# Babylonian Pythagoras' theorem, the early history of Zero and a polemic on the study of the History of Science

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*A difference between fancy and fact is that fancies may be as you please but facts are as the universe pleases.* Robert Kaplan, The Nothing that is – A natural history of zero.

## 1 Introduction

An irksome feature in the study of the history of science, mathematics, and society formation is encountering 'Eurocentrism' at practically every level. The notion that everything 'civilised' originated in Europe is an enterprise which began around two hundred years ago, at a time when the world was divided into the 'dark' continents and their 'enlightened' colonial masters. Besides strengthening the view that the 'dark' continents were indeed pitch dark, Eurocentrism was a reflex of the colonizers quest for legitimacy as the font of all things civilised. Many sociologists find in this Eurocentrism the seed which later bore the bitter fruit of Hitler's Aryan supremacy theory. Lately, however historians cutting across the North-South and East-West divides have come to realize the folly of Eurocentrism and the ways it has hindered the development and a critical study of history, be it of science or otherwise[1].

Unfortunately, some of us in India have not learned from these mistakes, and stubbornly continue to engage in an 'Indocentric' view of science, mathematics, society, language, etc. Thus we are told that "Sanskrit is the mother of all languages", "Indus valley civilisation is Vedic in origin", "India gave the world zero", or more ludicrously as Mukherjee [1991] writes "the mathematical conception of zero ... was also present in the spiritual form from

[1] Bernal [1991] gives a detailed account of this fabrication of Eurocentrism.

17000 years back in India". The immediate need for this myth making may be political, but as with Eurocentrism, these new myths will undoubtedly retard any serious study of our heritage.

This is not to suggest that important and path breaking developments in science and mathematics did not take place in ancient India, but claiming exclusive ownership rights or dismissing and trivialising work done in other civilisations is not conducive to a proper understanding of our common human heritage.

In this article we elaborate on two topics. We first discuss the tablets from the Babylonian civilisation which deal with the Pythagoras' theorem and next we discuss the origin and history of zero. We would like to place this article in the context of Professor Amartya Datta's article in Resonance [April 2002] where discussing on the Pythagoras' theorem found in the Sulbasutra he writes "Pythagoras theorem was known in other civilizations like the Babylonian, but the emphasis there was on the numerical and not so much on the proper geometric aspect, ..."; also regarding zero he writes " India gave to the world a priceless gift – the decimal system ... (it) derives its power mainly from two key strokes of genius: the concept of place-value and the notion of zero as a digit".

Most mathematics, e.g. Pythagoras' theorem and other geometric and algorithmic calculations, stem from utilitarian reasons, be they constructing religious altars, measuring lands, calculating the positions of stars and planets for astrological and astronomical purposes or constructing calendars. Thus it would not be a heresy to say that quite a few mathematical methods may have developed independently in many civilisations. Indeed, all available evidence seem to suggest against any monogenesis theory.

## 2 By the rivers of Babylon

The Babylonian civilization of Mesopotamia dates to around 2000 BCE and was located in the region between the two rivers Tigris and Euphrates. Prior to this the Sumerian civilization flourished here from around 3500 BCE. During this civilization cities were built and administration mechanism, legal systems, irrigation channels and even a postal service were developed. The Babylonians invaded Mesopotamia around 2000 BCE and after defeating the Sumerians established their capital at Babylon around 1900 BCE.

This Sumerian civilization introduced writing and also had a method of counting, which was subsequently further developed by the Babylonians. The cuneiform script was used to inscribe on wet clay, which was baked in the sun (incidentally the cuneiform fonts for LaTeX are also available nowadays!). This script was deciphered by Sir Henry Creswicke Rawlinson. The famous Code of Hammurabi dates from the Babylonian period and lists out around 280 laws for governance.

From the excavations at the Babylonian sites, many clay tablets with inscriptions have been found. Of these we will discuss some which are connected with mathematics. The number system used by the Babylonians was a positional system with a base of 60. There are various reasons put forth by historians for the use of this sexagesimal system. They had also divided the day into 24 hours, each hour having 60 minutes and each minute having 60 seconds. This sexagesimal system for measurement of time survives even now[2].

