Other-Regarding Preferences and Concerns for Procedure

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Abstract

We provide decision-theoretic foundations for other-regarding preferences, i.e., preferences of decision makers who care about others' outcomes in addition to their own. What appears paradigmatic about the choice behavior of such decision makers is a propensity to care about the 'process' or 'procedure' by which allocations are determined. Consider, for instance, Machina's famous example of a mother who has a single indivisible treat which she can give either to her daughter or her son. She is indifferent between her daughter getting the treat or her son getting it, but *strictly* prefers *tossing a coin* to determine who gets the treat. Indeed, experimental evidence corroborates such insights. For instance, it is not uncommon to find decision makers giving another person some chance of getting an indivisible good, though they *strictly* prefer that they, rather than the other person, receive the good. Such behavior, although compelling, cannot be accommodated by existing models of decision making under risk, expected utility and non-expected utility alike, because they violate the core consequentialist property of *stochastic dominance* (or *monotonicity*) that is at the very heart of these theories. The decision model that we develop accommodates such evidence. We consider a set up in which a decision maker (DM) has preferences over lotteries defined on a space of allocations. Within this set up, we elicit, from observable choice behavior, the influence that the procedure (determining others' outcomes) has on DM's ranking of ex-post allocations. We provide a sharp characterization of how DM evaluates lotteries over allocations by expressing 'payoffs' as a weighted average of the expected utility of the lottery and another component that captures her concerns for procedure. The weight used in evaluating this weighted average is uniquely determined from choice behavior and it quantifies the relative importance of procedural concerns. The decision model is therefore only one parameter richer than expected utility, and reduces to it in the special case in which procedural concerns are absent. We use our decision model to provide an 'expressive' theory of voting that sheds some light on the subject of people voting against their self-interest.

Who we are, our persona, is shaped by both the private and social consequences of our choices. In contrast, decision theory has been mainly concerned with the private side of economic choices.

- Fabio Maccheroni, Massimo Marinacci and Aldo Rustichini (2008)

1 Introduction

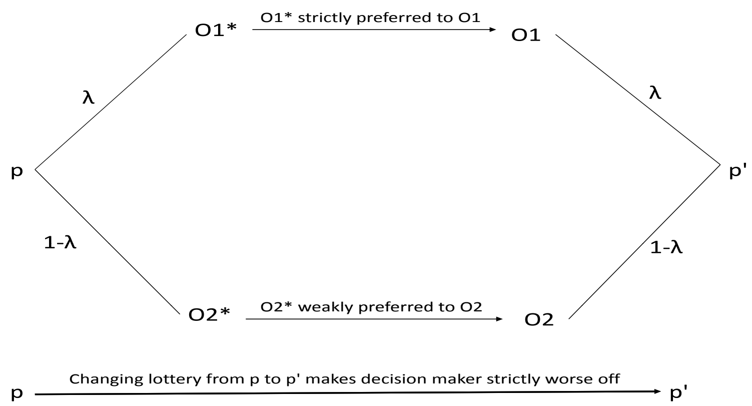
A decision maker has *other-regarding preferences* if she cares not just about her own outcomes, but others' outcomes as well. Consider, for example, Machina's famous example[1] of a mother who cares for the outcome of her two children, Abigail and Benjamin, and therefore has preferences that are other-regarding. Mom has a single indivisible treat, say, a ticket for a day at the theme park for one person, that she could give either to Abigail or Benjamin. She is indifferent between giving the ticket to her daughter or her son, but *strictly* prefers tossing a coin to determine who gets it. Mom's method of determining who gets the ticket by tossing a coin is illustrative of a tendency that seems paradigmatic of decision makers with other-regarding preferences, namely, that apart from caring about *what* outcomes or allocations are, they also care about *how* these are determined. We may think of this latter concern as one for 'process' or 'procedure'.

In the example, mom's (narrowly defined) outcomes are held fixed, but the concern for procedure that she demonstrates may be all too present even when that is not the case. Consider, for instance, the following experiment, involving two treatments of a (two person) dictator game, run by Krawczyk and LeLec (2008). The first treatment was a standard one conducted within a deterministic setting. A decision maker (the dictator) was endowed with 20 euros and given the option of giving some of that money to the other person in the game. Many decision makers in this setting chose to give small amounts of money to the other person.[2] In the second treatment, they considered a probabilistic analogue of this game. In this setting no division of the 20 euros itself was allowed (that is, all of the 20 euros would either be received by the dictator or the other person). The dictator was now given the option, if she chose, of assigning some probability to the other person receiving the money. A substantial portion of these decision makers (about 30%) chose to assign positive probability (on average of about 0.1) to the other person receiving the money.

[1] This example draws on a long literature on interpersonal fairness and equity under risk/uncertainty that has involved many distinguished participants, amongst others, Broome (1982, 1984), Diamond (1967), Harsanyi (1955, 1975, 1978), Keeney (1980), Sen (1985) and Strotz (1958, 1961).

[2] This conclusion from their experiment is in line with a fairly robust finding in the context of such dictator games that decision makers, in 'lab settings,' do choose to share some amount of money with others.

Figure 1: Stochastic Dominance.



The experimental subjects of Krawczyk and LeLec were behaving in a manner much like mom. They seem to care about ex-post fairness of outcomes (as demonstrated by the results from the deterministic treatment), but given that this is not possible to achieve under the probabilistic treatment, they compensate by sharing ex-ante chances of getting the endowment. In other words, procedural fairness acts as a substitute for allocation fairness for these decision makers. In other experimental settings (Bolton, Brandts, Ockenfels (2005)), it has been reported that ex-post resistance to unfair allocations, which is often observed in 'lab settings,' is mitigated if they are deemed to have been generated by a fair procedure. At any rate, the question of procedures, or *how* allocations are determined, is central to understanding the choices made by decision makers with other-regarding preferences as there appears to be subtle connections between such decision maker's concerns for outcomes and their concerns for procedure. What's more, such concerns for procedure at the individual level may, in turn, influence methods for allocating scare goods that are relied upon in real world economies.[3] Therefore, it is disconcerting that it may not be possible to accommodate such procedural concerns within existing models of decision making. We shall now elaborate on this important point.

Consider Figure 1. It illustrates a central principle of decision making under risk. Suppose there are two complementary events E and E^C that occur with probability λ and $1-\lambda$, respectively.[4] Consider a lottery p that gives outcome $O1^*$ if event E occurs, and $O2^*$

[3]For example, in countries where a military draft has been instituted, lotteries were often the means through which the determination of who gets drafted was made. In a similar vein, the US State department conducts lotteries in several countries to determine who gets a fixed allotment of Green Cards.

[4]The case of only two events is considered for simplicity, and the argument can easily be extended to the case of more than two events.

if event E^C occurs. Now consider transforming the lottery from p to p' in the following way. Replace the outcome in event E with something that the decision maker considers strictly worse, whereas in event E^C, replace the outcome with something that she considers no better. It stands to reason that such a transformation would leave our decision maker strictly worse off, and she would never prefer p' over p. That is the exact conclusion that the principle of *stochastic dominance* proposes.[5] Viewed abstractly, this principle appears almost unchallengeable – after all, if in every event our decision maker is at least weakly worse off, and in at least one event she is strictly worse off, how could it not be that she is strictly worse off overall? What's important, though, is to recognize the key behavioral content of this principle; it requires a decision maker's behavior to conform to a basic form of 'consequentialism,' namely, that her ranking of outcomes should be independent of the stochastic process that generates these outcomes. Stochastic dominance is central to how economists think about modeling environments featuring risk, and almost any model of decision making that economists use satisfies this property. This is true of *expected utility*, as well as of various models of *non-expected utility* (for instance, rank dependent utility, 'betweenness-based' theories, generalized prospect theory etc.) that have been proposed in the literature as generalizations of expected utility.[6]

Next consider Figure 2. It presents the preferences/choices of some of the experimental subjects of Krawczyk and LeLec. Under lottery q, the decision maker gets 20 euros for sure.[7] On the other hand, under lottery q', the other person gets the money with probability .1, whereas with complementary probability the decision maker gets the money. Note that changing the lottery from q to q' is along the lines of the transformation from p to p' mentioned above. The allocation that is realized in the .1 probability event under q, namely $(20, 0)$, is considered strictly better by the decision maker than the corresponding allocation in this event under lottery q', namely $(0, 20)$.[8] On the other hand, the allocation in the .9

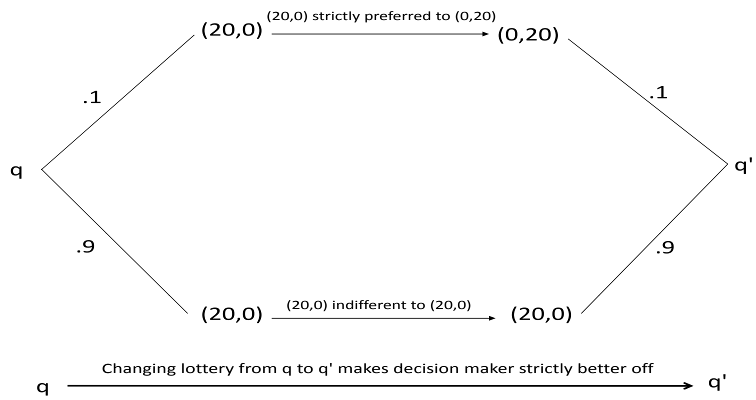
[5]Formally, suppose X is a set of outcomes, $\Delta(X)$ is the set of lotteries on X, and the decision maker has a preference relation $\succcurlyeq$ over $\Delta(X)$. Then we will say that a lottery p in $\Delta(X)$ first order stochastically dominates another lottery p' with respect to $\succcurlyeq$ if for all $x \in X$, the probability that p assigns to outcomes that are at least as good as x (according to $\succcurlyeq$) is higher than the corresponding probability under p', and for some x, it is strictly higher. *Stochastic dominance* says that if p first order stochastically dominates p', then p is strictly preferred to p'.

[6]It is worth noting that stochastic dominance is a more fundamental assumption than the famous *independence* condition of expected utility theory. Violation of stochastic dominance implies a violation of independence, but not vice versa. For instance, in Allais' famous example independence is violated but not stochastic dominance. The non-expected utility models mentioned above relax the independence condition, but retain stochastic dominance.

[7]The pair $(20, 0)$ denotes the ex-post allocation in which the decision maker gets 20 euros and the other person gets nothing, whereas the pair $(0, 20)$ denotes the ex-post allocation in which the other person gets the 20 euros and the decision maker gets nothing.

[8]The content of the argument that we make below continues to hold even if it were the case that the allocation $(0, 20)$ is strictly preferred by the decision maker to the allocation $(20, 0)$.

Figure 2: Violation of Stochastic Dominance.



probability event under q and q' are the same and hence deemed indifferent. Stochastic dominance would then dictate that the decision maker should strictly prefer the lottery q over q'. However in the experiments, some of the decision maker's (revealed) preferences went in the opposite direction, namely, they strictly preferred q' over q. Their preferences therefore violate stochastic dominance, and accordingly, neither expected utility nor any of the models of non-expected utility mentioned above can accommodate their choices. Arguably, the source of this violation is their concern for procedural fairness, which makes their ranking of ex-post allocations intrinsically dependent on the stochastic process that generates these outcomes. As we highlighted above, this is something that stochastic dominance does not allow.

There has been a great deal of interest in the last two decades or so in incorporating other-regarding concerns like fairness, equity and envy into economic models. The discussion above contains two important insights for this research program. First, it highlights the fact that models of other-regarding preferences that focus exclusively on outcomes[9] may not adequately capture all the concerns of decision makers with such preferences. In particular,

[9] A leading example of such models would be 'social utility models' like those of Bolton (1991), Fehr and Schmidt (1999), Bolton and Ockenfels (2000) and Charness and Rabin (2002). In these models, a decision maker's utility is defined over her own outcomes and those of others. For instance, Fehr and Schmidt propose the following social utility function to evaluate the utility derived by individual 1 in a two individual world. Suppose individual 1 receives the outcome x_1 and individual 2 receives the outcome x_2 (x_1, $x_2 \in \mathbb{R}_+$). Then individual 1's utility is given by:

$$U_1(x_1, x_2) = x_1 \text{ - } \mu.\max\{x_2 - x_1,\, 0\} \text{ - } \mu'.\max\{x_1 - x_2,\, 0\},\ \mu,\ \mu' > 0.$$

The basic idea behind this functional form is to incorporate a notion of inequity aversion. The decision maker receives 'utility' from her own outcome x_1, but receives 'dis-utility' from inequities in the final allocation.

such models will not be able to account for the fact that the procedure by which outcomes are determined may itself influence how these outcomes are ranked by decision makers. Second, and perhaps more importantly, it challenges an implicit assumption that seems to have been made all along within this research program, namely, that existing decision theoretic foundations that economists rely on are adequate to model preferences which encompass concerns for others. What the examples discussed here, and others of their kind (eg. Kircher, Ludwig and Sandroni (2010)) seem to suggest is that other-regarding preferences, or social preferences, more generally, may require a new set of theoretical foundations.

This paper makes an attempt to provide new decision-theoretic foundations for other-regarding preferences. We propose tractable and parsimonious 'utility representations' that clarifies the interaction between a decision maker's concern for outcomes and her concern for procedures. In addition, we identify axioms on behavior that are equivalent to the proposed representations. We now provide a brief sketch of some of the key ideas underlying our decision model. We first explain, using a 'basic representation,' how it can rationalize the choices in the probabilistic dictator game discussed above.

1.1 Explaining Choices in the Probabilistic Dictator Game

Consider a decision maker (DM) who is faced with the problem of deciding what probability $\lambda \in [0, 1]$ she wants to assign to the other person getting the money in the probabilistic dictator game discussed above. For any choice of λ, we get a lottery over allocations, namely,

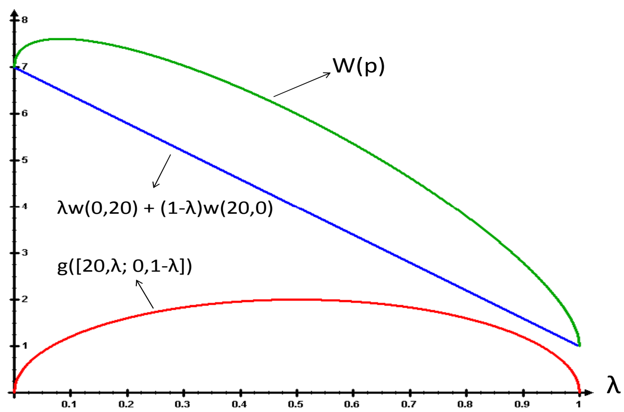
$$p = [(0, 20),\ \lambda;\ (20, 0),\ 1 - \lambda],$$

under which the allocation $(0, 20)$ (the other person gets the 20 euros, DM gets nothing) realizes with probability λ, and the allocation $(20, 0)$ (DM gets the 20 euros, the other person gets nothing) realizes with probability $1 - \lambda$. Under our **basic representation**, the decision maker evaluates a lottery like p by the following function:

$$W(p) = \{\lambda w(0, 20) + (1 - \lambda)w(20, 0)\} + g([20,\ \lambda;\ 0,\ 1 - \lambda])$$

First, note that under the representation there exists a function w that represents DM's ranking of ex-post allocations. The expression, $\{\lambda w(0, 20) + (1 - \lambda)w(20, 0)\}$, may be interpreted as DM's 'expected utility' from the lottery p. What about the second expression? The function g represents DM's concerns for procedure, in particular, the procedure by which the other person's outcomes are determined. Under our representation, for any allocation-lottery p, DM considers the marginal distribution over the other person's outcomes under p, to be the procedure determining her outcomes (Note that the lottery $[20,\ \lambda;\ 0,\ 1 - \lambda]$ is

Figure 3: Choices in the Probabilistic Dictator Game.



the marginal over the other person's outcomes under the allocation-lottery $p = [(0,20),\ \lambda;\ (20,0),\ 1-\lambda])$.

Suppose, as functions of λ, the expressions $\{\lambda w(0,20) + (1-\lambda)w(20,0)\}$ and $g([20,\ \lambda;\ 0,\ 1-\lambda])$ take the form shown in Figure 3. It is straightforward to verify that the expected utility payoffs, namely $\{\lambda w(0,20) + (1-\lambda)w(20,0)\}$, decreases linearly in λ (since $w(20,0) > w(0,20)$). On the other hand, her payoffs from the procedural component is shown to be concave in λ, with it first increasing, attaining a maximum at $\lambda = \frac{1}{2}$, and then decreasing. Heuristically, this may be so, if by DM's subjective evaluation, the lottery (over the other person's outcomes) in which the other person gets the 20 euros with even chance is deemed the "fairest" procedure by which the other person's outcomes can be determined, in this situation. Given that there is risk in the environment, DM may reason, that the procedure which gives the other person equal chances of getting the good and the bad outcome is the best one. Any procedure, then, that deviates from this ideal procedure are considered inferior by DM. Observe that, under these assumptions, as λ increases from 0 to something slightly larger, a tradeoff emerges. By the expected utility component, as λ increases, she gets worse off, but by the procedural consideration she is better off. The overall payoff is determined by the interaction of these two opposing influences. It is indeed possible that for small values of λ, on increasing λ slightly, the incremental improvement in the procedural payoffs, which is "non-linear" in λ, outweighs the drop-off in the expected utility payoffs, which goes down linearly in λ. That is why under our representation the payoff $W(p)$ from a lottery like $p = [(0,20),\ \lambda;\ (20,0),\ 1-\lambda]$ may be increasing in λ for small values of λ, and hence, DM may strictly prefer giving the other person some chance of getting the money, even though she strictly prefers the allocation in which she gets the money, namely $(20,0)$, to the allocation in which the other person receives the money, namely, $(0, 20)$.

