Optimal Efficiency-Wage Contracts with Subjective Evaluation*

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Abstract

We study a T-period contracting problem where performance evaluations are subjective and private. We find that the principal should punish the agent if he performs poorly in the future even when the evaluations were good in the past, and, at the same time, the agent should be given opportunities to make up for poor performance in the past by performing better in the future. Thus, optimal incentives are asymmetric. Conditional on the same number of good evaluations, an agent whose performance improves over time should be better rewarded than one whose performance deteriorates. Punishment is costly, and the surplus loss increases in the correlation between the evaluations of the two contracting parties. As the correlation diminishes, the loss converges to that of Fuchs (2007).

Keywords: subjective evaluation, relational contract.
JEL classification codes:

1 Introduction

Incentive contracts that explicitly ties compensation to objective performance measures are rare. According to MacLeod and Parent (1999), only about one to five percent of U.S. workers receive performance pay in the form of commissions or piece rates. Far more common, especially in positions that require team work, are long-term relational contracts that reward or punish workers on the basis of subjective performance measures that are not verifiable in court. Early work in the literature of subjective evaluation (Bull 1987, MacLeod 1989) has showed, using standard repeated games arguments, that efficient contracts can be self-enforcing so long as the contracting parties are sufficiently patient and always agree on some subjective performance measure.

Efficiency loss, however, becomes inevitable when the contracting parties disagree on performance. MacLeod 2003 and Levin (2003) are the first to make this point. To understand their arguments, consider a worker who can choose either to work or shirk, and suppose good job performance is more likely when the workers works. In order to motivate the worker to work, the employer needs to promise the worker a performance bonus. Since performance is subjective, the employer may falsely claim poor performance. To deter cheating, the worker must threaten to punish the employer through sabotage or quitting—if quitting harms the employer—when he feels that his performance is good but the employer does not pay a

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bonus. If the employer and worker always agree on performance, then the outcome will be efficient—the worker will exert effort, the employer will pay a bonus when performance is good, and the worker will never have to take revenge on the employer. But if the employer and worker sometimes have conflicting views over performance, then some efficiency loss due to sabotage will occur.

While MacLeod (2003) shows that this type of bonus-plus-sabotage contract, if properly constructed, could theoretically be optimal, many employers would be wary of giving disgruntled employees a chance to damage the firm. Instead, they might prefer to pay a high wage and use the threat of dismissal to motivate a worker. Compared to a bonus-plus-sabotage contract, the main advantage of an efficiency-wage contract—as this type of contract is known in the literature—is that dismissed workers can be prevented from taking revenge on the firm. Since the employer does not benefit from terminating a worker, he has no incentive to cheat. But efficiency loss will still occur when a productive worker is fired by mistake.

Fuchs (2007), adapting the results of Abreu, Milgrom, and Pearce (1991), shows that employers may substantially reduce the expected efficiency loss in an efficiency-wage contract by linking the dismissal decisions across time periods. Specifically, he studies a contracting game between an employer and a worker and shows that within the class of T review contracts, optimal termination occurs only at the end of every T periods and only when evaluations in all preceding T periods are bad. He shows that the resulting expected efficiency loss per T periods is independent of T. As a result, the per-period efficiency loss goes to zero as T goes to infinity and the discount factor of the contracting parties goes to one.

Fuchs (2007) assumes the worker's self evaluations are uncorrelated with the employer's evaluations of the worker. This is obviously a restrictive assumption. When the employer and worker share similar beliefs about performance, a worker who feels that he has been performing well would have little incentives to continue to work if he would be terminated only when his evaluations are poor in every period. In this paper we extend Fuchs (2007) to the case of positively correlated evaluations. We find that it remains optimal in this case for the employer to wait till the end of T periods to punish the worker. To prevent the worker from becoming complacent, the employer should punish the worker if he performs poorly in the future even when his evaluations were good in the past. But at the same time, the employer should allow the worker to make up for poor evaluations in the past by performing better in the future. The efficiency loss is increasing in the correlation between the evaluations of the two contracting parties. As the correlation diminishes, the loss converges to the one-period loss as in Fuchs (2007). When the correlation goes to one, the efficiency loss converges to the efficiency loss associated with the T repetition of the stationary contract.

2 Model

We consider a T-period contracting game between a Principal and an Agent. In period 0 the Principal offers the Agent a contract ω. If the Agent rejects the offer, the game ends with each player receiving zero payoff. If the Agent accepts the contract, he is employed for T periods. In each period $t \in \{1, ..., T\}$ of his employment the Agent decides whether to work ($e_t = 1$) or shirk ($e_t = 0$). The Agent's effort is private and not observed by the Principal.

Output is stochastic with the expected output equal to e_t. The effort cost to the Agent is $c(e_t)$, with $c(1) = c > 0$ and $c(0) = 0$.

Both the Principal and the Agent are risk neutral and discount future payoffs by a discount factor $\delta < 1$. Let $e^T \equiv (e_1, ..., e_T)$ denote the Agent's effort choices. Let Q denote the present value (evaluated at $t = 1$) of the Principal's labor expenditure and R the present value of the Agent's labor income. The Principal's expected payoff is

$$-Q + \sum_{t=1}^{T} \delta^{t-1} e_t,$$

and Agent's is

$$R - \sum_{t=1}^{T} \delta^{t-1} c\left(e_t\right).$$

We do not require that $Q = R$. When $Q > R$, the balance is "burnt". Intuitively, money-burning represents inefficient labor practice that harms the Agent without benefiting the Principal. We assume that $c < 1$ so that given any Q and R, the total surplus is maximized when the Agent works in every period.

There is no objective output measure that is commonly observed by the Principal and the Agent. Instead, each player observes a private binary performance signal at the end of each period t. Let $y_t \in \{H, L\}$ and $s_t \in \{G, B\}$ denote the period-t signals of the Principal and Agent, respectively. Neither y_t nor s_t are verifiable by a court. Let $\pi(.|e_t)$ denote the joint distribution of (y_t, s_t) conditional on e_t and $\pi(.|e_t, s_t)$ denote the distribution of y_t conditional on e_t and s_t.[1] Both the Principal and the Agent know π. We assume π satisfies the following assumptions:

Assumption 1. $\pi(H|1) > \pi(H|0)$.

Assumption 2. $\pi(H|1, G) > \max\{\pi(H|1, B), \pi(H|0, G), \pi(H|0, B)\}$.

We say that the Principal considers the Agent's output/ performance in period t as high/good when $y_t = H$ and low/bad when $y_t = L$, and that the Agent considers his own output/performance as high/good when $s_t = G$ and low/bad when $s_t = B$. Assumption 1 says that the Principal's evaluation is positively correlated with the Agent's effort. Assumption 2 requires that the correlation between Principal's and Agent's evaluation be positive correlated when $e_t = 1$ and that the Agent's evaluation be not "too informative" on the Principal's when $e_t = 0$.[2]

Since both players are risk neutral, were the Principal's signals contractible, the maximum total surplus could be achieved by a standard contract that pays the Agent a high wage when $y_t = H$ and a low wage when $y_t = L$. The problem here is that y_t is privately observed and non-verifiable. If the Principal were to pay the Agent less when he reports L, then he would have an incentive to always report L regardless of the true signal. In order to ensure the Principal reporting truthfully, any amount that the Principal does not pay the Agent when $y_t = L$ must be either destroyed or diverted to a use that does not benefit the Principal.

[1] Both y_t and s_t are uncorrelated over time.

[2] The first requirement is not restrictive as we can relabel the signals. The second requirement will hold if, for example, the Agent's evaluation correlates only with the Principal's evaluation and not with effort.

In this paper we call contracts that involve the Principal burning money "efficiency-wage" contracts since they resemble standard efficiency-wage contracts whereby workers are paid above-market wage until they are fired. Formally, an efficiency-wage contract $\omega\left(B, W, Z^T\right)$ contains a legally enforceable component (B, W) and an informal punishment agreement Z^T. The enforceable component stipulates that the Principal make a payment an up-front payment B before period 1 and a final payment $W \geq 0$ after period T.[3] The Agent will receive B in full. But the Principal reserves the right to deduct any amount $Z^T \leq W$ from the final payment and burn it in case he finds the Agent's overall performance unsatisfactory. The exact value of Z^T is governed by an informal punishment strategy $Z^T : \{H, L\}^T \rightarrow [0, W]$ that maps the Principal's information into an amount less than W. Note that the Principal has no incentive to renege on Z^T even though it is not legally enforceable.

In each period t, the Agent must decide whether to work. The Agent's history at date t for $t > 1$ consists of her effort choices and the sequence of signals observed in the previous $t-1$ periods, $h^t \equiv e^{t-1} \times s^{t-1}$, where $e^{t-1} \equiv (e_1, \ldots, e_{t-1})$ and $s^{t-1} \equiv (s_1, \ldots, s_t)$. Let H^t denote the set of all period-t histories. The Agent's history at the first period $h^1 = \emptyset$. A strategy for the Agent is a vector $\sigma \equiv (\sigma_1, ..., \sigma_T)$ where $\sigma_t : H^t \rightarrow \{0, 1\}$ is a function that determines the Agent's effort in period t.

