNN	-	-	223.84
6-series B (Inexperienced) Human	174	General	0.70(0.28)**	1.00(0.02)**	130.24
		PW	1.37(0.47)**	-	131.08
		RD	-	0.85(0.35)**	139.64
		RNN	-	-	159.91
9 (Inexperienced) Human	248	General	1.28(0.45)**	1.00(0.001)**	196.03
		PW	2.28(0.66)**	-	203.31
		RD	-	1.00(0.001)**	204.55
		RNN	-	-	644.96
4 (Inexperienced) Human	306#	General	1.51(1.00)	1.00(0.01)**	67.10
		PW	3.03(1.61)	-	67.27
		RD	-	1.00(0.01)**	85.32
		RNN	-	-	293.89
4 (Experienced) Human	371#	General	1.16(0.09)**	1.00(0.01)**	65.81
		PW	2.32(0.90)**	-	66.00
		RD	-	1.01(0.03)**	68.31
		RNN	-	-	253.01
4 (Experienced) Nash	268~	General	1.02(1.61)	0.91(0.89)	156.89
		PW	1.70(0.95)	-	162.98
		RD	-	0.91(0.37)*	156.91
		RNN	-	-	248.01

Notes: (1) The General model is based on Proposition 1;allows Nonlinear Probability Weighting and Loss-aversion (2) The PW model allows for Nonlinear Probability Weighting (no Loss-aversion) (3) The LA model allows loss-aversion defined in assumption B (linear Probability Weighting) only (4) The RNNE model is based on linear probability weighting where $\beta = 1$ and no loss-aversion (5) Asymptotic Standard Errors in brackets (6) SSE: Sum of squared errors based on the difference between actual and predicted bid (7) The estimates are based on search algorithms developed using MATLAB for the data described in Cox, Roberson and Smith (1982) (8)#Overbids beyond Private Values removed; ~Overbids beyond (3/4)*1000=Highest possible RNN bid removed (9) ** denotes significance at 1% level and * denotes significance at 5% level

Table 6: Hypothesis Tests			
Cox, Roberson and Smith(1982)			
No. of bidders (Observations) Experience Levels (Bidders)	Test	Estimated Log- likelihood ratio	p-value
4 (240) Inexperienced and Experienced (Human)	$k_l^{in\exp} = k_l^{\exp}$	0.2073	0.9015
	$\beta^{in\exp} = \beta^{\exp}$	2.0052	0.1568
	$\beta^{in\exp} = \beta^{\exp} \mid k_l^{in\exp} = k_l^{\exp}$	1.9244	0.1654
5 (300) Inexperienced and Experienced (Human)	$k_l^{in\exp} = k_l^{\exp}$	1.6639	0.4352
	$\beta^{in\exp} = \beta^{\exp}$	0.1457	0.7026
	$\beta^{in\exp} = \beta^{\exp} \mid k_l^{in\exp} = k_l^{\exp}$	0.0328	0.8542
Harrison (1989)			
4 (708) Inexperienced and Experienced (Human)	$k_l^{in\exp} = k_l^{\exp}$	0.0465	0.9770
	$\beta^{in\exp} = \beta^{\exp}$	16.2169**	0.0010
	$\beta^{in\exp} = \beta^{\exp} \mid k_l^{in\exp} = k_l^{\exp}$	14.5204**	0.0010
4 (584) Inexperienced against Human bidders and Experienced against Nash bidders	$k_l^{in\exp} = k_l^{\exp}$	15.9527**	0.0030
	$\beta^{in\exp} = \beta^{\exp}$	30.7884**	0.0000
4 (660) Experienced against Human bidders and Experienced against Nash bidders	$k_l^{in\exp} = k_l^{\exp}$	23.1346**	0.0000
	$\beta^{in\exp} = \beta^{\exp}$	9.2458**	0.0024
Note: (1) ** denotes significance at 1% level.			