1.2 The General Representation and Its Interpretation

We now provide an overview of our general representation. Assume that there are n individuals, denoted $1, \ldots, n$, about whose outcomes our decision maker may care. Denote the set of DM's outcomes by the set Z, individual i's outcomes by the set Z_i, $i = 1, \ldots, n$, and let $A = \prod_{i=1}^{n} Z_i$. Let p be a simple lottery on the allocation space $Z \times A$, and let p_A denote the marginal probability measures of p over A. Under our **general representation**, DM evaluates the lottery p by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)\{(1 - \sigma)w(z, a) + \sigma w(z, p_A)\},$$

where $\sigma \in [0, 1]$, and $p(z, a)$ denotes the probability that p assigns to the allocation (z, a).[10]

The representation provides a tractable and parsimonious account of the interaction between DM's concerns for outcomes (allocations) and concerns for procedure in determining her choices over allocation-lotteries. As noted above, for any allocation-lottery p, DM considers the marginal probability measure p_A over the others' outcomes, A, under the lottery p, to be the *procedure* by which others' outcomes are determined. Consider any allocation (z, a) in the support of p; DM's evaluation of this allocation is conditioned on the procedure p_A. The term inside the parentheses,

$$(1 - \sigma)w(z, a) + \sigma w(z, p_A)$$

reflects her payoffs from the allocation (z, a) when the procedure determining others' outcomes is p_A. This payoff is a weighted average of two terms. The first term is DM's payoffs from the allocation (z, a), whereas the second term is DM's payoffs from the procedure p_A given that she receives outcome z. The same function w is used to evaluate DM's concern for outcomes as well as procedures.[11] The weight σ used in evaluating this weighted average is subjective; that is, we derive this weight from DM's choice behavior, and this derivation is *unique.* We refer to σ as the *procedural weight*, and it quantifies the relative strength of concerns for procedure relative to concerns for outcome in determining DM's choices.

[10] Under our basic representation, which we used above to explain choices in the probabilistic dictator game, the lottery p is evaluated by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z, a)w(z, a) + g(p_A).$$

[11] The domain of the function w is $Z \times \Delta_A$, where Δ_A refers to the set of simple lotteries on the set A. We abuse notation by not distinguishing between the outcome a and the degenerate lottery that gives a with probability 1.

Finally, once these payoffs for all the allocations (z', a') that are possible under p have been evaluated, DM's overall payoff from the lottery p is determined by an 'expected utility criterion' over these payoffs.

There are three key ideas embedded in our representation. First, the representation specifies that a decision maker's evaluation of an allocation (z, a) depends on the procedure by which others' outcomes are determined. In particular, when an allocation (z, a) is realized under some lottery p, we may think of the triple (z, a, p_A) as representing the 'things DM cares about' in this situation. We call such a triple a **procedure-contingent allocation**. We use information about DM's ranking of lotteries on the allocation space $Z \times A$, which is a primitive of our model, to elicit her ranking over procedure-contingent allocations. Second, the representation provides a simple expression for how these procedure-contingent allocations are evaluated. It says that the concern for allocations and procedures interact linearly. The strength of the procedural concern is captured by the parameter σ, which, as we mentioned above, is subjective. In particular, decision makers for whom the parameter σ turns out to be 0 are expected utility maximizers. Our model is therefore parsimonious in the sense of being one parameter richer than expected utility. Third, in our representation, once DM's concerns for procedure have been accounted for by expanding the notion of outcomes to that of procedure-contingent allocations, 'event-separability' holds over this expanded space of outcomes. This property helps to keep the representation tractable.

1.3 Connections to the Literature

Our paper relates to the decision theoretic literature on *nonseparable* models of preferences in environments of risk. In such models decision makers' preferences are *nonseparable* across mutually exclusive events in the sense that their evaluation of a prospect in a given event may be intrinsically tied to considerations relating to other events that could have occurred but did not. In the words of Machina (1989), "An agent with nonseparable preferences feels (both ex ante and ex post) that risk which is borne but not realized is gone in the sense of having been *consumed* (or "borne"), rather than gone in the sense of *irrelevant*."[12] These are decision makers who violate the *independence* axiom of expected utility theory. It is worth highlighting here that the condition of independence/event-separability can be derived from another, more fundamental, normative principle of behavior. Hammond (1988) shows that decision makers whose preferences conform with independence satisfy the property of 'consequentialism'; that is, their behavior is entirely explicable by its consequences. Informally, this says that a decision maker's preference for some 'sub-lottery' p over some other 'sub-

[12]The emphases in the quote are as in the original.

lottery' q does not depend in any way on the form or content of 'parent lotteries,' in which p and q may be embedded.[13]

There is a large decision theoretic literature that accommodates preferences that are nonseparable and hence violate the independence axiom. Prominent examples include rank dependent utility, *betweenness* based theories (like implicit expected utility), and generalized prospect theory.[14] The key feature of these models is that although event-separability of preferences is not required to hold on the space of all lotteries, each of these models identifies a subset of lotteries over which preferences are separable (see Chew and Epstein (1988) for an illustration of this point). Decision makers whose preferences are accommodated by any of these models may then be thought to conform with consequentialism in a 'restricted sense.' In particular, under all these models behavior retains the following *minimal notion* of consequentialism: Suppose x and y are elements in some underlying set of outcomes and p is a lottery on that set. Then if the decision maker prefers x to y, she must prefer the compound lottery that gives x with some positive probability $\lambda > 0$, and p with probability $1 - \lambda$ to the compound lottery that gives y with probability λ and p with probability $1 - \lambda$. This condition, which has been called the *axiom of degenerate independence* (ADI) by Grant, Kajii and Polak (1992), is formally equivalent to stochastic dominance and implies that the decision maker has a ranking over outcomes that is independent of the particular lottery in which such outcomes are realized. In contrast, the decision model we present in this paper does not require behavior to conform to this minimal notion of consequentialism. The important point to recognize is that consequentialism (defined with respect to a *given* set of outcomes) and a concern for procedures are fundamentally conflicting notions, and therefore, consequentialism has to be given up at a very *basic level* to accommodate concerns for procedure. Viewed in this light, the reason that existing models of nonseparable preferences cannot accommodate concerns for procedure is that they retain consequentialism in the minimal sense of ADI/stochastic dominance. We should point out here that Karni and Safra (2002) is another paper that accommodates violations of consequentialism in the aforementioned minimal sense.

There is also a literature on procedures that our work relates to. This line of research

[13]More formally, let X be a set of outcomes and $\Delta(X)$ the space of lotteries defined on X. Let Ξ be a 'rich class of decision problems.' Let $F : \Xi \rightrightarrows \Delta(X)$ denote a feasibility correspondence that specifies the feasible set of lotteries that can possibly result from the decision maker's choices in any decision problem. Let $B : \Xi \rightrightarrows \Delta(X)$ be a behavior correspondence that specifies the decision maker's choice behavior in any decision problem. The decision maker is a consequentialist if there exists a choice correspondence $C : 2^{\Delta(X)} \setminus \emptyset \rightrightarrows \Delta(X)$ such that for all $\xi \in \Xi$, $B(\xi) = C(F(\xi))$. In other words, changes in the structure of a decision problem should have no bearing on choices unless they change the feasible set. It is critical to recognize that we provide this definition for a *given* set of outcomes that we hold fixed.

[14]Refer to Starmer (2000) for a comprehensive survey of non-expected utility models.

seeks to highlight the fact that in addition to outcomes or consequences, the way decisions are made may itself influence an individual's well being. Sen (1997) explains it thus: "Maximizing *behavior* differs from nonvolitional *maximization* because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over *comprehensive* outcomes (including the choice process) have to be distinguished from the conditional preferences over *culmination* outcomes *given* the acts of choice."[15] In order to model such concerns for procedure, Sen suggests using "menu dependent" models of choice behavior, which allow a decision maker's preferences over outcomes to depend on the set (menu) from which the choice is made. The vast majority of work that highlights concerns for procedure have been conducted within an empirical/experimental setting. Some examples are Kahneman, Knetsch and Thaler (1986), Bies, Tripp and Neale (1993), Frey and Pommerehne (1993), Bolton, Brandts and Ockenfels (2005) and Krawczyk and Le Lec (2008).

Finally, there is a large literature on other-regarding preferences that our work is a part of. A vast portion of this research has been undertaken within experimental settings, and this work has played an important role in demonstrating that other-regarding concerns matter to many decision makers. This experimental literature is too vast to adequately document here. For a recent survey of this literature, the reader may refer to Cooper and Kagel (2009). Inspired by this evidence from the 'lab,' many researchers have sought to incorporate other-regarding concerns into economic models. 'Social utility' models are a leading example of this endeavor. In these models a decision maker's utility is defined over not just her own outcomes but others outcomes as well. Different forms of other-regarding concerns like fairness, envy, altruism, etc. are incorporated into these models by writing functional forms for utility that intuitively correspond to the particular 'emotion' that is sought to be modeled. Some leading examples of such models are Bolton (1991), Fehr and Schmidt (1999), Bolton and Ockenfels (2000) and Charness and Rabin (2002). The difference between these works and ours is twofold. First, these models are outcome based and do not consider concerns for procedure. Second, these models are not based on an axiomatic treatment of choice behavior. This is an observation that holds true in general about how other-regarding preferences have been treated in the literature. Despite the interest in such preferences, there are not that many papers studying the choice theoretic foundations of such preferences. Some notable exceptions include Ok and Kockesen (2000), Gilboa and Schmeidler (2002), Karni and Safra (2002), Neilson (2006), Saito (2008), Sandbu (2005), Maccheroni, Marinacci and Rustichini (2008) and Rohde (2009).

[15]The emphases in the quote are as in the original.

2 The Framework

2.1 Preliminaries

We assume that our stylized society comprises of a decision maker (DM) and n other individuals, denoted $1, \ldots, n$, about whose outcomes DM may care. Associated with each individual is a well defined set of outcomes. We denote the set of DM's outcomes by Z and those of individual i by Z_i, $i = 1, \ldots, n$. Further, we let $A = \prod_{i=1}^{n} Z_i$ denote the set of outcome-vectors for the others (others' outcomes, for short). Accordingly, $Z \times A$ may be referred to as the set of allocations. We denote the set of simple probability measures (simple lotteries, or just lotteries, for short) on the sets $Z \times A$ and A by Δ and Δ_A respectively. We will denote elements of Δ by p, q etc., and for any $p \in \Delta$, we will denote the marginal probability measure of p over A by p_A.

We define a convex combination of lotteries in the standard way.[16] Following standard notation, we shall at times denote lotteries by explicitly listing the elements in the support along with their respective probabilities. For instance,

$$p = [(z_1, a_1), \lambda_1; \ldots; (z_K, a_K), \lambda_K]$$

shall denote a lottery in Δ that gives the outcome (z_k, a_k) with probability λ_k, $k = 1, \ldots, K$. Finally, note that we will abuse notation throughout by not distinguishing between an outcome and a lottery that gives that outcome with probability 1. For instance, $(z, a) \in Z \times A$ shall stand both for an outcome as well as the lottery that gives this outcome with probability 1.

2.2 Preference

DM's preferences are given by a weak order (a binary relation that is complete and transitive) $\succcurlyeq$ on the set Δ. The symmetric and asymmetric components of $\succcurlyeq$ are defined in the usual way and denoted by $\sim$ and $\succ$ respectively.

3 A Basic Representation

Before developing our general representation, we introduce a basic representation. The reason we present this representation first is because it is relatively straightforward to develop. Further, it can rationalize choice behavior like that in Machina's mom example, as well as the

[16] For instance, if $p^1, \ldots, p^K \in \Delta$, and $\lambda^1, \ldots, \lambda^K$ are constants in $[0, 1]$ that sum to 1, then $\sum_{k=1}^{K} \lambda^k p^k$ denotes an element in Δ that gives the outcome $(z, a) \in Z \times A$ with probability $\sum_{k=1}^{K} \lambda^k p^k(z, a)$.

probabilistic dictator game that we mentioned in the introduction. Under this representation, DM evaluates a lottery $p \in \Delta$ by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A),$$

where $p(z,a)$ denotes the probability that the lottery p assigns to the allocation (z,a), and $w : Z\times A \to \mathbb{R}$, $g : \Delta_A \to \mathbb{R}$ are real-valued functions that are unique up to common positive affine transformation.[17] The function w represents DM's ranking of ex-post allocations. On the other hand, the function g captures DM's evaluation of the procedure by which others' outcomes are determined. As we mentioned in the introduction, under our representations, for any lottery $p \in \Delta$, DM considers the marginal probability measure p_A over the others' outcomes, A, under the lottery p, to be the procedure by which others' outcomes are determined. We next provide the axioms that characterize this representation.

3.1 Axioms

We require that DM's preferences are complete and transitive.

- **AXIOM: Weak Order**
  $\succcurlyeq$ *is complete and transitive.*

Next, consider lotteries in which we fix DM's outcome at a particular $z \in Z$. Then the only risk borne in these lotteries are by the other members of society. We can think of how DM ranks these lotteries as a reflection of her 'values'. Our next axiom states that DM's values are contingent on the particular outcome that she receives.

- **AXIOM: Contingent Values**
  For any (z,a), $(z,a') \in Z\times A$, if $(z,a) \succ (z,a')$, then there exists $z' \in Z$ such that $(z',a') \succ (z',a)$.

Our next axiom puts certain restrictions on how good or bad lotteries can be in DM's subjective evaluation.

- **AXIOM: Bounded Archimedean**

  1. *Archimedean: For any p, q, $r \in \Delta$, such that $p \succ q \succ r$, and $p_A = r_A$, there exists λ, $\gamma \in (0,1)$, such that*

  $$\lambda p + (1-\lambda)r \succ q \succ \gamma p + (1-\gamma)r.$$

[17]That is, if $\tilde{w} : Z\times A \to \mathbb{R}$ and $\tilde{g} : \Delta_A \to \mathbb{R}$ are functions that also represent DM's preferences in the sense of our representation, then there exists constants $\alpha > 0$ and β such that $\tilde{w} = \alpha w + \beta$ and $\tilde{g} = \alpha g$.

2. Boundedness: There exists $(\overline{z}, \overline{a})$, $(\underline{z}, \underline{a}) \in Z \times A$, *such that for any* $p \in \Delta$,

$$(\overline{z}, \overline{a}) \succcurlyeq p \succcurlyeq (\underline{z}, \underline{a}).$$

The first part of the axiom is the standard Archimedean condition of the vonNeumann-Morgenstern (vNM) setting (see Kreps (1988)), with the qualification, though, that it is required to hold only when the "extreme" lotteries p and r have the same procedure of determining outcomes for the others, i.e., $p_A = r_A$. For any two such lotteries, p and r, it imposes bounds on how good or bad these can be in relation to each other. That is, p cannot be so good that, a small probability γ of p and a large probability $1 - \gamma$ of r is always better than q. Similarly, r cannot be so bad that, a large probability λ of p and a small probability $1 - \lambda$ of r is always worse than q. What is the motivation behind the second part of the axiom? Observe that if we were in the standard expected utility setting (or, in the setting of any of the standard models of non-expected utility), where preferences satisfy stochastic dominance, then the following would have been true: for any two lotteries p', $q' \in \Delta$, if $p' \succ q'$, then for any $\lambda \in [0, 1)$, $p' \succ \lambda p' + (1 - \lambda)q'$. However, in our current setting when we mix lotteries p' and q', the procedure determining (others') outcomes also changes. For instance, it is possible that that when p' and q' are mixed with $\lambda{:}1-\lambda$ probabilities, in DM's subjective evaluation, the procedure of this resulting lottery is deemed highly desirable and she ends up considering the resulting lottery to be better than p'. The second part of the axiom therefore puts bounds on how good (or bad) lotteries can be as a result of procedural considerations.

Our next axiom specializes the independence condition of expected utility theory to our current setting. The basic logic underlying it is the following. The reason why the standard independence axiom may not hold in our setting is because of DM's concerns for procedure, which, to reiterate, for any lottery $p \in \Delta$, she takes to be the marginal measure p_A on A. When we mix two lotteries in our setting, the underlying procedure determining outcomes may change, which in turn undermines the "substitution" logic of the independence condition. However, if we were to take two lotteries p, q for which the procedures are the same, i.e., $p_A = q_A$, then any probability mixture of the two, say $\lambda p + (1 - \lambda)q$, also has the same procedure (since, $(\lambda p + (1-\lambda)q)_A = p_A = q_A$). Accordingly, for probability mixtures of such lotteries, we contend that the logic of independence should still survive. The next axiom formalizes this idea.

- **AXIOM: Procedure Contingent Independence**
  Let p, q, p', $q' \in \Delta$ *be such that* $p_A = q_A$, $p'_A = q'_A$. *Then for any* $\lambda \in (0, 1]$

$$[p \succ p',\ q \sim q'] \Rightarrow \lambda p + (1 - \lambda)q \succ \lambda p' + (1 - \lambda)q'$$

3.1.1 Revealed Cardinality

The expected utility model, in particular, the independence condition, provides us with a 'cardinal' notion of the 'preference difference' between two lotteries. For instance, if p and q are two lotteries, then the lottery $\frac{1}{2}p + \frac{1}{2}q$, is precisely the lottery that lies half-way between p and q in DM's subjective preference evaluation. This is true for any probability mix – the lottery $\lambda p + (1 - \lambda)q$, is the lottery that is the λ-weighted preference average of p and q. In our current setting, although independence fails to hold over the entire domain Δ, the notion of preference average can still be recovered. The following definition formalizes this. First, consider the following notation. For any $a \in A$, let,

$$\Delta(a) = \{p \in \Delta : p_A = a\}^{18}$$

denote the set of lotteries in Δ which gives the others the outcome vector a for sure.

Definition 1. *Given p, $q \in \Delta$, $\lambda \in [0, 1]$, we say that $r \in \Delta$ is a λ-weighted preference average of p and q, denoted $r = \lambda p \oplus (1 - \lambda)q$, if there exists p_1, p_2, $p_3 \in \Delta(a)$, q_1, q_2, $q_3 \in \Delta(a')$, a, $a' \in A$, such that:*

1. $p \sim p_1$, $q \sim q_1$, $r \sim p_2 \sim q_2$, $p_3 \sim q_3$
2. $p_2 = \lambda_1 p_1 + (1 - \lambda_1)p_3$, $q_3 = \lambda_2 q_1 + (1 - \lambda_2)q_2$, *and*
3. $\lambda = \frac{\lambda_1}{1-(1-\lambda_1)(1-\lambda_2)}$.

Further, if r is a λ-weighted preference average of p and q, and r' is a γ-weighted preference average of p and r, that is,

$$r = \lambda p \oplus (1 - \lambda)q \text{ and } r' = \gamma p \oplus (1 - \gamma)r,$$

then r' is a $(\gamma + (1 - \gamma)\lambda)$-weighted preference average of p and q. That is,

$$r' = (\gamma + (1 - \gamma)\lambda)p \oplus (1 - (\gamma + (1 - \gamma)\lambda))q.$$

Note a special case of the definition, when $a = a'$ and $\lambda_2 = 1$ (that is $q_1 \sim q_3$). In this case we have $\lambda = \lambda_1$. To understand this, observe that *procedure-contingent independence* implies that preferences over the set of lotteries, $\Delta(a)$ which give the others the fixed outcome-vector, $a \in A$, satisfies the standard independence condition. Therefore, for lotteries p and q which are indifferent to two lotteries, p_1 and q_1 in such a set, the notion of preference average should coincide with the one in the expected utility set up that we mentioned above, namely,

[18] As stated above, we abuse notation by not distinguishing between a degenerate lottery and the outcome on which that degenerate lottery puts probability 1.

