Goods, Education and Health: A Combined Model for Evaluating PMGSY

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1 Introduction

A new, all-weather rural road will bring various benefits to the villagers along its route. As producers, they will enjoy higher net prices for their marketed surpluses; as consumers, they will pay less for urban goods. If there is no school in the village itself, those children who already attend one elsewhere will spend less time travelling to and fro, and those who did not attend earlier may do so now. If there is a village school, it is the teachers themselves who may appear more regularly. Much the same holds for medical treatment. Children and those adults with chronic ailments are more likely to make regular visits to the clinic; and in an emergency, those in need of medical attention will be able to reach the clinic sooner, which may make the difference between life and death.

Measuring the road's effects on these movements of goods and people is, in principle at least, relatively straightforward. Valuing the resulting benefits is another matter altogether. For the new road affects not just the decisions of what to produce and consume in the sphere of what might be called 'textbook goods', but also those having to do with the formation and maintenance of human capital, including life itself. These decisions are not, moreover, readily separable, which calls for their analysis within a unified framework. More is at stake, however, than consistency and rigour. It will be argued that valuing the benefits that arise in connection with more favourable prices of goods, improved educational attainment and lower morbidity involves a common (money) metric which is directly related to effects that are fairly readily measurable. In contrast, the benefits of reduced mortality, even if such reductions can be measured with some confidence, do not fit into this convenient scheme of things. How, then, are they to be estimated in practice? The unified framework provides one way of answering this question. For one can set up the model so that, counterfactually, the road lowers mortality but nothing else; or, at the other extreme, everything else but mortality. Granted that the model can be numerically – and persuasively – calibrated, one can use the equivalent variation (EV) for each setting, relative to the benchmark of 'no-road', to establish the size of the benefit arising from lower mortality to that arising from other effects. My object here is to do precisely this, with preliminary but not, I hope, outlandish numerical illustrations. At all events, there is also a check in the form of a completely independent estimate of the so-called value of a statistical life provided by Simon *et al.* (1999).

The plan of the paper is as follows. The model is set out and analysed in Section 2, the essential difficulty with valuing the benefits of lower mortality being addressed in Section 2.3. The numerical set-up follows in Section 3, which is divided up into subsections dealing with functional forms, parameter values and calibration under perfect foresight. This is the basis for the exact welfare analysis in Section 4, treating in sequence the benchmark of 'no-road', the world with the road and the contribution of lower mortality to the whole resulting benefit. The conclusions are drawn together in Section 5.

2 The Model

The basis is the model of Bell, Bruhns and Gersbach (2006), which deals with human capital formation and growth when there is premature adult mortality. To summarize, an extended family comprises three overlapping generations, with all surviving adults caring for all related children in each period, which stretches over a generation.[1] At the end of each generation, some of the surviving young adults die just before reaching old age, all surviving old adults die, and the children become young adults in their turn. The young adults are assumed to decide how current resources are to be allocated between consumption and the children's education. The level of current resources available to the family is heavily determined by the level of the parents' human capital and their survival rate through their offspring's early childhood and school years, but the children themselves can also work instead of attending school.

How much of childhood, if any, is spent at school depends not only on the family's available resources, but also on three further factors. First, there is the parents' desire to provide for their old age and their children's future, motives which express themselves in the parents' willingness to forego some current consumption in favour of investment in their children's schooling, and hence of the children's human capital when they attain adulthood in the next period. Second, there is the efficiency with which schooling is transformed into human capital, which arguably depends on the quality of the school system and child-rearing within the family, whereby the latter ought to improve with the parents' human capital (if they survive this phase of life). Third, the returns to the investment in any child will be effectively destroyed if that child dies prematurely in adulthood. This implies that the expected returns to education depend on parents' (subjective) assessments of the probability that their children will meet an untimely death.

For present purposes, we need to extend this framework in two ways. First, the transportation of goods and persons must be brought into the picture. Instead of the aggregate consumption good in Bell, Bruhns and Gersbach (2006), there are now two consumption goods, one of which the household produces; the other is an 'urban' good, which the household can obtain only through exchange. The resulting trade necessarily involves transportation. The same holds for education and health, insofar as the children must travel to school and the sick to a clinic, which may lie some distance off and, in the absence of an all-weather road, be inaccessible at times. Second, there is a place for morbidity, which, as formulated below, reduces individuals' capacities to go about their daily business.

[1] This arrangement is admittedly a rather idealized description of the social structure even in Kenya, let alone in rural India, but the pooling of the risks of premature mortality among the adults does greatly simplify the analysis.

2.1 Human capital and output

We begin by introducing some notation.

N_t^a: the number of individuals in the age-group $a\,(=1,2,3)$ in period t,

λ_t^a: the human capital possessed by an adult in age-group $a\,(=2,3)$,

γ: the human capital of a school-age child,

α_t: the output produced by a unit of human capital input in year t,

e_t: the proportion of their school-age years actually spent in school by the cohort of children $(a=1)$ in period t.

Human capital is formed through a process that involves the adults' human capital and the educational technology. The human capital attained by a child on becoming an adult in period $t+1$ depends, in general, on the numbers and human capital of the adults, the level of schooling that child received, and the number of siblings of school-going age, who were presumably competing for the adults' attention, care and support – all in the previous period. Formally,

$$\lambda_{t+1}^2 = \Phi(e_t, \boldsymbol{\lambda}_t, \mathbf{N}_t), \tag{1}$$

where $\boldsymbol{\lambda}_t = (\lambda_t^2, \lambda_t^3)$ and $\mathbf{N}_t = (N_t^1, N_t^2, N_t^3)$. It is plausible that Φ is increasing in all its arguments, except for N_t^1.

Establishing a specific functional form with an eye on the need to apply the model is quite another matter. We make the following assumptions. Φ is multiplicatively separable in: (i) the educational technology, which involves only e_t; (ii) the contribution of the parents' human capital; and (iii) the degree of competition among siblings. This form implies that formal education and the parents' human capital are complements in producing their children's human capital, which is intuitively quite plausible.[2] Parents in rural India have most of their children when they are in their twenties, and in their thirties, they are busy rearing them to adulthood. Normalizing the structure to a representative couple within the extended family, these assumptions yield the following specialization of (1):

$$\lambda_{t+1}^2 = f_t(e_t) \cdot \frac{2(N_t^2\lambda_t^2 + N_t^3\lambda_t^3)}{N_t^2 + N_t^3} \cdot \psi\left(\frac{N_t^1}{N_t^2 + N_t^3}\right) + 1, \tag{2}$$

[2]Becker, Murphy and Tamura (1990) and Ehrlich and Lui (1991) pioneered the approach based on the direct transmission of potential productivity from parent to child.

where $f_t(\cdot)$ represents the educational technology, whose efficiency may vary with time, and the function $\psi(\cdot)$ the effects of competition among siblings for their parents' time and attention. These functions are assumed to have the following properties: $f_t(\cdot)$ is continuous and increasing $\forall e_t \in [0, 1)$, with $f_t(0) = 0$; and $\psi(\cdot)$ is continuous and decreasing in the number of children per adult, and goes to zero as that number becomes arbitrarily large. The assumption $f_t(0) = 0$ implies that a child who receives no schooling will attain only some basic level of human capital, which, without loss of generality, may be normalized to unity – hence the '1' on the RHS of (2). The assumption that $\psi(\cdot)$ is a decreasing function implies that, *cet. par.*, an increase in mortality among parents that outweighs any reduction in fertility will hinder the formation of human capital among their children. Let there be no depreciation of human capital.