We restrict ourselves to the tablets which are related to the Pythagoras' theorem, in particular the tablets YBC 7289 (from the Yale University collection), Plimpton 322 (from the Columbia University collection), the more recently excavated Susa tablet (discovered at Shoosh, Iran) and Tell Dhibayi tablet (discovered near Baghdad, Iraq). All these tablets are dated between 1900 BCE and 1600 BCE. However, before we proceed further, here is a translation of a problem and its solution inscribed on a Babylonian tablet kept at the British Museum, London.

> *4 is the length and 5 the diagonal. What is the breadth?*
> *Its size is not known.*
> *4 times 4 is 16,*
> *5 times 5 is 25.*
> *You take 16 from 25 and there remains 9.*
> *What times what shall I take in order to get 9?*
> *3 times 3 is 9,*
> *3 is the breadth.*

Of course, it does not need any elucidation that the geometric aspect of the Pythagorean triplet is being discussed here.

For further confirmation of the fact that the Babylonians were quite familiar with the geometry behind Pythagorean triples we look at the tablet YBC 7289.

[2]For a comprehensive discussion of the mathematics found in the Babylonian tablets see Ifrah [2000].

Figure 1: Diagram on the tablet YBC 7289.

The diagram on this tablet is shown in Figure 1. A note of explanation is needed for this diagram. On one side of the square is inscribed 30, while on the horizontal diagonal are inscribed two numbers 1 24 51 10 and 42 25 35. Recalling that the Babylonians had a sexagesimal system[3] and assuming that the space between the digits represent position values of the (sexagesimal) digits and also assuming that the first number is $1.\bar{2}4\ \bar{5}1\ \bar{1}0$ (mod 60) and the second number is $\bar{4}2.\bar{2}5\ \bar{3}5\ \bar{4}2\ \bar{2}5\ \bar{3}5$ (mod 60), then translating these numbers gives $1.\bar{2}4\ \bar{5}1\ \bar{1}0$ (mod 60) $= 1.41421296$ (mod 10) and $\bar{4}2.\bar{2}5\ \bar{3}5\ \bar{4}2\ \bar{2}5\ \bar{3}5$ (mod 60) $=$ 42.4263888 (mod 10) $= 30 \times 1.41421296$ (mod 10). Note that $\sqrt{2} \approx 1.41421356$ (mod 10).

Although this confirms that the Babylonians were familiar with the geometry behind Pythagorean triplets, it raises the question as to how did they compute $\sqrt{2}$ so accurately.

Two among the many possibilities postulated are (i) the Babylonians were familiar with the Heron's method and (ii) they used an algorithmic method based on calculating squares. Heron's method is as follows:- take a guess $x$ for $\sqrt{2}$ and let $y = x^2 - 2$ denote the error. Now consider $(x - \frac{y}{2x})^2 = x^2 - y + (\frac{y}{2x})^2 = 2 + (\frac{y}{2x})^2$; for small $y$, $(\frac{y}{2x})^2$ is even smaller, thus $(x - \frac{y}{2x})^2$ is a better approximation of 2. By this method, staring with $x = 1$ in only two steps one obtains the approximation $1.\bar{2}4\ \bar{5}1\ \bar{1}0$ (mod 10). The algorithmic method is as follows:- take two guesses $x$ and $y$, one larger and the other

[3]Notation:- $0.\bar{2}4\ \bar{5}$ (mod 60) denotes the number $(24/60) + (5/3600)$ (mod 10), while $0.\bar{2}\ \bar{4}5$ (mod 60)denotes $(2/60) + (45/3600)$ $(mod 10)$. In the next section we discuss how the Babylonians used 0 as a place marker.

smaller than $\sqrt{2}$; if the number $(\frac{x+y}{2})^2$ is larger (respectively, smaller) than 2 then iterate the process with $\frac{x+y}{2}$ and $\min\{x, y\}$ (respectively, $\max\{x, y\}$ instead of the original guesses $x$ and $y$ – continue till the desired accuracy. By this method, taking $x = 1$ and $y = 2$, one obtains the approximation $1.\bar{24}\ \bar{51}\ \bar{10}$ for $\sqrt{2}$ in 19 steps[4].