$\lambda p \oplus (1-\lambda)q \sim \lambda p_1 + (1-\lambda)q_1$. The (general) definition extends this special case by connecting together elicitation about preference averages made in the context of the special case, to make inferences about preference averages of elements, for which the special case is not applicable. For instance, if we were to know that p' is the mid-point on the preference scale between p and p'' ($p \succ p''$), and p'', in turn, is the mid-point on the preference scale between p' and p''' ($p' \succ p'''$), that is, $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = \frac{1}{2}$, then it stands to reason that p' is the $\frac{2}{3}$-weighted preference average of p and p''' ($\lambda = \frac{\lambda_1}{1-(1-\lambda_1)(1-\lambda_2)} = \frac{\frac{1}{2}}{1-(1-\frac{1}{2})(1-\frac{1}{2})} = \frac{2}{3}$). The following lemma provides further implications of the above definition.

Lemma 1. *Suppose $\succcurlyeq$ is a weak order that satisfies bounded Archimedean, contingent values and procedure-contingent independence. Then*

1. *For any p, $q \in \Delta$ and $\lambda \in [0,1]$, there exists $\lambda p \oplus (1-\lambda)q \in \Delta$.*
2. *If $p \succcurlyeq q \succcurlyeq r$, then there exists a unique $\lambda \in [0,1]$ such that $q = \lambda p \oplus (1-\lambda)r$.*
3. *For any p, $q \in \Delta$, λ_1, $\lambda_2 \in [0,1]$, $\lambda_1 > \lambda_2 \Leftrightarrow \lambda_1 p \oplus (1-\lambda_1)q \succ \lambda_2 p \oplus (1-\lambda_2)q$.*

We can now formalize a notion of preference difference that we shall use in the next axiom. Let p_k, $k = 1, \ldots, 4$, be such that for some p, q,

$$p_k = \lambda_k p \oplus (1-\lambda_k)q,\ k = 1, \ldots, 4.$$

Then we define the binary relations $=^*$ on $\Delta \times \Delta$ as follows:

$$(p_1, p_2) =^* (p_3, p_4), \text{ if } \lambda_1 - \lambda_2 = \lambda_3 - \lambda_4.$$

In other words, according to DM's subjective evaluation, the preference difference between the lotteries p_1 and p_2 is the same as the preference difference between the lotteries p_3 and p_4.

The final axiom for our basic representation asserts that for lotteries p, q with the same procedure, i.e., $p_A = q_A$, the preference difference between them must be explicable by the preference difference of their constituent outcomes (allocations). Formally, it states:

- **AXIOM: Procedure Contingent Consequentialism**
  Let $p = [(z_1, a_1), \lambda_1; \ldots; (z_M, a_M), \lambda_M]$, $q = [(z'_1, a'_1), \lambda'_1; \ldots; (z'_N, a'_N), \lambda'_N] \in \Delta$ be such that $p_A = q_A$. Then

$$(p, q) =^* (\lambda_1(z_1, a_1) \oplus \ldots \oplus \lambda_M(z_M, a_M),\ \lambda'_1(z'_1, a'_1) \oplus \ldots \oplus \lambda'_N(z'_N, a'_N)).$$

3.2 Representation

We can now state the following representation of $\succcurlyeq$.

Theorem 1. *Suppose contingent values hold. Then* $\succcurlyeq$ *on* Δ *satisfies the axioms of weak order, bounded Archimedean, procedure-contingent independence and procedure-contingent consequentialism if and only if there exists bounded functions* $w : Z \times A \to \mathbb{R}$, $g : \Delta_A \to \mathbb{R}$, *such that the function,*

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A),$$

represents $\succcurlyeq$, *and range*$(W) \subseteq$ *range*(w).

Further, if $(\widetilde{w} : Z \times A \to \mathbb{R}, \widetilde{g} : \Delta_A \to \mathbb{R})$ *represents* $\succcurlyeq$ *in the above sense, then there exists constants* $\alpha > 0$, β, *such that:*

$$\widetilde{w} = \alpha w + \beta \text{ and } \widetilde{g} = \alpha g.$$

4 The General Representation

This section presents axioms on choice behavior that are necessary and sufficient for representing $\succcurlyeq$ by a function $W : \Delta \to \mathbb{R}$ of the form

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma)w(z,a) + \sigma w(z,p_A)\},$$

where the function $w : Z \times \Delta_A \to \mathbb{R}$ is unique up to positive affine transformation and $\sigma \in [0,1]$ is unique.

In order to understand this representation better, we first introduce some new notation. Consider any lottery $p \in \Delta$ in which DM gets some outcome $z \in Z$ for sure, that is, the marginal measure of p on the set Z is degenerate. We will denote such a lottery by $p = [z, p_A]$. Note that for any $z \in Z$, we can use the primitive preference relation $\succcurlyeq$ to define a weak order $\succcurlyeq_z$ on Δ_A as follows: for any p_A, $q_A \in \Delta_A$,

$$p_A \succcurlyeq_z q_A \text{ if } [z, p_A] \succcurlyeq [z, q_A].$$

For any $z \in Z$, the function $w(z,.) : \Delta_A \to \mathbb{R}$ in the representation above represents the preference relation $\succcurlyeq_z$. The representation therefore emphasizes that DM's ex-post evaluation of the procedure determining others'outcomes is contingent on the outcome that she herself receives.

We will assume in this section that each of the sets Z, Z_i, $i = 1, \ldots, n$, are connected topological sets. We define the symmetric and asymmetric components of $\succcurlyeq_z$ in the usual way, and denote them by $\sim_z$ and $\succ_z$ respectively. We will assume that the indifference surfaces of $\succcurlyeq_z$ restricted to A are connected. That is, for any $\succcurlyeq_z$, and for any $a \in A$, $\{a' \in A : a' \sim_z a\}$ is a connected subset of A.

4.1 Axioms

We continue to assume that DM's preference satisfies the axioms of *weak order*, *bounded Archimedean* and *procedure-contingent independence*. We state here the axioms that need modification, as well as two new axioms. We first strengthen the *contingent values* axiom that we introduced above by adding an extra condition to it.

- **AXIOM: Strong Contingent Values**

  1. *For any (z, a), $(z, a') \in Z \times A$, if $(z, a) \succ (z, a')$, then there exists $z' \in Z$ such that $(z', a') \succ (z', a)$.*
  2. *For all $z \in Z$, with $\succ_z \neq \emptyset$, there exists $z' \in Z$, with $\succ_{z'} \neq \emptyset$, such that for all $a \in A$, there exists a', $a'' \in A$ satisfying:*

  $$(z', a') \sim (z', a) \sim (z', a''), \text{ and } (z, a') \succcurlyeq (z, a) \succcurlyeq (z, a'').$$

  Further, if a is not a maximal (resp. minimal) element of $\succcurlyeq_z$, then $(z, a') \succ (z, a)$ (resp. $(z, a) \succ (z, a'')$).

We now introduce the important notion of comparable lotteries. This concept of comparable lotteries is important for us because it provides a way of eliciting the impact that procedures have on DM's evaluation of ex-post allocations.

Definition 2. *Two lotteries, $p = [(z, a), \lambda; (z_1, a_1), \lambda_1; \ . \ . \ . \ ; (z_M, a_M), \lambda_M]$ and $q = [(z, a'), \lambda; (z_1, a'_1), \lambda_1; \ . \ . \ . \ ; (z_M, a'_M), \lambda_M]$, are comparable in the λ-probability event, if in all the λ_m-probability events, $m = 1, \ . \ . \ . \ , M$, we have*

$$a_m \sim_{z_m} a'_m \text{ and } p_A \sim_{z_m} q_A.$$

Observe that in each of the λ_m-probability events, $m = 1, \ . \ . \ . \ , M$, the outcome that others get under p (namely, a_m), belongs to the same indifference class of $\succcurlyeq_{z_m}$ as the outcome that they get under q (namely, a'_m). Further, the procedure that determines others' outcome under p (namely, p_A), belongs to the same indifference class of $\succcurlyeq_{z_m}$ as the procedure that determines their outcome under q (namely, q_A). Accordingly, in each of these λ_m-probability events ($m = 1, \ . \ . \ . \ , M$), if DM independently evaluates the outcome that others get and the procedure that determines this outcome using the preference relation $\succcurlyeq_{z_m}$, then, from a preference perspective, we may conclude that DM considers the lotteries p and q to be identical in these events. Alternatively, the only place where DM may consider p and q to differ is in her assessment of the λ-probability event. Based on her choice between p and q, we may therefore elicit an inference about how she evaluates the "consequence" in the λ-probability event under p and q. It is in this sense p and q are comparable in the λ-probability event.

Our next axiom requires that the evaluation of comparable lotteries be consistent with the 'cardinal' preference differences that can be elicited from DM's choice behavior. To state it, we need to introduce some new notation that builds on the concept of preference differences that we stated above. Let p_k, $k = 1, \ldots, 4$, be such that for some p, q,

$$p_k = \lambda_k p \oplus (1 - \lambda_k)q,\ k = 1, \ldots, 4.$$

Then, for $\lambda > 0$, we will write

$$(p_1, p_2) \geq^* \lambda(p_3, p_4),\text{ if } \lambda_1 - \lambda_2 \geq \lambda(\lambda_3 - \lambda_4).$$

- **AXIOM: Consistent Revealed Cardinality**
  Suppose $p = [(z, a),\ \lambda;\ (z_1, a_1),\ \lambda_1;\ \ldots;\ (z_M, a_M),\ \lambda_M]$ *and* $q = [(z, \widetilde{a}),\ \lambda;\ (z_1, \widetilde{a}_1),\ \lambda_1;\ \ldots;\ (z_M, \widetilde{a}_M),\ \lambda_M]$, *are comparable in the* λ*-probability event, and* $p' = [(z, a'),\ \lambda';\ (z'_1, a'_1),\ \lambda'_1;\ \ldots;\ (z'_N, a'_N),\ \lambda'_N]$ *and* $q' = [(z, \widetilde{a}'),\ \lambda';\ (z'_1, \widetilde{a}'_1),\ \lambda'_1;\ \ldots;\ (z'_N, \widetilde{a}'_N),\ \lambda'_N]$, *are comparable in the* λ'*-probability event. If* $p \succcurlyeq q$*, and, for some* $\lambda > 0$,

  $$((z, a'),\ (z, \widetilde{a}')) \geq^* \lambda((z, a),\ (z, \widetilde{a}))\ \text{and}\ ([z, p'_A],\ [z, q'_A]) \geq^* \lambda([z, p_A],\ [z, q_A]),$$

  then $p' \succcurlyeq q'$.

In our set up, DM's evaluation of an ex-post outcome $(z, a) \in Z \times A$ may be contingent on her evaluation of the procedure by which others' outcomes are determined. In other words, for any outcome (z, a) that is in the support of two lotteries $p, p' \in \Delta$, the ex-post evaluation of this outcome may differ depending on the respective procedures p_A and p'_A. We define the following notion to account for this difference: For any $p \in \Delta$ and (z, a) in the support of p, we will refer to the triple $(z, a, p_A) \in Z \times A \times \Delta_A$ as a **procedure-contingent allocation**. We next propose a method of eliciting DM's evaluation of procedure-contingent allocations by mapping procedure-contingent allocations into the space of allocations in a manner that is informed by her preferences. To that end consider the following definition.

Definition 3. *For any two procedure-contingent allocations,* (z, a', p'_A) *and* $(z, \widetilde{a}', q'_A)$*, we will write* $(z, a', p'_A) \sim^* (z, \widetilde{a}', q'_A)$*, if there exists two lotteries* $p = [(z, a),\ \lambda;\ (z_1, a_1),\ \lambda_1;\ \ldots;\ (z_M, a_M),\ \lambda_M]$ *and* $q = [(z, \widetilde{a}),\ \lambda;\ (z_1, \widetilde{a}_1),\ \lambda_1;\ \ldots;\ (z_M, \widetilde{a}_M),\ \lambda_M] \in \Delta$ *that satisfy:*

1. p and q are comparable in the λ-probability event,

2. $p \sim q$, and

3. $a' \sim_z a$, $p'_A \sim_z p_A$, $\widetilde{a}' \sim_z \widetilde{a}$, $q'_A \sim_z q_A$

*We will say that a procedure-contingent allocation (z, a^1, p^1_A) is **revealed indifferent** to a a procedure-contingent allocation (z, a^K, p^K_A), if there exists procedure contingent allocations (z, a^k, p^k_A), $k = 2, \ldots, K-1$, such that for each $k = 1, \ldots, K-1$, $(z, a^k, p^k_A) \sim^* (z, a^{k+1}, p^{k+1}_A)$*

Since the lotteries p and q are comparable in the λ-probability event, as discussed above, this is the 'only place' where the two lotteries can differ. Since, DM is indifferent between p and q, an outside observer, on seeing her choice, can infer that DM's evaluation of the procedure-contingent allocations $(z,\ a,\ p_A)$ and $(z,\ \widetilde{a},\ q_A)$ must be equivalent. Further, if we are to assume that DM makes an independent evaluation of the outcome that others get in this event, and the procedure determining their outcome, using the preference relation $\succsim_z$, then it stands to reason that DM's evaluation of the procedure-contingent allocations (z, a', p'_A) and $(z, \widetilde{a}', q'_A)$ are also equivalent. The definition of revealed indifferent then hypothesizes a 'transitivity' like consistency property on such equivalences.

The following definition proposes a way of accounting for the impact of procedures in DM's evaluation of lotteries.

Definition 4. *Let $p = [(z_1, a_1),\ \lambda_1; \ldots; (z_M, a_M),\ \lambda_M]$. We say that, $\pi(p) = [(z_1, \widetilde{a}_1),\ \lambda_1; \ldots; (z_M, \widetilde{a}_M),\ \lambda_M]$ is the **procedure-contingent equivalent** of p if for $m = 1, \ldots, M$, $(z_m,\ a_m,\ p_A)$ is revealed indifferent to $(z_m,\ \widetilde{a}_m,\ \widetilde{a}_m)$.*

Our next axiom imposes a consequentialist restriction similar in spirit to the axiom of *procedure-contingent consequentialism.* Informally, it says that once DM's concerns for procedure have been accounted for by "transforming" lotteries into their procedure-contingent equivalents, a consequentialist evaluation of these transformed lotteries should guide DM's preference ranking over the corresponding lotteries in Δ.

- **AXIOM: Procedure-Contingent Consequentialism***
  Let $\pi(p) = [(z_1, a_1),\ \lambda_1; \ldots; (z_M, a_M),\ \lambda_M]$, $\pi(q) = [(z'_1, a'_1),\ \lambda'_1; \ldots; (z'_N, a'_N),\ \lambda'_N]$ be the procedure-contingent equivalent of p and $q \in \Delta$, respectively. Then

$$p \succsim q \Longleftrightarrow \lambda_1(z_1, a_1) \oplus \ldots \oplus \lambda_M(z_M, a_M) \succsim \lambda'_1(z'_1, a'_1) \oplus \ldots \oplus \lambda'_N(z'_N, a'_N).$$

Finally, we impose the following regularity conditions on DM's preferences over the class of lotteries in which she gets some outcome $z \in Z$ for sure.

- **AXIOM: Regularity**

  1. *Continuity: For any* $z \in Z$*, and* $q \in \Delta$*, the sets*

     $$\{((a', a''), \lambda) \in A{\times}A \times [0,1] : [(z,a'), \lambda;\ (z,a''), 1-\lambda] \succcurlyeq q\},$$

     and,

     $$\{((a', a''), \lambda) \in A{\times}A \times [0,1] : q \succcurlyeq [(z,a'), \lambda;\ (z,a''), 1-\lambda]\}$$

     are closed in $A \times A \times [0,1]$.

  2. *Local Monotonicity: If* $q = [(z, a_1),\ \lambda;\ (z, a_2),\ 1-\lambda]$*, then there exists neighborhoods* $N(a_k)$ *of* a_k*,* $k = 1, 2$*, such that*

     (a) $[a' \in N(a_1),\ a'' \in N(a_2),\ a' \succcurlyeq_z a_1,\ a'' \succcurlyeq_z a_2] \Rightarrow [(z,a'),\lambda;\ (z,a''),1-\lambda] \succcurlyeq q$

     (b) $[a' \in N(a_1),\ a'' \in N(a_2),\ a_1 \succcurlyeq_z a',\ a_2 \succcurlyeq_z a''] \Rightarrow q \succcurlyeq [(z,a'),\lambda;\ (z,a''),1-\lambda]$.

  3. *Boundedness: For any* $p_A \in \Delta_A$*, there exists* $a,\ a' \in A$*, such that* $a \succcurlyeq_z p_A \succcurlyeq_z a'$.

4.2 General Representation – Contingent Procedural Weights

We can now state the following representation of $\succcurlyeq$.

Theorem 2. *Suppose regularity and strong contingent values hold. Then* $\succcurlyeq$ *on* Δ *satisfies the axioms of weak order, bounded Archimedean, procedure-contingent independence, consistent revealed cardinality and procedure-contingent consequentialism* if and only if there exists a bounded function* $w : Z \times \Delta_A \to \mathbb{R}$*, and constants* $\sigma_z \in [0,1]$*,* $z \in Z$*, such that the function* $W{:}\Delta \to \mathbb{R}$*, given by*

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma_z)w(z,a) + \sigma_z w(z,p_A)\}$$

represents $\succcurlyeq$*, and range*$(W) \subseteq$ *range*(w).
In addition, another pair $(\tilde{w} : Z \times \Delta_A \to \mathbb{R},\ (\tilde{\sigma}_z)_{z\in Z})$ *represents* $\succcurlyeq$ *in the above sense if and only if there exists constants* $\alpha > 0$ *and* β *such that* $\tilde{w} = \alpha w + \beta$*, and* $\tilde{\sigma}_z = \sigma_z$ *for all* $z \in Z$ *with* $\succ_z \neq \emptyset$.