Given contract $\omega\left(B, W, Z^T\right)$, a strategy σ induces a probability distribution over the effort and signal sequences e^T and y^T. Let

$$v(B, W, Z^T, \sigma) \equiv E\left(B + W - Z^T(y^T) + \sum_{t=1}^{T} \delta^{t-1} e_t \middle| \sigma \right).$$

be the Agent's expected payoff as a function of σ under contract $\omega(B, W, Z^T)$. An Agent's strategy σ^* is a best response against $\omega(B, W, Z^T)$ if for all strategies $\sigma \neq \sigma^*$,

$$v(B, W, Z^T, \sigma^*) \geq v(B, W, Z^T, \sigma).$$

The Agent accepts a contract $\omega\left(B, W, Z^T\right)$ if and only if there exists a best response σ^* against $\omega(B, W, Z^T)$ such that $v(B, W, Z^T, \sigma^*) \geq 0$.

A contract $\omega\left(B, W, Z^T\right)$ is optimal for the Principal if there exists an Agent's strategy σ such that $\left(B, W, Z^T, \sigma\right)$ is a solution to the following maximization problem:

$$\begin{aligned} & \max_{B,W,Z,\sigma} E\left(-B - W + \sum_{t=1}^{T} \delta^{t-1} e_t \middle| \sigma \right), \\ s.t. \quad & \sigma \in \arg\max v(B, W, Z^T, \sigma), \\ & v(B, W, Z^T, \sigma) \geq 0. \end{aligned}$$

The Agent works in every period according to σ if for all $t \in \{1, ..., T\}$ and all $h^t \in H^t$, $\sigma\left(h^t\right) = 1$. We say a contract ω induces maximum effort if working in every period (after any history) is a best response against ω. We say a contract is efficient in inducing maximum effort if it has the lowest money-burning loss among all contracts that induce maximum effort. We shall mostly focus on efficient maximum-effort contracts in the following. Such contracts are optimal when effort cost c is sufficiently small.

[3]Throughout, all payments regardless when they actually occur are in terms of present value evaluated at $t = 1$.

3 Optimal Efficiency-Wage Contract

A drawback of using money burning as a way to motivate the Agent is that a positive amount will be destroyed with positive probability even when the Agent works in every period. We can see this by considering the one-period case.

Proposition 1. *When $T = 1$, any contract that motivates the Agent to work must destroy an amount equal to $\pi(L|1)c/\left(\pi(L|0) - \pi(L|1)\right)$ or greater in expectation. It is optimal for the Principal to induce the Agent to work only if*

$$1 - c\left(1 + \frac{\pi(L|1)}{\pi(L|0) - \pi(L|1)}\right) \geq 0.$$

Proof. Working is a best response for the Agent (assuming that the contract has been accepted) if the sum of the effort and money-burning cost is lower when he works; that is, if

$$-\left(\pi(H|1)Z^1(H) + \pi(L|1)Z^1(L)\right) - c \geq -\left(\pi(H|0)Z^1(H) + \pi(L|0)Z^1(L)\right). \tag{1}$$

Minimizing the expected money-burning loss,

$$\pi(H|1)\, Z^1(H) + \pi(L|1)\, Z^1(L),$$

subject to (1) yields the solution

$$Z^{1*}(H) = 0 \text{ and } Z^{1*}(L) = \frac{c}{\pi(L|0) - \pi(L|1)}.$$

In this case, the expected money-burning loss is

$$C(Z^1) = \frac{\pi(L|1)c}{\pi(L|0) - \pi(L|1)}.$$

Since the Principal must compensate the Agent for both the effort and money-burning costs in order to induce the Agent to accept the contract, it is optimal for the Principal to induce the Agent to work if the expected output is greater than the sum of the effort and money-burning costs. □

MacLeod (2003) and Levin (2003) are the first to point out that, when evaluations are private, resources must be destroyed in order to motivate the Agent to exert effort. Fuchs (2007) shows that when $T > 1$ the Principal can save money-burning cost by linking the money-burning decisions across periods.

Define

$$\rho \equiv 1 - \frac{\pi(L|1, G)}{\pi(L|1)}$$

as the correlation coefficient of the Principal's and Agent's evaluations conditional on the Agent working. The coefficient is between 0 and 1. It equals 0 when the evaluations are uncorrelated and 1 when they are perfectly correlated. Let $\mathbf{L}^t$ denote a t-vector of L's.

Proposition 2. *When $T > 1$ and $\rho \leq 1 - \delta$, it is efficient to induce maximum effort through the punishment strategy*

$$\widehat{Z}^T(y^T) = \begin{cases} \left(\frac{c}{\pi(H|1)-\pi(H|0)}\right) \frac{1}{\pi(L|1)^{T-1}} & \text{if } y^T = \mathbf{L}^T, \\ 0 & \text{if } y^T \neq \mathbf{L}^T, \end{cases}$$

with money-burning cost $\pi(L|1)c/\left(\pi(H|1) - \pi(H|0)\right)$. It is optimal to induce maximum effort if

$$(1-c)\,\frac{1-\delta^T}{1-\delta} - \frac{\pi(L|1)c}{\pi(H|1) - \pi(H|0)} \geq 0.$$

Proof. See Appendix. □

Proposition 2 says that when the correlation between evaluations of the Principal and Agent are sufficiently low, the Principal should destroy resources only when his evaluations of the Agent are low in all T periods, and that, surprisingly, the money-burning loss is independent of T and always equal to the money-burning loss in the one-period case. This means that the optimal efficiency-wage contract is asymptotically efficient—as δ goes to zero and T to infinity, the per period money-burning loss converges to zero .

Fuchs (2007) proves Proposition 2 for the case $\rho = 0$. In that case, since the Agent is not learning anything about the Principal's evaluations over time, his dynamic decision problem is equivalent to a static one in which he is choosing whether to work in all T periods simultaneously. Hence, if the punishment is chosen such that it is not optimal for the Agent to shirk in only period 1, then it is not optimal to shirk in any single period. Furthermore, since the punishment is convex in the number of shirking periods, it is not optimal to shirk in multiple periods as well.

When $\rho > 0$, the Agent's problem cannot be treated as a static one. Consider the case $T = 2$. Any Z^2 that induces maximum effort must satisfy the following two incentive compatibility constraints:

$$\pi(H|1)(Z(LH) - Z(HH)) + \pi(L|1)(Z(LL) - Z(HL)) \geq \frac{c}{\pi\left(H|1\right) - \pi\left(H|0\right)}; \qquad (IC(e^0, s^0))$$

$$\pi(H|1,G)(Z(HL) - Z(HH)) + \pi(L|1,G)(Z(LL) - Z(LH)) \geq \frac{\delta c}{\pi\left(H|1\right) - \pi\left(H|0\right)}. \qquad (IC(1,G))$$

The first constraint requires that the Agent be better off working in both periods than working only in the second. The second constraint requires that the Agent be better off working in the second period after he has worked and observed a G signal in the first. It is straightforward to check that $\widehat{Z}^2$, while satisfying $IC\left(e^0, s^0\right)$, fails $IC\left(1, G\right)$ when $\rho > 1 - \delta$. Intuitively, when ρ is large, an Agent who has worked and received a G signal in the first period is quite sure that he has already passed the Principal's test and, hence, has little incentive to work in the second period. Since the Agent discounts the likelihood that $y_1 = L$ after a history of $(1, G)$, it is more effective for the Principal to motivate the Agent to work after $(1, G)$ through raising $Z\left(HL\right)$ than $Z\left(LL\right)$. As a result, an efficient maximum-effort strategy will no longer take the form of $\widehat{Z}^T$.

To determine the optimal contract scheme when $\rho > 1-\delta$, we first identify the lower bound on expected money-burning loss $C(Z^T)$ in a T-period contract. We next define a contract scheme $\bar{Z}^T$ with expected efficiency loss just equals the lower bound $C(Z^T)$.

For any $y^T \in Y^T$, let y^T_{-t} denote the Principal's signals in periods other than t. Let $\left(e^0, s^0\right)$ denote the null history for the Agent. Consider an Agent in period t, $t = 1, ..., T$, who has chosen e^{t-1} and observed s^{t-1} in the first $t-1$ periods, and who is planning to choose $e_k = 1$ in all future periods $k = t+1, ..., T$. His posterior belief that the outputs in periods other than t is y^T_{-t} is denoted by

$$\mu_t(y^T_{-t}|e^{t-1}, s^{t-1}) \equiv \prod_{k=1}^{t-1} \pi(y^T_k|e_k, s_k) \prod_{k=t+1}^{T} \pi(y^T_k|1).$$

His expected payoff if he works in period t and all subsequent periods is

$$B + W - \sum_{y^T \in Y^T} \mu_t(y^T_{-t}|e^{t-1}, s^{t-1})\pi(y^T_t|1)Z^T(y^T) - \sum_{k=1}^{t-1} e_k\delta^{k-1}c - \sum_{k=t}^{T} \delta^{k-1}c.$$

His expected payoff if he shirks in period t and works in all subsequent periods is

$$B + W - \sum_{y^T \in Y^T} \mu_t(y^T_{-t}|e^{t-1}, s^{t-1})\pi(y^T_t|0)Z^T(y^T) - \sum_{k=1}^{t-1} e_k\delta^{k-1}c - \sum_{k=t+1}^{T} \delta^{k-1}c.$$

The Agent, therefore, prefers working in all remaining periods to shirking in period t and working in all periods after t if

$$\sum_{y^T \in Y^T} \mu_t(y^T_{-t}|e^{t-1}, s^{t-1})I(y_t)Z^T(y^T) \geq \frac{\delta^{t-1}c}{\pi\left(H|1\right) - \pi\left(H|0\right)}, \tag{$IC(e^{t-1}, s^{t-1})$}$$

where

$$I(y_t) = \begin{cases} -1 & if \quad y_t = H, \\ 1 & if \quad y_t = L. \end{cases}$$

Let $\mathbf{1}^t$ denote a t-vector of 1's.

