Challenges and frustrations of being a mathematician[1]

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It is a great honour and pleasure for me to have had this opportunity of addressing this distinguished gathering. I express my gratitude to the members of the Indian Mathematical Society for having elected me to the office of the President, in the centenary year of the Society. I am also thankful to the local organizers for undertaking a task of such magnitude towards a successful and productive conference.

As mathematicians, we belong to a minority. Most people are turned off by mathematics. Very few have some liking for the subject and even a smaller number choose it as a profession. In this talk I have tried to give way to some feelings about the nature of our profession and the problems that we face. We all agree that it is a pleasure to do mathematics and so I dwell mostly on things that tend to obstruct that pleasure. The views expressed here have evolved largely out of my personal experiences and perspectives. It is hoped that they might induce a discussion which might result in further inputs and exchange of ideas. The tone of the writing is definitive, but since it is meant to generate a debate, it is justified.

We are mathematicians by choice. We chose the profession because we love the subject. Reading and assimilating deep results of masters and then solving some of our own small problems brings us pleasure to which nothing else compares much. Yet we live in a world populated mostly be non-mathematicians. We must survive and thrive in their midst. This brings forth its own challenges and frustrations.

Our professional activity is divided mainly into teaching and research, except for some administrative duties. A lot has been said about mathematics education and I will confine myself to a few comments. It is a fact that most

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of us like to do our own thing. We enjoy teaching if it is a course of our choice and the class consists of a few eager, motivated, well-behaved students. That is only a dream. Often we must teach large classes of uninterested students who are there only for completing the requirements. But in spite of all this we must strive to teach, giving it our best and at the same time maintaining the standard of our subject. Compromises have no place here. We believe in teaching in a certain way and it can be fine-tuned depending upon the reactions of the students. But it should not prevent us from communicating the basic spirit of mathematics, especially the importance of logical enquiry. All aspects of mathematics, including history, biographies, motivation, definitions, lemmas, theorems, corollaries, proofs, examples, counterexamples, conjectures, construction, computation and applications can and should find a place in the classroom.

The views expressed in this talk pertain to mathematics in general and not particularly to the Indian context. But with special reference to the situation in India it must be remarked that the bifurcation of undergraduate teaching and research has not served us well. Our best researchers are concentrated in research institutes and do not teach undergraduate courses in mathematics. This is one of the reasons for a steady decline in the quality of undergraduate mathematics education in the country.

Examinations and tests are a major part of the teaching process and let me say something about them. I find that in physics, students are routinely asked questions which are not in any of their prescribed books and require some extra application of thought. In mathematics however, we are supposed to stick to routine questions, except in olympiad type exams. This is true at the high school board examinations level as well as the college level. Submitting to this requirement makes the subject dull for gifted students. Examination boards need to be persuaded to consider providing for ten percent marks for nonroutine questions, without changing the syllabus. We will then continue to have students securing 100 percent marks but that will mean something much more than in the present system.

Mathematics has been projected as a dry and difficult subject. Being poor in mathematics is considered natural and in fashion, whereas being good in mathematics is taken as being eccentric and queer. This perception has been created by all, including press and popular media. Television interviews of celebrities invariably include the "I was awful in maths, just hated it ..." bit. All this has its effect on students. The forces that are at work pulling talented

students away from mathematics and towards other sciences are too powerful and the only thing that works in favour of mathematics is the enjoyment it provides to some of the students who cannot think of doing anything else. Why can't mathematics be both enjoyable and fashionable, the *in-thing*, to do?

We are repeatedly asked to provide real life examples, motivation, while teaching or while lecturing to a general audience. This is justified to some extent, but mathematics manifests its beauty only when it is stripped off of its worldly connections. It flourishes in the abstract and then again turns messy when brought back to apply. The messy part is the one that engineers and scientists (those who use the mathematics) are to deal with. Why should a mathematician be required to constantly make this back and forth transition? To protect our interests let us make this feeling known, after debating it among ourselves.