For any lottery $p \in \Delta$ and (z, a) in the support of p, the representation provides a 'valuation' of the procedure-contingent allocation (z, a, p_A). This valuation is given by the expression

$$(1-\sigma_z)w(z,a) + \sigma_z w(z,p_A)$$

It is a weighted average of DM's concern for outcomes and her concerns for procedures. The subjective weight σ_z is uniquely determined in our representation as long as $\succcurlyeq_z$ is non-trivial. We call σ_z a **procedural weight**. It quantifies the strength of procedural concerns relative to concerns for outcome in determining DM's choice behavior. Under the representation, once all the procedure-contingent allocations have been appropriately evaluated, the aggregation criterion across events is just like under expected utility. The proof of the theorem is available in the Appendix.

4.3 General Representation – Unique Procedural Weight

In Theorem 2 the procedural weights are a function of the outcome that DM receives. We now provide a representation in which there is a unique procedural weight independent of DM's outcomes; that is, DM evaluates a lottery $p \in \Delta$ by the function:

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma)w(z,a) + \sigma w(z,p_A)\}, \text{ where } \sigma \in [0,1].$$

It should be intuitively clear that to axiomatize this case we need to impose some form of symmetry on DM's preferences. We now make precise this notion of symmetry.

- **AXIOM: Symmetry**
  If $p = [(z,a), \frac{1}{2};\ (z',a'), \frac{1}{2}]$, and $q = [(z,a'), \frac{1}{2};\ (z',a), \frac{1}{2}] \in \Delta$ are such that $((z,a), (z,a')) =^* ((z',a), (z',a'))$, then $p \sim q$.

We then have the following representation result:

Theorem 3. *Suppose regularity and strong contingent values hold. Then $\succcurlyeq$ on Δ satisfies the axioms of weak order, bounded Archimedean, procedure-contingent independence, consistent revealed cardinality, procedure-contingent consequentialism* and symmetry if and only if there exists a bounded function $w : Z \times \Delta_A \to \mathbb{R}$, and a constant $\sigma \in [0,1]$, such that the function $W{:}\Delta \to \mathbb{R}$, given by*

$$W(p) = \textstyle\sum_{(z,a)\in Z\times A} p(z,a)\{(1-\sigma)w(z,a) + \sigma w(z,p_A)\}$$

represents $\succcurlyeq$, and range$(W) \subseteq$ range(w).
In addition, another pair $(\tilde{w} : Z \times \Delta_A \to \mathbb{R},\ \tilde{\sigma})$ represents $\succcurlyeq$ in the above sense if and only if there exists constants $\alpha > 0$ and β such that $\tilde{w} = \alpha w + \beta$, and $\tilde{\sigma} = \sigma$ whenever there exists some $z \in Z$ with $\succ_z \neq \emptyset$.

The proof is available in the appendix.

5 Application

Do people vote against their 'self-interest' in large elections? That is a question that has been at the forefront of many recent discussions on voting behavior. In this section, we want to provide some novel insights into the question using our decision model. In particular, we want to highlight the role that procedural considerations play in influencing people's voting behavior. This role may become particularly prominent in large elections where people's ability to influence the outcome of the election with their vote is rather small, and so their rationale behind who they vote for may be driven to a large extent by procedural considerations. We want to elaborate on this theme using our decision model.

The first question that we need to address here is a definitional one – what does it mean for someone to vote against their self-interest? To help us address this question, we begin by considering some experimental evidence from a recent paper by Feddersen, Gailmard and Sandroni (2009). The basic hypothesis that their work proposes is that large elections may exhibit a moral bias, namely, alternatives understood by voters to be morally superior are more likely to win in large elections than in small ones. To make this point, they conduct an experimental election with two alternatives – call these the *moral option* and the *selfish option*. The basic details of their experiment are as follows. First, subjects were divided into two groups, one consisting of voters and the other of non-voters. Then the voters cast their votes. Finally, after all voters had cast their vote, one voter was randomly picked, and the choice she reported became the outcome of the election. Observe that under this particular method of determining the outcome of the election, the probability that any given voter's vote is pivotal, i.e., that her vote determines the outcome of the election, is given by the reciprocal of the number of voters. The experimenters varied the number of eligible voters across different trials of the experiment and, by so doing, the probability of a voter being pivotal was directly controlled as a treatment variable in the experiment. As far as payoffs went, the selfish option gave a higher monetary reward to the voters than the moral option. On the other hand, the moral option was better for the non-voters than the selfish option. An interesting pattern of choice exhibited by a non-trivial number of voters is the following. When the probability of their vote being pivotal was high, in particular when it was 1 (i.e., they were dictatorial), these voters chose the selfish option. On the other hand, when the pivot probability was low, their vote switched to the moral option. Overall, the data from different trials of the experiment showed a strong (statistically significant) positive relationship between the probability of the moral option being the electoral outcome and the size of the electorate.

One may make the case that the voters mentioned above voted against their self-interest based on the following kind of argument. When these voters were dictatorial, they chose the selfish option. This choice *reveals* that they *prefer* the selfish option to the moral one. At the same time, in elections where the probability that their vote is pivotal was low, they ended up voting for the moral option which, if one were to go by their revealed preference inferred from the first choice, is their less preferred alternative. It is important to recognize though that such an argument is based on consequentialism. That is, it assumes that voters have a ranking over the electoral outcomes independent of the process by which these outcomes are generated. Such a consequentialist argument is not appropriate for voters who care about the process by which the electoral outcome is generated. For instance, such voters may derive 'utility' from the very *act* of voting for a particular choice owing to motivations like a sense of civic obligation or a desire to act 'morally' by making certain choices. Such individuals may be said to derive an *expressive* value from voting. We will now use our decision model to sketch out such an expressive theory of voting. We will show how concerns for procedure can rationalize voting behavior which, when viewed from a consequentialist standpoint, appear to be against one's self-interest. Our primary goal here is to highlight in the simplest possible terms how procedural considerations that are embedded in our decision model influence voting behavior, and how such voting behavior differs from that in 'standard models' where such considerations are absent. To that end we are going to make the specification of the electoral process extremely simple.

We consider an election with two alternatives, 1 and 2, in a society consisting of n voters. We think of each of the alternatives as determining an outcome for each of the individuals in society. Accordingly, the alternatives can be thought of as determining the allocation for this society. We treat n as a parameter of the model. We assume that there are no costs to voting. This will ensure that everyone votes in the election. Further, the result of the election will be determined by the following mechanism, which mimics the one used by Feddersen et al. First, all voters cast their votes. After all voters have reported their choice, one voter is drawn at random, and the choice she reported determines the outcome of the election.

We make the extreme assumption that all voters are identical in terms of their preferences. This greatly simplifies the analysis, since it allows us to conduct it in the context of a 'representative voter.' Let us now describe what the problem looks like when viewed from the perspective of one such representative voter (RV). As mentioned above, she can vote for either alternative 1 or alternative 2. If alternative 1 is the group choice, the resulting allocation is $(z^1, a^1) \in Z \times A$,[19] where z^1 refers to the outcome for RV, and a^1 refers to

[19] We continue using the notation that the set Z denotes the outcomes of the decision maker (who in this case is the representative voter under consideration), A_i, $i \neq RV$, denotes the set of outcomes of individual i, and $A = \prod_{i \neq RV} A_i$.

the vector of outcomes for everyone else. Similarly, if alternative 2 is the group choice, the resulting allocation is $(z^2, a^2) \in Z \times A$, where again z^2 refers to the outcome for RV, and a^2 the outcomes for everyone else.

Note that the probability that RV is pivotal is given by $\lambda = 1/n$. Further, let γ denote the probability that alternative 1 is the outcome of the election when RV is not pivotal.[20] Then the probability distribution over final allocations generated by RV choosing alternative 1 is given by:

$$p^1 = [(z^1, a^1),\, \lambda + (1-\lambda)\gamma;\, (z^2, a^2),\, 1 - \lambda - (1-\lambda)\gamma],$$

and that by choosing alternative 2 is given by:

$$p^2 = [(z^1, a^1),\, (1-\lambda)\gamma;\, (z^2, a^2),\, 1 - (1-\lambda)\gamma]$$

Note that if RV's preferences satisfy stochastic dominance, then her vote choice is independent of pivot probabilities or, equivalently, of the number of voters. To understand this claim, suppose, she prefers the allocation (z^2, a^2) to (z^1, a^1), that is, she would choose alternative 2 if the choice *were* completely left to her. Now consider any situation in which she is pivotal with probability $\lambda = 1/n$. In this case, taking the other voters' choices as given (that is, taking γ as given), her vote for alternatives 1 and 2 generates respectively the lotteries p^1 and p^2 over final allocations (listed above). Since she prefers the allocation (z^2, a^2) to (z^1, a^1), stochastic dominance requires that she must prefer the lottery p^2 to the lottery p^1, and hence must vote for alternative 2 irrespective of what λ and γ are. Accordingly, assuming that RV has a strict preference for one of the alternatives (in the above sense) we have:

Proposition 1. *If voters' preferences satisfy stochastic dominance, then there exists a unique Nash equilibrium (in dominant strategies) that is independent of n in which either everyone votes for alternative* 1 *or everyone votes for alternative* 2*.*

We now contrast this result with one that is implied by our decision model in which decision makers may have procedural concerns. To simplify the analysis, we will assume that the function w that we derived in the representation is separable across DM's outcomes and others' outcomes. In particular any lottery p is evaluated by the function:

$$W(p) = \textstyle\sum_z p(z, a)[u(z) + (1-\sigma)v_z(a) + \sigma v_z(p_A)]$$

where $\sigma \in [0, 1]$. Here the function v_z represents $\succcurlyeq_z$. We will assume that RV considers alternative 1 to be better on grounds of her values or morals. That is,

[20] Of course, γ is an 'endogenous object'

$$v_H = v_z(a^1) > v_z(a^2) = v_L, \text{ for } z = z^1,\ z^2.$$

Further, we assume that the preference relations $\succcurlyeq_z$, $z = z^1$, z^2, are identical, and that v_z takes what is called a biseparable form. This means that there exists a probability weighting function, that is, a strictly increasing bijection $\varphi : [0, 1] \to [0, 1]$ that satisfies $\varphi(0) = 0$, $\varphi(1) = 1$, such that a lottery of the type $[a^1,\ r;\ a^2,\ 1 - r]$ is evaluated as,

$$v_z([a^1,\ r;\ a^2,\ 1 - r]) = \varphi(r)v_z(a^1) + (1 - \varphi(r))v_z(a^2), \text{ for } z = z^1,\ z^2.$$

The probability weighting function has the interpretation that it transforms objective probabilities into decision weights. These decision weights capture the attitude that DM has toward the chance or risk faced by others. We will assume that the procedural weight σ is equal to $\frac{1}{2}$. Further, define,

$$\nu = \tfrac{u_H - u_L}{v_H - v_L}$$

and assume that:

- $[V1]$ $\nu > 1$.
- $[V2]$ There exists $\underline{\lambda}$, $\overline{\lambda} \in (0, 1)$, such that for all $\widetilde{\lambda} \in (0,\underline{\lambda}) \cup (\overline{\lambda}, 1)$, φ is differentiable, and $\varphi'(\widetilde{\lambda}) > 2\nu - 1$. Further, φ is concave on the interval $[0,\underline{\lambda})$.

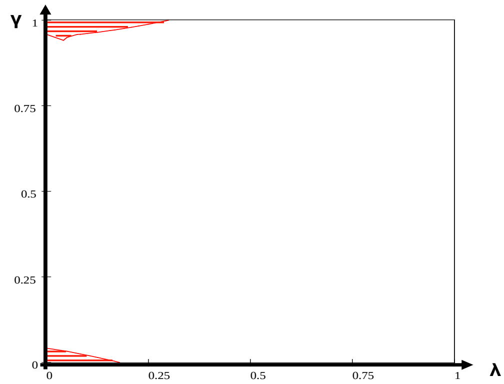
[V1] can be rewritten as

$$u_H + v_L > u_L + v_H$$

The left-hand side gives RV's payoffs under our representation from the allocation (z^2, a^2), whereas the right-hand side gives her payoffs from the allocation (z^1, a^1). This condition therefore states that RV prefers the allocation (z^2, a^2) to the allocation (z^1, a^1), when these allocations are considered by themselves (that is, each is viewed as realizing with probability 1). Accordingly, if RV were a dictator who could decide the election outcome on her own, she would choose alternative 2.

Assumptions [V1] and [V2] together imply that for all $\widetilde{\lambda} \in (0,\underline{\lambda}) \cup (\overline{\lambda}, 1)$, $\varphi'(\widetilde{\lambda}) > 1$. It follows that there exists a neighborhood of 0 in which $\varphi(\widetilde{\lambda}) > \widetilde{\lambda}$, and there exists a neighborhood of 1 in which $\varphi(\widetilde{\lambda}) < \widetilde{\lambda}$. in other words, the representative voter tends to overweight small probabilities and underweight large probabilities of her morally preferred outcome for others, a^1, being realized. This phenomenon of over-weighting small probabilities, and under-weighting large ones, which is referred to as regressive probability weighting, has been extensively documented in the literature on decision making under risk, starting with the important contribution of Kahneman and Tversky (1979).

Figure 4: Payoff difference between voting for alternatives 1 and 2.



Proposition 2. *Under assumptions [V1] and [V2], there exists positive integers $\underline{n}$ and $\overline{n}$, $\underline{n} < \overline{n}$, such that for all $n \leq \underline{n}$, everyone voting for alternative 2 is a Nash equilibrium, and for all $n \geq \overline{n}$, everyone voting for alternative 1 is a Nash equilibrium.*

The proof is available in the Appendix. Here, we briefly go over the reasoning that drives the result. Consider Figure 4, which has been constructed by taking particular values of u_H, u_L, v_H, v_L and functional form for the probability weighting function that are consistent with assumptions [V1] and [V2]. The figure shows the payoff difference for our representative voter from voting for alternatives 1 and 2 as a function of λ, the pivot probability, and γ, the probability that alternative 1 will be chosen when RV is not pivotal. The shaded area represents those values of λ and γ for which the payoff of voting for alternative 1 exceeds that of voting for alternative 2. The incentives that RV has for voting for alternative 2 for high values of λ is quite apparent given that she prefers alternative 2 to alternative 1. The interesting feature of our model is that for low values of λ, and for suitable values of γ, her vote choice shifts from alternative 2 to 1. In particular, there are two regions in the λ-γ box of the figure in which the payoff of voting for alternative 1 exceeds that of voting for alternative 2. This vote switch is brought about by the role that procedures play in her evaluation of prospects.

Consider first the lower south-west region where both λ and γ are small. In this scenario, RV knows that alternative 2 is the likely electoral outcome. Further, this is true irrespective of which way she votes, since the probability λ that her vote is pivotal is small. Thus, her vote is relatively insignificant in terms of determining actual *outcomes*. But given that she cares about procedures, her vote holds a significance beyond its ability to influence the outcome of the election. Observe that since alternative 1 is her morally preferred outcome, she can be made better off in the event that alternative 2 is the electoral outcome if alternative 1 had a higher ex-ante chance of being realized. So by voting for alternative 1 she can increase this ex-ante chance and receive higher payoffs with respect to her procedural concerns. What makes this increase in 'procedural payoffs' significant (relative to the increase in 'outcome payoffs' if she votes for alternative 2) is the fact that she over-weights small chances of her morally preferred outcome for others, a^1, being realized. So to sum up, voting for alternative 1 is almost identical to voting for alternative 2 via her concerns for outcomes. On the other hand, voting for alternative 1 is comparatively much better than voting for alternative 2 via her concerns for procedure. Accordingly, under this scenario, she votes for alternative 1.

Now consider the north-west corner of the λ-γ box. In this scenario alternative 1 is the likely electoral outcome, and given that λ is small, this is true irrespective of which way RV votes. Therefore, voting for alternative 1 is almost identical, once again, to voting for alternative 2 in terms of outcomes. On the other hand, voting for alternative 1 is relatively better than voting for alternative 2 via her concerns for procedure. To see this, note that if she were to vote for alternative 2, it would reduce the ex-ante chance of alternative 1 being realized by λ. Given that the chance of alternative 1 being realized is close to 1, the regressive nature of probability weighting close to 1, namely, that probabilities are under-weighted, makes this reduction in ex-ante chance unattractive for her. Accordingly, under this scenario, she votes for alternative 1

Given the structure of payoff differences, it should now be obvious why our result follows. In particular, note that when everyone else is voting for alternative 1 ($\gamma = 1$), for small pivot probabilities, RV's best response is to vote for alternative 1.

6 Concluding Remarks

In many settings, decision makers care about procedures. Decision makers with other-regarding preferences fall into this category, since they may care not just about the outcomes of others but also about the procedure by which others' outcomes are determined. Existing theories of decision making cannot incorporate such concerns for procedures. The reason is that these theories preserve a form of consequentialism (stochastic dominance) that requires a decision maker's ranking over outcomes to be independent of the stochastic process that produces these outcome. On the other hand, concerns for procedure imply that a decision maker's ranking over outcomes is contingent on the outcome-generating process. In this paper, we provided a tractable and parsimonious decision model that accommodates such concerns for procedure. In our representations, we showed how a decision maker's choices emerge out of a linear interaction between her concerns for outcome and her concerns for procedure. We were able to identify from behavior the precise subjective weight that such decision makers put on concerns for procedure relative to concerns for outcomes.

At a conceptual level, if choices are influenced both by a concern for outcomes as well as a concern for procedures, then economic models that ignore the role of procedures may provide us with misleading deductions. For instance, in the voting model we considered, we showed that when procedural concerns are present, the conclusion that we arrive at differs vastly from the one when such concerns are ignored. Further, if the welfare of economic agents is influenced by procedural concerns, then ignoring such concerns may lead to biases in the ranking of economic policies.[21] It therefore stands to reason that economic analysis should incorporate procedural considerations in its purview and study how individual and social choices are influenced by the interactions and tradeoffs that exist between concerns for outcomes and concerns for procedure.

7 Appendix

7.1 Proof of the Basic Representation

Observe that the boundedness condition under the *bounded Archimedean* axiom states that there exists (z^1, a^1), $(z_2, a_2) \in Z \times A$, such that for all $p \in \Delta$,

$$(z^1, a^1) \succcurlyeq p \succcurlyeq (z_2, a_2).$$

Let (z^2, a_2) be such that $(z^2, a_2) \succcurlyeq (z, a_2)$ for all $z \in Z$. Further, let (z_1, a^1) be such that $(z, a^1) \succcurlyeq (z_1, a^1)$ for all $z \in Z$. We can now state the following lemma.

