Monetary Policy Switching to Avoid a Liquidity Trap

Siddhartha Chattopadhyay
Vinod Gupta School of Management
IIT Kharagpur

Betty C. Daniel*
Department of Economics
University at Albany – SUNY

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Abstract

We propose a monetary-policy-switching Taylor Rule, which would allow the economy to avoid a liquidity trap. In the event of a demand shock, large enough to send the nominal interest rate below zero under a Taylor Rule with a fixed long-run inflation target, the monetary authority switches to a higher short-run inflation target which decays toward the long-run target over time. If the short-run target is sufficiently persistent, then the increase in inflationary expectations is large enough to raise inflation and output even though the nominal interest rate does not fall below zero. The switching regime imparts an inflation bias to policy, but avoids indeterminacy created by the fixed nominal interest rate in a liquidity trap.

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1 Introduction

The world-wide recession and financial crisis, which began in 2007, created a severe fall in demand in many countries. Monetary authorities responded by reducing nominal interest rates close to zero and announced plans to keep nominal interest rates low for a considerable period of time. The combination of the adverse demand shocks and the monetary policy response to them sent these economies into liquidity traps, in which monetary authorities lost the ability to stimulate demand with further nominal interest rate reductions.

Policy-makers do not like liquidity traps. The most often stated reason is that conventional monetary policy looses its effectiveness. However, in the context of the New Keynesian macroeconomic model with a Taylor Rule for the nominal interest rate, there is another reason to avoid liquidity traps. The Taylor Rule requires that the nominal interest rate rise in response to an increase in inflation and/or the output gap. When these responses are large enough, the model has two unstable roots, yielding a unique determinate equilibrium. When these responses are too small, as they are in a liquidity trap with the interest rate fixed at zero, there is a single unstable root, creating indeterminacy. The second reason to avoid a liquidity trap is that the interest rate is no longer free to respond to inflation and/or the output gap, leaving a role for sunspot equilibria.

This paper proposes a monetary-policy switching rule, under which the economy can always avoid a liquidity trap. To motivate our proposal, think carefully about monetary policy which yields the liquidity trap. The linear Taylor Rule is feasible only as long as the nominal interest rate is positive. When the Taylor Rule becomes infeasible, monetary policy must "switch" to something else. The standard assumption is that it switches to a policy of fixing the nominal interest rate at a value close to zero.[1] Nakov (2008) explicitly considers several "truncated" Taylor Rules. We propose an alternative switching rule, which stimulates the economy in response to an adverse demand shock and keeps it out of a liquidity trap.

Woodford (2003) emphasizes that the Taylor Rule for the nominal interest rate should contain a time-varying intercept. The intercept has the interpretation as the natural rate of interest, defined as the value for the real interest rate in the flexible price equilibrium, plus the inflation target. Woodford (2003) focuses on allowing the natural interest rate to vary, thereby allowing the nominal rate to follow the natural rate.

We propose that the inflation target also be allowed to vary, thereby differing from the

[1] Benhabib, Schmitt-Grohe and Uribe (2002) let the responsiveness be non-linear, thereby avoiding the need for policy switching.

fixed long-run inflation target. And we assume that the time-varying short-run inflation target is a choice variable for the monetary authority. Ireland (2007) emphasizes the importance of allowing the inflation target to vary over time in explaining the US inflation experience. We propose a policy-switching rule whereby the monetary authority can allow the short-run inflation target to switch from its long-run value in response to a large adverse demand shock. We show that this policy-switching allows the economy to avoid a liquidity trap.

Consider the effect of an increase in the inflation target on equilibrium values of economic variables. An increase in the inflation target is a reduction in the time-varying intercept for the nominal interest rate. The reduction in the inflation target has a direct effect and an indirect effect on the nominal interest rate. The direct effect reduces the nominal interest rate; a fall in the nominal interest rate stimulates demand, increasing inflation and the output gap. The indirect effect operates through inflationary expectations. An increase in the inflation target raises inflationary expectations, reducing the real interest rate, further stimulating demand and increasing inflation and the output gap. When persistence in the short-run inflation target is strong enough, the indirect effect dominates. That is, with strong enough persistence, an increase in the inflation target increases inflation and the output gap sufficiently that the nominal interest rate actually rises. Therefore, with strong enough persistence, an increase in the inflation target in response to an adverse demand shock stimulates output and inflation and keeps the nominal interest rate above zero, allowing the economy to avoid a liquidity trap.

Our proposal for monetary policy switching to avoid a liquidity trap is the following. If an adverse demand shock is strong enough to send the nominal interest rate to or below zero under a conventional Taylor Rule with a fixed long-run inflation target, then the monetary authority switches from the fixed inflation target Taylor Rule. It announces and implements an increase in the short-run inflation target above the long-run target and sets persistence high enough that an increase in the inflation target actually increases the nominal interest rate. If the monetary authority has sufficient credibility that the public believes the announcement, then the economy never enters a liquidity trap.

This proposal is closest to the Krugman (1998) and Svensson (2001, 2003) proposals to exit a liquidity trap. Krugman emphasizes the need for a permanent monetary expansion, one that would not be reversed in the future, to create an increase in inflationary expectations, thereby stimulating demand.[2] Svensson's "foolproof" policy fixes the exchange rate at a depreciated rate to increase inflationary expectations. Both policies work

[2] Auerbach and Obstfeld (2005) also make this point.

because they raise inflationary expectations, reducing real interest rates. Eggertson and Woodford (2003), Adam and Billi (2006), and Nakov (2008) demonstrate that optimal monetary policy relies on an increase in inflationary expectations to exit a liquidity trap. Eggertson and Woodford (2003) propose a policy rule similar to a Taylor Rule, but with a price-level target instead of an inflation target. As the price level falls in a liquidity trap, inflationary expectations rise. The problem with truncated Taylor Rules and with optimal policies in a liquidity trap is implementation. For truncated Taylor Rules, while the economy is in the liquidity trap, prices are actually indeterminate – sunspot equilibria are possible because the interest rate cannot respond to eliminate them. For optimal policy, it is not generally clear how to implement the policy to assure unique equilibria while interest rates cannot respond.[3]

These policies work within the confines of a simple New Keynesian model, in which the effects of monetary policy are transmitted through the real interest rate. Much of the literature on monetary policy in a liquidity trap expands policy to unconventional methods, which are effective to the extent that financial-market arbitrage is imperfect and/or the quantity of money has an effect on the economy independent of its effect on the real interest rate. These policies are interesting and potentially useful, but the simple New Keynesian model is not complex enough to provide a role for them.[4]

This paper is organized as follows. The next section presents optimal monetary policy in the simple three-equation New Keynesian model. We begin with a demonstration that the Taylor Rule with a time-varying intercept can be used to implement optimal policy as long as the implementation does not imply that the nominal interest rate falls below zero. Section 3 presents our proposal for monetary-policy switching to avoid a liquidity trap in the New Keynesian model. Section 4 replaces the sticky-price Phillips Curve with a sticky-information Phillips Curve and shows that the proposed policy-switching model is robust to the specification of the Phillips Curve. Section 5 concludes.

[3] Woodford (2003, p. 590) demonstrates how optimal policy can be expressed as an implementable interest rate rule when there is no concern for a liquidity trap.

[4] Examples of unconventional monetary policy include Auerbach and Obstfeld (2004), Blinder (2000, 2010), Bernanke (2002), Bernanke and Reinhart (2004), Bernanke, Reinhart and Sack (2004), Clouse *et.al.* (2003) and Gurkaynak, Sack and Swanson (2004,2005).