Now I turn to the second component of our job, research. Writing our work and then managing to get it published is an important part of our profession. In a subject that is nearly two thousand years old, and in areas that are more than two hundred years old, getting a drop of something original is not easy. The situation is perhaps different in experimental sciences. But a comparison of publication record with other sciences is always made for all policy decisions. The recent debate about the inappropriate use of *citation index* in mathematics is a case in point. This leads to many ills of our profession. There is too much pressure to publish, quality suffers in the process. Refereeing is a challenging task with no apparent reward except a feeling of satisfaction towards contributing to the health of your area. It is nearly impossible to track all that is published and that results in further narrowing of one's interests.

In this context I wish to propose a scheme, which addresses the question of decreasing the number of publications to some extent. There can be free, possibly electronic, journals, run by well-established societies or academies, in which all papers are by invitation only. Papers must be refereed but the role of the referee must be limited mainly to checking for correctness, style etc. To fix ideas, suppose one such journal is called *The Free Journal of Combinatorics.* The job of the editorial board of this journal would be to identify promising mathematicians in Combinatorics, at all levels, and invite them to contribute one paper to the journal per year for the next five years. The author in turn should agree to (i) submit the best of his

work to the journal and (ii) to restrict his publications in other journals to either zero or a very small number. If the journal acquires enough prestige, then being an invited author of the journal will carry lot of value and then there is no need to publish more. Also in the course of a few years the Free Journal of Combinatorics will be a reflection of the best work in the area of Combinatorics and give a fair picture of the development of the subject.

I am proposing this scheme after giving it some thought from my personal perspective. I would welcome a situation where a reputed publisher will publish one paper of mine per year for a certain number of years. That will then contain the best I have to offer. Apart from that I can stick mainly to expository writing or survey papers. In any event it is true that barring exceptions, the average professional mathematician does not have enough research to report and generate more than one or two papers per year after the initial burst of activity has subsided. In practice however, the number increases since more publications means better salary and more grants, among other things.

The very idea of limiting the role of a referee may appear drastic. Mathematics is an art as much as it is a science. Can one imagine the painting of an accomplished painter being subjected to a refereeing process, before it is exhibited? And with all the stringent refereeing regimen in place in the present system, has it really eliminated erroneous papers or duplication of results? Let us recognize the best talent amongst us and *invite* them to write for us. There are invited papers at present but an invited paper often means a paper which the author would never care to publish otherwise.

Mathematicians, like other scientists, need support for their research and hence must write proposals for research grants. The mechanics of seeking research grants is designed by and suited to experimental scientists. A natural scientist wishes to propose a theory about an enzyme or a drug and must conduct experiments to test the claims. This requires some equipment, graduate students, site visits and these constitute the bulk of the proposal requirements. This process does not quite suit us mathematicians. We like to think about a problem, and at the same time let our thoughts wander in a random fashion. If something along the way catches our attention then we may follow that route. Thus, in reality, we cannot indeed write an honest research proposal which gives too many specifics about what we are going to achieve. If someone claims in a proposal that he or she is going to investigate bounds for the eigenvalues of a certain class of $0-1$ matrices, very likely the

proposer has already obtained some such bounds which will only be written up when the proposal is approved. There cannot be any other way. If I do not have any eigenvalue bounds already obtained with me, there is a chance I will not get any, even if a very big grant is awarded.

This special nature of mathematical research needs to be kept in mind by one and all, particularly the funding agencies. Past achievements can be given more weight and a sketchy proposal should not necessarily mean automatic rejection. Travel and short visiting appointments are a big attraction to mathematicians. If a mathematician can spend a few months in a place with all facilities, an intellectually stimulating atmosphere, and no teaching or administrative duties, then he can achieve wonderful results. Needs of a mathematician are meagre compared to that of an experimental scientist but they need to be addressed sensitively.