Lemma 2. *One of the following cases hold.*

1. $(z_1, a^1) \sim (z_2, a_2)$, *and for any* $p \in \Delta$, $(z^1, a^1) \succcurlyeq p \succcurlyeq (z_1, a^1)$.

2. $(z^2, a_2) \sim (z^1, a^1)$, *and for any* $p \in \Delta$, $(z^2, a_2) \succcurlyeq p \succcurlyeq (z_2, a_2)$.

[21] The following observation by Dani Rodrik, in which he distinguishes between trade-induced changes and technology-induced changes in economic outcomes, illustrates the point: "Both [trade-induced changes and technology-induced changes] increase the size of the economic pie, while often causing large income transfers. But a redistribution that takes place because home firms are undercut by competitors who employ deplorable labor practices, use production methods that are harmful to the environment, or enjoy government support is *procedurally* different than one that takes place because an innovator has come up with a better product through hard work or ingenuity. Trade and technological progress can have very different implications for *procedural fairness*."
Refer to: *http://rodrik.typepad.com/dani_rodriks_weblog/*2007/04/*trade_and_proce.html*. The emphases in the quote are mine.

3. $(z^2, a_2) \succ (z_1, a^1)$*, and for any* $p \in \Delta$*, either,* $(z^1, a^1) \succcurlyeq p \succcurlyeq (z_1, a^1)$*, or* $(z^2, a_2) \succcurlyeq p \succcurlyeq (z_2, a_2)$*, or both.*

Proof. If $a^1 = a_2$, then the first two cases both hold. Otherwise suppose that the first two cases do not hold. That is, $(z_1, a^1) \succ (z_2, a_2)$, and $(z^1, a^1) \succ (z^2, a_2)$. Note that (by definition) $(z_2, a^1) \succcurlyeq (z_1, a^1)$, which then implies that $(z_2, a^1) \succ (z_2, a_2)$. It then follows from *contingent values* that there exists $z \in Z$ such that $(z, a_2) \succ (z, a^1)$. Hence,

$$(z^2, a_2) \succcurlyeq (z, a_2) \succ (z, a^1) \succcurlyeq (z_1, a^1).$$

That is, $(z^2, a_2) \succ (z_1, a^1)$ and we are in Case 3. □

We can now state the following representation result for $\succcurlyeq$

Lemma 3. *There exists a function* $W : \Delta \to \mathbb{R}$ *that represents* $\succcurlyeq$*, with the property that for any* p*,* $q \in \Delta$ *with* $p_A = q_A$*,*

$$W(\lambda p + (1 - \lambda)q) = \lambda W(p) + (1 - \lambda)W(q).$$

Further, the function W *is unique up to positive affine transformation*

Proof. Consider the space of lotteries that gives the others some outcome-vector a for sure:[22]

$$\Delta(a) = \{p \in \Delta : p_A = a\}$$

Note that, $\succcurlyeq$ restricted to any such set $\Delta(a)$ satisfies all the assumptions of the standard expected utility setting, and hence DM's preferences over lotteries in such a space can be provided with an expected utility representation. In particular consider the sets $\Delta(a^1)$, $\Delta(a_2)$ (where a^1, a_2 are as in the statement of lemma 2). It follows that there exists a function w^1 : $\{(z, a^1) : z \in Z\} \to \mathbb{R}$ such that the function $W^1 : \Delta(a^1) \to \mathbb{R}$, given by

$$W^1(p) = \textstyle\sum_z p(z, a^1).w^1(z, a^1)$$

represents $\succcurlyeq$ on $\Delta(a^1)$. Similarly, there exists a function $w_2 : \{(z, a_2) : z \in Z\} \to \mathbb{R}$, such that the function $W_2 : \Delta(a_2) \to \mathbb{R}$, given by

$$W_2(p) = \textstyle\sum_z p(z, a_2).w_2(z, a_2)$$

represents $\succcurlyeq$ on $\Delta(a_2)$.

Further, the functions W^1 and W_2 are unique up to positive affine transformation. Next note that we can recalibrate the functions W^1 and W_2 and define them in terms of a common function. Recall the 3 cases of lemma 2. In each of those 3 cases we can find lotteries p^1,

[22] As we stated in the text, we will abuse notation, by referring to a degenerate lottery by the corresponding outcome on which it puts probability 1.

$q^1 \in \Delta(a^1)$ and p_2, $q_2 \in \Delta(a_2)$, such that $p^1 \sim p_2 \succ q^1 \sim q_2$. Recalibrate W^1 and W_2 by setting,

$$W^1(p^1) = W_2(p_2) \text{ and } W^1(q^1) = W_2(q_2)$$

Now define a function $W : \Delta(a^1) \cup \Delta(a_2) \to \mathbb{R}$ by setting $W(p) = W^1(p)$ if $p \in \Delta(a^1)$ and $W(p) = W_2(p)$ if $p \in \Delta(a_2)$. *Procedure-contingent independence* guarantees that W can be consistently defined thus.

It follows from the *Archimedean* condition and lemma 2 that for any $p \in \Delta$ there exists $p^* \in \Delta(a^1) \cup \Delta(a_2)$ such that $p \sim p^*$. We can then extend W to the whole of Δ by setting $W(p) = W(p^*)$ for any such $p \in \Delta$. Clearly W represents $\succcurlyeq$. It is also straightforward to verify using procedure-contingent independence that for any p, $q \in \Delta$ with $p_A = q_A$,

$$W(\lambda p + (1-\lambda)q) = \lambda W(p) + (1-\lambda)W(q).$$

In addition, W is unique up to positive affine transformation. $\square$

It is also straightforward to verify that for any p, $q \in \Delta$, $\lambda \in [0,1]$, $\lambda p \oplus (1-\lambda)q$ exists, and

$$W(\lambda p \oplus (1-\lambda)q) = \lambda W(p) + (1-\lambda)W(q).$$

This fact establishes the statements of lemma 1.

Define a function $w : Z \times A \to \mathbb{R}$ by $w(z,a) = W(z,a)$. Next, note that, for any $p = [(z_1, a_1),\ \lambda_1;\ .\ .\ .\ ;\ (z_M, a_M),\ \lambda_M]$, $q = [(z'_1, a'_1),\ \lambda'_1;\ .\ .\ .\ ;\ (z'_N, a'_N),\ \lambda'_N] \in \Delta$ such that $p_A = q_A$, procedure-contingent consequentialism implies that

$$p \succcurlyeq q \Longleftrightarrow \lambda_1(z_1, a_1) \oplus\ .\ .\ .\ \oplus \lambda_M(z_M, a_M) \succcurlyeq \lambda'_1(z'_1, a'_1) \oplus\ .\ .\ .\ \oplus \lambda'_N(z'_N, a'_N)$$

That is,

$$p \succcurlyeq q \Longleftrightarrow \lambda_1 w(z_1, a_1) +\ .\ .\ .\ + \lambda_M w(z_M, a_M) \geq \lambda'_1 w(z'_1, a'_1) +\ .\ .\ .\ + \lambda'_N w(z'_N, a'_N)$$

Let $\Delta(q_A) = \{p \in \Delta : p_A = q_A\}$. Define the function $W_{q_A} : \Delta(q_A) \to \mathbb{R}$ as follows: for any $p = [(z_1, a_1),\ \lambda_1;\ .\ .\ .\ ;\ (z_M, a_M),\ \lambda_M] \in \Delta(q_A)$, let

$$W_{q_A}(p) = \lambda_1 w(z_1, a_1) +\ .\ .\ .\ + \lambda_M w(z_M, a_M).$$

It is straightforward to verify that that function is linear (in probabilities), and hence it follows that for any q_A, there exists constants $\alpha(q_A) > 0$ and $g(q_A)$ such that for any $p \in \Delta(q_A)$

$$W(p) = \alpha(q_A) W_{q_A}(p) + g(q_A).$$

It follows immediately from *procedure-contingent consequentialism* that $\alpha(q_A) = 1$. Hence, we can conclude that the function

$$W(p) = \sum_{(z,a)\in Z\times A} p(z,a)w(z,a) + g(p_A)$$

represents $\succcurlyeq$.

It is also straightforward to verify the necessity of the axioms, as well as, that (essential) uniqueness results for the functions w and g as stated in the statement of the theorem. We omit the details here.

7.2 Proof of the General Representations

7.2.1 A Binary Relation

We define here a binary relation. Consider the following definition.

Definition 5. *For any two procedure contingent allocations,* (z, a', p'_A) *and* $(z, \widetilde{a}', q'_A)$*, we will write* $(z, a', p'_A) \succcurlyeq^* $ *(resp.* $\succ^*$*)* $(z, \widetilde{a}', q'_A)$*, if there exists two lotteries* $p = [(z, a),\ \lambda;\ (z_1, a_1),\ \lambda_1;\ .\ .\ .\ ;\ (z_M, a_M),\ \lambda_M]$ *and* $q = [(z, \widetilde{a}),\ \lambda;\ (z_1, \widetilde{a}_1),\ \lambda_1;\ .\ .\ .\ ;\ (z_M, \widetilde{a}_M),\ \lambda_M] \in \Delta$ *that are comparable in the* λ*-probability event, and* $p \succcurlyeq$ *(resp.* $\succ$*)* q*, that satisfy:*

$$a' \sim_z a,\ p'_A \sim_z p_A,\ \widetilde{a}' \sim_z \widetilde{a},\ q'_A \sim_z q_A$$

We will say that a procedure-contingent allocation (z, a^1, p^1_A) *is* ***revealed better*** *(resp.* ***revealed strictly better****) to a a procedure-contingent allocation* (z, a^K, p^K_A)*, if there exists procedure contingent allocations* (z, a^k, p^k_A)*,* $k = 2, .\ .\ .\ , K-1$*, such that for each* $k = 1, .\ .\ .\ , K-1$*,* $(z, a^k, p^k_A) \succcurlyeq^*$ *(resp.* $\succ^*$*)* (z, a^{k+1}, p^{k+1}_A)

Since, the revealed indifferent, revealed better, revealed strictly better relations are uniquely defined only up to the indifference classes of the preference relations $\succcurlyeq_z$, we propose next a means of conveying the information contained in them in a more compact fashion. First consider the following notation. For any $p_A \in \Delta_A$ and $z \in Z$, the indifference class of p_A under $\succcurlyeq_z$ is denoted by

$$[p_A]_z = \{q_A \in \Delta_A : q_A \sim_z p_A\}$$

Further, $\Delta_A / \sim_z$ shall denote the set of all such indifference classes. We define the binary relations, $\widehat{\succcurlyeq}_z$, $\widehat{\succ}_z$, $\widehat{\sim}_z$ on $\Delta_A/\sim_z \times \Delta_A/\sim_z$ as follows:

Definition 6. $([p_A]_z,\ [q_A]_z)\ \widehat{\succcurlyeq}_z$ *(resp.* $\widehat{\succ}_z$*, resp.* $\widehat{\sim}_z$*)* $([p'_A]_z,\ [q'_A]_z)$ *if there exists procedure-contingent allocations* $(z, \tilde{a}, \tilde{p}_A)$ *and* $(z, \hat{a}, \hat{p}_A)$ *satisfying*

$$\tilde{a} \in [p_A]_z,\ \tilde{p}_A \in [q_A]_z \text{ and } \hat{a} \in [p'_A]_z,\ \hat{p}_A \in [q'_A]_z$$

such that $(z, \tilde{a}, \tilde{p}_A)$ *is revealed better than (resp. revealed strictly better than, resp. revealed indifferent to)* $(z, \hat{a}, \hat{p}_A)$.

Remark 1. The axiom of *consistent revealed cardinality* implies that $\widehat{\sim}_z$ and $\widehat{\succ}_z$ are respectively the symmetric and asymmetric components of $\widehat{\succcurlyeq}_z$. That is,

$$([p_A]_z,[q_A]_z)\ \widehat{\sim}_z\ ([p'_A]_z,[q'_A]_z) \text{ iff } ([p_A]_z,[q_A]_z)\ \widehat{\succcurlyeq}_z\ ([p'_A]_z,[q'_A]_z)\ \&\ ([p'_A]_z,[q'_A]_z)\ \widehat{\succcurlyeq}_z\ ([p_A]_z,\ [q_A]_z).$$

and,

$$([p_A]_z,[q_A]_z)\widehat{\succ}_z([p'_A]_z,[q'_A]_z) \text{ iff } ([p_A]_z,[q_A]_z)\ \widehat{\succcurlyeq}_z\ ([p'_A]_z,[q'_A]_z)\ \&\ \neg\ ([p'_A]_z,[q'_A]_z)\widehat{\succcurlyeq}_z\ ([p_A]_z,[q_A]_z).$$

Remark 2. The definition of revealed better (resp. revealed strictly better, resp. revealed indifferent) implies that if $([p_A]_z,\ [q_A]_z)\ \widehat{\succcurlyeq}_z$ (resp. $\widehat{\succ}_z$, resp. $\widehat{\sim}_z$) $([p'_A]_z,\ [q'_A]_z)$ and there exists procedure contingent outcomes $(z, \tilde{a}, \tilde{p}_A)$ and $(z, \hat{a}, \hat{p}_A)$ such that

$$\tilde{a} \in [p_A]_z,\ \tilde{p}_A \in [q_A]_z \text{ and } \hat{a} \in [p'_A]_z,\ \hat{p}_A \in [q'_A]_z$$

then $(z, \tilde{a}, \tilde{p}_A)$ is revealed better than (resp. revealed strictly better than, resp. revealed indifferent to) $(z, \hat{a}, \hat{p}_A)$.

In the way of notation, note that we will write $[p'_A]_z\ \widehat{\succcurlyeq}_z$ (resp. $\widehat{\succ}_z$, resp. $\widehat{\sim}_z$) $[p''_A]_z$ as a shorthand for $([p'_A]_z,\ [p'_A]_z)\ \widehat{\succcurlyeq}_z$ (resp. $\widehat{\succ}_z$, resp. $\widehat{\sim}_z$) $([p''_A]_z,\ [p''_A]_z)$.

7.2.2 A Topological Structure on $\Delta_A/\!\sim_z$

We next endow the sets $\Delta_A/\!\sim_z$, $z \in Z$, with a topology. For any $[p'_A]_z$, $[p''_A]_z \in \Delta_A/\!\sim_z$, let,

- $][p'_A]_z,\ [p''_A]_z[\ = \{[p_A]_z \in \Delta_A/\!\sim_z\ :\ p'_A \succ_z p_A \succ_z p''_A\}$,
- $][p'_A]_z,\ \rightarrow[\ = \{[p_A]_z \in \Delta_A/\!\sim_z\ :\ p_A \succ_z p'_A\}$, and
- $]\leftarrow,\ [p'_A]_z[\ = \{[p_A]_z \in \Delta_A/\!\sim_z\ :\ p'_A \succ_z p_A\}$.

Since $\succcurlyeq_z$ is a preference relation, it is natural to interpret these sets as preference intervals. Let $[q^{**}_A]_z$ and $[q^{*}_A]_z$ denote the maximal and minimal indifference classes respectively of $\succcurlyeq_z$ in $\Delta_A/\!\sim_z$, if such elements exist. That is,

$$[q^{**}_A]_z = \{p_A \in \Delta_A : p_A \succcurlyeq_z p'_A, \text{ for all } p'_A \in \Delta_A\},$$

and

$$[q_A^*]_z = \{p_A \in \Delta_A : p'_A \succcurlyeq_z p_A, \text{ for all } p'_A \in \Delta_A\},$$

If $[q_A^{**}]_z$ and/or $[q_A^*]_z$ exist, then for any $[p'_A]_z \in \Delta_A/\sim_z$ we shall write,

$$][p'_A]_z, \rightarrow [\ = \ ][p'_A]_z, [q_A^{**}]_z], \text{ and }] \leftarrow, [p'_A]_z[\ = [[q_A^*]_z, [p'_A]_z[$$

We endow the set $\Delta_A/\sim_z$ with the *order topology* of $\succcurlyeq_z$, i.e., the coarsest topology containing all sets of the form $][p'_A]_z, \rightarrow [$ and $] \leftarrow, [p'_A]_z[$, thus all sets of the form $][p'_A]_z, [p''_A]_z[$. We endow $[\Delta_A/\sim_z]^2$ with the product topology. A set of the type $C = I \times I' \subseteq [\Delta_A/\sim_z]^2$, where I and I' are of the form $][p'_A]_z, [p''_A]_z[$, or $][p'_A]_z, \rightarrow [$, or $] \leftarrow, [p'_A]_z[$ shall be referred to as a **cube** in $[\Delta_A/\sim_z]^2$. Our strategy in the proof of the representation results below shall be to first establish that $\widehat{\succcurlyeq}_z$ is a weak order 'locally' on such cubes, and then to extend this 'globally' by 'tying together' these cubes. Observe that if C, $C' \subseteq [\Delta_A/\sim_z]^2$ are cubes, then so is $C \cap C'$, if the intersection happens to be non-empty. Further, if we can establish that $\widehat{\succcurlyeq}_z$ is a weak order on C and C', then *consistent revealed cardinality* implies that the derived rankings must coincide on $C \cap C'$.

7.2.3 A Mixture Set Structure

Consider any $\succcurlyeq_z$. For any $[p_A]_z$, $[q_A]_z \in \Delta_A/\sim_z$, and $\lambda \in [0,1]$, we define a unique element $\lambda[p_A]_z \,\widehat{\oplus}_z\, (1-\lambda)[q_A]_z \in \Delta_A/\sim_z$ as follows. Let $[z, \widetilde{p}_A] \in \Delta$ be such that

$$[z, \widetilde{p}_A] = \lambda[z, p_A] \oplus (1-\lambda)[z, q_A].$$

We know from the *boundedness* and *continuity* conditions of the *regularity* axiom, and the mixture set structure imposed on Δ by $\oplus$ that such a $[z, \widetilde{p}_A]$ exists. We define $\lambda[p_A]_z \,\widehat{\oplus}_z\, (1-\lambda)[q_A]_z$ to be the element $[\widetilde{p}_A]_z \in \Delta_A/\sim_z$. We will abuse notation below, and write $\lambda p_A \,\widehat{\oplus}_z\, (1-\lambda)q_A$ to denote the element $\widetilde{p}_A$.