The difference equation (2) governs the system's dynamics. A brief remark will suffice on the asymptotic behaviour of λ_t^2 when there is full education. Observe that under stationary technological and demographic conditions, (2) may be written as

$$\lambda_{t+1}^2 = 2f(e_t)(a_2\lambda_t^2 + a_3\lambda_{t-1}^2) + 1,$$

where a_2 and a_3 are constants and $\lambda_{t-1}^2 = \lambda_t^3$. Suppose $e_t = 1\ \forall t$, so that the relevant characteristic root is $a_2[1+\sqrt{(}1+2a_3/(a_2^2 f(1)))]f(1)$. Then, starting from a sufficiently large value of the parents' combined human capital, when they would choose $e_t = 1$, unbounded growth of λ_t^2 is possible if $a_2[1 + \sqrt{(}1 + 2a_3/(a_2^2 f(1)))]f(1) \geq 1$, and the growth rate then approaches $a_2[1 + \sqrt{(}1 + 2a_3/(a_2^2 f(1)))]f(1) - 1$ from above. If, however, $1 > 2(a_2+a_3)f(1)$, λ_t^2 will approach the stationary value $1/[1-2(a_2+a_3)f(1)]$.

The household produces a single consumption good (1) solely by means of labor, measured in efficiency units, under constant returns to scale. A natural normalization is that a healthy adult who possesses human capital in the amount λ_t^a is endowed with λ_t^a efficiency units of labor, which he or she is assumed to supply completely inelastically. It is also assumed that human capital does not depreciate for any reason other than the death of the individual in question. What can affect the current supply of labor, however, is sickness or injury among the living during the course of the period. Denote the fraction of each period that an adult spends in disability by d_t^a $(a = 2, 3)$. Children, too, suffer ailments, which reduce the effective time left for schooling and work. Each child supplies $(1 - d_t^1 - (1 + \tau)e_t)\gamma$ efficiency units of labor when it spends e_t units of time in school and, unavoidably, τe_t units of time travelling to and from school, whereby $\gamma \in (0, 1)$, i.e., a full-time working child is less productive than an uneducated adult. The household therefore produces

$$y_{1t} = \alpha_t \left[N_t^1 (1 - d_t^1 - (1+\tau)) e_t) \gamma + (1 - d_t^2) N_t^2 \lambda_t^2 + (1 - d_t^3) N_t^3 \lambda_t^3 \right] \tag{3}$$

units of good 1 in period t.

2.2 The family's preferences and decisions

Some additional notation is needed.

x_{it}: the consumption of good $i\,(=1,2)$ by each young adult,

β: the proportion of a young adult's consumption received by each child,

ρ: the proportion of a young adult's consumption received by each old adult,

σ_t: the direct costs per child of each unit of full-time schooling,

n_t: the number of children born to a representative couple who survive to school age in period t,

${}_xq_a$: the probability that an individual aged a will die before reaching the age of $a+x$,

q_t: the probability that a young adult in period t will die before reaching the third phase of life.[3]

The parameters β and ρ are viewed as binding by all concerned through a social norm and they are enforced by appropriate social sanctions.

The extended family's expenditure-income identity involves, in principle, outlays on the two consumption goods, health care and education, where the latter include both the direct expenditures and the opportunity costs of the pupils' time. In what follows, we resort to the drastic simplification that there are no expenditures on getting to the clinic and being treated: mortality and morbidity rates are set exogenously, at levels that depend on the availability or otherwise of an all-weather road. This is not merely simplification in the interests of making the analysis more tractable. For the relationship between choosing treatment and the experience of disability over the whole stretch of a generation is not only difficult to model, but is also not well-established empirically: for example, better access to a clinic may induce timelier and heavier

[3] In the present structure, this statistic corresponds to ${}_{20}q_{20}$, the probability that an individual will die before 40, conditional on surviving until 20.

outlays on treating an acute ailment, and so save outlays on undoing even more damage later, should the condition go untreated at the outset.

It will be convenient to normalize the budget identity by the number of young adults. The said identity may then be written as:

$$P_t(\beta, \rho) \cdot \mathbf{p}_t \cdot \mathbf{x}_t + Q_t(\alpha_t, \gamma, \sigma_t, \tau) \cdot e_t \equiv p_{1t}\alpha_t \cdot (\Lambda_t + N_t^1(1 - d_t^1)\gamma)/N_t^2, \qquad (4)$$

where the household faces the price vector $\mathbf{p}_t$ for the two consumption goods and

$$\Lambda_t \equiv N_t^2(1 - d_t^2)\lambda_t^2 + N_t^3(1 - d_t^3)\lambda_t^3 \qquad (5)$$

is the adults' aggregate supply of efficiency units of labor. The term

$$P_t(\beta, \rho) \equiv (1 + (\rho N_t^3 + \beta N_t^1)/N_t^2) \qquad (6)$$

expresses the effect of the family's demographic structure on the 'price' of the consumption bundle $\mathbf{x}_t$ relative to education. Analogously, the 'price' of a unit of full education ($e_t = 1$) is

$$Q_t \equiv (p_{1t}\alpha_t(1 + \tau)\gamma + \sigma_t)N_t^1/N_t^2\,. \qquad (7)$$

The RHS of (4) is the level of (normalized) full income in year t.

The young adults' preferences are, in principle at least, defined over the following: the levels of consumption in young adulthood and old age, $\mathbf{x}_t$ and $\rho\mathbf{x}_{t+1}$, respectively, and the human capital attained by their school-age children on attaining full adulthood (λ_{t+1}^2), which they may appreciate in both phases of their own lives. Investment in the children's education therefore produces two kinds of pay-offs, namely, in the form of altruism, as expressed by the value directly placed on λ_{t+1}^2, and selfishly, inasmuch as an increase in λ_{t+1}^2 will also lead to an increase in $\rho\mathbf{x}_{t+1}$ under the said social rules.

Although the pooling arrangement implicit in the extended family structure eliminates the risk that orphaned children will be left to fend for themselves, others remain. A young adult still faces uncertainty about whether he or she will actually survive into the last phase of life, and whether his or her children will do likewise, conditional on their reaching adulthood in their turn. The incidence of morbidity is uncertain at the level of the individual, though if the extended family is large enough, its realized levels of morbidity will differ little from the population rates. There is also arguably uncertainty about future demographic developments, which will influence the realized level of $\rho\mathbf{x}_{t+1}$.

A young adult who survives the hazard of an untimely death at the very start of adulthood in period t chooses a like partner, produces n_t children, and then draws up a plan for current consumption and investment in the children's education based on the family's resources and expectations about its members' state of health and other relevant variables in the coming and future periods. Given such assortative mating, the pair will agree wholly on what is to be done. Appealing to the law of large numbers in order to rid the system of any uncertainty about the realized levels of morbidity in period t, so that full income in that period is non-stochastic, let the preferences of a young adult at time t be represented as follows:

$$E_t U = b_1 u(\mathbf{x}_t) + b_2(1 - q_t)E_t[u(\rho \mathbf{x}_{t+1})] + E_t[(1 - q_{t+1})] \cdot n_t \phi(\lambda^2_{t+1}), \tag{8}$$

where goods 1 and 2 are private goods in consumption, but the children's attainment of human capital is a public one within the union. It should be noted, first, that no account has been taken of the pain and suffering associated with morbidity, even though its level may change exogenously; second, that the 'pay-offs' in the event that the parent should die prematurely (with probability q_t), or that any of the children, in their turn, should die prematurely in adulthood (each with probability q_{t+1}), have been normalized to zero; and third, that conditional on surviving into old age at $t + 1$, the associated level of consumption, $\rho \mathbf{x}_{t+1}$, is also a random variable viewed at time t, for its level depends on a whole variety of future economic and demographic developments. Finally, observe also that the parents' altruistic motive makes itself felt only when they themselves are young and actually make the sacrifices, whereby λ^2_{t+1} is non-stochastic by virtue of e_t being non-stochastic.