Lemma 1. *If Z^T induces maximum effort, then $IC(\mathbf{1}^{t-1}, s^{t-1})$ must hold for all $t = 1, ..., T$, and all $s^{t-1} \in \{G, B\}^{t-1}$.*

Proof. Obviously, it is optimal for the Agent to work in all T periods only if after working in the first t periods it is optimal to continue working in the remaining periods. □

Lemma 2. *Z^T induces maximum effort if $IC(e^{t-1}, s^{t-1})$ holds for all $t = 1, ..., T$, $e^{t-1} \in \{1, 0\}^{t-1}$, and $s^{t-1} \in \{G, B\}^{t-1}$.*

Proof. It is optimal for the Agent to work in period T after history (e^{T-1}, s^{T-1}) if $IC(e^{T-1}, s^{T-1})$ holds. Suppose starting from period $t+1$ it is optimal for the Agent to work in all remaining periods regardless of his effort choices and signals during the first t periods. Then, it would be optimal for the Agent to work in period t after history of (e^{t-1}, s^{t-1}) if $IC(e^{t-1}, s^{t-1})$ holds. The lemma is true by induction. □

Lemma 3. *Suppose Z^T induces maximum effort in a T-period contracting game. Then*

$$C(Z^T) \geq \frac{(\pi(L|1))c}{\pi(L|0) - \pi(L|1)} \left[\delta^{T-1} + \rho \sum_{t=1}^{T-1} \delta^{t-1} \right].$$

Proof. By Lemma 1, Z^T must satisfy $IC(e^0, s^0)$ which can be written as

$$\sum_{y_{-1}^T \in \{H,L\}^{T-1}} \left(\prod_{k=2}^{T} \pi(y_k|1) \right) \left[Z^T(L \circ y_{-1}^T) - Z^T(H \circ y_{-1}^T) \right] \geq \frac{c}{\pi(L|0) - \pi(L|1)}. \tag{2}$$

with

$$x \circ y_{-1}^T \equiv (x, y_2, ...y_T)$$

denoting the T-period history that starts with $x \in \{H, T\}$ following by $y_{-1}^T \equiv (y_2, ..., y_T)$.

Define a $T-1$ period agreement Z^{T-1} as follows. For all $y^{T-1} \in \{H, L\}^{T-1}$

$$Z^{T-1}(y^{T-1}) \equiv \frac{1}{\delta}[\pi(H|1, G)Z^T(H \circ y^{T-1}) + \pi(L|1, G)Z^T(L \circ y^{T-1})]. \tag{3}$$

An Agent who has worked and observed G in period 1 is effectively facing Z^{T-1} from period 2 onward. Since Z^T, by supposition, induces maximum effort, it must be a best response for the Agent to work in all subsequent periods after working and observing G in the first. It follows that Z^{T-1} must induces maximum effort in a $(T-1)$-period contracting game. Using (2) and (3), we have

$$\begin{aligned} C(Z^T) &= \sum_{y^T \in \{H,L\}^T} \left(\prod_{k=1}^{T} \pi(y_k|1) \right) Z^T(y^T) \qquad (4) \\ &= \sum_{y^{T-1} \in \{H,T\}^{T-1}} \left(\prod_{k=2}^{T} \pi(y_k|1) \right) \left(\pi(H|1, G)Z^T(H \circ y^{T-1}) + \pi(L|1, G)Z^T(L \circ y^{T-1}) \right) \\ &= \delta C(Z^{T-1}) + \rho\pi(L|1) \sum_{y^{T-1} \in Y^{T-1}} \left(\prod_{k=2}^{T} \pi(y_k|1) \right) (Z^T(H \circ y^{T-1}) - Z^T(L \circ y^{T-1})) \\ &\geq \delta C(Z^{T-1}) + \frac{\rho\pi(L|1)c}{\pi(L|0) - \pi(L|1)}. \end{aligned}$$

This shows that the proposition will hold for T if it holds for $T-1$. Since the proposition holds for $T = 1$, by induction it holds for all T. □

We now define a punishment strategy that is efficient in inducing maximum effort when $\rho > 1 - \delta$. Set $\overline{Z}^1 \equiv Z^{1*}$. For $T \geq 2$, define recursively

$$\overline{Z}^T(y^T) \equiv \begin{cases} \delta\overline{Z}^{T-1}(\mathbf{L}^{T-1}) + \frac{\pi(H|1,G)}{\pi(L|1)^{T-2}} \left(\frac{c}{\pi(L|0) - \pi(L|1)} \right) & \text{if } y_1 = H \text{ and } y_{-1}^T = \mathbf{L}^{T-1}, \\ \delta\overline{Z}^{T-1}(\mathbf{L}^{T-1}) - \frac{\pi(L|1,G)}{\pi(L|1)^{T-2}} \left(\frac{c}{\pi(L|0) - \pi(L|1)} \right) & \text{if } y_1 = L \text{ and } y_{-1}^T = \mathbf{L}^{T-1}, \\ \delta\overline{Z}^{T-1}\left(y_{-1}^{T-1}\right) & \text{if } y_{-1}^T \neq \mathbf{L}^{T-1}, \end{cases} \tag{5}$$

where $y_{-1}^T \equiv (y_2, ..., y_T)$ is the Principal's signals in periods other than 1, and $\mathbf{L}^{T-1}$ is a $t-1$ vector of L's. For example, when $T = 2$,

$$\begin{aligned}
\overline{Z}^2 (LL) &= \delta Z^{1*}(L) + \frac{\pi(H|1,G)}{\pi(L|1)} \left(\frac{c}{\pi(L|0) - \pi(L|1)} \right) = \frac{c}{\pi(L|0) - \pi(L|1)} \left(\delta + \frac{\pi(H|1,G)}{\pi(L|1)} \right), \\
\overline{Z}^2 (HL) &= \delta Z^{1*}(L) - \frac{\pi(L|1,G)}{\pi(L|1)} \left(\frac{c}{\pi(L|0) - \pi(L|1)} \right) = \frac{c}{\pi(L|0) - \pi(L|1)} \left(\delta - \frac{\pi(L|1,G)}{\pi(L|1)} \right), \\
\overline{Z}^2 (LH) &= \overline{Z}^2 (HH) = \delta Z^{1*}(H) = 0.
\end{aligned}$$

It is straightforward to verify that $\overline{Z}^T(y^T) \geq 0$ for all T and all y^T.

Proposition 3. *When $\rho > 1 - \delta$, it is efficient to induce maximum effort through the punishment strategy $\overline{Z}^T$. The money burning cost of $\overline{Z}^T$ is*

$$C\left(\overline{Z}^T\right) = \frac{(\pi(L|1))c}{\pi(L|0) - \pi(L|1)} \left(\delta^{T-1} + \rho \sum_{t=1}^{T-1} \delta^{t-1} \right).$$

In addition, when $T = 2$, any punishment strategy that induces maximum effort has a strictly higher money-burning cost than Z^{*2}.

$\overline{Z}^T$ depends only on the time the Principal last observes a H signal. The Agent will receive the same compensation whether the Principal receives a G signal in every period or just the last period. More generally, his compensation will be higher when the last G signal is closer to the end of the game. For any $y^T, \tilde{y}^T \in \{H, L\}^T$

$$\overline{Z}^T(y^T) > (=) \overline{Z}^T(\tilde{y}^T) \text{ iff } \max(t|y_t = H) < (=) \max(t|\tilde{y}_t = H). \tag{6}$$

$\overline{Z}^T$ is more complex compared to $\widehat{Z}^T$. Whereas to implement $\widehat{Z}^T$ the Principal needs to know only whether any H signal has occurred, he needs to know the last time time a H signal occurred in order to implement $\overline{Z}^T$. The extra complication is needed in order to overcome the "learning problem" we mentioned earlier. The difference between $\overline{Z}^T$ and $\widehat{Z}^T$ diminishes as ρ converges to $1 - \delta$ (from above).