Further, for any $([p_A]_z, [q_A]_z)$, $([p'_A]_z, [q'_A]_z) \in [\Delta_A/\sim_z]^2$, and $\lambda \in [0,1]$, we define a unique element $\lambda([p_A]_z, [q_A]_z) \,\widehat{\oplus}_z\, (1-\lambda)([p'_A]_z, [q'_A]_z) \in [\Delta_A/\sim_z]^2$ as follows:[23]

$$\lambda([p_A]_z, [q_A]_z) \,\widehat{\oplus}_z\, (1-\lambda)([p'_A]_z, [q'_A]_z) = (\lambda[p_A]_z \,\widehat{\oplus}_z\, (1-\lambda)[p'_A]_z, \lambda[q_A]_z \,\widehat{\oplus}_z\, (1-\lambda)[q'_A]_z)$$

Any subset of $[\Delta_A/\sim_z]^2$ that is itself a mixture set shall be referred to as a mixture subset of $[\Delta_A/\sim_z]^2$. In particular, note that, any cube $C \subseteq [\Delta_A/\sim_z]^2$ is a mixture subset of $[\Delta_A/\sim_z]^2$. In addition, note the following result about mixture subsets of $[\Delta_A/\sim_z]^2$. (The proof is standard, and hence omitted).

Lemma 4. *Every mixture subset of $[\Delta_A/\sim_z]^2$, in particular $[\Delta_A/\sim_z]^2$ itself, is connected.*

[23]Formally, $\widehat{\oplus}_z : [\Delta_A/\sim_z]^2 \times [\Delta_A/\sim_z]^2 \times [0,1] \rightarrow [\Delta_A/\sim_z]^2$. Observe that we are abusing notation here by using the same notation $\widehat{\oplus}_z$ to denote 'mixture operations' on the sets $\Delta_A/\sim_z$ and $[\Delta_A/\sim_z]^2$. We do so because this should not cause any confusion, and it allows us to economize on notation.

7.2.4 Proof of Theorem 2

We shall first collect some useful notation to aid the exposition of the proof of Theorem 2. We shall denote the restriction of $\widehat{\succsim}_z$ to any set $\widehat{\Omega}$ in $[\Delta_A/\sim_z]^2$ by $(\widehat{\succsim}_z)_{\widehat{\Omega}}$. Further, let

$$int(\Delta_A/\sim_z) = \{[p_A]_z \in \Delta_A/\sim_z : [p_A]_z \neq [q_A^{**}]_z,\ [q_A^*]_z\}$$

$$D^* = \{([q_A]_z,\ [q_A]_z) \in [\Delta_A/\sim_z]^2 : [q_A]_z \in \Delta_A/\sim_z\}$$

$$D = \{([q_A]_z,\ [q_A]_z) \in [\Delta_A/\sim_z]^2 : [q_A]_z \in int(\Delta_A/\sim_z)\}$$

$$\Omega = \Delta_A/\sim_z \times int(\Delta_A/\sim_z), \text{ and } \Omega^* = \Omega \cup D^*.$$

Note that if $\widehat{\succsim}_z$ does not have any extremal elements then, $\Delta_A/\sim_z = int(\Delta_A/\sim_z)$ and $D^* = D$. In that case $D^* \subseteq int(\Delta_A/\sim_z) \times int(\Delta_A/\sim_z) = \Omega$ and so $\Omega^* = \Omega$.

Lemma 5. *Let $\succ_z \neq \emptyset$. For any $([p_A]_z,\ [q_A]_z) \in \Omega$ there exists a cube C containing $([p_A]_z,\ [q_A]_z)$ such that $\widehat{\succsim}_z$ restricted to C (denoted $(\widehat{\succsim}_z)_C$), satisfies the following.*

1. *Weak Order: $\widehat{\succsim}_z$ is complete and transitive on C.*

2. *vNM Continuity: Let $([p_A]_z,\ [q_A]_z)$, $([p'_A]_z,\ [q'_A]_z)$, $([p''_A]_z,\ [q''_A]_z) \in C$ be such that $([p_A]_z,\ [q_A]_z)\ \widehat{\succ}_z\ ([p'_A]_z,\ [q'_A]_z)\ \widehat{\succ}_z\ ([p''_A]_z,\ [q''_A]_z)$. Then there exists $\lambda,\ \lambda' \in (0,1)$ such that*

   $\lambda([p_A]_z,[q_A]_z)\ \widehat{\oplus}_z\ (1-\lambda)([p''_A]_z,[q''_A]_z)\ \widehat{\succ}_z\ ([p'_A]_z,[q'_A]_z)\ \widehat{\succ}_z\ \lambda'([p_A]_z,[q_A]_z)\ \widehat{\oplus}_z\ (1-\lambda')([p''_A]_z,[q''_A]_z)$.

3. *vNM Independence: Let $([p_A]_z,\ [q_A]_z)$, $([p'_A]_z,\ [q'_A]_z) \in C$ be such that $([p_A]_z,\ [q_A]_z)\ \widehat{\succ}_z\ ([p'_A]_z,\ [q'_A]_z)$. Then for any $([p''_A]_z,\ [q''_A]_z) \in C$, $\lambda \in (0,1]$,*

   $$\lambda([p_A]_z,[q_A]_z)\ \widehat{\oplus}_z\ (1-\lambda)([p''_A]_z,[q''_A]_z)\ \widehat{\succ}_z\ \lambda([p'_A]_z,[q'_A]_z)\ \widehat{\oplus}_z\ (1-\lambda)([p''_A]_z,[q''_A]_z).$$

4. *Monotonicity: for any $([p'_A]_z,\ [q'_A]_z)$, $([p''_A]_z,\ [q''_A]_z) \in C$,*

   $$[p'_A]_z\ \widehat{\succsim}_z\ [p''_A]_z \text{ and } [q'_A]_z\ \widehat{\succsim}_z\ [q''_A]_z \Rightarrow ([p'_A]_z,\ [q'_A]_z)\ \widehat{\succsim}_z\ ([p''_A]_z,\ [q''_A]_z).$$

5. *Non Degeneracy: $\widehat{\succ}_z \neq \emptyset$.*

Proof. We first consider the case of $([p_A]_z,\ [q_A]_z) \in \Omega$ for which $[p_A]_z \neq [q_A^{**}]_z$ or $[q_A^*]_z$.

- $(\widehat{\succsim}_z)_C$ is complete and transitive, for an appropriately defined cube C.

Pick any $([p_A]_z, [q_A]_z) \in \Omega$. There may be two possibilities. First, $p_A \nsim_z q_A$, and second $p_A \sim_z q_A$. For the first case assume without loss of generality that $p_A \succ_z q_A$. We can then find $a, a' \in A$ such that $a \sim_z p_A \succ_z q_A \succ_z a'$. The fact that we may find a as specified follows from the the *boundedness* and *continuity* condition of the *regularity* axiom. On the other hand a' exists as specified because $([p_A]_z, [q_A]_z) \in \Omega$ and so $q_A \notin [q_A^*]_z$. Further, it follows from the *continuity* condition that there exists $\lambda^* \in (0, 1)$ such that,

$$[a, \lambda^*; a', 1 - \lambda^*] \sim_z q_A$$

Now consider the case where, $p_A \sim_z q_A$. In this case pick $a, a' \in [q_A]_z$ that are "close" (It is possible that $a = a'$). Then it follows from the *local monotonicity* condition under *regularity* that there exists $\lambda^* \in (0, 1)$,

$$[a, \lambda^*; a', 1 - \lambda^*] \sim_z q_A$$

In either case therefore we can find $a, a' \in A$, and some $\lambda^* \in (0, 1)$ such that the above preference indifference condition holds. Henceforth, without loss of generality, we shall consider $q_A = [a, \lambda^*; a', 1 - \lambda^*]$.

It follows from the second condition of *strong contingent values* that there exists $\succcurlyeq_{z'} \neq \succcurlyeq_z$, with $\succ_{z'} \neq \emptyset$, such that, there exists $\overline{a}$, $\underline{a}$ and $\overline{a}'$, $\underline{a}'$ that satisfy,

$$\overline{a} \sim_{z'} a \sim_{z'} \underline{a} \text{ and } \overline{a} \succ_z a \succ_z \underline{a},$$

$$\overline{a}' \sim_{z'} a' \sim_{z'} \underline{a}' \text{ and } \overline{a}' \succ_z a' \succ_z \underline{a}'$$

In particular, *continuity* allows us to choose $\overline{a}$, $\underline{a}$ and $\overline{a}'$, $\underline{a}'$ in such a way that:

$$\overline{q}_A \equiv [\underline{a}, \lambda^*; \overline{a}', 1 - \lambda^*] \succ_z q_A \succ_z [\overline{a}, \lambda^*; \underline{a}', 1 - \lambda^*] \equiv \underline{q}_A.$$

We can now define the cube $C \subseteq \Omega$ that the statement of the lemma requires us to do. Define,

$$C =][\underline{a}]_z, [\overline{a}]_z[\times][\underline{q}_A]_z, [\overline{q}_A]_z[$$

Further, let,

$$I_a = \{\widehat{a} \in [a]_{z'} : \overline{a} \succcurlyeq_z \widehat{a} \succcurlyeq_z \underline{a}\}, \; \& \; I_{a'} = \{\widehat{a}' \in [a']_{z'} : \overline{a}' \succcurlyeq_z \widehat{a}' \succcurlyeq_z \underline{a}'\}.$$

Define a subset M of Δ as follows:

$$M = \{[(z, \widehat{a}), \lambda^*; (z', \widehat{a}'), 1 - \lambda^*] \in \Delta : \widehat{a} \in I_a, \widehat{a}' \in I_{a'}\}.$$

Consider any $p' = [(z, \widehat{a}), \lambda^*; (z', \widehat{a}'), 1 - \lambda^*] \in M$. Since, $\widehat{a} \in I_a \subseteq [a]_{z'}$, $\widehat{a}' \in I_{a'} \subseteq [a']_{z'}$, by appropriately choosing $\overline{a}$, $\underline{a}$ and $\overline{a}'$, $\underline{a}'$, we can establish using the *local monotonicity* condition of the *regularity* axiom that

$$p'_A = [\widehat{a}, \lambda^*; \widehat{a}', 1-\lambda^*] \sim_{z'} [a, \lambda^*; a', 1-\lambda^*] = q_A.$$

Therefore, for any $p' = [(z, a(z,p')), \lambda^*; (z', a(z',p')), 1-\lambda^*]$, $p'' = [(z, a(z,p'')), \lambda^*; (z', a(z',p'')), 1-\lambda^*]$ in M,

$$[a(z',p')]_{z'} = [a(z',p'')]_{z'} = [a']_{z'} \text{ and } [p'_A]_{z'} = [p''_A]_{z'} = [q_A]_{z'}.$$

That is, any p', $p'' \in M$ are comparable at the λ^*-probability event, and accordingly if $p' \succ p''$, then the procedure-contingent allocation $(z, a(z,p'), p'_A)$ is *revealed strictly better* than the procedure-contingent allocation $(z, a(z,p''), p''_A)$, and if $p' \sim p''$, then $(z, a(z,p'), p'_A)$ is *revealed indifferent* to $(z, a(z,p''), p''_A)$. Hence,

$$p' \succ p'' \Rightarrow ([a(z,p')]_z, [p'_A]_z) \mathrel{\widehat{\succ}_z} ([a(z,p'')]_z, [p''_A]_z)$$

$$p' \sim p'' \Rightarrow ([a(z,p')]_z, [p'_A]_z) \mathrel{\widehat{\sim}_z} ([a(z,p'')]_z, [p''_A]_z)$$

Consider any $([\widehat{p}_A]_z, [\widehat{q}_A]_z) \in C$. Since, $\overline{a} \succ_z \widehat{p}_A \succ_z \underline{a}$, it follows that there exists $\widehat{a} \in I_a$ such that $\widehat{a} \sim_z \widehat{p}_A$.[24] Further, by *local monotonicity*, it follows that

$$[\widehat{a}, \lambda^*; \overline{a}', 1-\lambda^*] \succ_z \overline{q}_A \succ_z \widehat{q}_A \succ_z \underline{q}_A \succ_z [\widehat{a}, \lambda^*; \underline{a}', 1-\lambda^*].$$

Continuity then implies that there exists $\widehat{a}' \in I_{a'}$ such that

$$\widehat{q}_A \sim_z [\widehat{a}, \lambda^*; \widehat{a}', 1-\lambda^*].$$

That is for any $([\widehat{p}_A]_z, [\widehat{q}_A]_z) \in C$, there exists

$$p' = [(z, \widehat{a}), \lambda^*; (z', \widehat{a}'), 1-\lambda^*] \in M$$

such that $\widehat{p}_A \sim_z a(z,p') = \widehat{a}$ and $\widehat{q}_A \sim_z p'_A = [\widehat{a}, \lambda^*; \widehat{a}', 1-\lambda^*]$. Accordingly, $\widehat{\succsim}_z$ is a weak order on C.

- $(\widehat{\succsim}_z)_C$ satisfies vNM Continuity.

First we introduce the following piece of notation: For any,

$$\widetilde{p} = [(z, \tilde{a}), \lambda^*; (z', \tilde{a}'), 1-\lambda^*], \widehat{p} = [(z, \hat{a}), \lambda^*; (z', \hat{a}'), 1-\lambda^*] \in M$$

and $\lambda \in [0,1]$, let

$$\lambda\widetilde{p} \oplus^* (1-\lambda)\widehat{p} = [(z, a_\lambda), \lambda^*, (z', a'_\lambda), 1-\lambda^*]$$

be any element in M that satisfies

[24]This follows since $[a]_{z'}$ is a connected subset of A. Note that

$$W_1 = \{\widetilde{a} \in [a]_{z'} : \widetilde{a} \succsim_z \widehat{p}_A\}, \ \& \ W_2 = \{\widetilde{a} \in [a]_{z'} : \widehat{p}_A \succsim_z \widetilde{a}\}$$

form a separation of $[a]_{z'}$, and hence their intersection must be nonempty.

$$(z, a_\lambda) = \lambda(z, \tilde{a}) \oplus (1-\lambda)(z, \hat{a}) \text{ and } [z,(\lambda\widetilde{p} \oplus^* (1-\lambda)\widehat{p})_A] = \lambda[z, \widetilde{p}_A] \oplus (1-\lambda)[z, \widehat{p}_A]$$

We will now establish that for any $\widetilde{p}$, $\widehat{p} \in M$, $\lambda \in [0,1]$, there exists $\lambda\widetilde{p} \oplus^* (1-\lambda)\widehat{p} \in M$. First, pick $a_\lambda \in I_a$ be such that,

$$(z, a_\lambda) = \lambda(z, \tilde{a}) \oplus (1-\lambda)(z, \hat{a}).$$

Next, let $\widetilde{q}_A \in \Delta_A$, be such that

$$[z, \widetilde{q}_A] = \lambda[z, \widetilde{p}_A] \oplus (1-\lambda)[z, \widehat{p}_A]$$

By *local monotonicity*, it follows that

$$[a_\lambda, \lambda^*; \overline{a}', 1-\lambda^*] \succcurlyeq_z \overline{q}_A \succcurlyeq_z \widetilde{q}_A \succcurlyeq_z \underline{q}_A \succcurlyeq_z [a_\lambda, \lambda^*; \underline{a}', 1-\lambda^*],$$

with strict preference holding at least somewhere. *Continuity* in conjunction with the fact that $[a']_{z'}$ is a connected subset of A implies that there exists, $a'_\lambda \in I_{a'}$, such that

$$[a_\lambda, \lambda^*; a'_\lambda, 1-\lambda^*] \sim_z \widetilde{q}_A.$$

Hence,

$$[(z, a_\lambda), \lambda^*; (z', a'_\lambda), 1-\lambda^*] = \lambda\widetilde{p} \oplus^* (1-\lambda)\widehat{p}.$$

We now establish that $(\widehat{\succcurlyeq}_z)_C$ satisfies the vN-M Continuity axiom. Note that this is equivalent to proving the following: For any p, p', $p'' \in M$ such that $p \succ p' \succ p''$, there exists λ, $\lambda' \in (0,1)$, such that:

$$\lambda p \oplus^* (1-\lambda)p'' \succ p' \succ \lambda' p \oplus^* (1-\lambda')p''$$

Suppose otherwise – say that $p' \succcurlyeq \lambda p \oplus^* (1-\lambda)p''$ for all $\lambda \in (0,1)$. We proved above that for all $\lambda \in [0,1]$ there exists $a_\lambda \in I_a$, $a'_\lambda \in I_{a'}$ such that,

$$[(z,\, a_\lambda),\, \lambda^*;\, (z',\, a'_\lambda);\, 1-\lambda^*] = \lambda p \oplus^* (1-\lambda)p''.$$

Denote,

$$p = [(z, \tilde{a}),\, \lambda^*;\, (z', \tilde{a}'),\, 1-\lambda^*]$$

We may then construct a sequence $(a_{\lambda_k},\, a'_{\lambda_k})_{k\in\mathbb{Z}_+} \subseteq I_a \times I_{a'}$ converging to $(\tilde{a},\, \tilde{a'}) \in I_a \times I_{a'}$, such that for all $k \in \mathbb{Z}_+$,

$$p' \succcurlyeq \lambda_k p \oplus^* (1-\lambda_k)p'' = [(z,\, a_{\lambda_k}),\, \lambda^*;\, (z',\, a'_{\lambda_k});\, 1-\lambda^*]$$

Let

$$\Xi = \{(a_{\lambda_k},\, a'_{\lambda_k}) \in I_a \times I_{a'} : p' \succcurlyeq [(z,\, a_{\lambda_k}),\, \lambda^*;\, (z',\, a'_{\lambda_k});\, 1-\lambda^*]\}$$

By *continuity* the set Ξ is closed in $I_a \times I_{a'}$. It then follows that $(\tilde{a},\, \tilde{a'}) \in \Xi$, that is $p' \succcurlyeq p = [(z, \tilde{a}),\, \lambda^*;\, (z', \tilde{a}'),\, 1-\lambda^*]$, which is absurd.

- $(\widehat{\succcurlyeq}_z)_C$ satisfies vNM Independence.

This follows from a straightforward application of *consistent revealed cardinality.* The details are omitted.

- $(\widehat{\succcurlyeq}_z)_C$ satisfies Monotonicity.

This again follows immediately from *consistent revealed cardinality.*

- $(\widehat{\succcurlyeq}_z)_C$ is Non Degenerate.

This follows immediately from the assumption made in the lemma that $\succ_z \neq \emptyset$.

