# 2017 Symposium on Mathematical Programming and Game Theory

# 2017 SMPGT

*Organized by*

*Indian Statistical Institute,*

*SQC & OR Unit, Delhi Centre*

## Contents

# *Welcome to SMPGT 17*

On behalf of the organizers of SMPGT17, I welcome you in the *2017 Symposium on Mathematical Programming and Game Theory during* January 9-11, 2017 at Indian Statistical Institute, Delhi Centre. The objective of this symposium is to provide a forum for new developments and applications of Mathematical programming and game theory. Leading scientists, experienced researchers and practitioners, as well as younger researchers will come together to exchange knowledge and to build scientific contacts.

This symposium will provide an excellent opportunity to disseminate the latest major achievements and to explore new directions and perspectives, and is expected to have a broad international appeal, dealing with topics of fundamental importance in Mathematical Programming and other related sciences (Economics, Physics, Management Science and Engineering).

The Symposium also intends to bring out a publication of selected and refereed papers.

The symposium topics include (but not limited to):

- Linear and Nonlinear Programming
- Global Optimization
- Game Theory
- Nonsmooth Optimization
- Complementarity and Variational Inequalities
- Combinatorial Optimization
- Optimization in Finance and Economics
- Multi-Objective Optimization
- Game Theoretical Applications
- Logistics, Traffic and Transportation
- Optimization Problems in Graph Theory

Information about social events will be available to you at the time of registration.

*S. K. Neogy*
*Organizing Committee Chair*

# Committees

## Organizing Committee

S. K. Neogy (Chair), Dipti Dubey, R.B. Bapat, Arunava Sen, Prabal Roy Chowdhury

## Programme Co-ordinating Committee

Dipti Dubey, R. Chakraborty and Praveen Pandey

## Facilities Committee

R. Chakraborty, R. C. Satija, Simmi Marwah, Praveen Pandey, Sujan Dutta, Parama Gogoi and Srinivas

## 2017 Symposium on
## Mathematical Programming and Game Theory

# Program Overview

## Inaugural Session Details

**January 9, 2017 Time: 10:00 -10:30 Venue: Auditorium**

**Welcome address, Opening Remarks, About symposium, Vote of Thanks**

## Sessions Details

**January 9, 2017 Time: 11:00 -13:00 Venue: Auditorium**

**Invited Session I**

**Chairman :** Reinoud AMG Joosten, University of Twente, The Netherlands.

| | |
|---|---|
| 1. | **T E S Raghavan (**University of Illinois at Chicago, USA) Graphical algorithms for locating the nucleolus of binary valued assignment games |
| 2. | **Sandeep Juneja** (Tata Institute of Fundamental Research, India) Rest in the lounge or directly join the queue |
| 3. | **Yasunori Kimura (**Toho University, Funabashi, Japan**)** Convex minimization problems and resonvents on geodesic spaces |

**January 9, 2017 Time: 14:00 -15:20 Venue: Auditorium**

**Invited Session II**

**Chairman:** T E S Raghavan, University of Illinois at Chicago, USA

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|---|---|
| 1. | **J. Dutta (**jointly with S. Dempe, N. Dinh, and T. Pandit ) (Indian Institute of Technology, Kanpur, India) Simple Bilevel Programming and Extensions : Theory and Algorithms |
| 2. | **Sunil Chandran. L.** (Department of Computer Science and Automation, Indian Institute of Science, Bangalore**)** Separation Dimension of Graphs and Hyper Graphs. |

**January 9, 2017 Time: 15.45 -17:30 Venue: Auditorium**

**Technical Session-I A**

**Chairman :** Agnieszka Wiszniewska-Matyszkiel, Institute of Applied Mathematics and Mechanics, University of Warsaw, Poland

| | |
|---|---|
| 1. | **Jitendra kumar Maurya** (Department of Mathematics, Institute of Science, Banaras Hindu University, Varansi, India), S. K. Mishra, Approximate Karush-Kuhn-Tucker condition in multiobjective smooth and nonsmooth pseudolinear optimization |
| 2. | **Prasenjit Mondal** (Mathematics Department, Government General Degree College, Ranibandh, Bankura-722135, India) Algorithms for Solving Some Structured Classes of semi-Markov Game |
| 3 | **Izhar Uddin** (Department of Mathematics, JMI, New Delhi), J.J. Nieto, Javid Ali, Image Recovery Problem in Hadamard Spaces |
| 4. | **Anusuya Ghosh** (Production and Operations Management, Indian Institute of Management Bangalore), Vishnu Narayanan, Construction of Semidefinite Representation of Convex Body in $\mathbb{R}^2$ |
| 5 | **Mansi Dhingra** (Department of Mathematics, University of Delhi, Delhi) C.S. Lalitha Set-valued optimization via improvement sets |
| 6 | **Meghana Dhayal,** Vidyottama Jain, S. Dharmaraja (Department of Chemical Engineering, IIT Delhi, New Delhi, India) Performance Analysis of Higher Education Institutions using DEA-TOPSIS Approach |

**January 9, 2017 Time: 15.45 -17:30 Venue: Conference Room**

**Technical Session-I B**

**Chairman :** T. Parthasarathy, Chennai Mathematical Institute, Chennai, India

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|---|---|
| 1. | **Pooja Gupta** (Department of Mathematics, Banaras Hindu University Varanasi, India), Vivek Laha, S.K. Mishra, Approximate efficiency via generalized approximate convexity |
| 2. | **Mamata Sahu** (Delhi Technological University, India), Anjana Gupta, Incomplete intuitionistic multiplicative preference relations |
| 3. | Haneefa Kausar, **Firoz Ahmad** (Department of Statistics and Operations Research Aligarh Muslim University, Aligarh, India), An Approach for Solving Bi-level Programming Problem of Banking Crisis Management under Uncertainty |
| 4. | **Arshpreet Kaur** (Thapar University, Patiala, Panjab, India), Mahesh Kr. Sharma, Navdeep Kailey, Duality for a class of multiobjective fractional programming problems |
| 5. | **Akhilesh Kumar (**Department of Applied Mathematics, Delhi Technological University, Delhi,India), Anjana Gupta, Aparna Mehra**,** Railways Decision Making Problem on Special Trains in a Bilevel Programming Framework |
| 6. | **Rubi Arya** (MNIT Allahabad), Pitam Singh, Generating fuzzy efficient and Ɛ- fuzzy efficient solution for multi-objective linear fractional programming problem |

**January 10, 2017 Time: 10:00 -11:20 Venue: Auditorium**

**Invited Session III**

**Chairman: R. B. Bapat**

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|---|---|
| 1. | **Masahiro Hachimori** (University of Tsukuba, Japan), Optimization on acyclic orientations of graphs, shellability of simplicial complexes, and related topics |
| 2. | **Tadashi SAKUMA (**Yamagata University, Japan), Similarities and dissimilarities between the blocking and anti-blocking polyhedra |

**January 10, 2017 Time: 11:40 -13:00 Venue: Auditorium**

**Invited Session IV**

**Chairman:** T E S Raghavan, University of Illinois at Chicago, USA

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|---|---|
| 1. | **Reinoud Joosten (**University of Twente, The Netherlands**),** Robin Meijboom, Stochastic game with endogenous transitions |
| 2. | **T. Parthasarathy (**Chennai Mathematical Institute, Chennai, India), Completely mixed stochastic games |

**January 10, 2017 Time: 14:00 -15:20 Venue: Auditorium**

**Invited Session V**

**Chairman:** Masahiro Hachimori, University of Tsukuba, Japan

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|---|---|
| 1. | **Aparna Mehra (**Indian Institute of Technology Delhi, India) Portfolio selection for index tracking and enhanced indexing |
| 2. | **S.K. Mishra** (Banaras Hindu University, India) Lagrange duality and saddle point optimality criteria for mathematical programming problem with equilibrium constraints via convexificators |

**January 10, 2017 Time: 15.45 -17:30 Venue: Auditorium**

**Technical Session-II A**

**Chairman :** Joydeep Dutta, Indian Institute of Technology, Kanpur, India

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|---|---|
| 1. | P. K Shukla, J. Dutta, K. Deb, **P. Kesarwani** (Department of Mathematics, Indian Institute of Technology Kanpur, India) On a Practical Notion of Geoffrion Proper Optimality in Multicriteria Optimization |
| 2. | **Dharini Hingu (**Department of Mathematics, Indian Institute of Technology Madras, Chennai), K.S. Mallikarjuna Rao, A.J. Shaiju, On Superiority and Weak Stability of Population States |
| 3. | **Hamidur Rahman** (Indian Institute of Technology Bombay, Mumbai, India Ashutosh Mahajan) On the split-rank of the facets for mixed-integer bilinear covering set and their effective separation |
| 4 | Suvra Kanti Chakraborty**, Sutanu Paul (**Department of Mathematics, Indian Institute of Technology Kharagpur, India), Geetanjali Panda, Modified Projected Newton Scheme for Nonconvex Function with Box Constraints |
| 5 | **Srikant Gupta** (Aligarh Muslim University, India) Interactive Fuzzy Goal Programming for Agricultural land allocation problem Under Uncertainity |
| 6 | **Kalpana Shukla (**Department Of Mathematics, GLA University Mathura, India) Properties of generalized Nondifferentiable multiobjective variational control problems with generalized convexity with its applications |