Proposition 4. *$\overline{Z}^T$ converges to $\widehat{Z}^T$ as the correlation coefficient decreases. That is, as $\rho \to 1 - \delta$,*

$$\lim_{\rho \to 1-\delta} \overline{Z}(y^T) = \begin{cases} \frac{c}{(\pi(L|0) - \pi(L|1))(\pi(L|1))^{T-1}} & \textit{if } y_t^T = L \;\; \forall t = 1, \ldots, T; \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$

An interesting feature of $\overline{Z}^T$ is that it rewards improvements in performance. Since under $\overline{Z}^T$ the Agent's compensation depends only on the time a H signal last occurs, an Agent with poor performance evaluations in the past will obtain a greater benefit for performing well in the future than an Agent whose past evaluations are better. There are two forces at work here. In order to prevent an Agent who has received a string of G signal in the earlier periods from shirking, the Principal needs to threaten to punish the Agent if his current evaluation is poor even when his past evaluations have been good. But since punishment is costly, he will forgive the punishment if the Agent performs well in the future. The need to reward improvements means that any punishment strategies that is either linear performance evaluations or depends only on the total number of high evaluations are unlikely to be efficient in inducing maximum efforts.

Proposition 5. *The expected cost $C(Z^T)$ increases with the the correlation between the Principal's and Agent's evaluations. It converges to $C(Z^1)\sum_{t=1}^{T}\delta^{t-1}$ as $\rho \to 1$ and $C(Z^1)$ as $\rho \to 1-\delta$.*

Thus, while the contract that uses only the Principal's evaluation in determining the Agent's compensation is optimal when the Agent is only moderately informed of the Principal's evaluations, it may not be optimal when the Agent's private information is quite informative. In this case, inducing maximum effort becomes extremely expensive. The Agent expects that the punishment cost would likely to be small conditional on observing G, and would likely to be large conditional on observing B. This is gives her more freedom in devising profitable shirking strategies. In particular, there exists information path where the Agent expects the the likelihood of bad evaluations by the Principal to be extremely low. To induce maximum effort at low probability situations requires extremely large punishment, which results in larger expected cost as the correlation increases.

4 Contract with Infinite Horizon

In reality the Principal does not burn cash in real. What the Principal can do is terminating the contract that generates surplus for the Agent. We consider T-period review contract as follows. The Principal pays a fixed wage w to the Agent every period, and the Agent exerts effort every period. At the end of the predetermined T periods, the Principal terminates the employment contract with probability $\psi(y^T)$ given the sequence of evaluations in the T periods. If the Agent is not fired, he continues working for the Principal with a clear record from period $T+1$ on.

With the T-period review contract, the Agent's expected payoff equals

$$v = \sum_{t=1}^{T}\delta^{t-1}w + \delta^T v - \delta^T \sum_{y^T\in Y^T}\mu(y^T_{-t}|e^{t-1},s^{t-1})\pi(y^T_t|1)\psi(y^T)v - \sum_{k=1}^{t-1}\delta^{k-1}c - \sum_{k=t}^{T}\delta^{k-1}c.$$

if he exerts effort in period t and all subsequent periods. Meanwhile, if he exerts effort in all remaining periods except period t, his expected payoff is

$$v' = \sum_{t=1}^{T}\delta^{t-1}w + \delta^T v - \delta^T \sum_{y^T\in Y^T}\mu(y^T_{-t}|e^{t-1},s^{t-1})\pi(y^T_t|0)\psi(y^T)v - \sum_{k=1}^{t-1}\delta^{k-1}c - \sum_{k=t+1}^{T}\delta^{k-1}c$$

The agent prefers exerting effort in period t and all subsequent periods to shirking in period t and exerting effort in periods after t if

$$\delta^{T-t+1}\sum_{y^T\in Y^T}\mu_t(y^T_{-t}|e^{t-1},s^{t-1})I(y^T_t)\psi(y^T)v \geq \frac{c}{p-q}. \tag{8}$$

In what follows we divide our discussion into two parts, depending upon the correlation coefficient ρ. If $\rho \leq 1-\delta$, we define

$$\Gamma^0(T) = \frac{(w-c)}{(1-\delta)} - \frac{(1-p)c}{p-q}\frac{1}{1-\delta^T}.$$

We may simply write it as Γ^0, while keeping in mind that it is a function of T instead of a fixed number. And we let

$$\psi(y^T) = \begin{cases} \frac{\delta^{-T}c}{(1-p)^{T-1}(p-q)\Gamma^0} & \text{if } y_t^T = L \;\forall\, t, \\ 0 \quad otherwise. \end{cases} \tag{9}$$

Proposition 6. *When $\rho \leq 1-\delta$, given the efficiency wage contract, if it is not profitable for the Agent to shirk in the first period, he has no incentives to shirk at all.*

Proof. See Appendix. □

Hence, if the expected payoff to the Agent v is such that $v \geq \Gamma^0$, the efficiency wage contract induces maximum effort every period before the Agent is fired. In equilibrium, it turns out that $v = \Gamma^0$.

Corollary 1. *When $\rho \leq 1-\delta$, the per period efficiency loss decreases in T.*

Proof. See Appendix. □

When $\rho > 1-\delta$, this efficiency wage contract may not induce maximum efforts because of the learning problem. As discussed before, when $\rho = 1$, dynamic contract does not improve upon one period contract. However, when $\rho < 1$, linking the punishment across period still leads to improvement.

In this case, we define $\Gamma'(T)$ as

$$\Gamma'(T) = \frac{(w-c)}{(1-\delta)} - \frac{(1-p)c}{p-q}\frac{(\rho\sum_{t=1}^{T-1}\delta^{t-1} + \delta^{T-1})}{1-\delta^T}.$$

Again, we will write $\Gamma'(T)$ as Γ'. When $\rho > 1-\delta$, we let

$$\psi(y^T) = \begin{cases} \frac{\delta^{-T}c}{(p-q)\Gamma'}\left[\frac{1}{(1-p)^{T-1}} + (\rho+\delta-1)\sum_{t=1}^{T-1}\delta^{T-1-t}(1-p)^{1-t}\right] & \text{if } y_t^T = L \;\forall t \\ \frac{\delta^{-T}c}{(p-q)\Gamma'}\left[(\rho+\delta-1)\sum_{t=1}^{T-\bar{t}}\delta^{T-1-t}(1-p)^{1-t}\right] & \text{if } y_T^T = L, \bar{t} = \max(t|y_t^T = H) \\ 0 \quad \text{if } y_T^T = H. \end{cases} \tag{10}$$

As in the previous case, it turns out that the ex ante expected payoff for the Agent payoff equals Γ' in equilibrium.

Lemma 4. *If the Agent exerts effort every period before he is fired, his expected payoff $v = \Gamma'$.*

If we let $\tilde{W}^T(y^T) \equiv \psi(y^T)v$ be the loss of continuation payoff the Agent suffers given the Principal's evaluations y^T, then the previous result indicates

$$\tilde{W}^T(y^T) = \begin{cases} \frac{\delta^{-T}c}{(p-q)} \left[\frac{1}{(1-p)^{T-1}} + (\rho+\delta-1)\sum_{t=1}^{T-1}\delta^{T-1-t}(1-p)^{1-t}\right] & \text{if } y_t^T = L\ \forall t \\ \frac{\delta^{-T}c}{(p-q)} \left[(\rho+\delta-1)\sum_{t=1}^{T-\bar{t}}\delta^{T-1-t}(1-p)^{1-t}\right] & \text{if } y_T^T = L, \bar{t} = \max(t|y_t^T = H) \\ 0 \quad \text{if } y_T^T = H. \end{cases} \tag{11}$$

Note that there is a one-to-one correspondence between $W^T(y^T)$ here and the punishment $\bar{Z}^T(y^T)$ in Section 3.

Proposition 7. *The efficiency wage contract $\psi(y^T)$ in (10) induces maximum effort from the Agent in any period before he is fired.*

Proof. Note that given the expected loss in continuation payoff $\tilde{W}(y^T)$, when $T = 1$, the Agent has no incentives to shirk, as

$$\delta\tilde{W}^1(L) = \frac{c}{p-q}.$$

Now suppose that the result is true for T, we show it is also true for $T+1$. By construction $\tilde{W}^{T+1}(y^{T+1})$ satisfies the Agent's IC at $t = 1$,

$$\delta^{T+1}(1-p)^T[\tilde{W}^{T+1}(L^{T+1}) - \tilde{W}^{T+1}(H \circ L^T)] \geq \frac{c}{p-q}.$$

For $t \geq 2$ and $\left(e^{t-1}, s^{t-1}\right) \in \{1,0\}^{t-1} \times \{G,B\}^{t-1}$,

$$\begin{aligned} &\sum_{y^{T+1}\in\{H,L\}^{T+1}} \left(\mu_{-t}\left(y_{-t}^{T+1}|e^{t-1}, s^{t-1}\right) - \mu_{-t}\left(y_{-t}^T|1\circ e_{-1}^{t-1}, G\circ s_{-1}^{t-1}\right)\right) I\left(y_t\right)\tilde{W}^{T+1}(y^{T+1}) \\ = &\left(\prod_{k=2}^{t-1}\pi(L|e_k,s_k)\right)\pi(L|1)^{T-t+1}\left(\pi\left(L|e_1,s_1\right) - \pi\left(L|1,G\right)\right)[\tilde{W}^{T+1}(L^{T+1}) - \tilde{W}^{T+1}(H\circ L^T)] \\ \geq\ & 0. \end{aligned}$$

This implies that for all $t \geq 2$ and for all $\left(e^{t-1}, s^{t-1}\right)$, the left-hand side of $IC\left(e^{t-1}, s^{t-1}\right)$ is greater than the left-hand side of $IC\left(1\circ e_{-1}^{t-1}, G\circ s_{-1}^{t-1}\right)$.