The proof for the case when $[p_A]_z$ is equal to either $[q_A^{**}]_z$, $[q_A^{*}]_z$ is exactly along similar lines. When $[p_A]_z = [q_A^{**}]_z$, take $\overline{a} = a$ in the above proof, and define the cube C as follows:

$$C =][\underline{a}]_z,\, [a]_z] \times][\underline{q}_A]_z,\, [\overline{q}_A]_z[\ .$$

The rest of the details are exactly identical. Similarly, when $[p_A]_z = [q_A^{*}]_z$, take $\underline{a} = a$ in the above proof, and define

$$C = [[a]_z,\, [\overline{a}]_z[\times][\underline{q}_A]_z,\, [\overline{q}_A]_z[\ .$$

$\square$

Lemma 6. $(\widehat{\succcurlyeq}_z)_{\Omega^*}$ *is a weak order. Further, there exists*
(i) a function $v_z : \Delta_A \to \mathbb{R}$ *that represents* $\succcurlyeq_z$ *and satisfies: for all* $\lambda \in [0,1]$, p_A, $q_A \in \Delta_A$,

$$v_z(\lambda p_A \mathbin{\widehat{\oplus}_z} (1-\lambda)q_A) = \lambda v_z(p_A) + (1-\lambda)v_z(q_A),\ \textit{and}$$

(ii) a constant $\sigma_z \in [0,\, 1]$,
such that the function $V_z : \Omega^* \to \mathbb{R}$ *given by*

$$V_z([p_A]_z,\, [q_A]_z) = (1-\sigma_z)v_z(p_A) + \sigma_z v_z(q_A)$$

represents $(\widehat{\succcurlyeq}_z)_{\Omega^*}$. *Further, another pair* $(\widetilde{v}_z,\, \widetilde{\sigma}_z)$ *represents* $(\widehat{\succcurlyeq}_z)_{\Omega^*}$ *in the above sense iff* $\widetilde{v}_z$ *is a positive affine transformation of* v_z *and* $\widetilde{\sigma}_z = \sigma_z$, *for all* $z \in Z$ *with* $\succ_z \neq \emptyset$.

Proof. First consider those $z \in Z$ for which $\succ_z \neq \emptyset$. From Lemma 5 it follows that for any $([q_A]_z,\, [q_A]_z) \in D$, there exists a cube containing $([q_A]_z,\, [q_A]_z)$, which we can take to be

$$C_{[q_A]_z} =][\underline{q}_A]_z,\, [\overline{q}_A]_z[\times][\underline{q}_A]_z,\, [\overline{q}_A]_z[\ \subseteq [\Delta_A/{\sim_z}]^2$$

such that $\widehat{\succsim}_z$ restricted to $C_{[q_A]_z}$ satisfies the five axioms of the Anscombe Aumann Theorem (for finite states) – *weak order*, *vN-M continuity*, *vN-M independence*, *monotonicity* and *non-degeneracy*. It follows that there exists a function $v_z^{q_A} :][\underline{q}_A]_z, [\overline{q}_A]_z[\to \mathbb{R}$ that is unique up to positive affine transformation, and a constant $\sigma_z^{q_A} \in [0, 1]$ that is unique, such that the function $V_z^{q_A} : C_{[q_A]_z} \to \mathbb{R}$ defined by,

$$V_z^{q_A}([p'_A]_z, [q'_A]_z) = (1 - \sigma_z^{q_A}) v_z^{q_A}(p'_A) + \sigma_z^{q_A} v_z^{q_A}(q'_A)$$

represents $(\widehat{\succsim}_z)_{C_{[q_A]_z}}$. That is, for all $([p'_A]_z, [q'_A]_z), ([p''_A]_z, [q''_A]_z) \in C_{[q_A]_z}$,

$$([p'_A]_z, [q'_A]_z) \mathrel{\widehat{\succsim}_z} ([p''_A]_z, [q''_A]_z) \text{ if and only if } V_z^{q_A}([p'_A]_z, [q'_A]_z) \geq V_z^{q_A}([p''_A]_z, [q''_A]_z)$$

Further note that the function v_z satisfies: for all $\lambda \in [0, 1]$, $[p_A]_z, [p'_A]_z \in][\underline{q_A}]_z, [\overline{q_A}]_z[$,

$$v_z(\lambda[p_A]_z \mathbin{\widehat{\oplus}_z} (1 - \lambda)[p'_A]_z) = \lambda v_z([p_A]_z) + (1 - \lambda) v_z([p'_A]_z).$$

It also follows that for any $([p'_A]_z, [q'_A]_z) \in C_{[q_A]_z}$, there exists $[\widehat{q}_A]_z \in][\underline{q}_A]_z, [\overline{q}_A]_z[$ such that $([p'_A]_z, [q'_A]_z) \mathrel{\widehat{\sim}_z} ([\widehat{q}_A]_z, [\widehat{q}_A]_z)$.

Note that $\widehat{\succsim}_z$ restricted to D^* is complete. This follows since, any two degenerate lotteries like $[(z, a), 1]$ and $[(z, a'), 1]$ are comparable (in the sure event), and accordingly

$$([a]_z, [a]_z) \mathrel{\widehat{\succ}_z} ([a']_z, [a']_z) \text{ if } (z, a) \succ (z, a'),$$

or,

$$([a]_z, [a]_z) \mathrel{\widehat{\sim}_z} ([a']_z, [a']_z) \text{ if } (z, a) \sim (z, a').$$ [25]

Now define $O = (\cup_{[q_A]_z \in D} C_{[q_A]_z}) \cup D^*$. We will next show that $\widehat{\succsim}_z$ restricted to O is a weak order. Pick any $([p'_A]_z, [q'_A]_z) \in C_{[q_A]_z}$, $([p''_A]_z, [q''_A]_z) \in C_{[p_A]_z}$. We know that there exists $[\widehat{q}_A]_z, [\widehat{p}_A]_z \in \Delta_A / \sim_z$ such that

$$([p'_A]_z, [q'_A]_z) \mathrel{\widehat{\sim}_z} ([\widehat{q}_A]_z, [\widehat{q}_A]_z) \text{ and } ([p''_A]_z, [q''_A]_z) \mathrel{\widehat{\sim}_z} ([\widehat{p}_A]_z, [\widehat{p}_A]_z).$$

Accordingly, it follows that

$$([p'_A]_z, [q'_A]_z) \mathrel{\widehat{\succ}_z} ([p''_A]_z, [q''_A]_z) \text{ if } ([\widehat{q}_A]_z, [\widehat{q}_A]_z) \mathrel{\widehat{\succ}_z} ([\widehat{p}_A]_z, [\widehat{p}_A]_z),$$

or,

$$([p'_A]_z, [q'_A]_z) \mathrel{\widehat{\sim}_z} ([p''_A]_z, [q''_A]_z) \text{ if } ([\widehat{q}_A]_z, [\widehat{q}_A]_z) \mathrel{\widehat{\sim}_z} ([\widehat{p}_A]_z, [\widehat{p}_A]_z).$$

Hence, $(\widehat{\succsim}_z)_O$ is a weak order.

Now consider any two cubes $C_{[q_A]_z}$ and $C_{[p_A]_z}$ that intersect. Pick $([q'_A]_z, [q'_A]_z), ([q''_A]_z, [q''_A]_z) \in C_{[q_A]_z} \cap C_{[p_A]_z}$, $[q'_A]_z \neq [q''_A]_z$, and recalibrate the function $v_z^{p_A}$ by setting

$$v_z^{p_A}([q'_A]_z) = v_z^{q_A}([q'_A]_z) \text{ and } v_z^{p_A}([q''_A]_z) = v_z^{q_A}([q''_A]_z)$$

[25]Note that $\{[a]_z \in \Delta_A / \sim_z : a \in A\} = \Delta_A / \sim_z$.

Note that by the uniqueness result of the Anscombe Aumann Theorem, the pair $(v_z^{p_A}, \sigma_z^{p_A})$ continues to represent $(\widehat{\succsim}_z)_{C_{[p_A]}}$. Further, $v_z^{p_A} = v_z^{q_A}$ on $][\underline{p}_A]_z, [\overline{p}_A]_z[\cap][\underline{q}_A]_z, [\overline{q}_A]_z[$. Hence it follows that $\sigma_z^{p_A} = \sigma_z^{q_A}$. Next consider $[q_A]_z$, $[p_A]_z$ such that cubes $C_{[q_A]_z}$ and $C_{[p_A]_z}$ do not intersect. Since the set D is connected, $([q_A]_z, [q_A]_z)$ and $([p_A]_z, [p_A]_z)$ can be *linked* by finitely many cubes; that is there are finitely many cubes $C_{[p_A^1]_z}, \ldots, C_{[p_A^m]_z}$, such that $C_{[p_A^1]_z} = C_{[q_A]_z}$, $C_{[p_A^m]_z} = C_{[p_A]_z}$, and each subsequent pairs of $C_{[p_A^j]_z}$'s intersect. Further, we can take $C_{[p_A^j]_z} \cap C_{[p_A^{j-k}]_z} = \emptyset$ for every $k \geq 2$. We can then repeat the above re-calibration exercise over pairs of intersecting cubes in the link. This exercise allows us to define a function v_z on $int(\Delta_A/\sim_z)$, as well as establish $\sigma_z^{q_A} = \sigma_z^{p_A} = \sigma_z$, for all $q_A \neq p_A$, $[q_A]_z, [p_A]_z \in int(\Delta_A/\sim_z)$. Finally, for $[\overline{p}_A]_z = [q_A^{**}]_z$, or $[q_A^*]_z$ define

$$v_z([\overline{p}_A]_z) = \lim_{\lambda \to 1} v_z(\lambda[\overline{p}_A]_z \widehat{\oplus}_z (1-\lambda)[p_A]_z),$$

where $[p_A]_z$ is any element of $int(\Delta_A/\sim_z)$.

We next establish the following claim: for any $([p_A]_z, [q_A]_z) \in \Omega^*$ there exists $([p'_A]_z, [p'_A]_z)$ in D^* such that $([p_A]_z, [q_A]_z) \widehat{\sim}_z ([p'_A]_z, [p'_A]_z)$. To that end, define the function $V_z : \Omega^* \to \mathbb{R}$ by

$$V_z([p_A]_z, [q_A]_z) = (1-\sigma_z)v_z([p_A]_z) + \sigma_z v_z([q_A]_z)$$

where v_z and σ_z are as defined above. For any $[\widehat{q}_A]_z \in int(\Delta_A/\sim_z)$, let

$$J_{\widehat{q}_A} = \{([p_A]_z, [q_A]_z) \in \Omega : V_z([p_A]_z, [q_A]_z) = V_z([\widehat{q}_A]_z, [\widehat{q}_A]_z)\}$$

We claim that for all $([p_A]_z, [q_A]_z)$, $([p'_A]_z, [q'_A]_z) \in J_{\widehat{q}_A}$, $([p_A]_z, [q_A]_z) \widehat{\sim}_z ([p'_A]_z, [q'_A]_z)$. To see this note that, Lemma 5 guarantees that for any $([p_A]_z, [q_A]_z) \in J_{\widehat{q}_A}$, there exists a cube C containing $([p_A]_z, [q_A]_z)$ such that $(\widehat{\succsim}_z)_C$ satisfies the three vN-M axioms of Weak Order, vNM Continuity and Independence on the mixture set $(C, \widehat{\oplus}_z)$. Accordingly $(\widehat{\succsim}_z)_C$ can be represented by a von Neumann-Morgenstern utility function. Consider two such cubes C_1 and C_2 that intersect. Because of the axiom of *consistent revealed cardinality*, it follows that for any $([p_A]_z, [q_A]_z)$, $([p'_A]_z, [q'_A]_z) \in C_1 \cap C_2$,

$$([p_A]_z, [q_A]_z)\ (\widehat{\succsim}_z)_{C_1}\ ([p'_A]_z, [q'_A]_z) \text{ iff } ([p_A]_z, [q_A]_z)\ (\widehat{\succsim}_z)_{C_2}\ ([p'_A]_z, [q'_A]_z).$$

Further note that if V_{C_1} and V_{C_2} are two vN-M utility functions that represent $(\widehat{\succsim}_z)_{C_1}$ and $(\widehat{\succsim}_z)_{C_2}$ respectively, these functions can be re-calibrated (in a manner similar to that used in Step 2) and set equal on $C_1 \cap C_2$.

Now, consider the cube $C_{[\widehat{q}_A]_z}$ around $([\widehat{q}_A]_z, [\widehat{q}_A]_z)$. We have already established above that $(\widehat{\succsim}_z)_{C_{[\widehat{q}_A]_z}}$ is represented by the function V_z. Further, $J_{\widehat{q}_A}$ is connected. Accordingly,

$([\widehat{q}_A]_z, [\widehat{q}_A]_z)$ can be *linked* to any $([p_A]_z, [q_A]_z) \in J_{\widehat{q}_A}$ using a finite number of cubes. On each pair of intersecting cubes $\widehat{\succsim}_z$ must coincide as suggested in the last paragraph. Furthermore the vN-M representations of $\widehat{\succsim}_z$ on these cubes can be re-calibrated and brought in line with V_z. Hence, we may conclude that for all $([p_A]_z, [q_A]_z), ([p'_A]_z, [q'_A]_z) \in J_{\widehat{q}_A}$, $([p_A]_z, [q_A]_z) \widehat{\sim}_z ([p'_A]_z, [q'_A]_z)$.

Note that if $\sigma_z \neq 1$, or if $[q^{**}_A]_z$ and $[q^*_A]_z$ do not exist, then we are done establishing our claim. However, if $\sigma_z = 1$, and either $[q^{**}_A]_z$ or $[q^*_A]_z$ exists then members of the set

$$B = \{([p_A]_z, [q_A]_z) \in \Omega : [p_A]_z = [q^{**}_A]_z \text{ or } [q^*_A]_z\}$$

are not indifferent to any element of D. In this case it is straightforward to verify that for any $([q^{**}_A]_z, [q_A]_z) \in B$, $([q^{**}_A]_z, [q_A]_z) \widehat{\sim}_z ([q^{**}_A]_z, [q^{**}_A]_z)$. Similarly, for any $([q^*_A]_z, [q_A]_z) \in B$, $([q^*_A]_z, [q_A]_z) \widehat{\sim}_z ([q^*_A]_z, [q^*_A]_z)$.

Now consider any $([p_A]_z, [q_A]_z), ([p'_A]_z, [q'_A]_z) \in \Omega^*$. From the argument just made, we know that there exists $([\widehat{q}_A]_z, [\widehat{q}_A]_z), ([\widetilde{q}_A]_z, [\widetilde{q}_A]_z) \in D^*$, such that $([p_A]_z, [q_A]_z) \widehat{\sim}_z ([\widehat{q}_A]_z, [\widehat{q}_A]_z)$ and $([p'_A]_z, [q'_A]_z) \widehat{\sim}_z ([\widetilde{q}_A]_z, [\widetilde{q}_A]_z)$. Hence,

$$([p_A]_z, [q_A]_z) \widehat{\succsim}_z ([p'_A]_z, [q'_A]_z) \text{ iff } ([\widehat{q}_A]_z, [\widehat{q}_A]_z) \widehat{\succsim}_z ([\widetilde{q}_A]_z, [\widetilde{q}_A]_z).$$

Clearly it also follows that,

$$([p_A]_z, [q_A]_z) \widehat{\succsim}_z ([p'_A]_z, [q'_A]_z) \Leftrightarrow V_z(([p_A]_z, [q_A]_z)) \geq V_z(([p'_A]_z, [q'_A]_z)).$$

Note that we may (with an abuse of notation) define the function v_z on Δ_A by simply giving all elements of an equivalence class, say $[p_A]_z$, the value $v_z([p_A]_z)$. It then follows that for all $\lambda \in [0, 1]$, p_A, $q_A \in \Delta_A$,

$$v_z(\lambda p_A \mathbin{\widehat{\oplus}_z} (1 - \lambda) q_A) = \lambda v_z(p_A) + (1 - \lambda) v_z(q_A).$$

The uniqueness statement is simply a re-statement of the essential uniqueness result in the first half of the proof. This then completes the proof for those $z \in Z$ for which $\succ_z \neq \emptyset$.

The proof for those $z \in Z$ for which $\succ_z = \emptyset$ is trivial. Note that for this case $[\Delta_A/\sim_z \times \Delta_A/\sim_z]$ is a singleton. We can take v_z to be any constant function, and σ_z to be any number in [0, 1]. □

We can therefore establish that any procedure-contingent outcome (z, a, p_A) is revealed indifferent to a procedure contingent outcome $(z, \widetilde{a}, \widetilde{a})$ in which the outcome and procedure are the same.

Lemma 7. *For any $p \in \Delta$, and (z, a) in the support of p, the procedure-contingent allocation (z,a,p_A) is revealed indifferent to a procedure-contingent allocation $(z, \widetilde{a}, \widetilde{a}) \in \Delta$ that is unique in the following sense: if $(z, \widehat{a}, \widehat{a})$ is another procedure contingent outcome that is revealed* *indifferent to (z,a,p_A), then $\widetilde{a} \sim_z \widehat{a}$. Further, there exists a function $v_z : \Delta_A \to \mathbb{R}$, and a constant $\sigma_z \in [0,1]$ such that*

$$v_z(\widetilde{a}) = (1-\sigma_z)v_z(a) + \sigma_z v_z(p_A)$$

The function v_z is unique up to positive affine transformation, and the constant σ_z is unique for all z such that $\succ_z \neq \emptyset$.

Proof. The proof is immediate and we omit the details. □

The following corollary then follows.

Corollary 1. Every lottery $p \in \Delta$ has a procedure-adjusted equivalent $\pi(p) \in \Delta$.