These adults take all features of the environment in periods t and $t + 1$ as parametrically given. It will be helpful to distinguish between what they know and what they must forecast. At the time of decision, the current endowment and environment are described by the vector

$$Z_t \equiv (\mathbf{N}_t, n_t, \boldsymbol{\lambda}_t, P_t, Q_t, \mathbf{p}_t, \tau, \alpha_t, q_t, \mathbf{d}_t). \tag{9}$$

This is assumed to be known.[4] What is unknown are the (future) realizations of $\mathbf{x}_{t+1}$ and q_{t+1}. Under the social norm expressed by ρ, the parents at t must form expectations about how their surviving children will allocate full income in period $t + 1$, a decision that depends, not only on Z_{t+1}, which will have been revealed at that time, but also

[4] It can be argued that there is uncertainty about q_t at the point of decision at time t, the (indivisible) unit period being rather long. This possibility is addressed below.

on all future constellations thereafter, to the extent that these influence $(\mathbf{x}_{t+1}, e_{t+1})$.

For simplicity, all individuals' forecasts of all elements of the future environment are assumed to be point estimates, so that whilst there is uncertainty about an individual's personal fate, there is none about the future mortality profile itself or future fertility. Indeed, we go farther down this path, and assume not only that all individuals share the same forecasts of $\{Z_{t+1}\}_{t=1}^{t=\infty}$, but also that these forecasts are unerring: that is to say, there is perfect foresight about everything – with the vital exception of whether a particular individual will die prematurely. Under this assumption, $\rho\mathbf{x}_{t+1}$ becomes non-stochastic, conditional on surviving into old age at time $t+1$, so that (8) may be written

$$E_t U = b_1 u(\mathbf{x}_t) + b_2(1-q_t)u(\rho\,\mathbf{x}_{t+1}^0) + (1-q_{t+1})\cdot n_t\phi(\lambda_{t+1}^2(e_t)), \tag{10}$$

where the superscript '0' denotes the optimal choice of those making decisions at time $t+1$, which their parents forecast unerringly at t. The functions u and ϕ are assumed to be strictly concave.

A young adult's decision problem therefore takes the following form:

$$\max_{(\mathbf{x}_t,\, e_t | \{Z_{t+t'}\}_{t'=0}^{t'=\infty})} E_t U \quad \text{s.t. } \mathbf{x}_t \geq \mathbf{0}, e_t \in [0,1], (2), (4). \tag{11}$$

If $f_t^1(\cdot)$ is concave, $E_t U(\cdot)$ will be strictly concave in $(\mathbf{x}_t,\ e_t)$. Hence, problem (11) has a unique solution and the first-order necessary conditions are also sufficient. Observe that the optimum always involves $\mathbf{x}_t > \mathbf{0}$. The corner solution in which the children are not educated at all can also be ruled out when $f(\cdot)$ satisfies the lower Inada condition, since $\partial\lambda_{t+1}^2/\partial e_t$ is then unbounded at $e_t = 0$.

2.3 Comparative statics

The associated Lagrangian is, omitting the non-negativity constraints for brevity,

$$\begin{aligned}\mathcal{L} &= b_1 u(\mathbf{x}_t) + b_2(1-q_t)u(\rho\,\mathbf{x}_{t+1}^0) + (1-q_{t+1})\cdot n_t\phi(\lambda_{t+1}^2(e_t)) \\ &\quad + \mu\left[p_{1t}\alpha_t\cdot(\Lambda_t + N_t^1(1-d_t^1)\gamma)/N_t^2 - P_t(\beta,\rho)\cdot\mathbf{p}_t\cdot\mathbf{x}_t - Q_t(\alpha_t,\gamma,\sigma_t)\cdot e_t\right]. \end{aligned} \tag{12}$$

The Envelope Theorem yields the following results, all of which accord with elementary intuition. Where the movement of goods and school children is concerned, we have

$$\frac{\partial E_t U^0}{\partial p_{1t}} = \mu\left[\alpha_t \cdot (\Lambda_t + N_t^1(1-d_t^1)\gamma)/N_t^2 - P_t(\beta,\rho)\cdot x_{1t} - (\alpha_t(1+\tau)\gamma N_t^1/N_t^2)e_t\right] > 0, \tag{13}$$

and

$$\frac{\partial E_t U^0}{\partial p_{2t}} = -\mu P_t(\beta,\rho)\cdot x_{2t} < 0, \tag{14}$$

by virtue of the fact that the household is a net seller of good 1 and a net buyer of good 2, and $e_t \leq (1-d_t^1)/(1+\tau)$. An increase in the travel-time to school is likewise damaging:

$$\frac{\partial E_t U^0}{\partial \tau} = -\mu p_{1t}\alpha_t\gamma(N_t^1/N_t^2)e_t\,. \tag{15}$$

An improvement in the educational technology, which, broadly construed, might arise from more regular attendance by teachers, yields more capital accumulation without additional investment:

$$\frac{\partial E_t U^0}{\partial z_t} = (1-q_{t+1})n_t\phi'(\lambda_{t+1}^2(e_t))\cdot\frac{\partial f_t}{\partial z_t}\cdot\lambda_t, \tag{16}$$

where z_t is an efficiency parameter. Morbidity acts only to reduce the family's productive endowments, pain and suffering having been ruled out by assumption:

$$\frac{\partial E_t U^0}{\partial d_t^1} = -\mu p_{1t}\alpha_t\gamma\cdot(N_t^1/N_t^2), \tag{17}$$

$$\frac{\partial E_t U^0}{\partial d_t^2} = -\mu p_{1t}\alpha_t\cdot\lambda_t^2\,, \tag{18}$$

$$\frac{\partial E_t U^0}{\partial d_t^3} = -\mu p_{1t}\alpha_t\cdot(N_t^3/N_t^2)\lambda_t^3\,. \tag{19}$$

Turning at last to mortality, we have

$$\frac{\partial E_t U^0}{\partial q_t} = -b_2 u(\rho\,\mathbf{x}_{t+1}^0); \tag{20}$$

$$\frac{\partial E_t U^0}{\partial q_{t+1}} = -b_2\cdot\nabla u(\rho\,\mathbf{x}_{t+1}^0)\cdot\left(\rho\frac{\partial \mathbf{x}_{t+1}^0}{\partial q_{t+1}}\right) - n_t\phi(\lambda_{t+1}^2(e_t)), \tag{21}$$

which reflects the fact that an increase in mortality among the children on reaching adulthood will also adversely affect the parents' consumption in old age, should they survive to enjoy it.