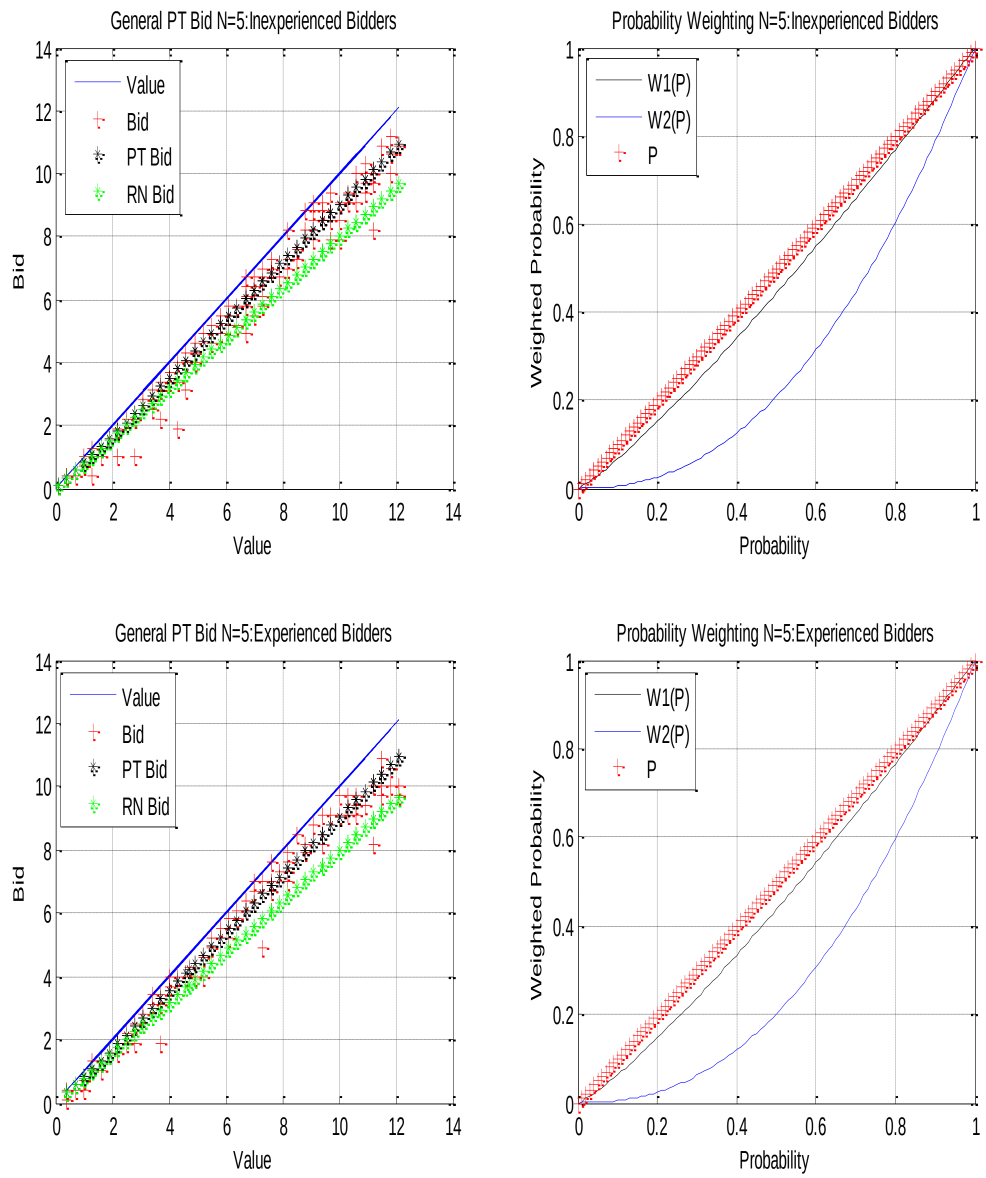
Appendix C: Figures

Figure 2: General PT Bid and Probability Function; CRS(1982); n=4 (Inexperienced and experienced bidders)