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**January 10, 2017 Time: 15.45 -17:30 Venue: Conference Room**

**Technical Session-II B**

**Chairman:** S. K. Neogy, Indian Statistical Institute, Delhi Centre

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|---|---|
| 1. | **Anil Kumar Yadav** (Chhatrapati Shahu Ji Maharaj University Kanpur India) Analysis of Randomized Maximal Independent Set Problem |
| 2. | **Sonali Sethi**, N. Kailey, V. Sharma Madhu (School of Mathematics, Thapar University, Patiala, India) Nonsymmetric mixed-higher order multiobjective dual programs with support functions involving cone constraints |
| 3. | **Ekta Jain (**Department of Mathematics, Panjab University, Chandigarh, India), Kalpana Dahiya, Vanita Verma, Integer Quadratic Fractional Programming Problems with Bounded Variables |
| 4 | **Khushboo (**Department of Mathematics, University of Delhi, Delhi, India)**,** C.S. Lalitha Scalarizations for a unified vector optimization problem based on the order representing and the order preserving properties |
| 5. | **Mohd. Rizwanullah (**Manipal University, Jaipur, India), K.K. Kaanodiya, Distribution System using Recourse Model |
| 6 | **Bhawna Kohli (**Department of Mathematics, University of Delhi, Delhi, India) Sufficient Optimality Conditions for Optimistic Bilevel Programming Problem Using Convexifactors |

**January 11, 2017 Time: 10:00 -11:20 Venue: Auditorium**

Invited **Session VI**

**Chairman:** Yasunori Kimura, Toho University, Funabashi, Japan

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|---|---|
| 1. | **Agnieszka Wiszniewska-Matyszkiel** (Institute of Applied Mathematics and Mechanics, University of Warsaw, Poland) (jointly with Rajani Singh) Linear quadratic game of exploitation of common renewable resources with inherent constraints |
| 2 | **B. K. Mohanty (**Indian Institute of Management Lucknow, India**)** Use of Fuzzy MCDM in Product Ranking |

**January 11, 2017 Time: 11.40 -12.20 Venue: Auditorium**

Invited **Session VII**
**Chairman:** Tadashi SAKUMA**,** Yamagata University, Japan

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|---|---|
| 1. | **K.C. Sivakumar (**Department of Mathematics, Indian Institute of Technology Madras Chennai , India.**)** Complementarity Properties of Singular M-matrices |

**January 11, 2017 Time: 12.20 -13:00 Venue: Auditorium**

**Technical Session III**

**Chairman:** S. Dharmaraja, Indian Institute of Technology Delhi, India

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|---|---|
| 1. | **Aditya Aradhye (**Chennai Mathematical Institute (CMI), Chennai, India), Nagarajan Krishnamurthy, Pramod Mane, Kapil Ahuja**,** Stable Social Clouds |
| 2. | **Shilpi Verma (**Department of Mathematics, Shivaji College, New Delhi, India), Mukesh Kumar Mehlawat, An Integrated analytical hierarchy process approach for COTS component selection |

**January 11, 2017 Time: 14:00 -15.20 Venue: Auditorium**

**Invited Session VIII**

**Chairman:** T. Parthasarathy**,** Chennai Mathematical Institute, Chennai, India

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|---|---|
| 1. | **C. S. Lalitha (**Department of Mathematics, University of Delhi, Delhi, India) Stability of Solutions of Set Optimization Problems |
| 2. | **Pankaj Gupta (**Department of Operational Research, Faculty of Mathematical Sciences, University of Delhi, Delhi, India) Possibilistic Optimization |

**January 11, 2017 Time: 15:45 -17:30 Venue: Auditorium**

**Technical Session IV**

**Chairman:** Pankaj Gupta**,** Department of Operational Research, University of Delhi, India

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|---|---|
| 1. | **Mahima Gupta** (IMT Ghaziabad, India) A Multistage Multiobjective Production Planning Problem-A Goal Programming Approach |
| 2. | **Khushboo Verma (**Department of Mathematics and Astronomy, University of Lucknow, Lucknow, India**),** Pankaj Mathur, T. R. Gulati, Higher order symmetric duality in multiobjective programming over cone-invex function |
| 3. | **Kunwar V. K. Singh** (Department of Mathematics, Institute of Science, Banaras Hindu University Varanasi, India), S. K. Mishra, Duality for multiobjective mathematical programming problems with equilibrium constraints |
| 4 | **Vinod Kumar Mishra** (Department of Computer Science and Engineering, Bipin Tripathi Kumaon Institute of Technology, Dwarahat, Almora, Uttarakhand, India) Inventory model for substitutable deteriorating items with stock out based substitution under joint replenishment |
| 5. | **Shalabh Singh** (IIM Lucknow), Sonia**,** Multistage Stochastic Decision Making in Dynamic Multi-level Bi-criteria multi-choice assignment problem |
| 6. | **Vinod Kumar Chauhan** (Computer Science & Applications Panjab University Chandigarh, INDIA), Kalpana Dahiya, Anuj Sharma, Trust Region Levenberg-Marquardt Method for Support Vector Machines |

# ABSTRACT OF THE PAPERS

**Graphical algorithms for locating the nucleolus of binary valued assignment games**.

**T E S Raghavan**
University of Illinois at Chicago, USA.

*Abstract:* Assignment games with side payments are models of certain two-sided markets. It is known that prices which competitively balance supply and demand correspond to elements in the core. The nucleolus, lying in the lexicographic center of the nonempty core, has the additional property that it satisfies each coalition as much as possible. The corresponding prices favor neither the sellers nor the buyers, hence provide some stability for the market.

Here we present the recent algorithm for the special case where the matrix defining the game is a 0-1 matrix. If say n boys and n girls specify acceptable candidates among members of the opposite sex among them for marriage, potential marriages depend on mutually acceptable matching. If the rows of a 0-1 matrix represent the boys and columns represent the girls then a 0-1 matrix where a 1 in the i-th row j-th column correspond to mutually acceptable mates and where a 0 in the i-th row j-th column correspond to not a mutually acceptable mate. Suppose the game admits full matching, namely n mutually acceptable mates . The real issue is what is the relative strength between each married couple. We propose the nucleolus of the game as a measure of the relative power split (balance of power) between the husband and wife of each matched couple in the full matching. Here the algorithm exploits quickly to initiate at the south west corner of the core based on Noltmier's algorithm to locate the longest path in a graph and avoids several intermediate steps in Solymosi_Raghavan algorithm for general assignment games and uses simply the induced graph of the given 0-1 matrix to reach the nucleolus iteratively. The algorithm specializes Maschler-Peleg-Shapley's approach to lexicographic center for just the graph G=(V, E) where V consists of the main diagonal (i, i') where prime is used to refer to columns and an edge e of the edge set E corresponds to the mutual acceptability of boy i (row player) with j' ( column player j).

## Rest in the lounge or directly join the queue

**Sandeep Juneja**
Tata Institute of Fundamental Research, India

In this talk we first review the concert queueing game with a finite number of customers, where the queue opens at a specified time and the customers are allowed to queue up earlier. Customers would like to get serviced quickly as and wait in the queue as little as possible. We consider a variant of this game where the customers relax in a lounge before deciding to join the queue and can see the queue length at all times. The service times are assumed to be exponentially distributed. In this setting we arrive at a symmetric Nash equilibrium dynamic policy for each customer. We also discuss a related setting of an M/M/1 queue where customers first arrive at a lounge as a Poisson process and then decide to join the queue at an opportune equilibrium time.

## Convex minimization problems and resonvents on geodesic spaces

**Yasunori Kimura**
Toho University
Funabashi, Japan
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The notion of resolvent for proper lower semicontinuous convex functions defined on a Banach space plays an important role in the theory of convex analysis because it properties are strongly connected to the fixed point thoery for nonexpansive mappings. It is known that the resolvent is firmly nonexpansive and the set of its fixed points coincides with the set of minimizers of the corresponding convex function. These facts imply that the approximation techniques of fixed points of nonexpansive mappings can be applied to the convex minimization problems through the concept of resolvent.

In this talk, we propose the notion of resolvent in the setting of complete geodesic spaces with curvature bounded above. Then we apply the shrinking projection method to the problem of approximating minimizers of the function. We also show some recent developments related to this topic.

**Simple Bilevel Programming and Extensions : Theory and Algorithms**

S. Dempe, N. Dinh, **J. Dutta** and T. Pandit

Indian Institute of Technology, Kanpur
Kanpur-208016, India

In this talk we focus on the minimization of a convex function over the solution set of another convex optimization problem. Such a problem is termed as a simple bilevel programming problem. The problem of course has an inherent bilevel structure and we call it simple since it can be viewed as a simplified version of the standard bilevel programming problem. Further we also consider a more general problem where we minimize a convex function over the solution set of a monotone variational inequality.

Our aim here is first provide examples where such problems naturally arises and then provide some new approaches to develop necessary and sufficient optimality conditions for such class of problems. Then we shall discuss first a solution approach to the simple bilevel programming problem where the problem data is nonsmooth and the lower-level problem has unbounded feasible set. We end the talk by developing an algorithm for minimizing a convex function over the solution set of a variational inequality.