3 Setting up the System Numerically

The main aim is to estimate the benefits flowing from an all-weather road, which stem from more favorable prices facing the household as producer and consumer, from reduced time for the children to go to and from school, and from lower morbidity and mortality due to timelier treatment. Under the above assumptions, it is seen from (13) - (19) that, with the exception of reduced mortality, sufficiently small changes in each of these features of the 'environment' yield benefits that, in money-metric utility, are equal to the gains or savings calculated at the allocation ruling before the said change and valued at the corresponding opportunity cost.[5] This is exactly the basis for the short-cut proposed in Bell (2009), which deals only with the prices of goods: with plausible preferences, technologies and the size of changes in unit transport costs, the associated error is small. Intuition suggests that the same will hold with the extensions to cover the travel-time to school and the levels of morbidity – though it would be as well to do exact calculations using numerical examples, as in Bell (2009). Inspection of (20) and (21), however, reveals at once that there is no such ready simplification where mortality is concerned; for the sub-utility functions u and ϕ appear explicitly, as does the next bundle $\mathbf{x}_{t+1}^0$ in the perfect-foresight sequence $\{\mathbf{x}_t^0\}_{t=0}^{t=\infty}$. It follows that there is no avoiding the need to explore some numerical examples in order to obtain some feel for the size of the value placed on reduced mortality relative to that of other benefits. We now take up this task, which involves the construction of the whole perfect-foresight sequence.

3.1 Functional forms

There is no hope of estimating more than a tiny part of this system econometrically; and even 'calibration' for the system as a whole is ruled out for want of suitable data. The approach, therefore, is to choose functional forms that are both tractable and plausible, if only through common usage in other contexts, and then constellations of associated parameter values such that certain key magnitudes correspond to what are called the 'stylized facts'.[6]

1. Technologies. Let $f_t(e_t) = ze_t \ \forall t$, where z represents an inter-generational transmis-

[5]Observe that each of these expressions is scaled by the Lagrange multiplier μ.

[6]As Solow once wrote in the original connection with the character of growth in industrialized countries in the decades following WWII, they are certainly stylized, but whether they are facts is another matter.

sion factor, which reflects the quality of both child-rearing and the school system. This limiting form is certainly the simplest, and it causes no technical problems in view of the assumption that ϕ is strictly concave (see below). The absence of diminishing returns does not seem especially odd when one reflects on the need for children to spend some years in school before they have mastered the three R's, which form the basis of all other acquired abilities involving literacy. The great majority of school-children in India's villages now receive some education, moreover, so that this form of $f(e_t)$ can also be thought of as applying over the relevant range up to a full education. Turning to competition among siblings, rather little is known about its effects on human capital formation, so we adopt the agnostic position that $\psi = 1 \; \forall \, \mathbf{N}_t$. With these choices, (2) specializes to

$$\lambda^2_{t+1} = 2z \cdot e_t \cdot \frac{N^2_t \lambda^2_t + N^3_t \lambda^3_t}{N^2_t + N^3_t} + 1. \tag{22}$$

2. Preferences. In the macroeconomics literature, especially the empirical kind, the logarithm holds sway. There is almost invariably, however, an aggregate consumption good. For present purposes, therefore, form the Cobb-Douglas aggregate $x^a_{1t} \cdot x^{1-a}_{2t}$ $(0 < a < 1)$, which is homogeneous of degree one in $\mathbf{x}_t$. Applying the logarithm to this index of consumption, we obtain

$$u(\mathbf{x}_t) = a \ln x_{1t} + (1-a) \ln x_{2t} \quad \forall \, t.$$

There is much less guidance to be had about $\phi(\cdot)$. In their study of Kenya over the historical period 1950-1990, Bell, Bruhns and Gersbach (2006) employed the logarithmic form for u, but the data resisted their attempts to impose this on ϕ. More curvature was needed, and successful calibration was achieved with the iso-elastic form

$$\phi(\lambda^2_{t+1}) = 1 - (\lambda^2_{t+1})^{-\eta}/\eta \, ,$$

whereby the value of η lay in the range 0.35 – 0.65, with a clustering around 0.5. This form will be adopted here, too. The associated values of η will provide a useful point of departure.

3.2 Parameters and the values of exogenous variables

Given the vast array of parameters, and equally vast degree of under-identification, there is no call for great precision everywhere. We need some starting values, namely, for period $t = 1$. In view of India's demographic history and the prevailing state of affairs in rural areas, let $\mathbf{N}_1 = (3, 2, 0.75)$, with $n_1 = 3.5$. There is much illiteracy among the old, but less among their children, who are today's parents. Rising productivity over the past generation also suggests that λ_1^2 is substantially larger than λ_1^2. Hence, let $\boldsymbol{\lambda}_1 = (1.7, 1.2)$, with $\gamma = 0.65$.

Turning to expenditures, let the social norms demand $\beta = 0.6$ and $\rho = 0.8$. Households are still rather poor, so their taste for good 1 should be at least as strong as that for good 2: accordingly, let $a = 0.5$. Without loss of generality, set the prices of both goods in the town at unity in all periods. In the absence of an all-weather road, Bell (2009) employs unit transport costs of 0.2 and 0.15, respectively, so that households then face the price vector $\mathbf{p}_t = (0.8, 1.15)\ \forall\, t$. Under these conditions, the travel-time to school and back can be likewise rather long, easily an hour or more a day: allowing for sleeping, eating and bathing at home, let $\tau = 0.08$. The direct costs of state schooling are surely modest: recalling (7), let σ_t be 0.15 times the opportunity cost factor $p_{1t}\alpha_t(1 + \tau)\gamma$, whereby α_t has yet to be determined. For the moment, we also defer discussion of the inter-temporal taste parameters b_1 and b_2.

Coming by estimates of premature adult mortality is a far easier task than that of morbidity. In a setting of three overlapping generations, each generation corresponds to about 20 years, the age at which full adulthood is attained. The rate q_t therefore corresponds to ${}_{20}q_{20}$, the probability that an individual will die before reaching 40, conditional on reaching 20. For India in 2005, WHO (2007) gives ${}_{20}q_{20} = 0.065$ and ${}_{30}q_{20} = 0.123$. Something closer to the latter is better suited to our present purposes, first, to allow for some mortality in the first part of old age, which the rigid time structure of the model rules out, and secondly, to reflect higher mortality among rural middle-aged adults than the all-India average. Hence, let $q_1 = 0.125$. If history and international experience are any guides, this is sure to fall over the coming generation, PMGSY or no. It does not seem too much to hope that India will do as well then as China does now, so let $q_2 = 0.053$. Where morbidity and disability are concerned, I am rather lost for sources at the time of writing. School-age children typically suffer less sickness than their parents, who, in turn, are in better health than their aged parents. The vector $\mathbf{d}_1 = (0.05, 0.10, 0.20)$ represents a speculative stab at an estimate for period 1. Since it is easier to ward off premature death than morbidity, the associated improvement to $\mathbf{d}_2 = (0.04, 0.08, 0.16)$ in period 2 is less dramatic than that in mortality.

3.3 Setting up the sequence under perfect foresight

The system must be set up in such a way that it satisfies two requirements. First, the household must choose a plan that is in keeping with what we observe in the present. The key variable here is the level of investment in education, e_1. Children in India's rural areas typically start school at 6 years of age and complete about 6 years of schooling on average. Hence, with up to 12 years of schooling available, the model must be set up so as to yield $(1+\tau)e_1^0 = 6/12$. Second, the whole sequence must be anchored to some plausible configuration in the future. In this connection, there is much talk of meeting the so-called Millenium Development Goals by 2015, and suchlike. Let us suppose, therefore, that a full education for all is attainable within one complete generation, so that the model must be set up such that parents in period 2 do choose $e_2^0 = 1/(1+\tau)$. If the general environment described by Z_t does not deteriorate thereafter, it will then follow that $e_t^0 = 1/(1+\tau)\ \forall\, t \geq 3$. For once $e_t^0 = 1/(1+\tau)$ is attained, the young adults in that period can then be certain that all future generations will continue this policy, with its corresponding effects on their consumption in old age, as formulated in (11). The desired anchoring of the system will have been accomplished.