2 Monetary Policy in the Simple New Keynesian DSGE Model

2.1 Simple New Keynesian Model

Following Walsh (2010) and Woodford (2003), we represent the simple standard linearized New Keynesian model as an IS curve, derived from the Euler Equation of the representative agent, and a Phillips Curve, derived from a model of Calvo pricing (Calvo, 1983). The linearization is about an equilibrium with a long-run inflation rate of zero.[5]

$$y_t = E_t\left(y_{t+1}\right) - \sigma\left[\hat{\imath}_t - E_t\left(\pi_{t+1}\right)\right] - u_t \tag{1}$$

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \kappa y_t. \tag{2}$$

In these equations y_t denotes the output gap with $y_t = \hat{Y}_t - \hat{Y}_t^n$, where $\hat{Y}_t = \log\left(\frac{Y_t}{\bar{Y}_t}\right)$ with the bar denoting long run equilibrium at an inflation rate of zero, and the superscript n denoting the flexible price value (natural) for output; inflation is the deviation about a long-run value of zero, with $\pi_t = \log\left(\frac{1+\pi_t}{1+\bar{\pi}}\right)$, where $\bar{\pi} = 0$; the nominal interest rate variable is defined as $\hat{\imath}_t = \log\left(\frac{1+i_t}{1+i}\right)$, where $i = r = \frac{1-\beta}{\beta}$, with r defined as the long-run real interest rate; u_t represents the combination of shocks associated with preferences, technology, fiscal policy, etc.; σ represents the intertemporal elasticity of substitution with $\sigma \geq 1$, κ represents the degree of price stickiness;[6] and $\beta \in (0,1)$ denotes the discount factor. The shock in the Euler equation (u_t) is assumed to follow an AR(1) process with parameter ρ_u. Following Woodford (2003, Chapter 4), we do not add an independent shock to inflation in the Phillips Curve.[7] This restricts the analysis to the case where monetary policy faces no trade-off between inflation and the output gap.

[5] This does not require that the inflation rate be zero in the long run, only that it not be so far from zero to make the linearization inappropriate (Woodford 2003, p. 79).

[6] $\kappa = \frac{(1-s)(1-\beta s)}{s}\frac{\sigma^{-1}+\omega}{1+\omega\varepsilon}$, where $s \in (0,1)$ represents the fraction of randomly selected firms that cannot adjust their price optimally in a given period. Therefore, $s = 0 \Rightarrow \kappa \to \infty \Rightarrow$ complete flexibility and $s = 1 \Rightarrow \kappa = 0 \Rightarrow$ complete stickiness. Hence, $\kappa \in (0,\infty) \Rightarrow$ incomplete flexibility. $\omega > 0$ is the elasticity of firm's real marginal cost with respect to its own output, $\varepsilon > 0$ is the price elasticity of demand of the goods produced by monopolistic firms. See, Adam and Billi (2006) and Woodford (2003) for details.

[7] Adam and Billi (2006) demonstrate that the supply shock is not important for consideration of the zero lower bound.

2.2 Policy to Choose Nominal Interest Rate

2.2.1 Optimal Policy

The model is completed with determination of the nominal interest rate. We consider two alternative methods to specify the nominal interest rate. The first follows Woodford (2003), and chooses values for the time paths of inflation and the output gap to minimize the loss function,

$$L_t = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \lambda y_{t+j}^2 \right), \ \lambda \in [0, \infty). \tag{3}$$

Woodford derives equation (3) as a linear approximation to the utility function of the representative agent when equilibrium inflation is zero and the flexible-price value for output is efficient.[8] When the only shock is to the Euler equation, it is optimal to set $\pi_t = y_t = 0$. Given these values, it is straightforward to show that the optimal value for the nominal interest rate is

$$\hat{\imath}_t = -\sigma^{-1} u_t. \tag{4}$$

According to equation (4), a reduction in the demand for current output (a rise in u_t) should be offset by a reduction in the nominal interest rate. The interest rate should remain lower as long as demand is lower. An interest rate which fully offsets demand shocks keeps inflation and the output gap both at their target values of zero. A nominal interest rate, set according to equation (4), is compatible with the target values of zero for inflation and the output gap. Woodford (2003) shows that the optimal interest rate, given by equation (4), is also equal to the natural rate of interest (r_t^n), defined as the real interest rate which sets the output gap at zero.

However, if equation (4) is used as the interest rate rule, then there are also many other equilibrium values for inflation and the output gap in addition to the target values. An interest rate rule like equation (4) leaves the price level indeterminate. Sargent and Wallace (1981) were the first to raise the issue of indeterminacy in the context of a policy which fixes the nominal interest rate. Hence, the monetary authority cannot implement optimal policy using equation (4) as an interest rate rule. Equation (4) determines the equilibrium value of the optimal interest rate, but it does not explain how the monetary authority can achieve it.

[8] The government can subsidize firms to increase production to the perfectly competitive level.

2.2.2 Taylor Rule

The method, typically employed in New Keynesian models for determining the nominal interest rate, is to assume that the monetary authority follows a Taylor rule. In Taylor's original rule, the nominal interest rate is set to equal a fixed real rate plus a fixed inflation target and to respond positively to deviations of inflation and output from fixed target values. The Taylor Rule, log linearized about long-run equilibrium values of zero, can be expressed as

$$\hat{\imath}_t = \hat{r}_t + \pi^* + \phi_\pi \left(\pi_t - \pi^*\right) + \phi_y \left(y_t - y^*\right), \ \phi_\pi > 0, \ \phi_y \geq 0, \tag{5}$$

where $\hat{r}_t$ in Taylor's original formulation is zero. We allow the monetary authority to choose a target value for inflation (π^*) greater than the long-run value of zero about which we log linearize. When the inflation target is positive, solution of equation (2) implies that the output gap target is given by $y^* = \frac{1-\beta}{\kappa}\pi^*$.

Allowing the interest rate to respond strongly enough to endogenous variables solves the problem of indeterminacy which arises if equation (4) is treated as an interest rate rule. Specifically, Bullard and Mitra (2002) demonstrate that if ϕ_π and ϕ_y are large enough such that equations (1) and (2), with equation (5) substituted for the interest rate, yields a dynamic system with two unstable roots, corresponding to the two forward-looking variables, then the equilibrium is unique. This condition has been labeled the Taylor Principle.[9]

2.2.3 Implementation of Optimal Policy with a Taylor Rule

Woodford (2003) demonstrates that is possible to use the Taylor Rule to implement optimal monetary policy by allowing the intercept in the Taylor Rule ($\hat{r}_t + \pi^*$) to be time-varying. Erceg, Henderson, and Levine (2000) and Woodford (1993, pp. 246) also use Taylor Rules in which a time-varying intercept can be chosen by the monetary authority. Woodford sets $\pi^* = 0$, and lets $\hat{r}_t$ be time-varying. Optimal policy can be implemented with

$$\hat{r}_t = -\sigma^{-1} u_t. \tag{6}$$

Substituting equation (6) for $\hat{r}_t$ into equation (5), setting $\pi^* = 0$, and substituting the Taylor Rule with this optimal policy into equations (1) and (2) sets inflation and the

[9] The Taylor Principle originally referred to requiring $\phi_\pi > 1$, but has been generalized to allow the nominal interest rate to respond to both inflation and the output gap.

output gap at their target values of zero.[10] At equilibrium values for the output gap and inflation of zero , the interest rate equals the optimal interest rate in equation (4), Woodford's (2003) natural rate of interest.