By assumption $\tilde{W}^T(y^T)$ induces maximum effort every period when the Agent is reviewed every T periods,

$$\delta^{T-t+1}\sum_{y^T\in Y^T}\left(\prod_{k=1}^{t-1}\pi(y_k^T|e_k,s_k)\right) I(y_t^T)\pi(L|1)^{T-t+1}\tilde{W}^T(y^T) \geq \frac{c}{p-q}.$$

This implies that

$$\sum_{y^T\in Y^T}\left(\prod_{k=2}^{t}\pi(y_k^{T+1}|e_k,s_k)\right)I(y_t^T)\pi(L|1)^{T-t+1}[\pi(H|1,G)\tilde{W}^{T+1}(H\circ y^T)+\pi(L|1,G)\tilde{W}^{T+1}(L\circ y^T)] \tag{12}$$

$$= \sum_{y^T\in Y^T}\left(\prod_{k=1}^{t-1}\pi(y_k^{T}|e_k,s_k)\right)I(y_t^T)\pi(L|1)^{T-t+1}\tilde{W}^{T}(y^T)$$

$$\geq \frac{\delta^{-T-1+t}c}{p-q}$$

Thus, the Proposition will hold for $T+1$ if it holds for T.

To see the equality in (12), we note that when $y^T \neq L^T$,

$$\tilde{W}^{T+1}(H\circ y^T)=\tilde{W}^{T+1}(L\circ y^T)=\frac{\delta^{-T-1}c}{(p-q)}\left[(\rho+\delta-1)\sum_{t=1}^{T-\bar{t}}\delta^{T-t}(1-p)^{1-t}\right].$$

Thus,

$$\pi(H|1,G)\tilde{W}^{T+1}(H\circ y^T)+\pi(L|1,G)\tilde{W}^{T+1}(L\circ y^T)=\frac{\delta^{-T-1}c}{(p-q)}\left[(\rho+\delta-1)\sum_{t=1}^{T-\bar{t}}\delta^{T-t}(1-p)^{1-t}\right]$$
$$=\tilde{W}^{T}(y^T).$$

When $y^T = L^T$,

$$\pi(H|1,G)\tilde{W}^{T+1}(H\circ L^T)+\pi(L|1,G)\tilde{W}^{T+1}(L\circ L^T)$$
$$=\frac{\delta^{-T}c}{p-q}\left\{\frac{1}{(1-p)^{T-1}}\left[\frac{\delta^{-1}\pi(L|1,G)}{\pi(L|1)}+\delta^{-1}(\rho+\delta-1)\right]+(\rho+\delta-1)\sum_{t=1}^{T-1}\delta^{T-t-1}(1-p)^{1-t}\right\}$$
$$=\tilde{W}^{T}(L^T).$$

Hence, we have demonstrated that the Proposition holds for $T=1$, and it holds for $T+1$ if it holds for T. This concludes the proof that it true for all T. □

Having a longer review phase is clearly optimal with the review contract. However, the length of review phase T is bounded by the need to have $\psi(y^T)\leq 1$ given any sequence of evaluations $y^T \in Y^T$.

Proposition 8. *The optimal review length T is*

i Increasing in δ;

ii Decreasing in q and c;

ii Equal to one if $p=1$.

This result is in line with Fuchs (2007). As the Agent becomes more patient, δ increases, future payoff is more valued and there is more cash to burn. A longer review phase can be used.

5 Self Evaluation

It is common practice for supervisors and subordinates to exchange opinions during periodic performance appraisals. Under our set-up, the Principal will have no incentive to reveal his signals to the Agent. Here we consider one-sided communication from the Agent to the Principal. Specifically, we assume that at the end of each period t after the realization of s_t, the Agent sends the Principal a message m_t from a message set M_t that is sufficiently rich to encompass the Agent's private information at that time. The Agent's history at date t for $t > 1$ now includes the messages he sent, as well as his effort choices and private evaluations observed in the previous $t-1$ periods. A message strategy is a vector $\rho \equiv (\rho_1, ..., \rho_T)$ where $\rho_t : H^t \to M_t$ is the Agent's period-t message strategy. By the end of period T, the Principal will have observed T messages $m^T \equiv (m_1, ..., m_T)$ in addition to his T private signals $y^T \equiv (y_1, \ldots, y_T,)$. A punishment strategy for the Principal is now $Z^T : \{H, L\}^T \times \{M_t\}_{t=1}^T \to [0, W]$. An Agent's strategy (σ^*, ρ^*) is a best response against $\omega(B, W, Z^T)$ if for all strategies (σ, ρ),

$$v(B, W, Z^T, \sigma^*, \rho^*) \geq v(B, W, Z^T, \sigma, \rho).$$

Proposition 9. *When $T = 1$, the optimal contract is*

$$\begin{aligned} Z(L,G) &= Z(L,B) = \frac{c}{\pi(L|0) - \pi(L|1)}, \\ Z(H,G) &= Z(H,B) = 0. \end{aligned}$$

Proof. The optimal contract solves the optimization problem

$$\min_{y \in \{H,L\}, s \in \{G,B\}} C(Z^1) \equiv \pi(y, s|1) Z(y, s)$$

subject to the following constraints:

i. The Agent's incentive to report truthfully conditional on $(e = 1, G)$

$$\pi(L|1,G)Z(L,B) + \pi(H|1,G)Z(H,B) \geq \pi(L|1,G)Z(L,G) + \pi(H|1,G)Z(H,G), \tag{13}$$

ii. the Agent's incentive to report truthfully conditional $(e = 1, B)$

$$\pi(L|1,B)Z(L,G) + \pi(H|1,B)Z(H,G) \geq \pi(L|1,B)Z(L,B) + \pi(H|1,B)Z(H,B), \tag{14}$$

iii. the Agent's incentive constraint (IC) not to shirk and report G

$$\begin{aligned} &\pi(L|0)Z(L,G) + \pi(H|0)Z(H,G) - \pi(L,G|1)Z(L,G) - \pi(L,B|1)Z(L,B) - \\ &\pi(H,G|1)Z(H,G) - \pi(H,B|1)Z(H,B) \\ &\geq c, \end{aligned} \tag{15}$$

iv. the Agent's IC not to shirk and report B

$$\begin{aligned} &\pi(L|0)Z(L,B) + \pi(H|0)Z(H,B) - \pi(L,G|1)Z(L,G) - \pi(L,B|1)Z(L,B) - \\ &\pi(H,G|1)Z(H,G) - \pi(H,B|1)Z(H,B) \\ &\geq c. \end{aligned} \tag{16}$$

Solving the minimization problem subject to the four constraints gives

$$Z(H,B) = Z(H,G) = 0, \qquad Z(L,B) = Z(L,G) = \frac{c}{\pi(L|0) - \pi(L|1)}.$$

$\square$

When $T = 1$, communication does not bring about improvement as the expected money-burning loss remains $C(Z^1) = \pi(L|1)c/(\pi(L|0) - \pi(L|1))$. It turns out this remains true for $T > 1$ when correlation is not very high.

Proposition 10. *The no communication contract is optimal among all communication contracts when* $\pi(L|0) > \pi(L|1,B)$.

We establish the proposition is two steps.

Lemma 5. *Consider the minimization problem*

$$\min_{q(H),q(L)} \pi(L|1,B)q(L) + \pi(H|1,B)q(H)$$

such that

$$\begin{aligned} \pi(H|1,G)q(H) + \pi(L|1,G)q(L) &\geq \lambda, \\ (\pi(H|0) - \pi(H,B|1))q(H) + (\pi(L|0) - \pi(L,B|1))q(L) &\geq c + \pi(G|1)\lambda. \end{aligned}$$

Suppose $\pi(L|0) > \pi(L|1,B)$. *The solution to this problem* q^* *satisfies the equation*

$$\begin{aligned} \pi(H|1,B)q(H) + \pi(L|1,B)q(L) &= \frac{(\pi(L|1,B) - \pi(L|1,G))c}{\pi(L|0) - \pi(L|1)} + \lambda, \\ q(L) - q(H) &= \frac{c}{\pi(L|0) - \pi(L|1)}. \end{aligned}$$

Proof. Note that

$$\frac{\pi(L|0) - \pi(L,B|1)}{\pi(H|0) - \pi(H,B|1)} > \frac{\pi(L|1,B)}{\pi(H|1,B)} > \frac{\pi(L|1,G)}{\pi(H|1,G)}.$$

(The first inequality follows from $\pi(L|0) > \pi(L|1,B)$.) It is straightforward to show that both constraints are binding at the optimal solution.