The simplest way of ensuring all this where Z_t is concerned is to impose stationarity from period 2 onwards. On the assumption that fertility will fall to replacement levels by 2030, and allowing for premature mortality among adults, let $\mathbf{N}_t = (2, 2, 1.5)\ \forall t \geq 2$. Also stationary are prices, tastes and costs, a road being built, if at all at the start of period 1 (see below). Recalling the discussion of mortality and morbidity in Section 3.2, we also have $q_t = 0.053$ and $\mathbf{d}_t = (0.04, 0.08, 0.16)\ \forall t \geq 2$.

The final step is to choose the productivity parameters z and α and the intertemporal taste parameters b_1 and b_2 so as to yield $e_2^0 = 1$ with as little to spare as possible. With the exception of z, this must be accomplished by trial and error as part of the whole process of computation. The only prior restriction to be imposed is that with pure impatience for consumption, $b_1 > b_2$. Since premature mortality already appears in connection with preferences over dated consumption, the pure discount rate arguably should not greatly exceed 15 per cent per generation of 20 years.

One can arrive at an appropriate value of z by imposing the assumption that individual productivity, λ_t, will grow without limit if, after some point, all generations are fully educated. As noted in Section 2.1, by choosing $z > 0.5$ and otherwise setting up the system so that $e_t^0 = 1\ \forall t \geq 2$, we ensure that λ_t^2 will indeed grow without bound. Recall that the relevant characteristic root is $a_2[1 + \sqrt(1 + 2a_3/(a_2^2 f(1))]f(1)$, whereby

$a_2 = 2/3.5$ and $a_3 = 1.5/3.5$. We set $z = 0.88$, which implies that output per head will grow at the rate of 50 per cent per generation, or 1.7 per cent a year. This seems defensible for an horizon many generations off.

4 A New Road: Exact Welfare Measures

Imagine two islands, each inhabited by a representative extended family. The first is characterized by the constellation of numerical values set out in Section 3. The second is identical, except for the happy event that a road is provided free of charge at the very start of period 1. As a result, unit transport costs, travel-times, morbidity and mortality all become lower than those on island 1. In this more benign environment, it is not only certain that $e_t^0 = 1\ \forall t \geq 2$, but also highly likely that e_1^0 will be higher than on island 1. The latter being so, the path $\{\lambda_t^2\}_{t=2}^{\infty}$ will lie everywhere above its counterpart on island 1, albeit both paths will exhibit the same asymptotic rate of growth by virtue of the common value of z. This 'level-effect' will continue to hold, moreover, even if the road falls into utter disrepair at the end of period 1; for $e_t^0 = 1\ \forall t \geq 2$ is still ensured thereafter, under the hypothesis that it holds on island 1, which will have no road in any period. If, however, the road should increase the transmission factor z, there will be the additional advantage of a permanent 'growth-effect'.

Island 1, therefore, provides the benchmark for the following thought-experiment. At the very start of period 1, the inhabitants of island 2 are given the choice between having the road and making do with the conditions ruling on island 1, but receiving a lump-sum payment instead. If the latter sum is such that they are indifferent between these two alternatives, we will have found the equivalent variation (EV) corresponding to the provision of the road, as assessed by the young adults in period 1 on both islands. The road will, of course, yield further benefits in later periods, even if it falls into utter disrepair at the end of period 1. For the level-effect alluded to above will come into play, and to the associated increase in the family's full income there will correspond an EV for that generation of young adults. In what follows, however, we will confine our attention to the EV for young adults in periods 1 and 2, leaving the task of estimating the whole sequence thereof to another paper.

4.1 The benchmark: no road

Recalling Section 3.3, the task here is to choose α, b_1 and b_2 so that $e_2^0 = 1$ is barely attained. It is easily seen that the whole sequence $\{\lambda_t^2\}_{t=1}^{\infty}$ can be derived on the hypothesis that $e_t^0 = 1\ \forall t \geq 2$ without any reference to α, b_1 and b_2. The parameter values, the initial conditions and (22) yield

$$\lambda_2^2 = 2 \times 0.88 \cdot \frac{6/12}{1.08} \cdot \frac{2 \times 1.7 + 0.75 \times 1.2}{2 + 0.75} + 1 = 2.2742\,.$$

Continued recursion using $\{N_t\}_{t=2}^{\infty}$ yields, upon introducing morbidity explicitly,

$$\lambda_3^2 = 2 \times 0.88 \cdot \frac{1 - 0.04}{1.08} \cdot \frac{2 \times 2.2742 + 1.5 \times 1.7}{2 + 1.5} + 1 = 4.1729\,,$$

$$\lambda_4^2 = 2 \times 0.88 \cdot \frac{1 - 0.04}{1.08} \cdot \frac{2 \times 4.1729 + 1.5 \times 2.2742}{2 + 1.5} + 1 = 6.2552\,,$$

and so forth.

Having thus determined that $\lambda_2^2 = 2.2742$, the hypothesis $e_2^0 = 1$ leaves the household in period 1 with almost all the information needed to derive $\mathbf{p}_2 \cdot \mathbf{x}_2^0$ from (4), whereupon $\mathbf{x}_2^0$ would follow from the assumption that the sub-utility function u is (transformed) Cobb-Douglas. The missing element is the value of α. Moving back to calibration, therefore, it follows that by hazarding a guess at α, we are then left to find a pair (b_1, b_2), with $b_2 \approx 0.85 b_1$, such that the solution to problem (11) in period 1 indeed involves $e_1^0 = (6/12)/1.08$, whereby α may be varied until the desired result is obtained. Mindful of the need to fulfill the hypothesis $e_2^0 = 1$, but not too comfortably, some experimenting yielded $\alpha = 5$, albeit with quite strong impatience ($b_2 = 0.8 b_1 = 17.001$) and modest curvature of the sub-utility function ϕ ($\eta = 0.1$).

The resulting sequence of the main endogenous variables when there is no road is set out in the upper panel of Table 1. Three generations on, in some 60 years, a young adult is almost four times more productive and enjoys a level of real consumption likewise almost four times higher than his or her great-grandparents were in young adulthood.

4.2 Life with the road

We need to specify precisely how the road improves the household's environment in period 1, as described by the vector Z_1. Let it halve the unit transport costs for goods

Table 1: The sequences of the main variables, with and without the road

Period		1	2	3	4
No Road	x_{1t}	2.5632	3.4882	5.8843	9.4206
	x_{2t}	1.7831	2.4266	4.0934	6.5534
	e_t^0	0.4630	0.8889	0.8889	0.8889
	λ_t^2	1.7000	2.2742	4.1729	6.2552
Road: V1a	x_{1t}	2.5437	3.7028	6.3739	10.3822
	x_{2t}	2.1296	3.1000	5.3363	8.6921
	e_t^0	0.5220	0.9269	0.9269	0.9269
	λ_t^2	1.7000	2.4365	4.4599	6.8611
Road: V1b	x_{1t}	2.5221	3.7856	6.5930	10.8482
	x_{2t}	2.1115	3.1693	5.5197	9.0822
	e_t^0	0.5383	0.9269	0.9269	0.9269
	λ_t^2	1.7000	2.5150	4.6134	7.1968
Road: V2a	x_{1t}	2.5324	3.7276	6.4142	10.4372
	x_{2t}	2.1201	3.1208	5.3700	8.7381
	e_t^0	0.5305	0.9269	0.9269	0.9269
	λ_t^2	1.7000	2.4600	4.4819	6.8980
Road: V2b	x_{1t}	2.5108	3.8110	6.6348	10.9063
	x_{2t}	2.1020	3.1906	5.5547	9.1308
	e_t^0	0.5468	0.9269	0.9269	0.9269
	λ_t^2	1.7000	2.5391	4.6364	7.2359