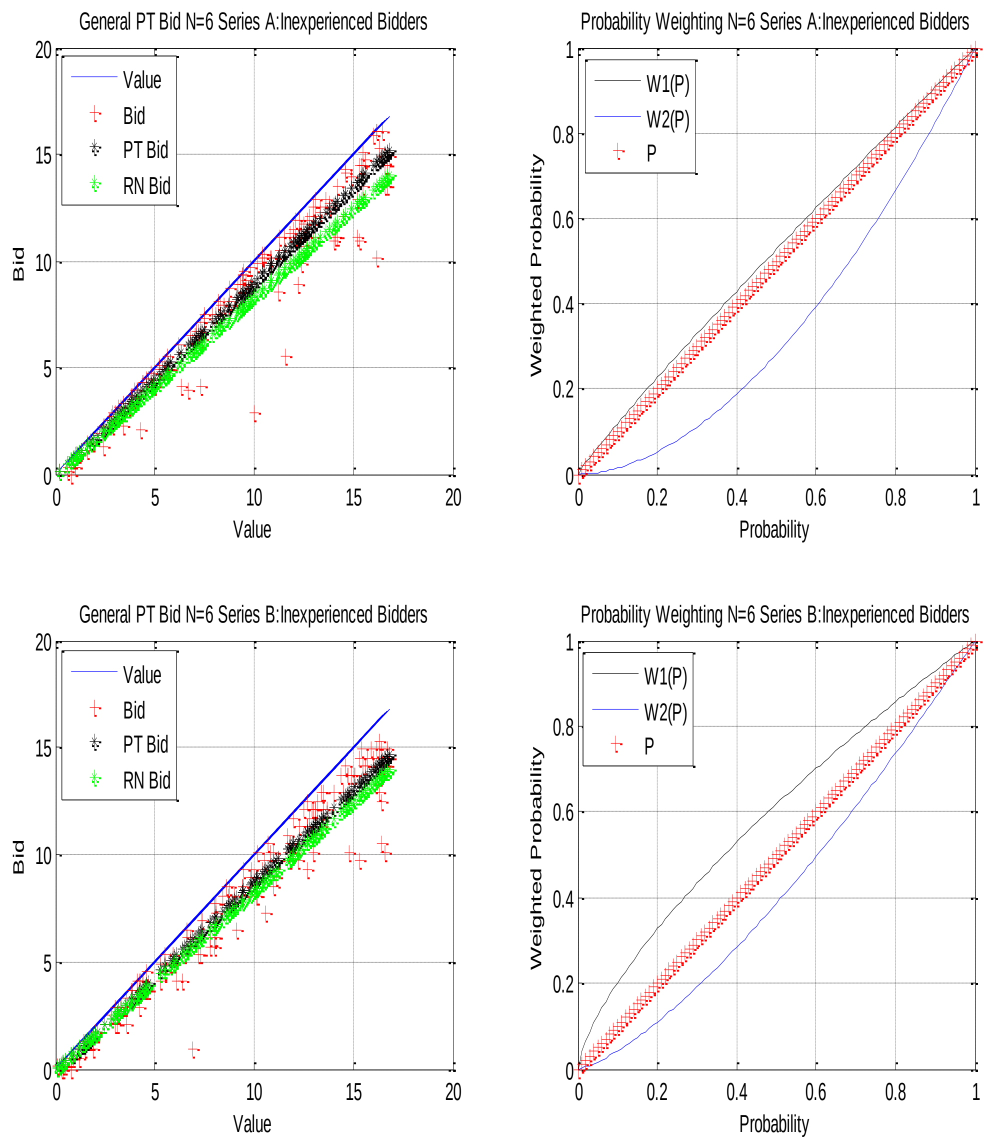
Note: (1) The right column is a plot of the probability weighting function with (W1 (P)) and without (W2 (P)) loss aversion.

Figure 3: General PT Bid and Probability Function; CRS(1982); n=5 (Inexperienced and experienced bidders)



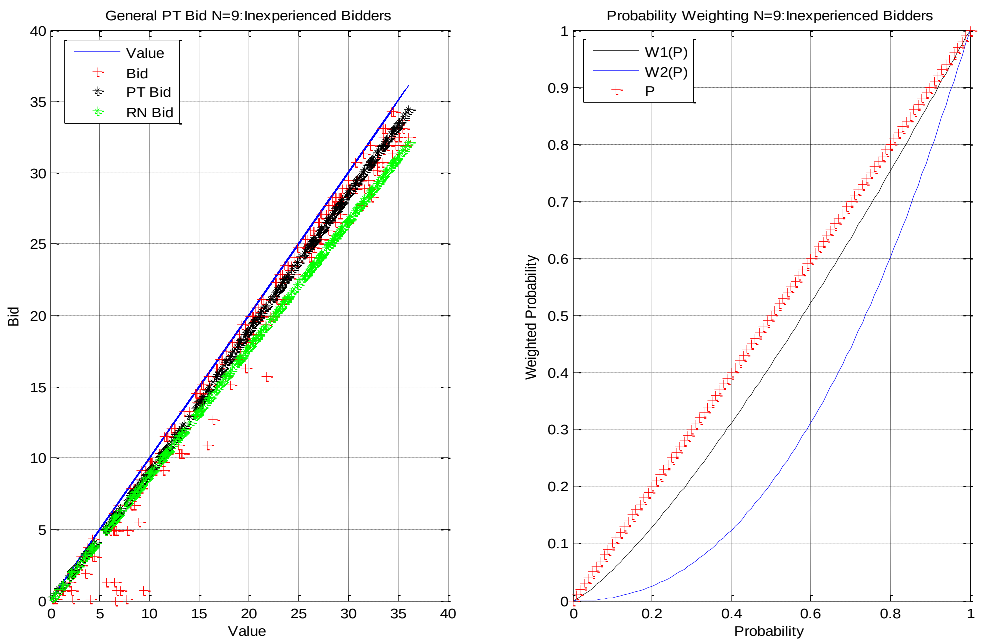
Note: (1) The right column is a plot of the probability weighting function with (W1 (P)) and without (W2 (P)) loss aversion.