**Separation Dimension of Graphs and Hyper Graphs.**

**Sunil Chandran. L.**

Department of Computer Science and Automation

Indian Institute of Science, Bangalore**.**

The separation dimension of a hyper graph G is the smallest natural number k for which the vertices of G can be embedded in R_k such that any pair of disjoint edges in G can be separated by a hyperplane normal to one of the axes. Equivalently, it is the smallest possible cardinality of a family F of total orders of the vertices of G such that for any two disjoint edges of G, there exists at least one total order in F in which all the vertices in one edge precede those in the other. It can be shown that the separation dimension of a hyper graph G equals the boxicity of the line graph of G, where boxicity of a graph H is defined as the smallest integer d such that H can be represented as the intersection graph of d-dimensional axis parallel boxes. In this talk we discuss the relation of separation dimension with various other graph theoretic parameters. We will also mention some of the recently introduced variants of separation dimension: Induced separation dimension (from the team of Martin Golumbic) and fractional separation dimension (D. B. West and S. Loeb).

## Optimization on acyclic orientations of graphs, shellability of simplicial complexes, and related topics

**Masahiro Hachimori**

University of Tsukuba, Japan

In this talk, we consider an optimization problem in which the value of the objective function is determined by the way of orientations of each edges on a given graph, and we want to optimize under the restriction that the whole orientation is acyclic on the graph. This kind of optimization problem first appears in Kalai's new proof (1988) to the reconstruction of simple polytopes from their graphs, shown by Blind and Mani (1987), and, as a kind of its generalization, used as a characterization of shellability of simplicial complexes by Hachimori and Moriyama (2008). A general setting is proposed by Hachimori (2006, unpublished manuscript) and a similar application is given for cubical complexes. In this talk we will introduce these results with several related topics.

## Similarities and dissimilarities between the blocking and anti-blocking polyhedra

**Tadashi SAKUMA**

Yamagata University, Japan

The study of similarities and dissimilarities between the blocking and anti-blocking polyhedra began with a series of celebrated papers by Fulkerson (1970, 1971, 1972), and it has grown up a mature theory by significant contributions of Lehman, Lov\'{a}sz, Padberg, and others in 1970s and 1980s. Even today, this theory still shows a big progression such as the perfect graph theorem of Seymour et al. (2006). In this paper, we survey the current status of this research field with a focus on the conjecture of Conforti \& Cornu\'{e}jols and the conjecuture of Grinstead.

**Stochastic games with endogenous transitions**

**Reinoud Joosten**

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Robin Meijboom

IEBIS, School of Management & Governance
University of Twente.

We present and analyze a stochastic game in which transition probabilities between states are not fixed as in standard stochastic games, but depend on the history of the play, i.e., the players past action choices. For the limiting average reward criterion we determine the set of jointly-convergent pure-strategy rewards which can be supported by equilibria involving threats.

For expository purposes we analyze a stylized fishery game. Each period, two agents choose between catching with restraint or without. The resource is in either of two states, High or Low. Restraint is harmless to the .sh, but it is a dominated action at each stage. The less restraint shown during the play, the higher the probabilities that the system moves to or stays in Low. The latter state may even become `absorbing temporarily',.i.e., transition probabilities to High temporarily become zero while transition probabilities to Low remain nonzero.

Keywords Stochastic games, limiting average rewards, endogenous transition probabilities, temporarily absorbing states, hysteresis

**Completely mixed stochastic games**

**T. Parthasarathy**
Chennai Mathematical Institute, Chennai, India.

If the undiscounted stochastic game is completely mixed then discounted stochastic games are completely mixed for beta sufficiently close to one under the assumption transition probabilities are cotrolled by one player.

P.S. Talk is based on joint work with Purba Das and Ravindran

**Portfolio selection for index tracking and enhanced indexing**

**Aparna Mehra**

Indian Institute of Technology Delhi, India

More than forty years of index investing, since when the first index investment trust was established, exhibit that the investment portfolios for index investing are highly diversified portfolios allowing investors to obtain the average market return at an optimal costs. One needs to construct the portfolio which approximates the index for investing into the selected index. This process is known as index tracking, and the selected portfolio is called the tracking portfolio. Index tracking is considered a passive investment strategy. Investors considering a shift from the traditional active equity strategies to the passive index funds investments, in an effort to decrease high equity risk, find a more risk-efficient solution in enhanced index strategies. The enhanced indexing is somewhere in the middle of the active and the passive investments. The enhanced indexing strategy seeks to perform better than the benchmark index in the sense of higher return. It takes advantage of existence of weakly efficient markets - the less (more) efficient the market is, the greater (lesser) is the chance of beating it.

In this talk, we shall investigate various portfolio models proposed in the literature for index tracking and enhanced indexing, highlighting their advantages and disadvantages, and also propose a model for enhanced indexing which considers second order stochastic dominance constraints (to achieve higher utility for rational risk averse investors) with some tolerable relaxation. Empirical evidences indicate the superior performance of the proposed model over the traditional models in certain financial parameters.

## Lagrange duality and saddle point optimality criteria for mathematical programming problem with equilibrium constraints via convexificators

**S.K. Mishra**
Departyment of Mathematics
Institute of Science
Banaras Hindu University
Varanasi, India
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**Abstract**: We consider a mathematical programming problem with equilibrium constraints (MPEC).We formulate Lagrange type dual model for MPEC and establish weak and strong duality results under convexity assumptions in terms of convexificators. Further, we investigate the saddle point optimality criteria for the mathematical programming problem with equilibrium constraints. We also illustrate our results by an example.

**Keywords**: Mathematical programming problems with equilibrium constraints; Convexificators; Duality.

## Linear quadratic game of exploitation of common renewable resources with inherent constraints

Rajani Singh
rajanibabu7@gmail.com,
**Agnieszka Wiszniewska-Matyszkiel**
agnese@mimuw.edu.pl
Institute of Applied Mathematics and Mechanics, University of Warsaw, Poland

In this paper we present an analysis of a discrete time dynamic game of extraction of common renewable resources by many players with infinite time horizon and possibility of depletion. The game is defined such that increasing number of players does not mean introduction of additional users of the resource, but decomposition of the decision making structure of the same mass of users (into regions, countries, firms). The game is linear-quadratic with constraints which are inherent to the problem: player cannot extract negative amount or more than available. Such an obvious modification of the standard framework of linear quadratic game substantially changes both Nash equilibrium and social optimum. We look for Nash equilibria and social optima in feedback form (strategies dependent on the state of the resource).

After modification, we can calculate social optima for arbitrary number of players and prove that they are identical, while Nash equilibrium can be computed only for continuum of players, and we obtain piecewise linear equilibrium with surprisingly compound value function resulting from exhaustion: non-smooth, piecewise linear-quadratic with infinitely many intervals. For finitely many players a negative result can be proved, that the equilibrium is not even piecewise linear while the value function piecewise linear-quadratic with less than three intervals. In obtained results, we always have sustainability at social optima and exhaustion in finite time for reasonable interval of initial states for Nash equilibria. When applicational aspect is considered, we also calculate tax rate enforcing social optimum (and, therefore, sustainability) in the game with continuum of players. Besides, during the process of computation of social optimum, we discovered that this simple dynamic optimization problem constitutes a counterexample to correctness of skipping or relaxation of checking terminal condition -a simplification often used in applications.

Keywords: common renewable resources, social optimum, Nash equilibrium, linear- quadratic game with constraints, exhaustion.

### Decentralized Matching with Agent Preferences in Cab Networks

Manish Sarkhel, and **Nagarajan Krishnamurthy**

Operations Management and Quantitative Techniques

Indian Institute of Management (IIM) Indore, India

Ridesharing and taxi services like Ola Cabs, Uber, etc. have rapidly risen in many cities in India. However, local networks among taxi operators and drivers have been thriving too, especially in smaller cities and towns. In this paper, we attempt to model such networks and look at the decentralized peer-to-peer matching mechanism already existing in such places. We build a model where taxi drivers take into account the preferences of the customers and match them to drivers so as to maximize the utility of both the agents, with weights assigned to the utilities. Our model captures implicit ratings of agents for others as well as explicit factors like waiting time of customers.

Each driver has a preference order for drivers – that is the order in which the driver would choose to pass on customers. Each customer specifies a preferred set of drivers, a forbidden set of drivers and others or the indifferent set. A driver allocated from the preferred set of the customer gives more

utility as compared to the driver allocated from the indifferent set and subsequently the forbidden set. A customer gets a part of his/ her utility from the set to which the allocated driver belongs and the rest of it from the time the customer has to wait before the cab arrives. Increase in waiting time of the customer decreases his/her utility. A customer on requiring a ride initiates the matching process by calling a driver randomly from his/ her preferred set of drivers. Upon calling, if the driver is not free or does not agree to come, the customer conveys his/ her preferred set, forbidden set and location to the **driver. The driver takes all these factors into consideration and assigns a driver. The driver gets a part** of his/ her utility from the customer's utility for whom the driver arranges the ride and the rest of it from the rank of the driver allocated to the customer.

We look at finding optimal as well as stable matchings between drivers and customers. We also discuss the network formation game when there are measures of centrality that contribute to the utility of the driver.