V1a: 1%, $\Delta z = 0$; V1b: 1%, $\Delta z > 0$; V2a: 10%, $\Delta z = 0$; V2b: 10%, $\Delta z > 0$

and the travel-time to school, so that $\mathbf{p}_1 = (0.9, 1.075)$ and $\tau = 0.04$. It can hardly be expected that the road will have such a dramatic effect on morbidity and mortality. To obtain some feel for the sensitivity of the outcomes to such improvements, let these rates fall by just 1 per cent,[7] so that $\mathbf{d}_1 = (0.0495, 0.0990, 0.1980)$ and $q_1 = 0.1238$, a constellation henceforth labelled Variant 1. The sub-variants in which there is only a level-effect ($\Delta z = 0$) and a growth-effect ($\Delta z > 0$) are denoted by a and b, respectively. Let us also consider a more optimistic alternative, namely, that the road results in a uniform reduction of 10 per cent, so that $\mathbf{d}_1 = (0.045, 0.090, 0.180)$ and $q_1 = 0.112$, henceforth labelled Variant 2. Turning to future periods, let the road be perfectly durable, thereby maintaining these more favourable prices and travel-times indefinitely. As argued in Section 3.2, however, morbidity and mortality will surely fall over the next generation even in the absence of a road. Let the road continue to result in a 1 per cent improvement in Variant 1, so that $\mathbf{d}_2 = (0.0396, 0.0792, 0.1584)$ and $q_2 = 0.048$. Thereafter, all these remain stationary. The same holds, *mutatis mutandis*, for Variant 2.

The next step is to compute the sequence of perfect foresight equilibria that will arise under these more favorable sequences of $\{Z_t\}_{t=1}^{\infty}$. The values of the main variables for the first four periods are reported in the lower four panels of Table 1. It is seen that, three generations on in Variant 1a, the levels of real productivity and consumption[8] are about 9.7 and 20.9 per cent higher, respectively, than on island 1. With the lower levels of morbidity and mortality in Variant 2a, the corresponding figures are 10.3 and 24.2 per cent, respectively. In the presence of the growth-effect (sub-Variants b), the differences are naturally somewhat larger: 15.1 and 26.3 per cent, respectively, in Variant 1, and 15.7 and 27.0 per cent in Variant 2. For the chosen value of Δz ($= 0.02$), a comparison of V2a with V1b reveals that the growth-effect dominates those stemming from the reductions in morbidity and mortality.

4.3 The benefits generated by the road

The EV for young adults in period 1 is obtained by finding lump-sum payments such that they would attain the same level of expected utility were they confronted instead with the less favourable environment ruling on island 1. Since most will survive into old age and, in this model, there are no financial instruments beyond pooling and the

[7] At the time of writing, I have found nothing in the empirical literature to guide me. Any leads would be most welcome.

[8] Form and compare the Cobb-Douglas aggregates $(x_{1t} x_{2t})^{0.5}$.

social norm expressed by ρ, it is desirable to smooth the payments. For simplicity, let there be one payment in period 1, T_1^2, which each young adult will enjoy for sure, and another of equal size in period 2, conditional on the individual surviving into old age. The level of the family's (normalized) full income in period 1 is therefore augmented by the amount T_1^2, and its allocation remains subject to the social norms expressed by β and ρ. In keeping with the assumption that altruism disappears with the onset of old age, let surviving individuals keep the contingent payment in period 2 wholly for themselves. Under the social norms, these receipts have no effect on their children's decisions as adults in period 2, and hence introduce no further complications into the computation of the whole sequence. The task, therefore, is to find a T_1^2 such that young adults attain E_1U^0 with the road in period 1, given that they correctly forecast $\rho\,\mathbf{x}_2^0$ when solving problem (11) with (normalized) full income augmented by T_1^2 and so enjoy purchasing power $\mathbf{p}\cdot\rho\,\mathbf{x}_2^0 + T_1^2$ in old age, conditional on their surviving to enjoy it. Given Cobb-Douglas preferences, the sum $\mathbf{p}\cdot\rho\,\mathbf{x}_2^0 + T_1^2$ will be spent in proportions a and $1-a$ on goods 1 and 2, respectively, which completes the formulation of their decision problem, and so permits the joint computation of T_1^2 and the whole sequence.

The payments in question for Variants 1 and 2 are reported in the column so labelled in Table 2. To put them in perspective, recall that a young adult in period 1 can produce $5 \times 1.7 = 8.5$ units of good 1, with a farmgate price of 0.8 in the absence of the road. In Variant 1a, therefore, T_1^2 is 11.5 per cent of the value of this output. In the healthier environment of Variant 2a, it rises to 12.8 per cent thereof. Here, it should be recalled that those who survive into old age will receive the same payment once more on reaching that phase of life, and that in view of the fact that T_1^2 reflects both intertemporal substitution and pure impatience, there is no reason to discount the second (contingent) payment when summing up over both periods to yield the total benefit received by each adult who is young in period 1. The willingness to pay is only slightly larger with the growth effect in the sub-Variants b, which reflects the fairly strong concavity of U.

Later generations also benefit, not only because the road is assumed to be perfectly durable, but also because it promotes human capital formation in period 1, and so confers higher productivity on adults in all subsequent periods, even when there is only a level-effect. With the growth-effect, T_2^2 in Variants 1b and 2b is 12.9 and 14.0 per cent, respectively, of the value of a young adult's output in the absence of a road ($5 \times 2.2742 \times 0.8$). These benefits, and those accruing to subsequent generations, must be discounted back to period 1 at the appropriate rate; for those receiving them arrive progressively later on the scene under conditions of improving living standards. Even

Table 2: The EV and its decomposition, $t = 1$

	E_1U_1	T_1^2	$E_1U_1(\Delta q)$	$T_1^2(\Delta q)$	$E_1U_1(\Delta q^c)$	$T_1^2(\Delta q^c)$
No Road	67.7205					
$\Delta z = 0$						
Road: V1a	72.0855	0.7817	67.7680	0.0078	72.0348	0.7718
				(1.00%)		(1.26%)
Road: V2a	72.5424	0.8709	68.1968	0.0792	72.0348	0.7719
				(9.10%)		(11.38%)
$\Delta z > 0$						
Road: V1b	72.3480	0.7895	68.0277	0.0080	72.2969	0.7795
				(1.01%)		(1.26%)
Road: V2b	72.8083	0.8803	68.4604	0.0808	72.2969	.7795
				(9.17%)		(11.45%)

so, their contribution to the discounted sum of all benefits will be large: with an intergenerational discount rate of 100 per cent (almost 4 per cent per annum), for example, the present value of T_2^2 slightly exceeds T_1^2.