The Plimpton 322 tablet contains an array of numbers (sexagesimal) divided into 15 rows and 4 columns. The last column just gives the row numbers, while the numbers in the first three columns are very interesting. Taking $a$, $b$ and $c$ to be the numbers in the first, second and third columns respectively, we see that they satisfy the relation $a = \frac{c^2}{c^2-b^2}$ and that $c^2 - b^2$ is a perfect square[5]. While some historians postulate a relation between the secant function and the first column, the mathematician Zeeman observes that taking $b = m^2 - n^2$, $h = 2mn$ and $c = m^2 + n^2$ to construct the Pythagorean triple $(b, h, c)$, Plimpton 322 lists 15 of the 16 triples such that $n \leq 60$, $\pi/6 \leq \theta \leq \pi/4$ and $\tan\theta = h^2/b^2$ has a finite sexagesimal expansion.

The geometric problem given in the Susa tablet is to obtain the radius of the circumscribed circle on an isosceles triangle with sides 50, 50 and 60. A simple algebraic calculation employing Pythagoras' theorem immediately yields that the radius is $\bar{31}.\bar{15}$(mod 60).

The question on the Tell Dhibayi tablet is to obtain the sides of the rectangle whose area is $0.\bar{45}$ (mod 60) and diagonal is $\bar{1}.\bar{15}$ (mod 60). Setting up a quadratic equation one immediately obtains that the sides have to be of lengths 1 and $0.\bar{45}$ (mod 60). However, the solution given on the tablet merits a discussion. Using a hybrid of modern notation and sexagesimal arithmetic, let $x$ and $y$ denote the lengths of the two sides. The steps are (i) $2xy = \bar{1}.\bar{30}$, (ii) $x^2 + y^2 - 2xy = (\bar{1}.\bar{15})^2 - \bar{1}.\bar{30} = 0.\bar{3}\ \bar{45}$, (iii) $x - y = \sqrt{0.\bar{3}\ \bar{45}} = 0.\bar{15}$, (iv) $(x - y)/2 = 0.\bar{7}\ \bar{30}$, (v) $((x - y)/2)^2 = 0.\bar{0}\ \bar{56}\ \bar{15}$, (vi) $((x - y)/2)^2 + (xy) = 0.\bar{0}\ \bar{56}\ \bar{15} + 0.\bar{45} = 0.\bar{45}\ \bar{56}\ \bar{15}$, (vii) $(x + y)/2 = 0.\bar{52}\ \bar{30}$, (viii) $x = ((x+y)/2)+((x-y)/2) = 1$, (ix) $y = ((x+y)/2)-((x-y)/2) = 0.\bar{45}$. Hence the dimensions of the rectangle are 1 and $0.\bar{45}$.

This clearly demonstrates that the Babylonians were quite conversant with both the geometric and arithmetic properties of the Pythagorean triples. Of course, whether they had a proof of the Pythagras' theorem or, more

[4]The Babylonians were not afraid of taking squares as has been attested by the fact that many tablets have been discovered of squares. In fact their multiplication was based on the formula $xy = \frac{(x+y)^2-(x-y)^2}{4}$.

[5]Note that Ifrah [2000] makes a mistake in obtaining these relations.

importantly, whether there was any notion of proof in Babylonian times is a different question and we do not discuss it here.

## 3 Nada sera como antes

In this section we discuss the history of the decimal system and the notion of zero as a place-value as well as a digit. We will see that the decimal system, together with zero as a place value were known, not only in post-Vedic India, but also earlier in the Babylonian system and in the contemporary Mayan civilization. In all likelihood, there could have been independent development of this system in all these places. However the notion of zero as an integer is quite clearly Indian in origin and Brahmagupta appears to be the first person to consider zero as an integer.