## Use of Fuzzy MCDM in Product Ranking

**B. K. Mohanty**

Indian Institute of Management

LUCKNOW-226013, INDIA

This paper introduces a new methodology for product ranking (in business) as per the customers' choice. Initially, the product preferences of the consumers are taken which are often, expressed in linguistic terms. Te concepts of fuzzy sets are used to represent the consumers' qualitatively defined terms as fuzzy numbers. The notion behind linguistic-numeric conversion or vice versa helps us to express all the consumer preferences either linguistically or numerically. Note that a product may be good with respect one criterion and may be worst with respect to another. For instance, for the criterion "price", a car might be at best price and the same car may not be lucrative for the criterion "mileage". Corresponding to each criterion, the products are sequentially chosen from a least satisfied ($P^{min}$) to best satisfied ($P^{max}$) in an increasing direction. This makes us to obtain a network consisting of number alternative paths from $P^{min}$ to $P^{max}$.

Our work derives the shortest path between two products with respect to a criterion depending on how the product's criteria values conflicts with the other products. For example, taking the product CAR and the criterion price, we can have the distance between two different cars as follows. If the Cars, C1 and C2 have the selling prices, respectively as $p_1$ and $p_2$ with satisfaction levels as $\mu_{p1}$ and $\mu_{p2}$ and if $\mu_{p2} > \mu_{p1}$, the distance between C1 and C2 as:

$$[\mu_{p2} - \mathrm{Max}\ (\mu_{zi})]$$

Where, $\mu_{zi}$ is the satisfaction level of other attributes other than the attribute price. The other attributes may be "maintenance cost" , "mileage" etc.

The shortest path between two products with respect to a criterion helps us to obtain the cost of moving from a product p1 to product p2. In real terms this indicates if a consumer goes for a better product in terms of price , he /she may lose in other attributes, may be in maintenance cost or mileage. The shortest path or cost of moving from a product to another is nothing but the degree of conflict a product has amongst the available products. This helps us to develop a Binary integer Programming model for product selection to select the products having minimum degree of conflict with the other products. This also makes us to obtain the preference ranking of the products.

## Complementarity Properties of Singular M-matrices.

**K.C. Sivakumar**
Department of Mathematics
Indian Institute of Technology Madras
Chennai 600 036, India.

For a matrix A whose off-diagonal entries are nonpositive, its nonnegative invertibility (namely, that A is an invertible M-matrix) is equivalent to A being a P-matrix, which is necessary and sufficient for the unique solvability of the linear complementarity problem defined by A. This, in turn, is equivalent to the statement that A is strictly semimonotone. In this talk, an analogue of this result will be presented for singular symmetric Z-matrices. This is achieved by replacing the inverse of A by the group generalized inverse and by introducing the matrix classes of strictly range semimonotonicity and range column sufficiency. A recently proposed idea of P# -matrices plays a pivotal role.

## Stability of Solutions of Set Optimization Problems

**C. S. Lalitha**
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In this paper we study the stability of efficient solution sets of a set optimization problem. In particular, we consider a sequence of set optimization problems obtained by perturbing the feasible set and study the convergence of the solutions sets of perturbed problems to the solution set of the given problem. A formulation of external and internal stability of the solutions is considered in the image space and both Hausdorff and Kuratowski-Painleve convergences are established under appropriate continuity and compactness assumptions.

## Possibilistic Optimization

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Zadeh presented the theory of possibility, which is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable. Since then there has been a lot of research on the possibility theory in optimization problems. Possibilistic decision-making models provide an important aspect in handling real world decision-making problems. There are many possibilistic approaches to tackle the imprecise coefficients in the objective functions as well as the constraints of an optimization problem in the literature. We discuss the widely used possibilistic approach given by Lai and Hwang that convert the fuzzy objective with a triangular possibility distribution into three crisp objectives corresponding to the three critical values (the most possible, the pessimistic, and the optimistic values). The approach has the following advantages over the other approaches from the literature: (i) unlike the other approaches which are based on defuzzification method, this approach does not lose any information on uncertainty before solving the problem. To solve real-world problems, it is better to keep the uncertainty until an optimal solution is reached; (ii) it can easily solve the imprecise objective function and obtain the whole possibilistic distribution of the satisfying objective value. Furthermore, it can solve rather general imprecise problems through an interactive process with the decision maker. Further, we discuss a new possibilistic programming approach given by Gupta and Mehlawat for solving optimization problems in which the objective function coefficients are characterized by triangular possibility distributions. The solution approach simultaneously minimizes the best scenario, the likeliest scenario, and the worst scenario for the imprecise objective functions using α-level sets that define confidence level of the fuzzy judgements of the decision maker. The new possibilistic approach is shown to be more flexible and realistic than the widely used approach of Lai and Hwang.

## Stable Social Clouds

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In a recent work, P. Mane, K. Ahuja and N. Krishnamurthy (2014) model Social Cloud as a network formation game and provide conditions for stability of social Clouds. We look at a variant of this model, where the nodes in the network are consumers of resources in the social cloud, which are provided by an external set of suppliers. Resources are limited in quantity and this quantity is affected by a random event. These resources, due to their shortage, cannot be provided to all of the consumers but only to the most central ones, degree centrality being the measure of centrality. We also consider an alternate model, in which closeness centrality is the measure of centrality. Consumers, on the one hand, try to maximize their centrality by forming links with the other consumers and on the other hand, try to minimize the cost incurred for forming the links.

We study the stability of Social Clouds. In addition to the usual pairwise stability as defined by Jackson and Wolinsky (1996), we define two stability notions, Add-Stability and Delete-Stability, where instability can occur due to only adding the links or only deleting the links respectively. We first write the necessary and sufficient conditions for addition or deletion of links to be beneficial to the consumers. We then state the conditions for stability in these two notions and then prove that it takes (n2) time to determine the stability in the network of n agents.

## Algorithms for Solving Some Structured Classes of Semi-Markov Games

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Various structured classes of zero-sum two-person finite (state and action spaces) semi-Markov games, namely, AR-AT-AITT (Additive Reward-Additive Transition and Action Independent Transition Time), AR-AIT-ATT (Additive Reward-Action Independent Transition and Additive Transition Time) and SC (Switching Controller) have been studied. Solution (value and stationary optimals) to each class of games under the discounted payoff criterion can be derived from an optimal solution to appropriate bilinear program with linear constraints. For the undiscounted payoff games with unichain assumption, we provide analogous results for the above special structured classes. Moreover, such games have been formulated as a vertical linear complementarity problem (VLCP) which can be solved by a stepwise generalized principal pivoting algorithm.

## Image Recovery Problem in Hadamard Spaces

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Let $C_1$, $C_2, ..., C_n$ be nonempty closed convex subsets of a real Hilbert/Banach space $X$ such that $\bigcap_{i=1}^{n} C_i \neq \emptyset$. Then, the problem of image recovery may be stated as follows: the original unknown image $x$ is known a priori to belong to the intersection of $\{C_i\}_{i=1}^{n}$ given only the metric projections $P_{C_i}$ of $X$ onto $C_i$ for $i = 1, 2, ..., n$, recover $x$ by an iterative scheme. Such a problem is connected with the convex feasibility problem and has been investigated by a large number of researchers. Bregman [1] considered a sequence $\{x_n\}$ generated by cyclic projections, that is, $x_0 = x \in H$, $x_1 = P_{C_1}x$, $x_2 = P_{C_2}x_1$, $x_3 = P_{C_3}x_2$,...,$x_{n+1} = P_{C_{n+1}}x_n$. It was proved that $\{x_n\}$ converges weakly to an element of $\bigcap_{i=1}^{n} C_i$ for an arbitrary initial point $x_0$. Later, Kitahara and Takahashi [2] and Takahashi and Tamura [3] dealt with the problem of image recovery by convex combinations of nonexpansive retractions in a uniformly convex Banach space. This problem has been investigated by many authors via several iteration scheme in linear spaces. In this section, we study this in the setting of Hadamard spaces.

**Keywords**: CAT(0) space, Fixed point, $\Delta$-convergence and Opial's property.

**AMS Subject Classification**: 54H25, 47H10.

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## Construction of Semidefinite Representation of Convex Body in $\mathbb{R}^2$

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Semidefinite representation of convex sets draws a considerable attention in recent research on modern convex optimization. We develop a construction method to obtain semidefinite representation of convex set. Based on a method to reconstruct convex body from its projections on lower dimensional space, we contribute a procedure to derive approximate solution of reconstruction problem. Although the construction method gives approximate solution, but the method is very fast as we compared our method with all other construction methods which are available in recent literature. We analyse the output of the reconstruction algorithm. We prove that the algorithm terminates after finite number of steps. Further we prove that there is no convex body to the given input data, if we do not get an approximate solution.

Key words— Semidefinite representation, Convex set, Reconstruction problem, Approximate solution, Core set, Envelope set

## Set-valued optimization via improvement sets

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In this paper we introduce a notion of minimal solutions for set-valued optimization problem, unifying a set criterion notion introduced by Kuroiwa [13] for set-valued problems and a notion introduced by Chicco et al. [3] using improvement sets for vector optimization problems.