The equilibrium solution is independent of the values for ϕ_π and ϕ_y as long as they are large enough to assure two unstable roots.[11] Therefore, it is important to understand the role of these policy parameters. The promise to respond strongly to any sunspot shocks that raise inflation and/or output, in Cochrane's words, "to blow up the economy" (Cochrane, 2011) in the event of sunspot shocks, serves to rule out sunspot equilibria and to assure a unique equilibrium. Therefore, we can obtain a unique equilibrium in which the interest rate is given by equation (4) only if the monetary authority follows an interest rate rule like (5), which differs from equation (4) by this extraordinary promise. This requires that the monetary authority be completely transparent, communicating the intention to "blow up the economy" and that this threat be completely credible. This is because ϕ_π and ϕ_y do not show up in the equilibrium solution and therefore cannot be inferred from any observable evidence.[12]

3 Liquidity Trap

The above policy is feasible only if the demand shock is never large enough to send the nominal interest rate below zero. In the linearized model, the deviation of the nominal interest rate from its long-run equilibrium value ($\hat{\imath}_t$) plus its long-run value ($\bar{\imath}$) equals the nominal interest rate (i_t), which must be greater than or equal to zero, requiring

$$\hat{\imath}_t + \bar{\imath} = i_t \geq 0 \implies \hat{\imath}_t \geq -\bar{\imath} \tag{7}$$

For large values of u_t, the policy in equation (6) would send $\hat{\imath}_t$ below $-\bar{\imath}$, implying that the nominal interest rate would fall below zero. Since this is not feasible, a complete description of monetary policy must specify how the monetary authority would react in this event.

[10] Any other values yield an explosive equilibrium, which we rule out.

[11] The criteria for two unstable roots is: $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$.

[12] Cochrane (2011) emphasizes that at the optimal equilibrium, values for ϕ_π and ϕ_y do not affect the equilibrium. Woodford (2003, p. 288) makes the same point. If there were shocks to the Phillips Curve, or if the intercept to the Phillips Curve did not vary optimally, then we would have evidence on the values of ϕ_π and ϕ_y.

3.1 Liquidity Trap as Policy Switching

Consider a value for u_t so large that with policy given by equations (5) and (6), the nominal interest rate would become negative, an impossibility. A common assumption is that policy would switch, setting $\phi_\pi = \phi_y = 0$ and $\hat{r}_t = -\bar{\imath}$, such that the nominal interest rate is fixed at zero. Nakov (2008) considers several "truncated" Taylor Rules. The zero nominal-interest-rate policy would persist until the shock becomes small enough to allow policy to switch back to the original Taylor Rule. Monetary policy is characterized locally by a nominal interest rate fixed at zero, yielding a liquidity trap. The fixed interest rate violates the promise to respond strongly to deviations of inflation and the output gap from their target values of zero, yielding the possibility of sunspot equilibria.

3.2 Policy Switching to Avoid a Liquidity Trap

We propose an alternative type of policy switching in the event of a demand shock large enough to send the economy into a liquidity trap. We depart from standard analysis and allow the short-run target for inflation to differ from its long-run target. Specifically, we assume that monetary policy can switch from targeting a zero inflation rate to targeting a positive inflation rate as a way of preventing a liquidity trap.

Ireland (2007) argues that US inflation can be explained by a New Keynesian model with a Taylor Rule only if the inflation target is allowed to vary over time. Additionally, Kozicki and Tinsley (2001), Rudebusch and Wu (2004), Gurkaynak, Sack and Swanson (2005) and Dewachter and Lyrio (2006) provide evidence of a time-varying short-run inflation target for the US. Krugman (1998), Svensson (2003), Eggertson and Woodford (2003), Adam and Billi (2006), and Nakov (2008) all suggest policies which increase expected inflation in a liquidity trap.

We show that if the monetary authority follows a Taylor Rule, which allows switching in the short-run target inflation rate, then the economy never enters a liquidity trap. Monetary policy retains values for ϕ_π and ϕ_y, which satisfy the Taylor Principle, therefore eliminating the possibility of sunspot equilibria.

3.2.1 Short-run Inflation Target

To motivate the alternative policy, consider the Taylor Rule with time subscripts on both terms in the intercept $(\hat{r}_t + \pi_t^*)$ and on inflation and output targets (π_t^*, y_t^*) in the Taylor Rule (equation 5). We are allowing the short-run inflation target to differ from the long-run target of zero when we let π_t^* be time-varying. Following Woodford (2003), we interpret y_t^* as the value for the output gap from equation (2), when inflation takes on its target value, yielding

$$y_t^* = \frac{\pi_t^* - \beta E_t\left(\pi_{t+1}^*\right)}{\kappa}.$$

Assuming that the inflation target follows an $AR(1)$ process with ϵ_t a zero-mean iid disturbance,

$$\pi_t^* = \rho_\pi \pi_{t-1}^* + \epsilon_t$$

we have,

$$y_t^* = \left(\frac{1-\rho_\pi\beta}{\kappa}\right)\pi_t^*. \tag{8}$$

A monetary authority which fixes π_t^* at zero and sets the real interest rate according to equation (6) yields Woodford's (2003) optimal policy.

However, policy settings at these optimal values are feasible only as long as the implied nominal interest rate is positive, equivalently, as long as the natural rate of interest is positive. The non-negativity constraint requires a departure from unconstrained optimal policy.[13]

The Taylor Rule with a time varying inflation target and with equation (6) substituted for $\hat{r}_t$ becomes

$$\hat{\imath}_t = \pi_t^* - \sigma^{-1}u_t + \phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_y\left(y_t - y_t^*\right). \tag{9}$$

Substituting for $\hat{\imath}_t$ using equation (7), for y_t^* using equation (8), and collecting terms on π_t^* yields

$$i_t = \bar{\imath} - z\pi_t^* - \sigma^{-1}u_t + \phi_\pi\pi_t + \phi_y y_t, \tag{10}$$

where z is a constant given by

$$z = \phi_\pi + \phi_y\left(\frac{1-\rho_\pi\beta}{\kappa}\right) - 1 > 0.$$

The sign restriction necessary to assure two unstable roots.

3.2.2 Using the Short-run Inflation Target to Avoid a Liquidity Trap

Assume for now that we are able to restrict π_t^* to assure that equation (7) holds for any value of u_t.[14] Using equations (1), (2), and (10), and denoting the unstable roots of the system as λ_1 and λ_2, the rational expectations solutions for the output gap and

[13] Adam and Billi (2006) derive constrained optimal policy.

[14] We derive those restrictions below.

inflation under the proposed policy-switching rule are unique and are given by[15]

$$y_t = \frac{1 - \rho_\pi \beta}{\beta (\lambda_1 - \rho_\pi)(\lambda_2 - \rho_\pi)} \sigma z \pi_t^*, \tag{11}$$

and

$$\pi_t = \frac{\kappa}{\beta (\lambda_1 - \rho_\pi)(\lambda_2 - \rho_\pi)} \sigma z \pi_t^*. \tag{12}$$

Both the output gap and inflation respond positively to the inflation target. This is because an increase in the inflation target raises inflationary expectations, reducing the real interest rate, stimulating current spending. Note that the Taylor Rule, with a time-varying intercept dependent on the natural rate of interest, eliminates any effect of u_t, which does not operate through π_t^*.

Substituting equilibrium values for π_t and y_t from equations (11) and (12) into the Taylor Rule for the interest rate yields an equilibrium value for the nominal interest rate as

$$i_t = \bar{\imath} - \sigma^{-1} u_t + q z \pi_t^* \qquad \text{where } q = \left[\frac{\phi_\pi \kappa + \phi_y (1 - \rho_\pi \beta)}{\beta (\lambda_1 - \rho_\pi)(\lambda_2 - \rho_\pi)} \sigma - 1 \right] \tag{13}$$

If we set π_t^* such that the nominal interest rate is always positive, then we have avoided the liquidity trap.

The issue in a liquidity trap is how to stimulate output and inflation without reducing the nominal interest rate. Equations (11) and (12) reveal that stimulating requires raising the inflation target. Note that the coefficient on π_t^* in equation (13) is increasing in the degree of persistence of the short-run inflation target, given by ρ_π. In the New Keynesian model, the direct effect of an increase in the inflation target is a reduction in the nominal interest rate, and this stimulates demand and inflation. However, the increase in the inflation target also raises expectations of inflation, further stimulating demand, and through the Taylor Rule responses to inflation and the output gap, leads to an increase in the interest rate. For large enough persistence of the short-run inflation target, this indirect effect dominates, implying that an increase in the inflation target raises the nominal interest rate.[16] To assure that the monetary authority can escape a liquidity trap by stimulating the economy with an increase in the short-run inflation target, the monetary authority must set ρ_π high enough such that q in equation (13) is positive.