Table 3: The EV and its decomposition, $t = 2$

	E_2U	T_2^2	$E_2U_2(\Delta q)$	$T_2^2(\Delta q)$	$E_2U_2(\Delta q^c)$	$T_2^2(\Delta q^c)$
No Road	68.2814					
$\Delta z = 0$						
Road: V1a	74.4694	1.7744	68.3371	0.0146	74.4120	1.7564
				(0.82%)		(1.01%)
Road: V2a	74.9856	1.9368	68.8371	0.1468	74.4120	1.7564
				(7.58%)		(9.32%)
$\Delta z > 0$						
Road: V1b	75.5579	1.8264	69.4215	0.0151	75.5000	1.8078
				(0.83%)		(1.026%)
Road: V2b	76.0778	1.9948	69.9255	0.1522	75.5000	1.8079
				(7.63%)		(9.37%)

4.4 Decomposing the contributions

The EV measures the combined benefits generated by favourable changes in transport costs, morbidity and mortality. As established above, the first two lend themselves fairly readily to empirical estimation through observation of households' actual behaviour, but the last confronts us with severe problems. If the parametrisation of the model be accepted, however, we can seek enlightenment by answering the question, how big is the contribution of the reduction in mortality to the EV in comparison with that of the rest taken together? For with practical chances of measuring the latter independently and empirically, the model will then yield a rough, but defensible estimate of the former.

To recap from Sections 4.1 and 4.2, the road causes the constellation

$$\mathbf{p}_t = (0.8, 1.15)\ \forall t, \tau = 0.08, \mathbf{d}_1 = (0.05, 0.10, 0.20),\ \mathbf{d}_t = (0.04, 0.08, 0.16)\ \forall t \geq 2,$$
$$q_1 = 0.125,\ q_t = 0.053\ \forall t \geq 2$$

to become, in Variant 2,

$$\mathbf{p}_t = (0.9, 1.075)\ \forall t, \tau = 0.04, \mathbf{d}_1 = (0.045, 0.09, 0.18),\ \mathbf{d}_t = (0.036, 0.072, 0.144)\ \forall t \geq 2,$$
$$q_1 = 0.1125,\ q_t = 0.0477\ \forall t \geq 2.$$

The desired decomposition involves the changes in q_t. Here, the problem of path dependence cannot be avoided; for the EV corresponding to the change in the whole constellation is not, except by mere fluke, additively separable in the component parts. We therefore proceed as follows. Suppose, at one extreme, the road were to affect *only* q_t. As before, we can calculate the value of T_1^2 that corresponds to this hypothetical change in the original constellation, and then express it as a proportion of its counterpart under the complete change. The columns labelled $E_1U_1(\Delta q)$ and $T_1^2(\Delta q)$ in Table 2 report the corresponding levels of expected utility and the said payments. In Variant 2a, the value of the reduction in mortality by itself makes up 9.1 per cent of T_1^2, which is about 9 times its proportional contribution in Variant 1a. That this falls just short of the tenfold reduction in mortality rates relative to Variant 1 is due to the concavity of u. The introduction of the growth-effect makes virtually no difference to these proportions.

At the other extreme, one can calculate the payment that corresponds to all the changes in the original constellation *except* q_t, which is denoted by $T_1^2(\Delta q^c)$. The residual left after subtracting this from the value of T_1^2 corresponding to the complete change is then attributable to the change in q_t *plus* any interaction effects between changes in mortality and all other changes. In Variant 2a, the corresponding value is 11.38 per cent, some 2.28 percentage points larger than that when the reduction in mortality stands alone. The difference in the presence of the growth-effect is the same. This indicates that there are substantial positive interactions between mortality and all the other factors. The intuition for this result is that the latter, when taken alone, increase consumption in period 2, and so improve the pay-off to survivors.[9] At all events, we have bracketed the 'right' estimate with near certainty, and the band around it is not especially wide.

[9]Inspection of (20) and (21) reveals this at once.

By way of an independent check on this finding, one can compare the above estimates with those yielded by a completely different approach. The one that comes immediately to mind involves the derivation of the so-called Value of a Statistical Life (VSL) from variation in wage rates across occupations or industries and the associated variation, if any, in fatality rates, an approach vigorously promoted by Viscusi (for a survey, see Viscusi [1993]). Pursuing this line, Simon *et al.* (1999) employ data from Indian manufacturing firms and arrive at the rather startling estimate that the VSL is 20 to 48 times larger than the present value of lifetime foregone earnings. As the authors note, this greatly exceeds the estimate of 7 to 8 obtained by Liu *et al.* (1997) for Taiwan in the 1980s.

Be that as it may, what is the ratio implied by the approach adopted here? Consider Variant 2b in period 1. The equivalent sum $T_1^2(\Delta q)$ stems from a reduction in q_1 of 0.0125 (= 0.1 × 0.125). Since the said sum is paid twice, the implied VSL is 12.928 (= 2 × 0.0808/0.0125), which is not quite twice as large as the value of output in old age of a surviving young adult, namely, 6.8 (= 5 × 1.7 × 0.8). The implied VSL in period 2 is 57.434 (= 2 × 0.1522/0.0053), as against lost output in the value of 9.097 (= 5 × 2.2742 × 0.8), a ratio of just over six to one. In the present framework, however, $T_t^2(\Delta q)$ relates not to the risk that a young adult will fail to survive into old age and so not produce any output in that phase of life, but rather that he or she will not enjoy her claim on the common pot in old age, whose value is $\rho \cdot \mathbf{p}_{t+1}\mathbf{x}_{t+1}$. Young adults in period 1 face the risk of losing 4.465, which in relation to the VSL implies a ratio of almost three to one. Their counterparts in period 2 face the possible loss of 7.532, which implies a ratio of 7.6 to one. All in all, these ratios do seem to be rather on the low side, which suggests that the benefits stemming from the posited reductions in mortality may be somewhat higher than those reported in Tables 2 and 3.

5 Conclusions

The main object of this paper has been to develop a method to value the resulting reductions in mortality, if any, when the grievously sick and injured can be brought to a clinic or hospital more speedily. The rural road that makes this possible also generates other benefits for the villagers along its route, benefits that are much more readily measurable. By analysing the road's effects on production, consumption, human capital formation and health within a unified framework, one can establish, for each constellation of functional forms and parameter values, how large is the willingness to pay for the reductions in mortality, relative to the willingness to pay for the improvements in other spheres. Given that the latter can be independently estimated, the former then follows – for the constellation in question.

Arriving at a plausible constellation is no mean task, and that proposed here is admittedly provisional and tentative, based as much of it is on macro-economic work that employs a somewhat simpler form of the same framework. If, in addition to the other improvements in the villagers' economic environment, the road reduces the prevailing mortality rate among adults by 10 per cent (*not* 10 percentage points), the model when so parametrized here yields a corresponding benefit that is about 10 per cent of the whole benefit generated by the road. This estimate may seem rather modest, and the independent check provided by the approach using the Value of a Statistical Life, while by no means decisive, does suggest that the parametrization of the OLG-model should be re-examined. It is also possible that the reduction in transport costs posited here is seriously amiss, or that the road will have a more dramatic effect on such mortality. For empirical evidence that this is indeed so, I should be much indebted to the bearer.

Appendix

by Jochen Laps

This appendix describes the solution methods used to produce the results in Tables 1, 2 and 3. To that end, it is divided into two sections. The first summarizes the parameters and exogenous variables with and without the road. The second deals with calibration of the model, the calculation of the sequences reported in Table 1 and the equivalent variation of Tables 2 and 3. The computer package used is `MATLAB`[1,2].

A.1 Parameters and Variables

The road causes the household's demographic environment and its expectations concerning the latter to change as reported in Table A.1.