Keywords: Set-valued optimization,  Improvement set

# Performance Analysis of Higher Education Institutions using DEA-TOPSIS Approach

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Education plays a key role in development of every country. Especially, Technical education plays more important role in improvement of life quality of society. Hence, it's highly essential to evaluate their performance and to figure out how their performance can be improved further. Now a day's students who are entering in technical education are increasing and resources are limited. Output of Institutions has also become a prime concern today; hence performance evaluation is a research issue. It is very difficult to optimize limited resources without their proper analysis. There are many attributes which play critical role in evaluating the performance of Higher Education Institutions (HEIs). Hence, a mathematical model is needed which can handle multiple inputs and outputs. There are no specific inputs and outputs for performance evaluation. The study depends on country what attributes to choose. Different methods have been discussed by other researchers to evaluate performance in last thirty years like performance indicators, parametric method (ordinary least square and stochastic frontier analysis) and non-parametric methods (Data Envelopment Analysis). Each method is unique; no method can be prescribed as the complete solution to the question of ranking. Hence, selecting the best method for ranking or the way of combining different ranking methods for evaluation of HEIs is very important.

Stochastic Frontier Analysis (SFA) and Data Envelopment Analysis (DEA) (Charnes et al., 1978) have been used widely. SFA can't handle multiple inputs and outputs while Data Envelopment Analysis (DEA) is a very popular for performance evaluation, as it can handle multiple inputs and outputs. Shortcoming of DEA is that it can't distinguish between best performing alternatives. Hence, to overcome this, Multiple Criteria Decision Making (MCDM) approach has been chosen. Lots of MCDM tools like TOPSIS, ELECTRE and PROMETHEE have been used by researchers for ranking and performance evaluation. TOPSIS (Hwang (1981) is different from other MCDM methods because it depends on logical thinking, which is based on simultaneous evaluation of the nearest distance from the best alternative (positive ideal solution) and the longest distance from the worst alternative (negative ideal solution). For weight determination, different methods have been applied by researchers like Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), entropy method and Fuzzy Decision Maps (FDM).

In this paper, a combine approach of DEA and TOPSIS has been discussed for ranking and performance evaluation of Higher Education Institutions (HEIs). The main objective of this paper is to evaluate ranking of HEIs based on their efficiency score. For illustration purpose, a set of 15 HEIs is evaluated based on six attributes which are student intake, paper published per faculty, expenditure, faculty strength, placement and perception. Initially, by using entropy method, weight index of six attributes is calculated and followed TOPSIS is employed. For evaluating the attribute perception, fuzzy logic rule based methodology is applied. In TOPSIS, efficiency score is evaluated and based on the efficiency score, HEIs are evaluated. In future, sensitivity analysis can be performed on selected attributes to evaluate their effect on performance evaluation.

Keywords – Performance Analysis, TOPSIS, DEA, Entropy method.

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## Approximate efficiency via generalized approximate convexity

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In this paper, we study nonsmooth vector optimization problems involving locally Lipschitz generalized approximate convex functions and derive some relationships between approximate convex functions and generalized approximate convex functions. We also establish necessary and sufficient optimality conditions for approximate efficient solutions of the nonsmooth vector optimization problem.

**Incomplete intuitionistic multiplicative preference relations**

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In various decision-making problems, there are different types of preference relations indicated through matrices and their power. The preference relations are classified into two forms: fuzzy preference relations (FPR) and multiplicative preference relations (MPRs). In FPRs, the decision maker [1] expresses the information by using 0 to 1 ratio scale where as in the MPR, proposed by Satty, [2] the intensity of the pairwise comparison of objects is expressed using $\frac{1}{9}$ *to* 9 ratio scale. This is also called 1-9 ratio scale. A fuzzy preference relation $R$ defined on the set $X = \{x_1, x_2, \ldots, x_n\}$ is represented by a matrix $R = (r_{ij})_{n\times n}$ in which $r_{ij}$ lies between 0 and 1, $r_{ij} + r_{ji} = 1$, $r_{ii} = 0.5$, $\forall\ i,j\ \in N$, where $r_{ij}$ denotes the preferred degree of the objects $x_i$ over the objects $x_j$. Similarly an MPR, $P = (p_{ij})_{n\times n}$ over $X$ is defined under the condition: where $\frac{1}{9} \le p_{ij} \le 9$, $p_{ij}p_{ji} = 1$, $p_{ii} = 1$, $\forall\ i,j\ \in N$, where $p_{ij}$ is the preference information of alternative $x_i$ to alternative $x_j$. Additionally, the MPRs have been deeply studied and successfully applied in analytic hierarchy process (AHP). Due to lack of time and knowledge, often the preferential surveys received are incomplete. This results in incomplete data set, thus making it very difficult for the decision-makers act on these results. To express the survey-taker's preference information, interval-valued FPRs [3] and the interval-valued MPRs [4] are provided to allow the decision makers to use these interval numbers.

All the elements of both FPRs and MPRs are single values, only involving the intensities of preferences relation. It is possible that, in the decision making process, the decision maker is not sure about the preference information. For such an obstacle, the intuitionistic fuzzy preference relations (I-FPRs) [6] and the intuitionistic multiplicative preference relations (I-MPRs) [7] are defined to indicate that the positive information $x_i$ is preferred to $x_j$ and the degree of negative information $x_i$ is not preferred to $x_j$. Our work focuses on I-MPRs. To present a complete preference relation, a decision maker should make $\frac{n(n-1)}{2}$ judgements at each level and when $n$ is large it becomes an huge task.

Sometimes, the decision maker may not have a good understanding of a particular question and thus they are unable to make a direct comparison between two objects. In this case the whole process may slow down. In the decision making process, it is more appropriate and flexible to skip some comparisons and the decision maker may express their judgment with incomplete preference relations. In 2015 according to Jiang et.al [5], the incomplete I-MPR is split into two MPRs and the missing element can be calculated which involves two steps, "estimating step" and "adjusting step". In this paper, by using a new transitivity property of I-MPR, the missing element has been estimated. Sometimes the other pre-standing properties of I-MPR are not satisfied by the value obtained. To eradicate this discrepancy, an optimization algorithm has been developed and then been realized through a MATLAB model and hence minimizing the error. Subsequently, a method to check the consistency levels has also been developed.

**Keywords** Intuitionistic multiplicative preference relation, Consistency, Multiplicative preference relation (MPR).

**An Approach for Solving Bi-level Programming Problem of Banking Crisis Management under Uncertainty**

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In this paper, we have considered nonlinear bi-level programming problem consists of a manager at upper level (ULDM) and a follower at lower level (LLDM) in fully uncertain environment. The standard normal distribution functions have been used in order to convert the objective function of each level decision maker (DM) into its equivalent deterministic form. In order formulate the model, the individual best solution of the objective functions have been used to develop the nonlinear membership functions with equivalent deterministic constraints. The approximated linearization of nonlinear membership functions has been done by using first order Taylor series. An approach based on fuzzy goal programming model is presented to achieve the highest degree of membership goals. A numerical illustration is also discussed and solved by using Euclidean distance function to obtain the most compromise optimal solution.
***Keywords:*** Nonlinear Bi-level Programming, Decision Making, Fuzzy Goal Programming

# Duality for a class of multiobjective fractional programming problems

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**Abstract:** A class of nondifferentiation multiobjective fractional programming problems has been discussed. For a differentiable function, generalized convexity assumptions, i.e., higher order $(C, \alpha, \gamma, \rho, d)-$ convexity assumptions has been discussed. An example of $(C, \alpha, \gamma, \rho, d)-$ convex has also been formulated. Further, a higher order dual model has been formulated and duality results are proved under the above mentioned generalized convexity assumptions.

**Key words:** Dual, Multiobjective fractional programming problems, Efficient solutions, $(C, \alpha, \gamma, \rho, d)$-convexity.

## A Multistage Multiobjective Production Planning Problem-A Goal Programming Approach

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In this work, a methodology is introduced to solve a multi-stage production planning problem having multiple objectives, which are conflicting, non-commensurable and fuzzy in nature. Though the priority order of objectives is given, their targets are given in vague terms. The production process consists of multiple stages having one or more machines in each stage. Every stage produces work-in-process, semi-finished items as an output, which becomes input to the subsequent stage. Events of machine breakdowns and imbalances in input-output relations in one or more stages may affect both work-in-process (WIP) and final production targets. To maintain the desired balanced input-output relation at each stage and the targeted production output at the final stage, either procurement of work-in-process inventory (WIP) or installation of new machines at appropriate stages is recommended. The objectives or goals expressed in linguistic terms are represented as fuzzy sets and the prioritized goal programming approach is used to obtain the desired solution. The production planning problem is formulated as a Mixed Integer Programming (MIP) problem. The solution to MIP shows the degrees of achievements of the production process objectives. The methodology is illustrated with a real life application of crankshaft productions in the automobile industry.

Keywords: Multi stage production systems; Fuzzy Goals; Multiple Objectives; Inventory balance; New Machine Installation; Prioritized Goal Programming

## Generating fuzzy efficient and Ɛ- fuzzy efficient solution for multi-objective linear fractional programming problem

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In this article, a fuzzy based computational algorithm is developed to generate various fuzzy efficient and Ɛ- fuzzy efficient solutions. The proposed algorithm is based on fuzzy preferences which are randomly decided with the satisfaction of the given constraints. Some theoretical results are also established for the validation. This method gives the solution based on the decision maker's choice as satisfaction degrees. The algorithm is coded in matlab (version R2014b) and applied on numerical problem for measuring the efficiency of the developed algorithm.

## Approximate Karush-Kuhn-Tucker condition in multiobjective smooth and nonsmooth pseudolinear optimization

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In this paper we consider multiobjective smooth and nonsmooth pseudolinear optimization problems. We establish approximate Karush-Kuhn-Tucker ( AKKT ) sufficient optimality condition for the considered problems.