We propose the following policy switching regime to assure that the economy never

[15] These are the rational expectations solutions, ignoring the lower bound on the nominal interest rate. If we are able to manipulate π_t^* to avoid the lower bound, then these are the equilibrium solutions.

[16] This is why calibrated models fail to find a liquidity effect of a negative interest rate shock when persistence is high.

enters a liquidity trap. Define a threshold value for u_t as $\hat{u}$, such that for $u_t \leq \hat{u} = \sigma\bar{\imath}$, and $\pi_t^* = 0$, i_t in equation (13) is greater than or equal to zero.[17] Begin from a period in which $u_t = 0$ and $\pi_t^* = 0$. Follow the policy rule in equation (13) with $\pi_t^* = 0$ for as long as $u_t \leq \hat{u}$. We label this the zero inflation-target rule. Once $u_t \geq \hat{u}$, the short-run inflation target switches to a positive inflation-target rule with the target given by

$$\pi_t^* = \frac{\sigma^{-1} u_t}{zq} \tag{14}$$

where ρ_π must be set large enough to assure $q > 0$. To maintain equation (14) going forward, it is necessary that the autoregressive coefficient on the inflation target, given by ρ_π, equals ρ_u. Given the strong persistence of demand shocks[18], setting $\rho_\pi = \rho_u$ satisfies the restriction on q in equation (13).[19] Additionally, the monetary authority must continue to follow this policy until $\pi_t^* \leq 0$. Once $\pi_t^* = 0$, the monetary authority can switch back to the zero target inflation rule until the demand shock again exceeds the threshold value.

This policy is history-dependent with two trigger points, $\hat{u}$ and 0. The inflation target is zero until $u_t > 0$, whereupon it switches to the value in equation (14) and decays to zero at rate ρ_π. The policy with a positive inflation target cannot switch back to that with a zero inflation target once the demand shock falls below the threshold value ($\hat{u}$) because this would violate the promise of strong persistence in the inflation target, as implied by a high value of ρ_π. The strong persistence is needed for an increase in the inflation target to imply an equilibrium increase in the interest rate instead of a decrease. An interest rate reduction in a liquidity trap is not feasible.

We illustrate the quantitative effects of our proposal using the RBC parameterization from Adam and Billi (2006),

$$\sigma = 1,\ \beta = 0.99,\ \kappa = 0.057,\ \phi_\pi = 1.5,\ \phi_y = 0.5, \rho_u = 0.8.$$

All values are expressed at quarterly rates. The values for the elasticity of substitution and the discount factor are standard. The value of κ is consistent with 44% of firms adjusting their price each period. We set the persistence of the monetary policy response

[17] We could define a higher threshold value if we require the nominal interest rate to remain above some minimum value to enable the monetary authority to promise a response to sunspot shocks.

[18] Ireland (2004) provides an estimate of persistence of 0.95, Adam and Billi (2006) estimate persistence at 0.8, and Mankiw and Reiss (2006) provide an estimate of of 0.94

[19] If not, the restriction on q must be satisfied, and the inflation target must disappear more slowly than the demand shock, implying that it will not be possible to follow equation (14) going forward. The next policy we propose deals explicitly with this case.

$\rho_\pi = \rho_u$, yielding $q > 0$. With these values, $q = 0.0216 > 0$, as required for an increase in the inflation target to raise the interest rate. We let the adverse demand shock be large enough to imply a negative interest rate under optimal policy, were such a value possible, $u_0 = 1.04\%$.

Impulse responses to the demand shock, with a Taylor Rule given by equation (13), and a time-varying inflation target, given by equation (14), are shown in Figure 1.

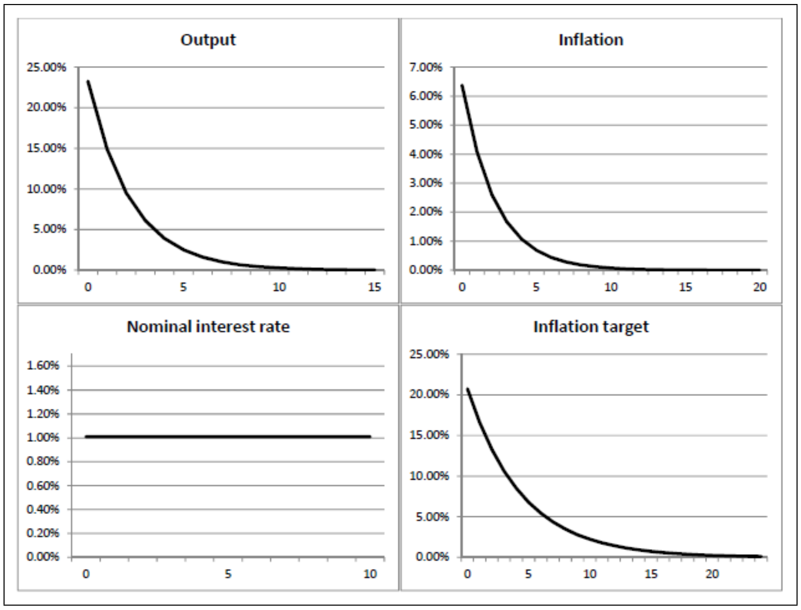

Figure 1: Impulse response under sticky prices with $i_t = \bar{\imath}$.

The demand shock itself has a negative effect on output and inflation. The monetary authority needs to stimulate by reducing the real interest rate. However, when the demand shock is sufficiently adverse, the Taylor Rule with a fixed long-run inflation target requires the nominal interest rate to fall below zero, implying that monetary policy looses its traditional nominal interest rate instrument.

Our policy provides an alternative way of manipulating the real interest rate. In response to the strong adverse demand shock, the monetary authority increases inflationary expectations by raising the time-varying inflation target and promising to keep it high for a long period of time by promising strong persistence. With sufficient persistence, the increase in inflationary expectations reduces the real interest rate, stimulating demand and inflation, even if the nominal interest rate does not actually fall. Output, inflation, and the inflation target all rise initially, and subsequently fall as the shock vanishes. Since persistence in the short-run inflation target and in the demand shock are both high and since the policy with a positive short-run inflation target must persist until the demand disturbance has vanished, inflation and the output gap remain above their long-run target values of zero for a long period of time.

This policy keeps the nominal interest rate at its long-run equilibrium value of $\bar{\imath}$. However, this is not a fixed interest rate policy. The nominal interest rate is allowed to respond to deviations of inflation and the output gap from their time-varying, short-run target values by ϕ_π and ϕ_y. Should sunspot shocks arise, the promise to offset them is credible, assuring that they do not arise in equilibrium.

Since the nominal interest rate does not fall, this policy generates very large increases in output and inflation with initial increases of 93.05% and 25.50%, respectively, at annual rates. We avoid the liquidity trap but at a substantial cost in terms of output and inflation deviations. However, there is no reason the monetary authority must keep the nominal interest rate this high. Under our policy proposal, the nominal interest rate must be above zero and it must retain the ability to respond, using the Taylor Principle, to sunspot deviations in inflation and output. The sunspot shocks it needs to rule out are positive ones since negative ones are ruled out by transversality conditions. Therefore, low positive nominal interest rates, responding with coefficients ϕ_π and ϕ_y to positive sunspot shocks, satisfy our criteria.