The importance of zero as a place marker can be understood from the fact that if we did not have it, then there would be no way of distinguishing between the numbers 201 and 21 (in any base). However, the Babylonians did not have it for over 1000 years and from all available evidence, it appears that they did not have any problems with the confusion that must have reigned.

A hybrid of the alphabetic system and the positional number systems appears in Babylonian tablets from around 2000 BCE – this system called the Mari system (named after the place where the tablets were found) was later refined to a positional system by the Babylonians by around 700 BCE. However, the system had its limitations, e.g., 36 had to be represented by three tens juxtaposed together followed by six 1's juxtaposed together.

In a tablet, dating from around 700 BCE, excavated at Kish, Iraq, the first evidence of using zero is found. The scribe Bêl-bân-aplu, who wrote the tablet used three hooks to denote zero. It is from tablets written between the sixth and third century BCE we find that the Babylonians used a variety of symbols to denote zero in a number. They used a single wedge or two wedge symbols to denote zero. Thus by around 400 BCE we find instances like 2“1 to distinguish it from 21. However, the use of zero in the units place of a number (e.g., 21“) or as the 'last' digits of a number is surprisingly absent. Probably one relied on the context to understand the missing zeros from the 'end' of a number[6].

[6]Note that if someone says that she spent *sarhe teen* (Hindi) to buy a car, then we'd immediately realise that she spent Rs. 3.5 lakhs for the car, on the other hand, if she

Significantly, a tablet found in Susa reads "*20 minus 20 comes to … you see?*" [Ifrah 2000] – this allusion to zero is quite different from the use of zero as a symbol in expressing numbers. However, this notion of zero as a number is not developed and neither is there anything which would suggest that this was not just a passing fancy of a scribe.

The Greek mathematicians did not have a positional number system and as such they did not need zero as a place-value symbol. Their number system, called the Attic system, dating to around 500 BCE had special symbols for each of the numbers 1, 5, 10, 50, 100, 500, 1000, 5000, 10000 and 50000. Thus the number 3202 was expressed as $XXXHHII$. The connection between the Attic system and the Roman numerals is immediate.

However, the Greek astronomers, who needed large numbers and were clearly hampered by the fact that large numbers were too cumbersome to be expressed in the Attic system, used the Babylonian sexagesimal system. Thus in Ptolemy's treatise on astronomy, *Almagest*, written around 130 AD, we find the use of a sexagesimal number system together with the symbol 0 to denote the 'empty place' in a number. This is probably the first occurrence of the symbol 0, although other symbols for zero were also used by the Greek astronomers during this period.

The oldest known writing in the Indian sub-continent is from the Indus valley civilization (2500 – 1500 BCE), however until the script is deciphered weare in no position to determine its relationship with the Indian number systems. It is from the Brahmi edicts during Ashoka's reign (273 – 235 BCE) that the earliest numerical notation is observed, that too rather fragmentary (only the numbers 1,2,4 and 6). Some more numbers appear in the 2nd century BCE during the Shunga and Magadha dynasties. However, by the first and second century AD, more complete number systems have been found in many places in India. There is some debate as to the origin of the Brahmi numerals and whether there were any influences from outside, however Ifrah [2000] taking into account the "universal constants of both psychology and paleography" demonstrates that the "Brahmi numerals were autochthonous, that is to say, their formation was not due to any outside influence"[7].

Since there had been a lot of influence of Greek astronomy in Indian astronomy and astrology (e.g., Indian names of zodiacal signs and various astronomical terms are Greek in origin, as well as the ratio of the length

spent *sarhe teen* for buying *samosas* then she bought only Rs 3.50 worth of *samosas*.