## On a Practical Notion of Georion Proper Optimality in Multicriteria Optimization

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Geoffrion proper optimality is a widely used optimality notion in multicriteria optimization that prevents exact solutions having unbounded trade-offs. As algorithms for multicriteria optimization usually give only approximate solutions, we analyze the notion of approximate Geoffrion proper optimality. We show that in the limit, approximate Geoffrion proper optimality may converge to solutions having unbounded trade-offs. Therefore, we introduce a restricted notion of approximate Geoffrion proper optimality and prove that this restricted notion alleviates the problem of solutions having unbounded trade-offs. Furthermore, using a characterization based on infeasibility of a system of inequalities, we investigate two convergence properties of different approximate optimality notions in multicriteria optimization. These convergence properties are important for algorithmic reasons. The restricted notion of approximate Geoffrion proper optimality seems to be the only approximate optimality notion that shows favourable convergence properties. This notion bounds the trade-offs globally and can be used in multicriteria decision making algorithms as well. Due to these, it seems to be a practical optimality notion.

**Keywords**: Geoffrion proper optimality; multicriteria optimization; trade-offs; approximate solutions.

# On Superiority and Weak Stability of Population States

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We revisit some concepts of superiority and weak stability of population states in evolutionary games with continuous strategy space. We prove a general stability result for replicator trajectories by introducing the concept of superiority with respect to a given closed set. Some important results in the literature regarding weak stability are special cases of our main result.

Keywords: Evolutionary Games; Replicator Dynamics; Population State; Weak Stability

Consider a two-player symmetric evolutionary game with the pure strategy set $S$, which is a Polish space, and a bounded measurable payoff function $u : S \times S \to \mathbb{R}$. The game is symmetric in the sense that the role of the players is not important: that is, $u(z, w)$ is the payoff for the strategy $z \in S$ when played against the strategy $w \in S$.

Let $\Delta$ be the set of all probability measures defined on the measurable space $(S, \mathcal{B})$, where $\mathcal{B}$ denotes the Borel sigma-algebra on $S$. A population state of the evolutionary game $G = (S, u)$ is nothing but a probability measure on $(S, \mathcal{B})$, that is it is an element of the set $\Delta$. The average payoff of a population state $P$ against population state $Q$ is given by,

$$E(P, Q) = \int_S \int_S u(z, w)\ Q(dw)\ P(dz).$$

In this note, the closeness of the population states is studied using the weak topology.

We study the evolution of the population with respect to the replicator dynamics as described below ([4, 5]). The average success (or lack of success) of a strategy $z \in S$ against a population $Q \in \Delta$ is given by

$$\sigma(z, Q) := E(\delta_z, Q) - E(Q, Q).$$

The replicator dynamics is derived based on the idea that the relative increment in the frequency of strategies in a set $B \in \mathcal{B}$ is given by the average success of strategies in $B$. That is, for every $B \in \mathcal{B}$,

$$Q'(t)(B) = \frac{dQ(t)}{dt}(B) = \int_B \sigma(z, Q(t))\; Q(t)(dz) \tag{1}$$

where $Q(t)$ denotes the population state at time $t$.

The weak stability of population states has been studied in [5, 1, 2, 3]. In these references, the weak stability has been studied for polymorphic population states, whereas the result regarding weak stability of a general population state assumes its Lyapunov stability. We study the stability of a general population state without the assumption of Lyapunov stability. To this end we introduce the concept of superiority of a population state with respect to a closed set.

Let $P^* \in \Delta$ and $F$ be a closed subset of $S$ containing support of $P^*$. Before defining the concept of superiority with respect to $F$, we let $\Lambda(P^*, F)$ denote the set of all probability measures satisfying the following three conditions.

(i) $\mathrm{supp}(Q) \subseteq F$ ; (ii) $P^* \ll Q$; (iii) $\log\left(\frac{dP^*}{dQ}\right)$ is integrable with respect to $P^*$.

**Definition 1.** *The population state $P^*$ is said to be* superior with respect to $F$ *if for every $Q \in \Lambda(P^*, F)$, $Q \neq P^*$, $E(P^*, Q) > E(Q, Q)$.*

The following theorem presents our main result.

**Theorem 1.** *Let $P^* \in \Delta$ be a rest point of the replicator dynamics and let $F \subset S$ be closed. If $P^*$ is superior with respect to $F$, then the replicator dynamics trajectory $Q(t)$ converges weakly to $P^*$ whenever $Q(0) \in \Lambda(P^*, F)$.*

Note that, $P^*$ being neighbourhood superior is equivalent to $P^*$ being superior with respect to all $F$'s which are sufficiently close to support of $P^*$ when $P^*$ is polymorphic. In view of this, our main result can be considered as a generalization of the weak stability of polymorphic states proved in [1, Theroem 2].

In [6, Theorem 4.8], the authors prove that a global $\mathcal{ER}$ rest point of replicator dynamics is weakly attracting. This becomes a special case of Theorem 1 when we take $F = S$ as the superiority property with respect to the set becomes nothing but the global $\mathcal{ER}$ property (see also [2]).

Hence, Theorem 1 unifies various weak stability results in the literature, and provides a more general perspective.

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# On the split-rank of the facets for mixed-integer bilinear covering set and their effective separation

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Over the past several years many classes of valid inequalities or cuts have been introduced for solving mixed-integer optimization problems. Some important classes of valid cuts are disjunctive cuts, split cuts etc. In particular, it is known that any valid cut for the convex hull of the feasible set of a mixed-integer linear programming (MILP) is a disjunctive inequality [7]. Split cuts [1] are one of the most important subclass of disjunctive cuts and are equivalent to Gomory mixed-integer (GMI) cuts [6], lift and project cut, mixed-integer rounding (MIR) cut etc. which are also very important classes of valid inequalities. A detailed tutorial on these different types of cuts can be found in [2]. Given a valid cut, one natural question is about finding whether it is a split cut and to determine its rank. For disjunctive programming, answer to this question is quite important.

Most of the literature on disjunctive cuts and split cuts covers only mixed-integer linear programming [2]. Recently, some results are available for convex mixed-integer nonlinear programming (MINLP) and for non-convex MINLP. Belotti [5] has discussed some aspects of disjunctive cuts for non-convex MINLP.

In this article, we consider the valid inequalities for the following mixed-integer bilinear covering set and study them as disjunctive cuts and their split-rank.

$$S^{MI} = \left\{ (x, y) \in \mathbb{Z}_+^n \times \mathbb{R}_+^n : \sum_{i=1}^{n} x_i y_i \geq r \right\}, r > 0.$$

This set appears in the celebrated non-convex formulation of trim loss problem [3]. The set $S^{MI}$ is a non-convex set, even the continuous relaxation $S$ of $S^{MI}$ is a non-convex set for $n \geq 2$. The description of $conv\left(S^{MI}\right)$ is derived by Tawarmalani et al. [4] using orthogonal restrictions of $S^{MI}$. It consists of countably infinite number of facet defining inequalities.

We study the split-rank of the facet defining inequalities for $conv\left(S^{MI}\right)$. We derive the disjunctions from which the facet defining inequalities can be constructed. We also show that the maximum split-rank of the facet defining inequalities of the set $S^{MI}$ is $n$ and the minimum split-rank is one. To derive these results, we have analyzed the description of extreme points of the convex hull of the intersection of the set $S$ with some other inequalities on the variable $x$. We have solved some global optimization problems to show that the facet defining inequalities can be derived from some specific disjunctions. In addition to that, we define the separation problem for the facet defining inequalities and present an efficient, in fact, a linear time algorithm to solve the separation problem.

In our analysis, we solve some global optimization problems using the following theorem to prove our results.

**Theorem 1.** *Let us consider the set $S$ along with some additional linear constraints on the variables $x$. Let us call it $S_X$. Also let $n \geq 2$. The additional constraints can be box constraints or bounds or some other linear constraints on the variable $x$. Let $(\bar{x}, \bar{y})$ be an extreme point of the set conv $(S_X)$. Then there exists an index $t \in N$ such that $\bar{x}_t\bar{y}_t = r, y_i = 0, \forall i \in N, i \neq t$, where $N = \{1, \ldots, n\}$.*

We provide an alternative proof of validity of the inequalities derived by Tawarmalani et al. [4] for the set $S^{MI}$. We do it by specifying the disjunctions that give the inequalities. In order to show that an inequality $c^T x + d^T y \geq b$ is valid for a given disjunction $D$, we solve a global optimization problem of the following form:

$$\begin{aligned} \zeta = \min_{x,y} \; & c^T x + d^T y \\ \text{s.t. } & (x, y) \in D \end{aligned}$$

Now, if the optimal value $\zeta$ of the above problem is at least $b$, i.e. $\zeta \geq b$, then we can say that the inequality $c^T x + d^T y \geq b$ is valid for $D$.

Here $D$ is a non-convex set, which is formed by adding a linear inequality (that comes from the disjunction $D$) to the set $S$. We use Theorem 1 to solve the above global optimization problem and then show the validity of the inequalities for their respective disjunctions that we derived. In general global optimization problem is NP-Hard. But, in our case we show that the above global optimization problem can be solved efficiently because of its special structure.