If we allow the nominal interest rate to fall, we can design a switching policy with smaller output and inflation fluctuations and therefore with lower welfare costs. This policy reduces the initial increase in the inflation target at the time of the shock; the short-run target subsequently decays at rate $\rho_\pi = 0.8$ over time. The time path for the short-run inflation target in response to a demand shock large enough to create a liquidity trap is given by

$$\begin{aligned} \pi_0^* &= \frac{\sigma^{-1}u_0 - \eta}{zq}, \\ \pi_t^* &= \rho_\pi^t \pi_0^*. \end{aligned} \tag{15}$$

where η is chosen to keep the nominal interest rate positive. This policy is not unique since feasible values for η are not unique. When $\eta = 0$, and $\rho_\pi = \rho_u$, this policy is identical to that proposed in equation (14). Additionally, this policy is feasible even when the demand shock decays more rapidly than the inflation target. The impulse responses based on equation (15) are graphed in Figure 2.[20] This policy yields much

[20] We generate the impulse response as follows. First, we calculate the impulse response with an initial

smaller output and inflation deviations, with initial values at annual rates of of 2.69% and 0.73%, respectively, while keeping the nominal interest rate positive.[21]

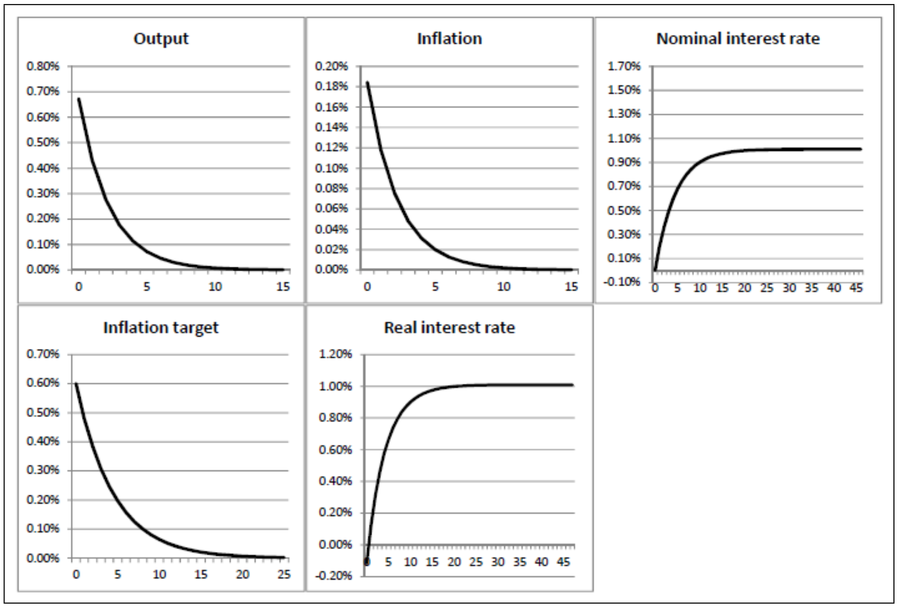

Figure 2: Impulse response under sticky prices letting interest rate fall

3.2.3 Implementation and Costs and Benefits

The policy we propose is dynamically inconsistent. Therefore, to implement it, the monetary authority must have the ability to commit to the interest rate rule with a time-varying target. The monetary authority must continue to keep the short-run inflation-target above its long-run level as long as the inflation target exceeds zero. This requires that the inflation target remain higher than its long-run optimal value, even after the demand shock has fallen in value sufficiently that the nominal interest rate with a zero

value for $\pi_0^* = 0.6\%$. Then we calculate the dynamics of the nominal and real interest rates. Then, π_0^* is recalculated using equation (15) to check the consistency of our result. From equations (13) and (15), $i_0 - \bar{\imath} = -\eta$. We choose η to be a little less than 1.0%, yielding a very tiny initial interest rate of $i_0 \approx 0.00000176\%$. The impulse response with an initial choice of π_0^* is equivalent with the impulse response with an initial choice of i_0 or η.

[21] We also calculated the impulse responses under the base line parameterization of Adam and Billi (2006), with values for $\sigma = 6.25$ and $\kappa = 0.024$ to check the robustness of our policy. The initial demand shock must be larger to reduce the nominal interest rate below zero under the normal Taylor Rule. We use $u_0 = 6.5\%$ large enough (so that $\sigma^{-1}u = 1.04\%$ as in the text for RBC parametrization) with $\pi_0^* = 0.40\%$ at an annual rate and $\rho_\pi = \rho_u = 0.8$. The nominal interest rate (i_0) falls from its initial annual rate of 4.0% to 0.07%, and the real interest rate (r_0) falls from 4.0% to 0.02%. Interest rates subsequently converge to initial long-run levels. Initial output and inflation rise at annual rates of 0.65% and 0.07% respectively and gradually converge to initial values.

target inflation rate would be positive. This is necessary to generate the strong increase in inflationary expectations required to keep the economy out of a liquidity trap following a large adverse demand shock.

Additionally, for implementation, the monetary authority must be able to communicate its policy to the public and its communication must have credibility. The public must know that the short-run inflation target has changed and that this change will be very persistent. An increase in the nominal interest rate, without this communication, is insufficient. A nominal interest rate increase could imply a policy reduction in the inflation target, together with low persistence; this would reduce inflationary expectations, raising the real interest rate, adding to the reduction in inflation and the output gap created by the adverse demand shock. The public needs to know more about policy than is revealed by the nominal interest rate alone to make correct expectations about future inflation.

Failure to establish credibility dooms the policy. However, we feel that it should be no more difficult to establish credibility for this policy than for policies like the promise to "blow up the economy" (Cochrane 2011) in the event of off-equilibrium paths for prices and/or output, Woodford's (2003) timeless perspective policy, or optimal policy (Eggertson and Woodford 2003, Adam and Billi, 2006, Nakov 2008).[22] Since our policy requires commitment to a rule, it is arguably easier to communicate than commitment to optimal policy. Svensson's (2003) devaluation policy has a credibility advantage because the exchange rate is an observable piece of data, but most countries no longer peg exchange rates. Perhaps a larger problem than getting the public to believe that the central bank would follow a rule would be public outrage over a policy to increase inflation, following the long and successful battle to reduce it.[23] The public would require re-education, countering the prevailing wisdom that inflation is always a "bad."

It is interesting to compare the two switching policies: 1) the traditional liquidity-trap policy whereby the interest rate is set to zero for $u_t \geq \hat{u}$ and is given by equation (9) with $\pi_t^* = 0$ otherwise; and 2) the inflation-target switching policy whereby the interest rate is set according to equation (9) with π_t^* time-varying and history dependent. In the region for which $u_t \geq \hat{u}$, the traditional policy allows sunspot equilibria while the inflation-target switching policy does not. In the region for which $u_t < \hat{u}$, the policies are identical if all values of u_t, occurring since the last time that $u_t = 0$, are less than $\hat{u}$. Otherwise, inflation and the output gap are higher under a policy of inflation-target switching. Under the conventional switching policy, inflation and the output gap become

[22] And optimal policy has the problem of implementabity in the liquidity trap, while interest rates cannot respond to sunspot shocks.

[23] Krugman (1998) made this point.

determinate and return to zero once the demand shock has become small enough to render the fixed-inflation-target nominal interest rate zero; in contrast, under our alternative policy, inflation and the output gap remain positive until the short-run inflation target has returned to zero. Therefore, the inflation-target switching policy imparts an inflation bias to policy.

Policy-makers would choose inflation-target switching if the gains of determinacy in some periods outweigh the loss created by inflation bias in others. Since we do not have any way to evaluate the welfare implications of determinacy, we cannot compare the welfare implications of our policy with those yielding a liquidity trap. Policy which yields a liquidity trap is optimal only under the assumption that in the liquidity trap, output and inflation are given by the fundamentals equilibrium. That is, optimality is obtained in these models only by ruling out sunspot equilibria, even though the interest rate response to sunspot shocks, which is necessary to yield determinacy, is missing.