The proof of Theorem 2 now follows. We know from the proof of lemma 3 that there exists a function $W : \Delta \to \mathbb{R}$ that represents $\succcurlyeq$, with the property that for any p, $q \in \Delta$,

$$W(\lambda p \oplus (1-\lambda)q) = \lambda W(p) + (1-\lambda)W(q).$$

Consider any $p = [(z_1,a_1),\ \lambda_1;\ .\ .\ .\ ;\ (z_M,a_M),\ \lambda_M]$, $q = [(z'_1,a'_1),\ \lambda'_1;\ .\ .\ .\ ;\ (z'_N,a'_N),\ \lambda'_N] \in \Delta$. Let $\pi(p) = [(z_1,\widetilde{a}_1),\ \lambda_1;\ .\ .\ .\ ;\ (z_M,\widetilde{a}_M),\ \lambda_M]$, $\pi(q) = [(z'_1,\widetilde{a}'_1),\ \lambda'_1;\ .\ .\ .\ ;\ (z'_N,\widetilde{a}'_N),\ \lambda'_N]$ be their procedure-contingent equivalents, then it follows from procedure-contingent consequentialism* that

$$p \succcurlyeq q \Longleftrightarrow W(\lambda_1(z_1,\widetilde{a}_1) \oplus\ .\ .\ .\ \oplus \lambda_M(z_M,\widetilde{a}_M)) \geq W(\lambda'_1(z'_1,\widetilde{a}'_1) \oplus\ .\ .\ .\ \oplus \lambda'_N(z'_N,\widetilde{a}'_N)).$$

That is,

$$p \succcurlyeq q \Longleftrightarrow \textstyle\sum_{m=1}^{M}\lambda_m W(z_m,\widetilde{a}_m) \geq \sum_{n=1}^{N}\lambda_n W(z'_n,\widetilde{a}'_n)$$

For any $(z,a) \in Z \times A$, define, as we did in the proof of Theorem 1, $w(z,a) = W(z,a)$. Therefore,

$$p \succcurlyeq q \Longleftrightarrow \textstyle\sum_{m=1}^{M}\lambda_m w(z_m,\widetilde{a}_m) \geq \sum_{n=1}^{N}\lambda_n w(z'_n,\widetilde{a}'_n)$$

Note that the function $w : Z \times A \to \mathbb{R}$ has the property that for any $z \in Z$, (z,a), $(z,a') \in Z \times A$, and $\lambda \in [0,1]$,

$$w(\lambda(z,a) \oplus (1-\lambda)(z,a')) = \lambda w(z,a) + (1-\lambda)w(z,a').$$

Therefore, for any z, w restricted to the set $\{(z,.) \in \{z\} \times A\}$ is an affine transformation of v_z that we specified in Lemma 7. Hence it follows that:

$$p \succcurlyeq q \Longleftrightarrow \textstyle\sum_{m=1}^{M}\lambda_m\{(1-\sigma_{z_m})w(z_m,a_m) + \sigma_{z_m}w(z_m,p_A)\} \geq \sum_{n=1}^{N}\lambda_n\{(1-\sigma_{z_n})w(z_n,a_n) + \sigma_{z_n}w(z_n,q_A)\}$$

The necessity of the axioms, and the essential uniqueness results are straightforward, and hence the details are omitted.

7.2.5 Proof of Theorem 3

Begin with a pair $(w, (\sigma_z)_{z\in Z})$ that represents $\succcurlyeq$ in the sense of Theorem 2. Consider any $z,\, z' \in Z$ with $\succ_z,\, \succ_{z'} \neq \emptyset$. There are two cases to consider. First suppose that there exists $\widehat{a} \in A$ such that

$$\{a \in A : a \sim_z \widehat{a}\} \neq \{a \in A : a \sim_{z'} \widehat{a}\}$$

In this case, there exists $a \in [\widehat{a}]_{z'}$, $a' \in [\widehat{a}]_z$ satisfying $a \succ_z \widehat{a}$ and $a' \succ_{z'} \widehat{a}$.

It is straightforward to verify by applying *continuity* that there exists $\widetilde{a} \in A$ such that

$$((z, \widehat{a}),\, (z, \widetilde{a})) = ((z', \widehat{a}),\, (z', \widetilde{a})).$$

Now define p, q as follows:

$$p = [(z, \widehat{a}), \tfrac{1}{2};\, (z', \widetilde{a}), \tfrac{1}{2}] \text{ and } q = [(z, \widetilde{a}), \tfrac{1}{2};\, (z', \widehat{a}), \tfrac{1}{2}].$$

It follows from the axiom of *symmetry* that $p \sim q$. Applying the representation in Theorem 2 gives us that $\sigma_z = \sigma_{z'}$.

On the other hand if

$$\{a \in A : a \sim_z \widehat{a}\} = \{a \in A : a \sim_{z'} \widehat{a}\}$$

for all $\widehat{a} \in A$, then by the *contingent values* assumption, there exists $\succcurlyeq_{z''}$, with $\succ_{z''} \neq \emptyset$ for which there exists $\widehat{a} \in A$ such that

$$\{a \in A : a \sim_z \widehat{a}\} \neq \{a \in A : a \sim_{z''} \widehat{a}\}$$

and,

$$\{a \in A : a \sim_{z'} \widehat{a}\} \neq \{a \in A : a \sim_{z''} \widehat{a}\}.$$

Based on the argument in the last paragraph, we can then conclude that $\sigma_z = \sigma_{z''}$, and $\sigma_{z'} = \sigma_{z''}$, and hence $\sigma_z = \sigma_{z'}$.

7.3 Proof of Proposition 2

Recall that the representative voter is pivotal with probability $\lambda = \frac{1}{n}$, and γ denotes the probability that alternative 1 is the outcome when she is not pivotal. Then the probability distributions over final allocations generated by the representative voter choosing 1 and 2 are respectively,

$$p^1 = [(z^1, a^1),\, \lambda + (1-\lambda)\gamma;\, (z^2, a^2),\, 1 - \lambda - (1-\lambda)\gamma],$$

$$p^2 = [(z^1, a^1),\ (1-\lambda)\gamma;\ (z^2, a^2),\ 1 - (1-\lambda)\gamma]$$

Under out representation these two lotteries are evaluated as:

$$U(p^1) = u_H + \frac{v_L}{2} - (\lambda + (1-\lambda)\gamma)[u_H - u_L - \frac{1}{2}(v_H - v_L)] + \frac{1}{2}[\varphi(\lambda + (1-\lambda)\gamma)v_H + (1 - \varphi(\lambda + (1-\lambda)\gamma))v_L]$$

and,

$$U(p^2) = u_H + \frac{v_L}{2} - (1-\lambda)\gamma[u_H - u_L - \frac{1}{2}(v_H - v_L)] + \frac{1}{2}[\varphi((1-\lambda)\gamma)v_H + (1 - \varphi((1-\lambda)\gamma))v_L]$$

Subtracting the two gives,

$$U(p^2) - U(p^1) = \lambda[u_H - u_L - \frac{1}{2}(v_H - v_L)] - \frac{1}{2}(v_H - v_L)[\varphi(\lambda + (1-\lambda)\gamma) - \varphi((1-\lambda)\gamma)]$$

Accordingly,

$$U(p^2) - U(p^1) \geq 0 \Leftrightarrow g(\lambda) = \lambda(2\nu - 1) - (\varphi(\lambda + (1-\lambda)\gamma) - \varphi((1-\lambda)\gamma)) \geq 0$$

Now suppose everyone other than RV votes for alternative 1; i.e., $\gamma = 1$. Then,

$$g(\lambda) = \lambda(2\nu - 1) - (1 - \varphi(1-\lambda))$$

and, for $\lambda \in (0, 1-\overline{\lambda})$,

$$g'(\lambda) = 2\nu - 1 - \varphi'(1-\lambda)$$

Let $\lambda' = \min\{1-\overline{\lambda}, \underline{\lambda}\}$. Then for all $\lambda \in (0, \lambda')$, $g'(\lambda) < 0$. Further, $g(0) = 0$. Hence, $g(\lambda) < 0$ for all $\lambda \in (0, \lambda')$. Let $\overline{n}$ be any integer greater than $\frac{1}{\lambda'}$. Then,for all $n > \overline{n}$, everyone voting for alternative 2 is a Nash equilibrium.

Now consider the case when everyone other than RV votes for alternative 2. That is $\gamma = 0$. Then,

$$g(\lambda) = \lambda(2\nu - 1) - \varphi(\lambda) = \lambda[2\nu - 1 - \frac{\varphi(\lambda)}{\lambda}]$$

Note that, for $\lambda < \lambda'$, $\varphi'(\lambda) > 2\nu - 1$, and since φ is concave over this range, $\frac{\varphi(\lambda)}{\lambda} > \varphi'(\lambda)$. Accordingly, for $\lambda < \lambda'$, $g(\lambda) < 0$, and everyone voting for alternative 2 can not be a Nash equilibrium. Hence, for all $n \geq \overline{n}$, everyone voting for alternative 2 is the unique symmetric Nash equilibrium (in pure strategies).

Further, note that when $\gamma = 0$, $g(1) = 2\nu > 0$. By continuity of g, there exists an interval $(\lambda_1, 1]$, such that for all $\lambda \in (\lambda_1, 1]$, $g(\lambda) > 0$, and accordingly everyone voting for alternative 2 is a Nash equilibrium.

Finally, note that when $\gamma = 1$, $g(1) = 2\nu - 2 > 0$. Once again by the continuity of g, there exists an interval $(\lambda_2, 1]$, such that for all $\lambda \in (\lambda_2, 1]$, $g(\lambda) > 0$, and accordingly everyone voting for alternative 1 is not a Nash equilibrium. Let, $\lambda'' = \max\{\lambda_1, \lambda_2\}$, and $\underline{n}$ be any integer less than $\frac{1}{n''}$. It follows that for all $n \leq \underline{n}$, everyone voting for alternative 2 is the unique symmetric Nash equilibrium (in pure strategies). $\square$

8 References

1. Arrow, Kenneth, *Social Choice and Individual Values*, John Wiley and Sons, 1966.

2. Bies, R.J., T.M. Tripp, M.A. Neale, Procedural Fairness and Profit Seeking: the Perceived Legitimacy of Market Exploitation, *Journal of Behavioral Decision-Making*, 6, 243 - 256, 1993.

3. Bolton, Gary. A Comparative Model of Bargaining: Effect Theory and Evidence, *American Economic Review*, 81, 1096 - 1136, 1991.

4. Bolton, Gary E., and Axel Ockenfels, ERC: A Theory of Equity, Reciprocity, and Competition, *American Economic Review*, 90, 166 - 193, 2000.

5. Bolton, Gary E., Jordi Brandts, and Axel Ockenfels: Fair procedures: Evidence from Games Involving Lotteries, *Economic Journal*, 115, 1054 - 1076, 2005.

6. Borah, Abhinash, 'Values Voters' in Large Elections, *Mimeo*, 2009.

7. Broome, John, Equity in Risk Bearing, *Operations Research*, 30, 412 - 414, 1982.

8. —— Uncertainty and Fairness, *Economic Journal*, 94, 624 - 632, 1984.

9. Charness, Gary, and Matthew Rabin, Understanding Social Preferences with Simple Tests, *Quarterly Journal of Economics*, 117, 817 - 869, 2002.

10. Chew, Soo Hong and Larry G. Epstein, A Unifying Approach to Axiomatic Non-expected Utility Theory, *Journal of Economic Theory*, 49, 207 - 40, 1989.

11. Chew, Soo Hong, Larry G. Epstein and Peter P. Walker, A Unifying Approach to Axiomatic Non-expected Utility Theory: Correction and Comment, *Journal of Economic Theory*, 59, 183 - 188, 1993.

12. Cole, Harold, George J. Mailath, and Andrew Postlewaite, Social Norms, Saving Behavior, and Growth, *Journal of Political Economy*, 100, 1092-1125, 1992.

13. Cooper, David, and John Kagel, Other-Regarding Preferences : A Selective Study of Experimental Results, *Mimeo*, 2009.

14. Diamond, Peter A., Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparison of Utility: Comment, *Journal of Political Economy*, 75: 5, 765-766, 1967.

15. Dillenberger, David, Sadowski, Philipp; Ashamed to be Selfish, *mimeo*, 2008.

16. Epstein, Larry G., and Uzi Segal, Quadratic Social Welfare Functions, *Journal of Political Economy*, 100, 691-712, 1992.

17. Feddersen, Timothy, Sean Gailmard and Alvaro Sandroni, Moral Bias in Large Elections: Theory and Experimental Evidence, *American Political Science Review*, 103, 175-192, 2009.

18. Fehr, Ernst and Urs Fischbacher, Why Social Preferences Matter - The Impact of Nonselfish Motives on Competition, Cooperation, and Incentives, *Economic Journal*, 112, 1-33, 2002.

19. Fehr, Ernst and Simon Gachter, Fairness and Retaliation: The Economics of Reciprocity, *Journal of Economic Perspectives*, 14(3), 159-181, 2000.

20. Fehr, Ernst and Klaus M. Schmidt, A Theory of Fairness, Competition and Cooperation, *The Quarterly Journal of Economics*, 114, 817 - 868, 1999.

21. Fishburn, Peter, *The Foundations of Expected Utility.* Dordrecht:Reidel, 1982.

22. Frey, B.S., W.W. Pommerehne, On the Fairness of Pricing - An Empirical Survey Among the General Population. *Journal of Economic Behavior and Organization*, 20, 295 - 307, 1993.

23. Ghirardato, Paolo and Fabio Maccheroni, Risk, Ambiguity and the Separation of Utility and Beliefs, *Mathematics of Operations Research*, 26, 864-890, 2001.

24. Ghirardato, Paolo; Maccheroni, Fabio; Marinacci, Massimo; Siniscalchi, Marciano, A Subjective Spin on Roulette Wheels, *Econometrica*, 71, 1897-1908, 2003.

25. Gilboa, Itzhak, and David Schmeidler, A Cognitive Model of Individual Well-Being, *Social Choice and Welfare*, 18, 269-288, 2001.

26. Grant, Simon, Atushi Kaji, and Ben Polak, Many Good Choice Axioms: When Can Many-Good Lotteries be Treated as Money Lotteries, *Journal of Economic Theory*, 56, 313 - 337, 1992.

27. Gul, Faruk, A Theory of Disappointment in Decision Making Under Uncertainty, *Econometrica*, 59, 667 - 86, 1991.

28. Gul Faruk, and Wolfgang Pesendorfer, The Canonical Type Space for Interdependent Preferences, *mimeo, Princeton University*, 2005.

29. Gul, Faruk, and Wolfgang Pesendorfer, The Case for Mindless Economics, in *The Foundations of Positive and Normative Economics*, Ed. Andrew Caplin and Andrew Schotter, New York: Oxford University Press, 2008.

30. Hammond, Peter, Consequentialist Foundations for Expected Utility, *Theory and Decisions*, 25, 25 - 78, 1988.

31. Harsanyi, John C., Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility, *Journal of Political Economy.* 63,309-321, 1955.

32. —— Nonlinear Social Welfare Functions: Do Welfare Economists Have a Special Exemption from Bayesian Rationality?, *Theory and Decisions*, 6, 311-32, 1975.

33. —— Bayesian Decision Theory and Utilitarian Ethics, *American Economic Review*, 68, 223 - 228, 1978.

34. Kahneman, D., J.L. Knetsch, R. Thaler, Fairness as a Constraint on Profit Seeking: Entitlements in the Market. *American Economic Review*, 76, 728 - 741, 1986.

35. Kahneman, Daniel and Amos Tversky, Prospect Theory: An Analysis of Decision Under Risk, *Econometrica*, 47, 263 - 291, 1979.

36. Karni, Edi, and Zvi Safra, Individual Sense of Justice: a Utility Representation, *Econometrica*, 70, 263-284, 2002.

37. ——- Moral Sentiments and Social Choice, *Social Choice and Welfare*, 30,427-446, 2008.

38. Keeney, Ralph, Equity and Public Risk, *Operations Research*, 28, 527 - 34, 1980.

39. Kircher, Philipp; Ludwig, Sandra and Sandroni, Alvaro, Procedural Fairness: A Note on the Difference Between Social and Private Goods, *mimeo*, 2010.

40. Krawczyk, Michal and Fabrice Le Lec, Social Decisions under Risk: Evidence from the Probabilistic Dictator Game, *Mimeo*, 2008.

41. Kreps, David M., *Notes on the Theory of Choice*, Westview Press / Boulder and London, 1988.

42. Levine, David, Modelling Altruism and Spitefulness in Game Experiments, *Review of Economic Dynamics*, 1, 593 - 622, 1998.

43. Li, Jing, The Power of Convention: A Theory of Social Preference, *Journal of Economic Behavior and Organization*, 65, 489 - 505, 2008.

44. Maccheroni, Fabio, Massimo Marinacci and Aldo Rustichini, Social Decision Theory: Choosing Within and Between Groups, mimeo, 2008.

45. Machina, Mark, J., Dynamic Consistency and Non-expected Utility Models of Choice Under Uncertainty, *Journal of Economic Literature*, 27,1622-1668, 1989.

46. Neilson, William S., Axiomatic Reference-Dependence in Behavior Toward Others and Toward Risk, *Economic Theory*, 28, 681-692, 2006.

47. Ok, Efe A., and Levent Kockesen, Negatively Interdependent Preferences, *Social Choice and Welfare*, 17, 533-558, 2000.

48. Postlewaite, Andrew, The Social Basis of Interdependent Preferences, *European Economic Review*, 42, 779-800, 1998.

49. Quiggin, John, A Theory of Anticipated Utility, *Journal of Economic Behavior and Organization*, 3, 323 - 43, 1982.

50. Rabin, M. (1993). Incorporating Fairness into Game Theory and Economics, *American Economic Review*, 83, 1281 - 302, 2005.

51. Rohde, Kirsten, A Preference Foundation for Fehr and Schmidt's Model of Inequity Aversion, *Social Choice and Welfare*, forthcoming, 2009.

52. Saito, Kota, Social Preference Under Uncertainty: Equality of Opportunity vs. Equality of Outcome, mimeo, 2008.

53. Sandbu, Martin E., Axiomatic Foundations for Fairness-motivated Preferences, *Social Choice and Welfare*, 31, 589 - 619, 2008.

54. Segal, Uzi, and Joel Sobel, Tit for tat: Foundations of Preferences for Reciprocity in Strategic Settings, *Journal of Economic Theory*, 136, 197-216, 2007.

55. Sen, Amartya, Rationality and Uncertainty, *Theory and Decisions*, 18, 109 - 27, 1985.

56. —— Maximisation and the Act of Choice. *Econometrica* 65, 745 - 779, 1997.

57. Sobel, Joel, Interdependent Preferences and Reciprocity, *Journal of Economic Literature*, 43, 392 - 436, 2005.

58. Starmer, Chris, Developments in Non-expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk, *Journal of Economic Literature*, 38, 332-382, 2000.

59. Strotz, Robert, How Income Ought to be Distributed: A Paradox in Distributive Ethics, *Journal of Political Economy*, 66, 189 - 205, 1958.

60. —– How Income Ought to be Distributed: Paradox Regained, *Journal of Political Economy*, 69, 271 - 78, 1961.