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## Modified Projected Newton Scheme for Nonconvex Function with Box Constraints

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In this paper a descent line search scheme is proposed to find a local minimum point of a nonconvex objective function with box constraints. The idea ensures that the scheme escapes the saddle point and finally settles for a local minimum point. Positive definite scaling matrix for the scheme is formed through symmetric indefinite matrix factorization of the Hessian matrix of the objective function at each iteration. Numerical illustration is provided and global convergence of the scheme is also justified.

Keywords Projected Newton Scheme , nonconvex function, Local minimum point

## Interactive Fuzzy Goal Programming for Agricultural land allocation problem Under Uncertainity

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In this paper, we have considered an agricultural production planning problem with multiple objectives and uncertainty in parameters. Uncertainty in parameters is handled by the concept of fuzzy set theory by considering parameters as a trapezoidal fuzzy number. $\alpha$ – cut and ranking function approach are used to get the crisp value of the parameters. We used an Interactive Fuzzy Goal Programming (IFGP) approach with linear, exponential and hyperbolic membership functions, which focuses on maximizing the minimum membership values to determine the optimum profit related to the vegetable crop under uncertain environment. A numerical example is given to illustrate how the approach is used.

**Properties of generalized Nondifferentiable multiobjective variational control problems with generalized convexity with its applications**

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In this article, the generalized convex functions are extended to variational control problem. By utilizing the new concepts, we obtain sufficient optimality conditions and prove Wolfe type and Mond Weir type duality results for multiobjective variational control programming problem.

Keywords: Optimality, Duality, Support Function, Variational Programming

**Analysis of Randomized Maximal Independent Set Problem**

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In this paper, we investigate the, a polynomial-time algorithm which, for an arbitrary input graph G. An algorithm is be considered "efficient" if its running time is polynomial: $O(n^c)$ for some constant c, where n is the size of the input. The proposed algorithms construct a random maximal independent set of a network modeled as graph. The computation of a maximal independent set of minimum total size is NP-hard for general graphs. Its expected running time is $O(|E| d_{max}) = O(\log n)$ where $d_{max}$ is the maximum degree in the graph. The size of the MIS found by our algorithm is at most $|opt|(\ln(\Delta + 1) + 1)$. The approximation ratio of the algorithm is $(\ln(\Delta + 1) + 1)$.

## Nonsymmetric mixed-higher order multiobjective dual programs with support functions involving cone constraints

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. The theory of duality for nonlinear programs is well established and has been successful in advancing both the theory and practice of nonlinear programming. In principle, much of this broad framework can be extended to mixed duality for nonlinear programs, but this has proven difficult, in part because mixed duality theory does not integrate well with current computational practice. In this paper, a pair of higher order mixed type dual program is formulated for a nondifferentiable multiobjective programming problem involving support functions. Various duality results are obtained under higher order $(F, \alpha, \rho, d)$ type I convexity assumptions.

**Keywords** Nondifferentiable multiobjective programming · Mixed higher order duality · Support function · Cone constraints · Higher order $(F, \alpha, \rho, d)$ type I convexity

## Integer Quadratic Fractional Programming Problems with Bounded Variables

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Quadratic fractional programming is an important class of non-linear fractional programming problem in which objective function is a ratio of two quadratic functions subject to a set of linear constraints. These type of problems arise in many fields such as production planning, financial and corporative planning, health care and hospital planning.

Khurana [1] studied QFPP when some of its constraints are homogenous. Nejmaddin [2] solved quadratic fractional programming problem (QFPP) by Wolfe's method and modified simplex method. Tantawy [3] used feasible direction method to solve QFPP. While considering real-world applications of QFPP it may occur that one or more unknown variables are constrained by lower and upper bound conditions. Therefore, dealing with quadratic fractional programming problems with bounded variables becomes more important. Also, in many practical situations, one is interested in only the integer optimal solutions of QFPP. In addition to this, sometimes an optimal integer solution is required, which apart from satisfying primary constraints, also satisfies some secondary requirements. In such cases, one needs to rank and scan the integer solutions of the problem to select the one which suits best under existing limitations.
In this paper, an algorithm for solving quadratic fractional integer programming problems with bounded variables (QFIPBV) has been developed. The method provides complete ranking and scanning of the integer feasible solutions of QFIPBV by establishing the existence of a linear or a linear fractional function, which acts as a lower bound on the values of the objective function of QFIPBV over the entire feasible set. The method involves ranking and scanning of the set of optimal integer feasible solutions of the linear or linear fractional program so constructed, which can be done by the technique developed by Dahiya et al [4].

The QFIPBV problem considered in this paper can be mathematically formulated as:

$$\text{(QFIPBV)} \qquad \min_{x\in\Omega} f(x) = \frac{C^T x + x^T D x + \alpha}{E^T x + x^T F x + \beta}$$

$$\text{where } \Omega = \{x \in R^n | Ax = b,\ l \leq x \leq u \text{ and integer vector}\}$$

Let $\bar{\Omega} = \{x \in R^n | Ax = b,\ l \leq x \leq u\}$,
D and F are $n \times n$ symmetric matrices; C, E $\in R^n$; $\alpha, \beta \in R$; $A \in R^{m\times n}$; $b \in R^m$; $l, u \in R^n$ such that $l, u \geq 0$ and $E^T x + x^T F x + \beta > 0 \ \ \forall x \in \bar{\Omega}$.
To ensure that there are finite number of integer points in $\Omega$, it is assumed that the set $\bar{\Omega}$ is closed and bounded.
In this paper, we have proved the following results:

**Theorem 1.** *Given a function $f$ as in Problem* (QFIPBV), *there always exists a linear or linear fractional function $g(x)$ such that $g(x) \leq f(x) \quad \forall\, x \in \Omega$.*

**Remark 1.** The linear integer programming problem with bounded variables (LIPBV) or the linear integer fractional programming problem with bounded variables (LFIPBV), which provides lower bounds on the objective function values of the QFIPBV problem is as follows:

$$\text{(LIPBV) or (LFIPBV)} \quad \min_{x\in\Omega} g(x)$$

**Notations:**
$g_i = g(x_j^i)$, $x_j^i \in X_i$, the set of the $i^{th}$ best integer feasible solutions of LIPBV or LFIPBV (the $i^{th}$ best integer feasible solutions of LIPBV or LFIPBV can be obtained as explained by Dahiya et al. [4]).
Here $i = 1, 2, ..., N$ where $g_N = \max\ \{g(x) : \ x \in \Omega\}$
$T^r = \cup_{i=1}^r X_i, \ r = 1, 2, ..., N.$
$f_k =$ The $k^{th}$ best objective function value in QFIPBV.
$L_k =$ The set of the $k^{th}$ best integer feasible solutions of QFIPBV.
The following proposition explains how an optimal integer feasible solution of QFIPBV is obtained from LIPBV or LFIPBV.

**Proposition 1.** If $g_k \geq \min\{f(x) : x \in T^k\} = f(\hat{x})$ (say), then $\hat{x}$ is an optimal solution of QFIPBV.

**Corollary.** If $g_1 = \min\{f(x) : x \in T^1\} = f(\hat{x})$, then $\hat{x}$ is an optimal solution of QFIPBV and $\{x \in T^1 : f(x) = g_1\}$ is the set of the optimal solutions of QFIPBV.

**Remark 2.** If $g_k < \min\{f(x) : x \in T^k\}$, then $g_k < f_1 \leq \min\{f(x) : x \in T^{k+1}\}$. This gives the current lower and upper bounds on the optimal objective function value of QFIPBV.

**Proposition 2.** If $g_p \geq \min\{f(x) : f(x) > f_{k-1}, \ x \in T^p\} = f(x^*)$ (say), then $x^*$ is an one of the $kth$ best integer feasible solutions of QFIPBV.

**Remark 3.** If $g_k < \min\{f(x) : f(x) > f_{k-1}, x \in T^p\}$, then
$g_p < f_k \leq \min\{f(x) : f(x) > f_{k-1}, \ x \in T^{p+1}\}$
Similar to Remark 2, this gives the current bounds on the $k^{th}$ best value of the objective function in QFIPBV.

**Remark 4.** Suppose the set of the last best integer feasible solutions of LIPBV or LFIPBV is reached and $f_1, f_2, ...., f_t$ have been computed. Then the next best values of $f(x)$ are given by

$$f_{t+a} = \min\{f(x) : f(x) > f_{t+a-1}, \ x \in T^N\}, \ a \geq 1$$
$$\text{and} \qquad L_{t+a} = \{x \in T^N : f(x) = f_{t+a}\}, \ a \geq 1$$

Based on these results, we established the following algorithm for ranking and scanning in QFIPBV problem.

## Algorithm

**Initial Step:** Find $g(x)$ according to the given $f(x)$ as explained in the proof of theorem 2 and construct the linear integer programming problem with bounded variables (LIPBV) or linear fractional integer programming problem with bounded variables (LFIPBV). Search for the best solutions of QFIPBV.

**I(a):** Solve LIPBV or LFIPBV and find $X_1 = T^1$, the set of its optimal integer feasible solutions (Dahiya et al. [4]). Compute $g_1$ and $f(x)$, $x \in T^1$.
If $g_1 = \min\{f(x) : \ x \in T^1\} = f(\hat{x})$ (say), then $\hat{x}$ is an optimal solution of QFIPBV and the corresponding optimal value is $f(\hat{x})$.
$L_1 = \{x \in T^1 : \ f(x) = f(\hat{x})\}$.
If $g_1 < \min\{f(x) : \ x \in T^1\}$, then set $s = 2$ and go to I(b).