4 Policy Robustness

In this section we examine the robustness of our policy proposal to an alternative Phillips curve. We replace the sticky price Phillips Curve with the sticky information Phillips Curve advocated by Mankiw and Reis (2002), and consider how the monetary authority could use the inflation target to keep the economy out of a liquidity trap.

4.1 Sticky Information Phillips Curve

The sticky information Phillips curve is derived using a monopolistically competitive market structure where a fraction of firms are allowed to update their information each period with a fixed probability.[24] The linearized sticky information Phillips curve can be expressed as

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\alpha y_t + (1-\theta)\sum_{j=0}^{\infty}\theta^j E_{t-1-j}\left[\pi_t + \alpha\left(y_t - y_{t-1}\right)\right] \tag{16}$$

where $1-\theta$ is the fraction of firms randomly selected to update their information each period, such that $(1-\theta)\,\theta^j$ represents the fraction of firms with updated information in period $t-j$, and $\alpha \in (0,1)$ represents the degree of nominal rigidity (Ball and Romer,

[24] Early ideas on sticky information are due to Friedman (1968), Phelps (1968), and Lucas (1972). Recent literature includes Mankiw and Reis (2002, 2006, 2010) and Reis (2009).

1990) as well as the degree of strategic complementarity (Cooper and John, 1988).

To understand the dynamics of this Phillips curve, consider the impulse response to a period 0 shock to the Euler equation beginning from a long-run equilibrium. We assume that this shock is $AR(1)$ with parameter ρ_u. Additionally, we assume all other past and future shocks, including sunspot shocks,[25] are zero and that the economy begins in a long-run equilibrium with $\pi_0 = y_0 = y_{-1} = 0$. We can write the period 0 values for inflation and the output gap as

$$\pi_0 = \left(\frac{1-\theta}{\theta}\right) \alpha y_0 \tag{17}$$

since expectations of inflation and output gap growth, dated prior to period 0, are zero.[26] Going forward one period, we obtain

$$\pi_1 = \left(\frac{1-\theta}{\theta}\right) \alpha y_1 + (1-\theta) E_0 \left[\pi_1 + \alpha \left(y_1 - y_0\right)\right],$$

since $E_{-j}\left[\pi_1 + \alpha \left(y_1 - y_0\right)\right] = 0$ for all $j \geq 1$. Continuing, we have

$$\pi_2 = \left(\frac{1-\theta}{\theta}\right) \alpha y_2 + (1-\theta) E_1 \left[\pi_2 + \alpha \left(y_2 - y_1\right)\right] + (1-\theta)\theta E_0 \left[\pi_2 + \alpha \left(y_2 - y_1\right)\right].$$

To compute the impulse-response, we take time-zero expectations to yield

$$E_0 \pi_2 = \left(\frac{1-\theta}{\theta}\right) \alpha E_0 y_2 + \left[(1-\theta) + \theta\right] E_0 \left[\pi_2 + \alpha \left(y_2 - y_1\right)\right].$$

Extending to t periods yields

$$E_0 \pi_t = \left(\frac{1-\theta}{\theta}\right) \alpha E_0 y_t + (1-\theta) \sum_{j=0}^{t-1} \theta^j E_0 \left[\pi_t + \alpha \left(y_t - y_{t-1}\right)\right]. \tag{18}$$

Define

$$\lambda_0 = 0$$

$$\lambda_t = (1-\theta) \sum_{j=0}^{t-1} \theta^j, \text{ for } t \geq 1,$$

[25] We discuss the restrictions necessary for this assumption later.

[26] Sunspot shocks could add arbitrary past expectations of inflation and changes in the output gap.

noting that $\lambda_t \to 1$ as $t \to \infty$. Using these definitions, we can write equation (18) as

$$E_0\pi_t = \left(\frac{1-\theta}{\theta}\right)\alpha E_0 y_t + \lambda_t E_0\left[\pi_t + \alpha\left(y_t - y_{t-1}\right)\right] \tag{19}$$

4.2 New Keynesian Model with Sticky Information Phillips Curve

We complete the model by adding the linearized Euler equation (1), and the Taylor Rule for the nominal interest rate, equation (5) with $y^* = 0$ since this model obeys the natural rate hypothesis. Substituting for the nominal interest rate, solving for $E_t y_{t+1,}$ and taking time zero expectations yields

$$E_0 y_{t+1} = E_0 y_t + \sigma\left[\phi_\pi\left(E_0\pi_t - \pi^*\right) + \phi_y E_0 y_t + E_0 x_t - E_0\pi_{t+1}\right] + E_0 u_t. \tag{20}$$

The dynamic system is comprised of equations (19) and (20). Given time-zero expectations of period t values, we can solve for time-zero expectations of period $t+1$ values.

Dropping time-zero expectational notation for convenience, setting $\pi^* = 0$, and updating equation (20) one period, a recursive expression of the model for $t \geq 0$ with a single shock in period 0 is given by:

$$y_{t+1} = \frac{\left[\left(1+\sigma\phi_y\right)\left(1-\lambda_{t+1}\right) + \sigma\alpha\lambda_{t+1}\right] y_t + \left(1-\lambda_{t+1}\right)\left(\sigma\phi_\pi\pi_t + \sigma x_t + u_t\right)}{1-\lambda_{t+1} + \sigma\alpha\left(\lambda_{t+1} + \frac{1-\theta}{\theta}\right)} \tag{21}$$

$$\pi_{t+1} = \frac{\left[\sigma\alpha\phi_y\left(\lambda_{t+1} + \frac{1-\theta}{\theta}\right) + \alpha\left(\frac{1-\theta}{\theta}\right)\right] y_t + \alpha\left(\lambda_{t+1} + \frac{1-\theta}{\theta}\right)\left(\sigma\phi_\pi\pi_t + \sigma x_t + u_t\right)}{1-\lambda_{t+1} + \sigma\alpha\left(\lambda_{t+1} + \frac{1-\theta}{\theta}\right)}, \tag{22}$$

where all variables should be understood as time zero expectations.

Equations (21) and (22) constitute a set of difference equations in y_t and π_t with a time-varying coefficient, λ_t. Therefore, we cannot use ordinary methods to solve it. However, we can understand the long-run stability properties of the model by considering the behavior of the system as $t \to \infty$. Setting $\lambda_{t+1} = 1$, and assuming that $\left(1-\lambda_{t+1}\right)\left(\pi_t\right) \to 0$ as $t \to \infty$, the model becomes[27],

$$y_{t+1} = \theta y_t \tag{23}$$

$$\pi_{t+1} = \left(\phi_y + \alpha\left(1-\theta\right)\right) y_t + \phi_\pi\pi_t. \tag{24}$$

These equations imply that in the long-run, this model has one root which becomes θ and is less than unity, and one which eventually equals the responsiveness of the monetary

[27] Variables should be understood as time zero expectations.

authority to inflation, ϕ_π. If the responsiveness is strong enough, $\phi_\pi \geq 1$, then the model has one unstable root. Since y_0 and π_0 must be related by equation (17), and since π_0 is anchored by past expectations and y_0, then y_0 can jump to nullify the unstable root, yielding a unique equilibrium.[28] The consequences of different initial values, equivalently of sunspot shocks, is hyperinflation or hyperdeflation with the output gap reaching its long-run equilibrium value of zero. Hyperdeflation is ruled out by transversality conditions, but hyperinflation is not. However, the typical assumption in the literature is that an unstable equilibrium assures that initial values will jump to rule out sunspot equilibria.[29] With these assumptions on ϕ_π and on initial values, equations (23) and (24) contain actual values, conditional on a single shock in the infinitely distant past, instead of expectations. Alternatively, if $\phi_\pi < 1$, then the model is globally stable. It reaches the unique long-run equilibrium of zero inflation and output gap no matter what initial values are. When the model is globally stable, initial values can be anything; the model admits sunspot equilibria.[30] Therefore, the Taylor Rule with the Taylor Principle has the same role in insuring uniqueness in this model as it does in the sticky price model. And the monetary authority has the incentive to avoid a liquidity trap, both to retain the effectiveness of policy and to eliminate sunspot equilibria.