[7]This argument being rather technical we do not reproduce it here.

of the longest day to that of the shortest day which is given as 3:2 – a ratio more true of Babylonia than anywhere in India) many historians have concluded that the place-value system of the Indian numerals have their origin in the Babylonian system (via the Greek astronomical texts). Although this hypothesis has not been ruled out, Ifrah [2000] raises a serious objection to it.

In the Surya Siddhanta (600 AD) we see numbers like 488,203 and 232,238 expressed, by just their digits spelt out one after the other, without taking recourse to the magnitude of the digit in the number. Zero finds its place in the place-value system, although not in the 'circular' symbol we know of it now. In fact, Lokavibhaga (458 AD), the Jain cosmological text is the earliest known Indian text to have used the place-value system together with zero. Aryabhata (500 AD) constructed an ingenious method of recording numbers based on consonants and vowels of the Devanagari alphabet. Moreover his method of calculating square and cube roots indicate a use of the place value system.

It is from here that the Indian mathematicians take the profound step to consider zero as an integer and carry on mathematical operations with it. While Varahamihira (575 AD) mentions the use of zero in mathematical operations, Brahmagupta (628 AD) elaborates on these operations in *Brahmasphutasiddhanta*. Brahmagupta defines zero as that quantity which is obtained when a number is subtracted from itself and he goes on to elaborate on the procedure of addition, subtraction, multiplication and division with zero. Thus he writes

> *"The sum of zero and a negative number is negative, the sum of a positive number and zero is positive, the sum of zero and zero is zero.*
> *A negative number subtracted from zero is positive, a positive number subtracted from zero is negative, zero subtracted from a negative number is negative, zero subtracted from a positive number is positive, zero subtracted from zero is zero."*

However, division by zero was problematic

> *"Positive or negative numbers when divided by zero is a fraction the zero as denominator. Zero divided by negative or positive numbers is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator. Zero divided by zero is zero."*

Mahavira (830 AD) in his work *Ganitasangraha* elaborates on Brahmagupta's work and realising the obvious inadequacy of Brahmagupta's explanation about division by zero states that *"A number remains unchanged when divided by zero"*. Later Bhaskara (1150 AD) tries to correct this mistake of Mahavira in his book *Lilavati*.

> *"A quantity divided by zero becomes a fraction the denominator of which is zero. This fraction is termed an infinite quantity. In this quantity consisting of that which has zero for its divisor, there is no alteration, though many may be inserted or extracted; as no change takes place in the infinite and immutable God when worlds are created or destroyed, though numerous orders of beings are absorbed or put forth."*

Here though we have a glimpse of the use of the mathematical notion on infinity, Bhaskara still couldn't arrive at the modern concept that division by zero is not allowed[8]. This would continue to plague Arab and subsequently European mathematicians who learnt from various translations of these Indian works the notion of zero as an integer[9].

Finally, to complete this brief history of zero, and reinforcing our argument against monogenetic origins of ideas in the sciences of ancient times, it should be noted that the Mayan civilization by 665 AD had a base 20 place-value number system with a symbol and use of zero. It is also recorded that they had used zero prior to having a place-value number system. The calendar system of the Mayans and their development of astronomy would seem to suggest that the Mayans must have been quite capable in their mathematics too. Even the wildest Euro/Indo centrist would have to admit that the Mayan civilization developed independently, without any influence from the 'old world'.

# 4 References


[1] Bernal, M., [1991]. *Black Athena*, Vintage, London.


[8]Many suggest that this could be because of the rejection by Indian mathematicians of this period of the Greek method of proof, which nowadays every school child knows as *reductio ad absurdum*. Indeed, assuming Bhaskara's use of division by zero to be correct we obtain that $m/0 = \infty = n/0$; hence $m = 0.\infty = n$ and so $m = n$ – a contradiction.

[9]In a sense, when L'Hôpital looks at $y/x$ with $x \to 0$ he is visiting this old conundrum

[2] Casselman, B, YBC 7289 – photograph from the Yale Babylonian collection, *http://www.math.ubc.ca/people/faculty/cass/Euclid/ybc/ybc.html.*
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