In this case, we have

$$\pi(H|1,G)q(H) + \pi(L|1,G)q(L) = \lambda,$$

$$(\pi(H|0) - \pi(H,B|1))q(H) + (\pi(L|0) - \pi(L,B|1))q(L) = c + \pi(G|1)\lambda.$$

Solving the equation system yields

$$q(H) = \frac{-\pi(L|1,G)c}{\pi(L|0) - \pi(L|1)} + \lambda, q(H) = \frac{\pi(H|1,G)c}{\pi(L|0) - \pi(L|1)} + \lambda.$$

$\square$

Lemma 6. *Suppose the minimum efficiency loss in the T period contracting game is C^T. Then the minimum efficiency loss in the $T+1$ period game is*

$$\delta C^T + \frac{\rho\pi(L|1)c}{\pi(L|0)-\pi(L|1)}.$$

Proof. Define for $y_1 \in \{H, L\}$ and $\widehat{s}_1\{G, B\}$

$$Q(y_1, \widehat{s}_1) \equiv \sum_{\widetilde{y}^T}\sum_{\widetilde{s}^T}\prod_{t=1}^{T}\pi(\widetilde{y}_t, \widetilde{s}_t|1)Z^{T+1}(y_1 \circ \widetilde{y}^T, 1^{T+1}, \widehat{s}_1 \circ \widetilde{s}^T).$$

$Q(y_1, \widehat{s}_1)$ is expected amount of money burnt if the period 1's output is y_1, and the Agent reports $(1, \widehat{s}_1)$ in the first period and exert effort and reports truthfully in all subsequent periods.

Note that an Agent who has exerted effort, received a G signal and reported truthfully in the first period is effectively facing the strategy

$$\pi(H|1,G)Z^{T+1}(H \circ y^T, 1 \circ \widehat{e}^T, G \circ \widehat{s}^T) + \pi(L|1,G)Z^{T+1}(L \circ y^T, 1 \circ \widehat{e}^T, G \circ \widehat{s}^T) \tag{17}$$

from period two onwards. It follows that

$$\pi(H|1,G)Q(H,G) + \pi(L|1,G)Q(L,G) \geq \delta C^T. \tag{18}$$

Incentive compatibility requires that at the end period 1 the Agent, conditional on $(e_1, s_1) = (1, G)$ prefers following the equilibrium strategy to reporting $(1, B)$ in that period and exerting effort and reporting honestly in all subsequent periods. This requires that

$$\pi(H|1,G)Q(H,B) + \pi(L|1,G)Q(L,B) \geq \pi(H|1,G)Q(H,G) + \pi(L,1,G)Q(L,G). \tag{19}$$

Inequalities (18) and (19) jointly implies

$$\pi(H|1,G)Q(H,B) + \pi(L|1,G)Q(L,B) \geq \delta C^T \tag{20}$$

In period 1, the Agent must prefer the equilibrium strategy to the strategy of shirking and reporting $(1, B)$ in period 1, followed by working and reporting truthfully in future periods. This requires that

$$\begin{aligned}&(\pi(H|0)Q(H,B) + \pi(L|0)Q(L,B)) - \\ &\quad(\pi(H,G|1)Q(H,G) + \pi(L,G|1)Q(L,G) + \pi(H,B|1)Q(H,B) + \pi(L,B|1)Q(L,B)) \geq c.\end{aligned} \tag{21}$$

Using (18) and rearranging terms, we have

$$\begin{aligned}&(\pi(H|0) - \pi(H,B|1))Q(H,B) + (\pi(L|0) - \pi(L,B|1))Q(L,B) \\ \geq\ & c + \pi(G|1)\delta C^T.\end{aligned} \tag{22}$$

With the two conditions, (20) and (23), it follows from Lemma 5 that

$$\pi(H|1,B)Q(H,B) + \pi(L|1,B)Q(L,B) \geq \delta C^T + \frac{(\pi(L|1,B) - \pi(L|1,G))c}{\pi(L|0) - \pi(L|1)}. \tag{23}$$

Combining conditions (18) and (23) gives

$$
\begin{aligned}
C(Z^{T+1}) =& \sum_{y_1 \in \{H,L\}, \widehat{s}_1 \in \{G,B\}} \pi(y,s|1) Q(y_1, \widehat{s}_1) \\
\geq& \delta C^T(\pi(B|1) + \pi(G|1)) + \frac{\pi(B|1)(\pi(L|1,B) - \pi(L|1,G))c}{\pi(L|0) - \pi(L|1)} \\
=& \delta C^T + \frac{(\pi(L|1) - \pi(L|1,G))c}{\pi(L|0) - \pi(L|1)} \\
=& \delta C^T + \frac{\rho \pi(L|1) c}{\pi(L|0) - \pi(L|1)}.
\end{aligned} \tag{24}
$$

□

Hence, when correlation of the Principal's evaluation and the Agent's self-evaluations is not high, $\pi(L|0) > \pi(L|1, B)$, the lower bound of expected efficiency loss is identical to that in Section 3 when own evaluations of the Agent are not used. Communication does not improve efficiency in this case.

Two caveats need to be applied. First, this result holds true only when correlation is not very high. When correlation of evaluations is high enough, the per period efficiency loss could be made arbitrarily small (approximate efficiency). Previously in a repeated game setting, Zheng (2008) has demonstrated how to obtain an approximate efficiency result when correlation of private informations of players is high. Similar trick can be applied here to get the same result. Second, we are not allowing the Principal to make transfers here. However, if transfer is allowed, communication will improve upon the no communication contract as we show in the Appendix.

6 Discussion

When correlation is high, dynamic contract that requires the Agent exert effort every period does not improve on static contract much, as we show previously. However, contracts that allow the Agent to shirk some time will fare better. It may get approximate efficiency, i.e., the per period cash-burning approaches zero as T goes to infinity. For example, such a contract, which we shall refer to as the unrestricted, can be structured such that, after a long sequence of good performance (Hs), the Agent will not be punished for a few bad outcomes (Ls). This essentially allows the Agent to shirk for a few periods after exerting effort and observing Gs for many periods.

The intuition for this result is as follows. The Principal essentially faces a trade-off between providing enough incentives to the Agent to exert effort along some small probability event and reduce the punishment cost. When $\rho > 1 - \delta$, providing incentives along the information path where the Agent observes a long sequence of good signals requires extremely high punishment cost because of the learning problem. As the chance of Agent observing a very long sequence of good signals is extremely small, the expected benefit for the Principal from inducing effort in such events is minimal; nonetheless, the expected cost for the Principal from inducing effort is huge. In this sense, it is not economical for the Principal to induce effort along information path where a long sequence of good outcome has been observed. The Principal would be better off by allowing the Agent to shirk in such small probability events.

Under the contract discussed in the previous two Sections, however, we restrict attention to contracts that inducing the Agent's effort every period, which explains the result of the increasing expected efficiency loss as correlation goes up. When unrestricted contract is allowed, the Principal can trade off punishment cost for shirking in events a long sequence of good outcomes has been observed. Though shirking is costly for the Principal, however, given the extremely low chance of its occurrence, the expected cost to the Principal will be negligible. Hence, the Principal will be much better off by allowing it to happen.

Appendix

Appendix A. Some Proofs in Section 3

Proof of Proposition 2. We demonstrate the first part of the Proposition in two steps. First, we show that given the punishment strategy $\widehat{Z}^T(y^T)$, the Agent has no incentives to shirk in the first period.

The Agent has no incentives to shirk in $t = 1$ and exerting effort in the subsequent periods if

$$\pi(L|1)^{T-1}(\pi(L|0) - \pi(L|1))\widehat{Z}^T(L^T) \geq c,$$

which holds given the construction of $\widehat{Z}^T(L^T)$ above.

Next, we show that if it it not profitable to shirk in the first period, the Agent has no incentives to shirk at all. The Agent has no incentive to deviate at period $t > 1$ at all if

$$\prod_{k=1}^{t-1} \pi(L|e^{t-1}, s_k^{t-1}) \prod_{k=t+1}^{T} \pi(L|e_k)\widehat{Z}^T(L^T) \geq \frac{\delta^{t-1}c}{p-q}. \tag{25}$$

Under Assumption 2 and the condition $\rho \leq \underline{\rho}(\delta)$,

$$\pi(L|e^{t-1}, s_k^{t-1}) \geq \delta(1-p) \qquad \forall \ e^{t-1} \quad \text{and } \forall s_k^{t-1}.$$

This implies

$$\prod_{k=1}^{t-1} \pi(L|e^{t-1}, s_k^{t-1}) \prod_{k=t+1}^{T} \pi(L|e_k)\widehat{Z}^T(L^T) \geq \delta^{t-1}(1-p)^{T-1}\widehat{Z}^T(L^T) \geq \frac{\delta^{t-1}c}{p-q}$$

if the Agent has no incentives to shirk in the first period.