4.2.1 Implementation of Optimal Policy with a Taylor Rule

Optimal policy in this model is identical to optimal policy in a sticky-price New Keynesian model. And it can be implemented with the Taylor Rule, given by equation (9) with $\pi_t^* = y^* = 0$. This requires that $\hat{r}_t$ be set according to equation (6), eliminating demand shocks from the system. A system that begins in long-run equilibrium with $\pi_0 = y_0 = y_{-1} = 0$ will remain there. A value for $\phi_\pi > 1$ rules out sunspot equilibria.

4.3 Policy Switching in a Sticky Information Model

As before, reducing the nominal interest rate to follow optimal policy is possible only if the optimal nominal interest rate is always positive. The nominal interest rate is given by equation (10) with

$$z = \phi_\pi - 1.$$

[28] The jump in y_0 will imply a unique jump in π_0, from equation (17).

[29] See Cochrane (2011) for a strong criticism of this assumption.

[30] Solution methods based on undetermined coefficients are designed to solve for a single equilibrium and do not admit the sunspot equilibria as candidates.

With π_t^* at its optimal value of zero, a large enough shock to u_t requires a correspondingly large reduction in the nominal interest rate to keep π_t and y_t at their optimal values of zero, yielding a negative nominal interest rate, an impossibility. A liquidity trap occurs when the nominal interest rate reaches zero and becomes unresponsive to π_t and y_t.

We can let the inflation target vary from its long-run value of zero and avoid the liquidity trap. We propose the same type of policy-switching policy as for the sticky-price model. However, since this model does not have a closed-form solution, due to the time-varying coefficient, λ_t, we must rely on numerical simulations to assure a sufficiently persistent short-run inflation target. Persistence must be strong enough to imply that the nominal rate rises in response to an increase in the inflation target, even though the direct effect of the increase in the target is to reduce the nominal interest rate.

We use the parameters estimated by Mankiw and Reis (2006)[31],

$$\sigma = 1,\ \beta = 0.99,\ \theta = 0.3,\ \rho_\pi = 0.92,\ \rho_u = 0.94.$$

We keep $\phi_\pi = 1.5$, $\phi_y = 0.5$ as in Taylor's original specification and in the sticky prices described above. In choosing a value for α, we follow Mankiw and Reis (2010) and set $\alpha = 0.2$.

Our solution method uses the method of undetermined coefficient proposed by Wang and Wen (2006) and modified for an interest rate rule given in (10) by Chattopadhyay (2011). The appendix shows that under the Taylor rule given in (10), the dynamics of output and inflation in the sticky information model can be expressed as,

$$\begin{aligned} y_t &= -a_{yt} z \pi_0^* \\ \pi_t &= -a_{\pi t} z \pi_0^*. \end{aligned}$$

Substituting these expressions into equation (10) and using $\pi_t^* = \rho_\pi^t \pi_0^*$ we have,

$$\begin{aligned} i_t &= \bar{\imath} - z\pi_t^* - \sigma^{-1} u_t - \phi_\pi a_{\pi t} z \pi_0^* - \phi_y a_{yt} z \pi_0^* \\ &= \bar{\imath} - z\pi_t^* - \sigma^{-1} u_t - \left(\phi_\pi a_{\pi t} + \phi_y a_{yt}\right) z \pi_0^* \\ &= \bar{\imath} - z\pi_t^* - \sigma^{-1} u_t - \left(\phi_\pi a_{\pi t} + \phi_y a_{yt}\right) z \rho_\pi^{-t} \pi_t^* \\ &= \bar{\imath} - \sigma^{-1} u_t + q_t z \pi_t^* \end{aligned} \qquad (25)$$

[31] We take ρ_u as the persistence in the demand, which is persistence in government spending in Mankiw and Reiss (2006) and ρ_π is the persistence in the shock to the Taylor Rule.

where,

$$q_t = -1 - \rho_\pi^{-t} \left(\phi_\pi a_{\pi t} + \phi_y a_{yt} \right)$$

Note, that in contrast to the sticky price model, the response of the nominal interest rate to the inflation target, given by q_t, is time varying.

Consider the monetary authority's choice for the inflation target in response to a large enough adverse demand shock to send the economy into a liquidity trap. The inflation target must increase enough to keep the nominal interest rate positive. Using equation (25), a policy response, designed to keep $i_t = \bar{\imath}, \forall t$, yields

$$\pi_t^* = \frac{\sigma^{-1} u_t}{z q_t}.$$

This rule for the target violates our assumption that it decays at rate ρ_π and is therefore inconsistent with our policy proposal.

We consider an alternative that keeps $i_t \geq \bar{\imath}, \forall t$. The initial short-run inflation target can be chosen to keep the nominal interest rate at its long-run value and then allowed to decay at rate ρ_π according to

$$\begin{aligned} \pi_0^* &= \frac{\sigma^{-1} u_0}{z q_0}, \\ \pi_t^* &= \rho_\pi^t \pi_0^*. \end{aligned}$$

We let $u_t = 1.04\%$ as in the sticky price model and find that this policy causes unreasonably large fluctuations in output and inflation. To reduce the fluctuations in output and inflation to acceptable values, it is necessary to allow the nominal interest rate to fall below its long-run equilibrium value, as in the sticky price case. A policy with

$$\begin{aligned} \pi_0^* &= \frac{\sigma^{-1} u_0 - \eta}{z q_0}, \\ \pi_t^* &= \rho_\pi^t \pi_0^*. \end{aligned} \tag{26}$$

is capable of keeping the nominal interest rate above zero and producing small short-run fluctuations in output and inflation.

We present impulse response functions in Figure 3 with $u_0 = 1.04\%$ as in the sticky price model. We let the initial inflation target be $\pi_0^* = 0.8\%$ at an annual rate, which we calculate as a small increase in the short-run target which keeps the nominal interest rate positive.[32]

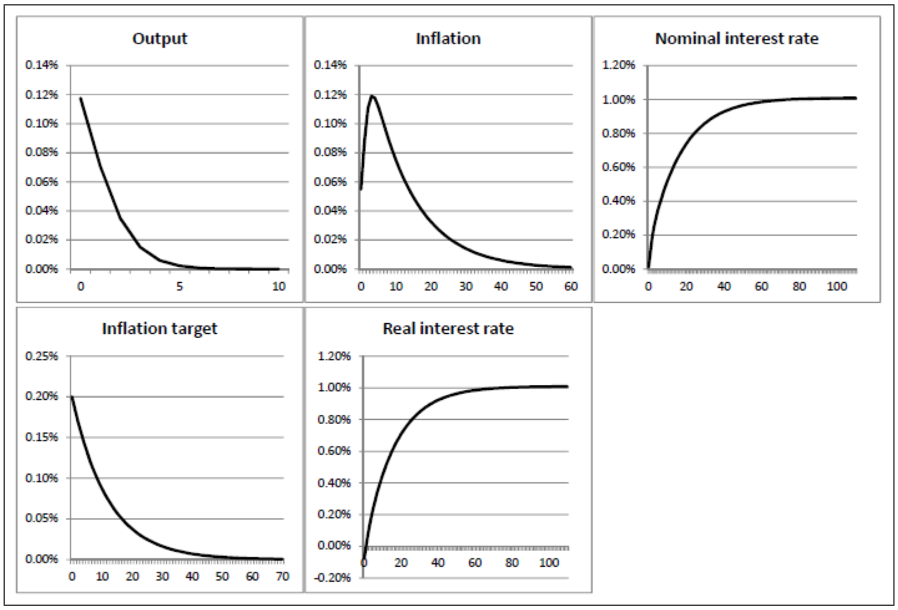

Figure 3: Impulse response under sticky information

An adverse demand shock, under a monetary policy to avoid a liquidity trap, requires an offsetting reduction in the nominal interest rate together with an increase in the inflation target, raising inflationary expectations and reducing the real interest rate. This stimulates inflation and the output gap. The peak response of inflation is delayed by exactly four quarters due to the slow dissemination of information about the shock. The initial deviations in output and the peak effect of inflation are both 0.48% at an annual rates. The policy successfully avoids the liquidity trap with modesty fluctuations in inflation and the output gap.