Table A.1: Demographic Environment

Period		1	$t \geq 2$
No Road	$(q_t^2, E_t q_{t+1})$	(0.125, 0.06)	(0.053, 0.06)
	$\mathbf{d}_t$	(0.05, 0.10, 0.20)	(0.04, 0.08, 0.16)
Road (V1)	$(q_t^2, E_t q_{t+1})$	(0.12375, 0.0594)	(0.05247, 0.0594)
	$\mathbf{d}_t$	(0.045, 0.09, 0.18)	(0.036, 0.072, 0.144)
Road (V2)	$(q_t^2, E_t q_{t+1})$	(0.1125, 0.054)	(0.0477, 0.054)
	$\mathbf{d}_t$	(0.045, 0.09, 0.18)	(0.036, 0.072, 0.144)

Table A.2 summarizes all remaining parameter values, including prices, time for travelling to and from school, and the efficiency factor in the educational technology. Some of these variables change with the road, though not across scenarios.

[1] The MathWorks, Inc., `http://www.mathworks.com`

[2] The complete `MATLAB` code is available from the author upon request.

Table A.2: Parameter Values

<table>
<tr><th></th><th>β</th><th>ρ</th><th>η</th><th>γ</th><th>a</th><th>$\mathbf{p}$</th><th>τ</th><th>z</th></tr>
<tr><td>No Road</td><td rowspan="2">0.6</td><td rowspan="2">0.8</td><td rowspan="2">-0.1</td><td rowspan="2">0.65</td><td rowspan="2">0.5</td><td>(0.8, 1.15)</td><td>0.08</td><td>0.88</td></tr>
<tr><td>Road</td><td>(0.9,1.075)</td><td>0.04</td><td>0.9</td></tr>
</table>

A.2 Algorithm

The anchoring of the system fleshed out in section 2 is achieved by appropriate choices for the productivity parameter α and the intertemporal taste parameters b_1 and b_2. Algorithm 1 calibrates the model such that, without the road,

$$(1+\tau)e_1^0 = 1/2 \text{ and } (1+\tau)e_t^0 = (1-d_t^1),\ t \geq 2. \tag{A.1}$$

That is to say, a young agent in period t correctly anticipates that the next generation's optimal choice involves full education in period $t+1$. This requirement has to be imposed in the maximization problem of the young agent in period $t+1$. If indeed $e_{t+1}^0 = (1-d_{t+1}^1)/(1+\tau)$ and if the general environment does not deteriorate thereafter, then the second part of equation (A.1) follows immediately. As the road encourages education, it is also clear that, with the road, the agents choose full education from period $t+1$ onwards. Note that maximal education level is higher with the road because

Algorithm 1 Calibration

for $it = 1 : maxit$ **do** ▷ $maxit$: prespecified # of iterations
    Given α and the ratio $b1/b2$, and some inititial guess for b_1, solve the young adult's decision problem
    if $(abs(fval) < tol)$ **then** ▷ $fval = e_1^0 - e_{1,it}^0$
        ▷ tol: prespecified tolerance level
        ▷ $tol = 1e-4$
        A solution found.
        Go to Algorithm 2
    else
        Adjust the taste parameter b_1, using the fact that e^0 is decreasing with b_1.

```
b1 = b1 - df * fval;
```

        ▷ df: some dampening factor
    end if
end for

both d_t^1 and τ are then lower. Algorithm 1 employs an iterative procedure to search for the taste parameter b_1, and results in

$$\alpha = 5, b_1 = 21.251, \text{ and } b_2 = 0.8 \cdot b_1 = 17.0008.$$

Algorithm 2 generates the sequences reported in Table 1. Special attention must be paid to the second period. If, without the road, e_2^0 is either smaller than, or unreasonably higher than its maximal level $(1 - d_t^1)/(1 + \tau) = .08889$, then one goes back to Algorithm 1 with appropriately adjusted parameter values; z is a promising candidate, as the efficiency factor directly affects individual productivity, λ_t. Note that a value for z that is too low results in a non-monotone sequence for expected utility, at least without the road. The reason is that the taste parameters b_1 and b_2 must fall dramatically with a decline in z in order to satisfy equation (A.1). As a consequence, the 'weight' n on the altruistic term in E_tU_t is relatively high and its decline along the presumed demographic transition outweighs the growth in human capital, at least in period $t = 2$. To complete the picture Table A.3 displays the sequences of E_tU_t for all scenarios.

Table A.3: The sequences of E_tU_t, with and without the road

Period		1	2	3	4
No Road		67.7205	68.2814	87.8659	105.3963
Road	V1a	72.0855	74.4694	94.8321	113.1481
	V1b	72.3480	75.5579	96.3669	115.1307
	V2a	72.5424	74.9856	95.3703	113.6989
	V2b	72.8083	76.0778	96.9106	115.6894

Algorithm 3 takes as inputs the sequences for E_tU_t with and without the road and solves for the equivalent variation in terms of the family's (normalized) full income, again by an iterative procedure. Over his life-cycle, an agent is assumed to receive two lump-sum payments of equal size, one as a young adult and one in old age. The young adult in period t enjoys the payment T_t^2 for sure, while the second payment is conditional on individual surviving into old age. Note that a sole payment as young adult results in an equivalent variation that is greater than $2 \cdot T_t^2$, a reflection of the desire to smooth out consumption over the life cycle. Algorithm 3 finds the transfer

Algorithm 2 The Sequences under Perfect Foresight

for $time = 1 : T$ **do** ▷ T: # of periods considered
  Given α, b_1 and b_2 as calculated with Algorithm 1, solve the young adult's decision problem
  if $e^0_{t,it} > (1 - d^1_t)/(1 + \tau), t \geq 2$ **then**
    Solve the decision problem, given $e^0_t = (1 - d^1_t)/(1 + \tau), t \geq 2$.
  else if $e^0_{t,it} < (1 - d^1_t)/(1 + \tau), t \geq 2$ **then**
    Return to Algorithm 1 using other parameter values
  end if
end for

Algorithm 3 Calculating the EV

for $time = 1 : T$ **do** ▷ T: # of periods considered
  for $it = 1 : maxit$ **do**
    Given α, b_1 and b_2 and some initial guess for the transfer T^2_t, solve the young adult's decision problem.
    if $(abs(fval) < tol)$ **then** ▷ $fval = E_t U^{road}_t - E_t U^{\text{no road}}_{t,it}$
      ▷ $tol = 1e - 4$
      A solution found.
    else
      Adjust the Transfer T^2_t, using the fact that $E_t U^{\text{no road}}_t$ increases with T^2_t.

```
T^2_t = T^2_t + dfev * fval;
```

      ▷ $dfev$: some dampening factor
    end if
  end for
end for

T^2_t that yields $E_t U^0_t$ with the road in period t when solving the young adult's decision problem without the road but with (normalized) full income augmented by T^2_t, given a correct forecast of his purchasing power in old age, $\mathbf{p}\rho\mathbf{x}^0_{t+1} + T^2_t$. The system is much simplified by the assumption that altruism disappears in old age, because the transfer received in this stage of life will not affect their children's future decisions. In order to find the equivalent variation for periods $t \geq 2$, however, one has to reset the $E_t U_t$–sequence to its original values; for there is path dependency with respect to the transfer payments. Algorithms 2 and 3 are applied to all scenarios and all decompositions of the combined benefits stemming from the road.

On commodity hardware (Intel Pentium processor 1300 MHz, 512 MB RAM), calibration takes 14 iterations and around 0.7 minutes to complete, even with a poor initial guess of $b_1 = 100$. Finding the equivalent variation takes around 15 iterations and 4.7 minutes.

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