Even though outsiders may recognize one of us simply as a mathematician, within our community there are many subdivisions. One is not just a mathematician, he is a *differential geometer* or a *quantum probabilist* or a *commutative ring theorist.* Mathematics is neatly divided into areas: algebra, geometry, analysis, topology are some of the respected ones and there are many others which do not enjoy a similar standing. A research mathematician must make his or her area known and then should stick to it, if he/she doesn't want the professional career to suffer. Contacts must be developed in that particular area, journals should be identified, one must become known to the editorial board members and then life may be easier. Except that if you get bored with the same type of problems and want to be adventurous and venture into new territory, you better be first rate and adapt quickly, otherwise getting a foothold in the new area is not easy. So if your Ph.D. thesis has been in *uniform bounded cohomology of sections of holomorphic vector bundles*, twenty years later you would at best be venturing into the *locally nonuniform* case. More specialization has created further divisions among the dwindling number of mathematicians.

In ground reality, however mathematicians are indeed divided, but these divisions are of a different kind. There are those who enjoy teaching and mentoring students, researchers who excel in what they do but are hardly comfortable or efficient in a classroom, good expositors who can make a difficult subject look simple in their writing, people who enjoy organizing conferences; they don't mind if their own area is far removed from the area of the conference, good Ph.D. guides; their number of students is in double

digits in a short time, those who are interested in foundations, those who like to write proposals and have several projects to their credit simultaneously, those who like to chair departments and so on.

Can we recognize this classification and take it into account in our decision making, rather than the narrow area-wise breakup? Some of our sister areas, notably Economics, are devoid of the sort of tight division that we mathematicians have imposed upon ourselves.

Now I come to a topic where mathematicians are not really at fault, but it is made to appear that they are doing something wrong, or rather, not doing something right. And here I am referring to the task of communicating our work to others, especially to a general audience. Explaining mathematics to non-mathematicians, even educated ones, is an ordeal. Our subject does not lend itself to explanation to nonspecialist, period. However most people, including mathematicians, believe that it is a shortcoming of mathematicians that they are not able to "explain" there subject to others. Explaining to a layman (an educated one) what a *group* means, appears interesting - talk about geometrical figures, rotations, reflections and so on. But that is just the beginning. Can we get to *normal subgroups* and still retain the same clarity? And then what about *trivial torsion units in G-adapted group rings*?

A recent book review by Daniel Bliss in the Notices of the AMS (June/July 2007) brings forth this dilemma in a very interesting fashion. I recommend reading it in original, but the highlight is that the author (John Stillwell) of the book under review (*Yearning for the Impossible,* A.K.Peters Limited, 2006) tries very sincerely and very hard to explain what an *ideal* is, develops the concept very patiently and finally has an explanation. But at the end of it one wonders whether it is worth the trouble. Why can't we be frank and say that, look, these concepts are really very abstract and cannot be explained but then it is also required that we justify why we are doing what we do. In any event whatever good nontechnical exposition of mathematics has been achieved has remained confined to a few areas, notably discreet mathematics, where more ground can be covered. Even the great Martin Gardner has had to restrict himself in terms of range of topics. But then we are not conveying a true picture of the vastness and depth of mathematics.

This difficulty is also faced by other areas of arts and science. But it is interesting to see how they get around the problem. We are made to believe that physics, chemistry or biology are easier to explain to the general audience. Nothing can be farther from the truth but this belief is implanted

successfully in our minds by the way scientists in these areas present their work. Their abstract concepts are presented in a way as if they are actual realities. Atoms, quarks, dark energy, strings, black holes, are all concepts but people believe in them. In comparison mathematicians are awfully shy of presenting anything for which they do not have a refereed proof. We need to be bold. Our ongoing investigations should be presented with a passion and sense of self-belief. Our deceptively simple terminology is also a culprit. How can anyone believe that simple sounding concepts like group, ring and field can have anything deep to connote?

I have tried to present some views about our profession and as remarked in the beginning, it will be helpful if it generates more exchange of ideas. Our subject, known as the *Queen of Sciences* and one with a long history, is losing its image in the eyes of the young student, policy makers and general public. Riding on the wave of computer science does not solve the problem, since the nature of mathematics is unique and computer science is at best a glimpse into a small portion of it. We must strive to effectively communicate the unique nature of our subject, its beauty as perceived by us, and its applications to the betterment of life, to the layman, as well as scientists and policy makers in order to create a positive feeling towards our profession.

I conclude by wishing all the members and delegates a very fruitful and memorable conference.