Our policy also works if we reduce persistence of the demand shock, to make the model more comparable to that in Adam and Billi (2006). With a lower persistence the dynamics of inflation, output, and the inflation target remain unchanged since they are independent of the dynamics of demand shock. There is a minor hump-shaped response in the nominal interest rate with a relatively higher peak value.

[32] Recall that the value for the initial increase in the inflation target is not unique.

5 Conclusion

The nominal interest rate cannot fall below zero. The economy enters a liquidity trap when a large adverse demand shock sends the nominal interest rate to zero as policy-makers try to stimulate the economy. Policy makers do not like liquidity traps for two reasons. Interest rates cannot be reduced to stimulate the economy in a liquidity trap. Second, since the nominal interest rate becomes fixed at zero, sunspot equilibria are possible. Inflation and the output gap become indeterminate.

We propose a monetary policy switching rule which would allow the monetary authority to avoid a liquidity trap. In the event of a large enough adverse demand shock, defined as one which sends the optimal nominal interest rate below zero, the monetary authority switches inflation targets. The short-run target rises above the long-run target, and the increase is highly persistent. In this event, the increase in the inflation target increases inflationary expectations, reducing the real interest rate and stimulating demand so much that the nominal interest rate actually rises. Sunspot equilibria are eliminated, but inflation and the output gap exceed their (unattainable) optimal values of zero. The economy is slow to return to the long-run equilibrium since the increase in the short-run inflation target must be persistent enough to stimulate demand sufficiently that the nominal interest rate rises. The costs of our policy are higher output and inflation fluctuations, while the benefits are determinacy under low interest rates.

The financial crisis which began in 2007 created a growth industry for papers dealing with liquidity traps. Most of them developed unconventional monetary policies, many of which were implemented. Yet, in the United States and Japan, we remain in liquidity traps. Our paper is about conventional monetary policy under a Taylor Rule. There is no role for unconventional monetary policy in simple New Keynesian models. It is also noteworthy that our policy of promising a sustained increase in short-run inflation has been adopted by countries in liquidity traps. The US policy of keeping nominal interest rates at zero for a substantial period of time could be interpreted as an increase in the inflation target if it were not accompanied by concerns about "exit strategies" once the economy recovers. Our analysis implies that positive inflation is not always bad policy for countries which choose to stay out of liquidity traps.

6 Appendix

To solve the New-Keynesian model with sticky information Phillips curve we assume that the demand shock follows the following $AR\left(1\right)$ process.

$$u_t = \rho u_{t-1} + \epsilon_t$$

We also assume that output and inflation follows the following $MA\left(\infty\right)$ process,

$$\begin{aligned} y_t &= \sum_{j=0}^{\infty} a_{yj} L^j \left(\epsilon_t\right) \\ \pi_t &= \sum_{j=0}^{\infty} a_{\pi j} L^j \left(\epsilon_t\right) \end{aligned} \tag{27}$$

Since, $\theta \in (0,1)$,Chattopadhyay (2011) shows a backward solution of the sticky information Phillips curve gives the coefficient of ϵ_{t-j} as,

$$a_{yj} = \frac{\theta^{j+1}}{\alpha\left(1-\theta^{j+1}\right)} \sum_{k=0}^{j} a_{\pi k},\ j \geq 0 \tag{28}$$

and with $\phi_\pi > 1$, a forward solution of the expectational IS schedule gives the coefficient of ϵ_{t-j} as,

$$a_{\pi j} = -\left[\left(\frac{1}{\sigma\phi_\pi}\right) Z_j + \frac{\rho^j}{\phi_\pi - \rho}\right],\ j \geq 0$$

where,

$$\begin{aligned} Z_j &= \sum_{k=j}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{k-j} b_k,\ b_j = \left(a_{yj} - a_{y(j+1)}\right),\ j \geq 0 \\ &= \phi_\pi^j \left[Z_0 - \sum_{k=0}^{j-1} \left(\frac{1}{\phi_\pi}\right)^k b_k\right],\ j \geq 1 \end{aligned}$$

$\Rightarrow$

$$
\begin{aligned}
a_{\pi j} &= -\left[\left(\frac{1}{\sigma\phi_\pi}\right) Z_j + \frac{\rho^j}{\phi_\pi - \rho}\right] \\
&= -\left[\left(\frac{\phi_\pi^j}{\sigma\phi_\pi}\right)\left[Z_0 - \sum_{k=0}^{j-1}\left(\frac{1}{\phi_\pi}\right)^k b_k\right] + \frac{\rho^j}{\phi_\pi - \rho}\right] \\
&= -\frac{\phi_\pi^{j-1}}{\sigma}\left[Z_0 - \sum_{k=0}^{j-2}\left(\frac{1}{\phi_\pi}\right)^k b_k\right] + \frac{1}{\sigma} b_{j-1} - \frac{\rho^j}{\phi_\pi - \rho} \\
&= \left[1 + \frac{A\left(j+1\right)}{\sigma}\right]^{-1} \left[\begin{array}{c} \frac{1}{\sigma}\left\{\chi + (1-\chi) - \frac{A(j+1)}{A(j)}\right\} a_{y(j-1)} - \frac{\rho^j}{\phi_\pi - \rho} \\ -\frac{\phi_\pi^{j-1}}{\sigma}\left\{Z_0 - \sum_{k=0}^{j-2}\left(\frac{1}{\phi_\pi}\right)^k b_k\right\} \end{array}\right]
\end{aligned}
\tag{29}
$$

where,

$$
\begin{aligned}
\chi &= 1 + \sigma\phi_y, \\
A\left(j+1\right) &= \frac{\theta^{j+1}}{\alpha\left(1 - \theta^{j+1}\right)},\ j \geq 0
\end{aligned}
$$

We have used the following algorithm to calculate $\{a_{yj}\}_{j=0}^{\infty}$ and $\{a_{\pi j}\}_{j=0}^{\infty}$ numerically,

1. Start with an initial guess of $Z_0 = Z_0^*$ and calculate

   $$a_{\pi 0} = -\left[\left(\frac{1}{\sigma\phi_\pi}\right) Z_0^* + \frac{1}{\phi_\pi - \rho}\right]$$

   and

   $$a_{y0} = A\left(1\right) a_{\pi 0}$$

2. Calculate $\{a_{\pi j}\}_{j=1}^{\infty}$,$\{a_{yj}\}_{j=1}^{\infty}$ recursively from (28) and (29) through a recursive calculation of $\{b_j\}_{j=0}^{\infty} = \{a_{yj}\}_{j=0}^{\infty} - \left\{a_{y(j+1)}\right\}_{j=0}^{\infty}$.

3. Obtain $Z_0^{cal} = \sum_{j=0}^{\infty}\left(\frac{1}{\phi_\pi}\right)^j b_j$ and calculate $\Delta = \left(Z_0^* - Z_0^{cal}\right)$.

4. If Δ is not sufficiently close to zero, change Z_0^* accordingly and follow above steps until $\Delta \to 0$.

5. When Δ is sufficiently close to zero, we get $\{a_{\pi j}\}_{j=0}^{\infty}$and $\{a_{yj}\}_{j=0}^{\infty}$.

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