In this case, the expected money-burning loss is

$$C(Z^T) = \frac{\pi(L|1)c}{\pi(L|0) - \pi(L|1)}.$$

Hence, it is optimal to induce maximum effort if

$$(1-c)\frac{1-\delta^T}{1-\delta} - \frac{\pi(L|1)c}{\pi(H|1) - \pi(H|0)} \geq 0.$$

□

Lemma 7. *If Z^T is efficient in inducing maximum effort in a T -period contracting game, then Z^{T-1} constructed from Z^T according to (3) must be efficient in inducing maximum effort in a $T-1$-period contracting game.*

Proof. By Proposition 3 that $\overline{Z}^T$ is efficient in inducing maximum effort for any $T \geq 1$, and, furthermore,

$$C\left(\overline{Z}^T\right) = \delta C\left(\overline{Z}^{T-1}\right) + \frac{\rho(\pi(L|1))c}{\pi(H|1) - \pi(H|0)}.$$

Following the argument in Lemma 3, we can write

$$\begin{aligned} C\left(Z^T\right) &= \delta C\left(Z^{T-1}\right) + \rho(\pi(L|1)) \sum_{y^{T-1} \in Y^{T-1}} \left(\prod_{k=2}^{T} \pi(y_k|1)\right) \left(Z^T(H \circ y^{T-1}) - Z^T(L \circ y^{T-1})\right) \\ &\geq \delta C\left(Z^{T-1}\right) + \frac{\rho(\pi(L|1))c}{\pi(H|1) - \pi(H|0)}. \end{aligned}$$

The last inequality follows from $IC\left(e^0, s^0\right)$. Since Z^T is efficient, $C\left(Z^T\right) \leq C\left(\overline{Z}^T\right)$. But $C\left(Z^{T-1}\right) \geq C\left(\overline{Z}^{T-1}\right)$ as $\overline{Z}^{T-1}$ is inefficient. It follows that

$$C\left(Z^{T-1}\right) = C\left(\overline{Z}^{T-1}\right).$$

We have already seen that in the two-period case any strategy Z^2 where $Z^2(HH)$ or $Z^2(LH)$ is strictly positive must be inefficient. Any Z^T where $Z^T(y^T) > 0$ for some y^T such that $y^T_T = H$ would imply that.

Proof of Proposition 6. The Agent has no incentive to shirk in $t = 1$ and exert effort in subsequent periods if

$$\delta^T (1-p)^{T-1} \psi(L^T) v = \frac{c}{p-q} \frac{v}{\Gamma^0} \geq \frac{c}{p-q}.$$

The Agent exerts effort in this period if $\Gamma^0 \leq v$.

The Agent has no incentive to deviate at period $t > 1$ at all if

$$\delta^{T-t+1} \prod_{k=1}^{t-1} \pi(L|e_k, s_k) \prod_{k=t+1}^{T} \pi(L|e_k) \psi(y^T) v \geq \frac{c}{p-q}. \tag{26}$$

Under Assumption 2 and the condition $\rho \leq 1 - \delta$,

$$\delta^{-1} \pi(L|e_k, s_k) \geq (1-p) \qquad \forall\ e_k \quad \text{and} \ \forall s_k.$$

This implies

$$\delta^{T-t+1} \prod_{k=1}^{t-1} \pi(L|e_k, s_k) \prod_{k=t+1}^{T} \pi(L|e_k) \psi(L^T) v \geq \delta^T (1-p)^{T-1} \psi(L^T) v \geq \frac{c}{p-q}$$

if the Agent has no incentives to shirk in the first period, i.e., $\Gamma^0 \leq v$. $\square$

Proof of Corollary 1. Given the probability of firing in (9), the expected payoff for the Agent from exerting effort every period when he is employed equals

$$v = \frac{(1-\delta^T)(w-c)}{1-\delta} + \delta^T v - \delta^T (1-p)^T \psi(L^T) v.$$

Rearranging terms and simplifying yield

$$v = \frac{w-c}{1-\delta} - \frac{1}{1-\delta^T}\frac{(1-p)c}{(p-q)} = \frac{1}{1-\delta}\left[w - c - \frac{1}{1+\delta+\cdots+\delta^{T-1}}\frac{(1-p)c}{(p-q)}\right].$$

□

Appendix B. Communication with Transfer

We let

$$\lambda \equiv \max\left\{\frac{\pi(L|1,G)c}{\pi(G|1)[\min\{\pi(L|1,B),\pi(L|0,G),\pi(L|0,B)\} - \pi(L|1,G)]}, \frac{c}{\pi(L|0)-\pi(L|1)}\right\}. \tag{27}$$

And also define

$$Z^T(\hat{s}^T, y^T) = \begin{cases} \lambda \prod_{t=1}^{T-1} \phi_t(y_t, \hat{s}_t) & \text{if } y_T = L \\ 0 & \text{if } y_T = H, \end{cases} \tag{28}$$

where

$$\phi_t(y_t, \hat{s}_t) = \begin{cases} \frac{1}{\pi(L|1,G)} & \text{if } y_t = L,\ \hat{s}_t = G \\ 0 & \text{if } y_t = H,\ \hat{s}_t = G \\ 1 & \text{if } \hat{s}_t = B. \end{cases} \tag{29}$$

Whenever the agent report a good signal "G" for a period $t \in \{1, 2, \ldots, T-1\}$, she is rewarded with a bonus b_t,

$$b_t(\hat{s}_t) = \begin{cases} \frac{c}{\pi(G|1)} & \text{if } \hat{s}_t = G \\ 0 & \text{if } \hat{s}_t = B. \end{cases} \tag{30}$$

The agent gets no bonus in period T, that is, $b_T(\hat{s}_T = 0$ irrespective her reported signal $\hat{s}_T$. By construction, the agent's report $\hat{s}_T$ for the last period does not affect the money to be burnt Z^T.

Given the construction of ϕ, we note that conditional on $e_t = 1$ and truthful reporting, $\hat{s}_t = s_t$, the expected value of ϕ equals one,

$$E[\phi_t(y_t, s_t)|e_t = 1] = \pi(L, G|1) \cdot \frac{1}{\pi(L|1,G)} + \pi(H, G|1) \cdot 0 + \pi(B|1) \cdot 1 = 1. \tag{31}$$

Therefore, even if the Agent can learn about the evaluation y_t of the Principal, she can not learn about the expected punishment $C(Z^T)$ at any date t before the end of the T periods. We have an "effective independence."

However, for any $(e_t, \hat{s}_t) \neq (1, s_t)$, the expected value is greater than one. We summarize this result into the following lemma.

Lemma 8. *For any* $(e_t, \hat{s}_t) \neq (1, s_t)$*,*

$$E[\phi_t(y_t, \hat{s}_t)|e_t, s_t] \geq 1.$$

Proof. This result follows from our construction of $\phi(y_t, \hat{s}_t)$. Conditional on $(e_t = 1, s_t = B)$, the expected value of ϕ_t would be

$$E[\phi_t(y_t, \hat{s}_t)|e_t, s_t] = \frac{\pi(L|1, B)}{\pi(L|1, G)} > 1$$

if the agent reports $\hat{s}_t = G$. Conditional on $(e_t = 0, s_t = G)$, the expected value of ϕ would be

$$E[\phi_t(y_t, \hat{s}_t)|e_t, s_t] = \frac{\pi(L|0, G)}{\pi(L|1, G)} > 1$$

if she reports $\hat{s}_t = G$. Conditional on $(e_t = 0, s_t = B)$, the expected value of ϕ would be

$$E[\phi_t(y_t, \hat{s}_t)|e_t, s_t] = \frac{\pi(L|0, B)}{\pi(L|1, G)} > 1$$

if she reports $\hat{s}_t = G$.

Moreover, the expected value of ϕ would be one whenever she reports $\hat{s}_t = B$. Hence we conclude that for any $(e_t, \hat{s}_t) \neq (1, s_t)$, $E[\phi_t(y_t, \hat{s}_t)|e_t, s_t] \geq 1$. □

This results states that we have an effective independence. Though the agent can learn about evaluations of the principal before the principal makes her evaluations known at the end of the T periods, she cannot update on the expected cost on the equilibrium path, i.e., when the agent chooses $e_t = 1$ and $\hat{s}_t = s_t$. This implies that the expected efficiency loss is independent of T and δ.

Proposition 11. *Given the principal's strategy* Z^T*, at any time* $t \in \{1, \ldots, T\}$ *and conditional on any history* $\{e^{t-1}, s^{t-1}, \hat{s}^{t-1}\}$*, it is a best response for the agent to choose* $e_t = 1$ *and send the message* $\hat{s}_t = s_t$*. Hence, the equilibrium efficiency loss is* λ*.*

We prove this result in two steps. As a first step, we will show that it is a best response for the agent to choose $e_t = 1$ and report truthfully at the last period T. In the next step, we demonstrate that if the agent will choose $e_{\hat{t}} = 1$ and $\hat{s}_t = s_t$ from period $\hat{t}$ on until the end of the T-stage game, it is a best response for her to have $e_{\hat{t}} = 1$ and $\hat{s}_{\hat{t}} = s_{\hat{t}}$.