Figure 4: General PT Bid and Probability Function; CRS(1982); n=6 (Inexperienced bidders)



Note: (1) The right column is a plot of the probability weighting function with (W1 (P)) and without (W2 (P)) loss aversion.

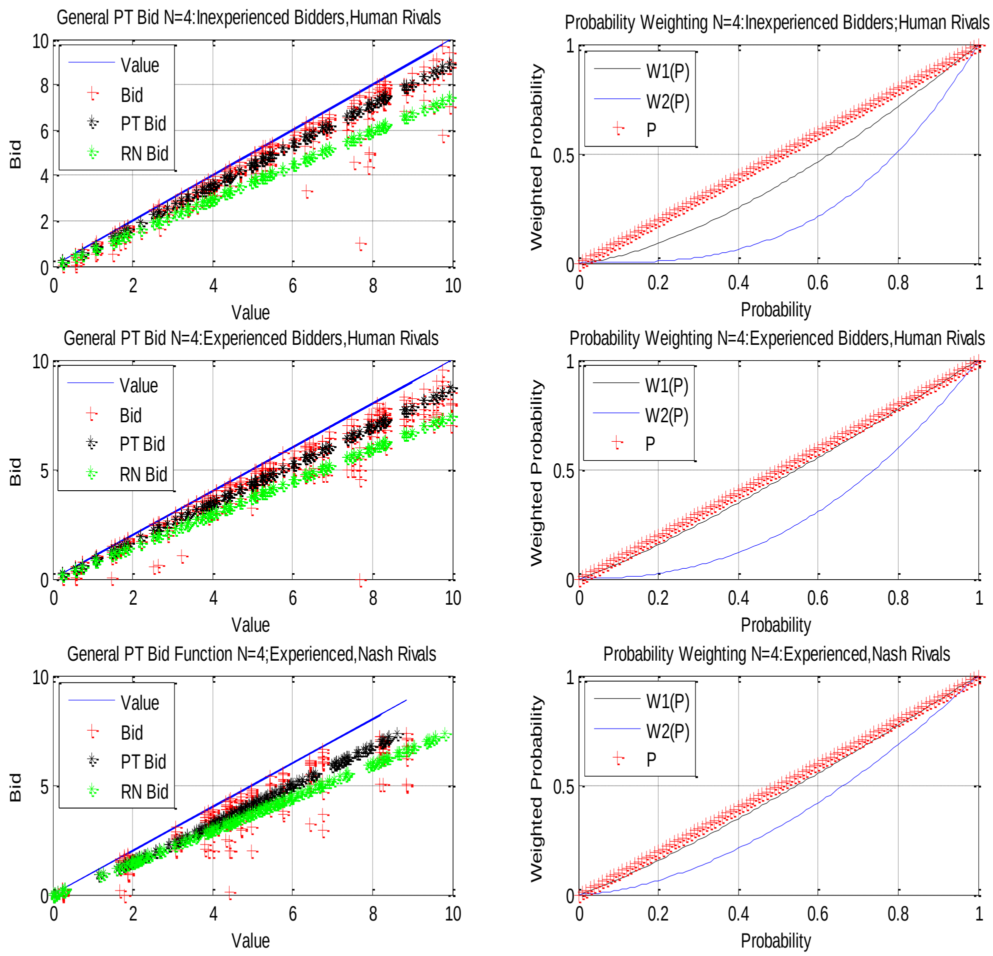
Figure 5: General PT Bid and Probability Function; CRS(1982); n=9 (Inexperienced bidders)



Note: (1) The right column is a plot of the probability weighting function with (W1 (P)) and without (W2 (P)) loss aversion.

Figure 6: General PT Bid and Probability Function; Harrison(1989); n=4

(Inexperienced and experienced bidders); against Human and Risk-neutral Nash bidders



Note: (1) The right column is a plot of the probability weighting function with (W1 (P)) and without (W2 (P)) loss aversion.