Lemma 9. *Given the principal's strategy* Z^T*, it is a best response for the agent to choose* $e_T = 1$ *and* $\hat{s}_T = s_T$ *for any history* $\{e^{T-1}, s^{T-1}, \hat{s}^{T-1}\}$*.*

Proof. First, it is optimal for the agent to report truthfully regardless of her effort choice e_T and history. This is so as her report does not affect the continuation payoff, that is, $Z^T + b(\hat{s}_T) + B_{-T}$, where B_{-T} denotes the total rewards the agent expects to get for reporting "G"s in previous periods. Given our construction, the message $\hat{s}_T$ does not affect Z^T and $b(\hat{s}_T)$; the agent has a weak incentive to report truthfully.

Second, it is a best response for the agent chooses $e_T = 1$. For any history $(y^{T-1}, \hat{s}^{T-1}, s^{T-1})$ and conditional on $e_T = 1$, the expected continuation payoff is

$$-\prod_{t=1}^{T-1} E[\phi_t(y_t, \hat{s}_t|e_t, s_t]\pi(L|1)\lambda - \delta^{T-1}c + B_{-T}.$$

However, if she chooses $e_T = 0$, the expected value value of Z^T would be

$$-\prod_{t=1}^{T-1} E[\phi_t(y_t, \hat{s}_t|e_t, s_t]\pi(L|0)\lambda + B_{-T}.$$

Hence, it is optimal to choose $e_T = 1$ if

$$\prod_{t=1}^{T-1} E[\phi_t(y_t, \hat{s}_t|e_t, s_t][\pi(L|0) - \pi(L|1)]\lambda \geq \delta^{T-1}c. \tag{32}$$

By construction, $\prod_{t=1}^{T-1} E[\phi_t(y_t, \hat{s}_t|e_t, s_t] \geq 1$ and $\lambda \geq c/[\pi(L|0) - \pi(L|1)]$, so the condition (32) hods true. It is optimal for the agent to choose $e_T = 1$. This concludes the proof for Lemma 9.

□

Lemma 10. *Given the principal's strategy Z^T, if it is optimal for the agent to follow the equilibrium strategy from period $\hat{t}+1$ on, i.e., $(e_t = 1, \hat{s}_t = s_t)$ for $t > \hat{t}$, then it is a best response for her to have $e_{\hat{t}} = 1$ and $\hat{s}_{\hat{t}} = s_{\hat{t}}$ in period $\hat{t}$.*

Proof. We first show that it is optimal for the agent to send message $\hat{s}_{\hat{t}} = G$ if and only if her private information is $(1, G)$, but it is optimal to send message $\hat{s}_{\hat{t}} = B$ otherwise. Next, we show that it is a best response to choose $e_{\hat{t}} = 1$.

For any history $(e^{\hat{t}-1}, s^{\hat{t}-1}, \hat{s}^{\hat{t}-1})$ and conditional on her private information $(e_{\hat{t}}, s_{\hat{t}})$, the expected continuation payoff for sending message $\hat{s}_{\hat{t}}$ is

$$-\lambda \left[\prod_{t=1}^{\hat{t}-1} E[\phi_t(y_t, \hat{s}_t)|e_t, s_t]\right] \left[\prod_{t=\hat{t}+1}^{T} E[\phi_t(y_t,\ s_t)|e_t = 1]\right] E[\phi_{\hat{t}}(y_t, \hat{s}_{\hat{t}}|e_{\hat{t}}, s_{\hat{t}}] + b(\hat{s}_{\hat{t}}) + B_{-\hat{t}}.$$

Here we use $B_{-\hat{t}}$ to represent the total bonus the agent expects to get for all periods except period $\hat{t}$. Note that condition (31) indicates the continuation payoff equals

$$-\lambda \left[\prod_{t=1}^{\hat{t}-1} E[\phi_t(y_t, \hat{s}_t)|e_t, s_t]\right] E[\phi_{\hat{t}}(y_t, \hat{s}_{\hat{t}})|e_{\hat{t}}, s_{\hat{t}}] + b(\hat{s}_{\hat{t}}) + B_{-\hat{t}}.$$

Given her private information $(e_{\hat{t}}, s_{\hat{t}})$, the agent's continuation payoff equals

$$-\lambda \left[\prod_{t=1}^{\hat{t}-1} E[\phi_t(y_t, \hat{s}_t)|e_t, s_t]\right] \frac{\pi(L|e_{\hat{t}}, s_{\hat{t}})}{\pi(L|1, G)} + \frac{c}{\pi(G|1)} + B_{-\hat{t}}$$

from reporting G, but is

$$-\lambda\left[\prod_{t=1}^{\hat{t}-1}E[\phi_t(y_t,\hat{s}_t)|e_t,s_t]\right]+B_{-\hat{t}}$$

from reporting B. It is optimal for the agent to send message $\hat{s}_{\hat{t}}=G$ if

$$\lambda\left[\prod_{t=1}^{\hat{t}-1}E[\phi_t(y_t,\hat{s}_t)|e_t,s_t]\right]\left(\frac{\pi(L|e_{\hat{t}},s_{\hat{t}})}{\pi(L|1,G)}-1\right)\leq\frac{c}{\pi(G|1)}. \tag{33}$$

Thus, conditional on the agent's private information $(e_{\hat{t}}=1,s_{\hat{t}}=G)$, the condition (33) holds strictly; it is optimal for her to report truthfully.

However, for any other cases of $(e_{\hat{t}},s_{\hat{t}})$, the condition (33) does not hold, and it is optimal for the agent to send a message $\hat{s}_{\hat{t}}=B$. To see the truth of latter part, note that $\prod_{t=1}^{\hat{t}-1}E[\phi_t(y_t,\hat{s}_t)|e_t,s_t]\geq 1$ for any history. Given the definition of λ in (27) and conditional on $(e_{\hat{t}}=1,s_{\hat{t}})=B$, the left-hand side (LHS) of (33) equals

$$LHS\geq\frac{\pi(L|1,G)c}{\pi(G|1)[\pi(L|1,B)-\pi(L|1,G)]}\frac{\pi(L|1,B)-\pi(L|1,G)}{\pi(L|1,G)}\geq\frac{c}{\pi(G|1)}.$$

Conditional on $(e_{\hat{t}}=0,s_{\hat{t}})=G$, the left-hand side (LHS) of (33) equals

$$LHS\geq\frac{\pi(L|1,G)c}{\pi(G|1)[\pi(L|0,G)-\pi(L|1,G)]}\frac{\pi(L|0,G)-\pi(L|1,G)}{\pi(L|1,G)}\geq\frac{c}{\pi(G|1)}.$$

Conditional on $(e_{\hat{t}}=0,s_{\hat{t}})=B$, the left-hand side (LHS) of (33) equals

$$LHS\geq\frac{\pi(L|1,G)c}{\pi(G|1)[\pi(L|0,B)-\pi(L|1,G)]}\frac{\pi(L|0,B)-\pi(L|1,G)}{\pi(L|1,G)}\geq\frac{c}{\pi(G|1)}.$$

Hence, we conclude that $LHS\geq c/\pi(G|1)$ for any $(e_{\hat{t}},s_{\hat{t}})\in\{(1,B),(0,G),(0,B)\}$.

As she prefers to report truthfully when exerting effort, her continuation payoff from choosing $e_{\hat{t}}=1$ is

$$\begin{aligned}&-\lambda\left[\prod_{t=1}^{\hat{t}-1}E[\phi_t(y_t,\hat{s}_t)|e_t,s_t]\right]E[\phi_{\hat{t}}(y_{\hat{t}},s_{\hat{t}})|e_{\hat{t}}=1]-\delta^{\hat{t}-1}+\pi(G|1)\frac{\delta^{\hat{t}-1}c}{\pi(G|1)}+B_{-\hat{t}}\\&=-\lambda\left[\prod_{t=1}^{\hat{t}-1}E[\phi_t(y_t,\hat{s}_t)|e_t,s_t]\right]+B_{-\hat{t}}.\end{aligned}$$

On the other hand, if she shirks, she strictly prefers to send message $\hat{s}_{\hat{t}}=B$, and her expected continuation payoff from choosing $e_{\hat{t}}=0$ is

$$-\lambda\left[\prod_{t=1}^{\hat{t}-1}E[\phi_t(y_t,\hat{s}_t)|e_t,s_t]\right]+B_{-\hat{t}}.$$

Thus, it is optimal for the agent to choose $e_{\hat{t}}=1$ for this period for any history □

Proof of Proposition 11. In above, we have first showed that the agent has no incentive to deviate from the equilibrium strategy in the last period T. We then demonstrated that if it is optimal for her to follow the equilibrium strategy for $t > \hat{t}$ for any $\hat{t} \in \{1, 2, \ldots, T-1\}$, then it is optimal for her to follow the equilibrium strategy at $\hat{t}$ for any history $(e^{\hat{t}-1}, \hat{s}^{\hat{t}-1}, s^{\hat{t}-1}$. Hence, the agent has no incentive to deviate from the equilibrium strategy for any t.

In equilibrium, the agent's expected transfer at the end of the contract period is

$$-\lambda + c\sum_{t=1}^{T}\delta^{t-1},$$

with the efficiency loss being λ, which is independent of T and δ.

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