**I(b):** Find $X_s$ and $g_s$ $(s \geq 2)$ (see Dahiya et al. [4]).
If $g_s \geq \min\{f(x) : x \in T^s\} = f(\hat{x})$, say, then $\hat{x}$ is an optimal solution of QFIPBV and the corresponding optimal value is $f(\hat{x})$ (by proposition 1).
$L_1 = \{x \in T^s : \ f(x) = f(\hat{x})\}$
If $g_s < \min\{f(x) : \ x \in T^s\}$, repeat **I**(b) for the next higher value of $s$.

**General Step:** Search for the $k^{th}$ best solutions of QFIPBV, $k \geq 2$.

**G(a)** Find $X_s$ and $g_s$ $(s \geq 2)$.
If $g_s \geq \min\{f(x) :\ f(x) > f_{k-1},\ x \in T^s\} = f(x^*)$, say, then $x^*$ is the $k^{th}$ best integer feasible solution of QFIPBV and $f(x^*)$ is the $k^{th}$ best objective function value (by proposition 2).
$L_k = \{x \in T^s : f(x) = f(x^*)\}$
If $g_s < \min\{f(x) :\ f(x) > f_{k-1},\ x \in T^s\}$, repeat this step for the next higher value of $s$.

**Terminal Step:**

**T(a):** Suppose $g_N$ and $X_N$ are reached and $f_1, f_2, ..., f_t$ have already been computed. Then the next best values $f_{t+a}$ $(a \geq 1)$ of $f(x)$ in QFIPBV are given by

$$f_{t+a} = \min\{f(x) : f(x) > f_{t+a-1},\ x \in T^N\},\ a \geq 1$$
$$\text{and} \qquad L_{t+a} = \{x \in T^N : f(x) = f_{t+a}\},\ a \geq 1$$

Numerical illustrations are also included in the paper in support of the theory.

**Keywords:** Fractional programming, Linear fractional programming, Quadratic fractional Programming, Integer feasible solution, Ranking.

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## Scalarizations for a unified vector optimization problem based on the order representing and the order preserving properties

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The aim of this paper is to study the characterizations of minimal and approximate minimal solutions of a unified vector optimization problem via scalarizations which are based on general order representing and order preserving properties. We show that an existing nonlinear scalariation, using the Gerstwitz function, is a particular case of the proposed scalarization. Furthermore, in case of normed space, using the well known oriented distance function, characterizations of minimal solutions are established. Also, we show that the oriented distance function satisfies the order representing and the order preserving properties under suitable assumptions.

## Distribution System using Recourse Model

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The decision taken in stage 0 is called the *initial decision*, whereas decisions taken in succeeding stages are called *recourse decisions*. In multi-objective, multi-level optimization problems, there often exist conflicts (contradictions) between the different objectives to be optimized simultaneously. Two objective functions are said to be in conflict if the full satisfaction of one, results in only partial satisfaction of the other. Multistage decision making under uncertainty involves making optimal decisions for a *T*-stage horizon before uncertain events (random parameters) are revealed while trying to protect against unfavorable outcomes that could be observed in the future.

The main objective of this work is to present some contributions for building general Multi-stage Stochastic multi-criteria Decision making models. With the help of Numerical problems and LINGO software (can solve linear, nonlinear and integer multistage stochastic programming problems), we tried to prove that Recourse decisions provide latitude for obtaining improved overall solutions by realigning the initial decision with possible realizations of uncertainties in the best possible way as compare with the heuristic model for the decision making.

*Keywords:* Recourse decision, Multicriteria dynamic decision making, optimization, decision tree, LINGO

## Sufficient Optimality Conditions for Optimistic Bilevel Programming Problem Using Convexifactors

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In this paper, we establish sufficient optimality conditions for optimistic bilevel programming problem with convex lower-level problem using the concept of convexifactors. For that, at first we give the notions of $\partial^u -$asymptotic pseudoconvex, $\partial^u -$asymptotic quasiconvex functions in terms of convexifactors. After that we prove sufficient optimality conditions for the problem using above mentioned functions.

**Keywords.** Bilevel Programming Problem, Convexifactors, Value function, Sufficient Optimality Conditions.

# Farmer Centric Crop Insurance Model: A Game Theoretic Approach

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All over the world agriculture is synonymous with risk and uncertainty. Agriculture production is frequently affected by the natural calamities like droughts, floods, cyclones, hail storms, earthquakes etc. and the man-made disasters such as fire, sale of spurious seeds, fertilizers and pesticides, prices crash etc. All these events are beyond the control of the farmers and these events directly affect the financial position of farmers in terms of loss of production and loss of income. Agricultural insurance is considered an important mechanism to effectively address the risks to agriculture production and income resulting from these various natural events

Agriculture contributes to 17% of the India's Gross Domestic Product (GDP) and any change in its production has a multiplier effect on the Indian economy as a whole. The government of India started offering crop insurance in 1985, with the Comprehensive Crop Insurance Scheme (CCIS). The CCIS has been replaced by the National Agriculture Insurance Scheme (NAIS). The NAIS is considered to be an improvement over the CCIS, but it has simply replaced one flawed scheme with another slightly less flawed one. One of the major issue related to NAIS is reduction of insurance unit to Village Panchayat which causes difficulty to decide the insurance premium due to lack of past records of land surveys, ownerships, tenancy and yields at individual farm level. Other issues are unawareness of the farmers regarding NAIS, mandatory to loanee farmers, adverse selection in the case of non-loanee farmers, the area approach etc. Consequently, even after 23 years of existence of NAIS, less than one fifth of the farmers are insured in the country, with only a few notable exceptions like Rajasthan, where about 50% of the farmers/holdings are insured. The scheme got upgraded as Modified National Agriculture Insurance Scheme (MNAIS) subsequently and even a specialized insurance company named Agriculture Insurance Company (AIC) was established to carry out these schemes with the help of state governments and the designated banks. However the schemes failed to deliver the expected results. The farmer centric approach was completely missing and it remained a scheme to help the banks recover their outstanding loans partially.

Recently, Government of India launched Prime Minister's Crop Insurance Scheme. This new scheme is prepared on the five basic premises, i.e., coverage to all, enhanced coverage in terms of sum insured & perils, high subsidization, use of latest technology and participation of more insurance companies. However like in the past, the path is not going to be easy even for this new scheme. As this scheme has a high level of in built subsidy to be borne by Central & State Govt. It requires a full participation and commitment from the State Govt. too which looks difficult because of political & financial reasons. In addition, the success of the scheme is heavily based on its acceptance by the non-loanee farmers. The distribution set-up of the insurance companies empaneled for the scheme are not up to the desired level and it requires huge effort in infrastructure building. The proposed claim settlement process of the loss assessment by Govt. agencies could also lead to the dispute. There is a possibility of such agencies assessing claim looking at the political benefit rather than on the sound technical footing. In this scheme, relying only on the latest technology can create data error. Also, there has not been any solution provided for the division of the indemnity between the landowner and the cultivator, therefore cultivator will incur the loss and the land owner will get compensation (officially) in this scheme too.

All these schemes have not really proved a significant risk mitigation tool for the farmers in many regions. This provides the motivation to focus over the crop insurance contracts which can be in the benefit for farmers and recover the major drawbacks of the previous existing insurance schemes. There are numerous research work has been proposed but to the best of our knowledge no existing crop insurance practices are for farmers' benefit and not much research is directed towards this. Thus, it motivates us to address the crop insurance problem from the farmers' perspective.

This paper aims to develop a farmer centric crop insurance mathematical model. Suppose that there is a group of farmers. Some of the farmers form an insurance group called as a risk sharing group (RSG). At the starting of each time period, all the members of this RSG will transfer some fixed amount to this RSG fund. At the end of the current period, all the farmers will save some amount for the next period depending on the yield outcome of this period. Assume that in between of the starting and end point of the current period, their crop yield as well as income will be measured. It is obvious that at each time period, farmers may have different yield due to uncertainty involve in crop production and consequently, income will be considered as a random variable. Based on their income realization, the group of farmers is classified into two groups: one is a group of rich farmers and the other is of poor farmers. Therefore, at every period, the number of rich farmers will take different values, and the RSG fund will be divided among poor farmers.

During any period, each farmer will consume a part of their income and hence, has an immediate reward (utility) for the same. It should be noted that the rest amount will be saved and obtained in the next period with some interest. Thus, how much a farmer consumes in a period becomes the decision variable which will collectively affect the overall wealth eventually possessed by the group. This group of farmers needs to be robust with respect to individual deviations as well as with respect to deviations by its subgroups. Our objective is to find out the optimal strategy for the farmers to maximize their overall value of their consumption provided the stability of coalition of farmers. The research is work-in-progress and the completed research and results obtained will be reported elsewhere.

Keywords: Crop Insurance, Dynamic Programming, Coalition.

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## An Integrated analytical hierarchy process approach for COTS component selection

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This paper deals with COTS evaluation and selection for developing a modular software system under single application development task. We consider both quantitative and qualitative criteria which fulfills the specific needs of a software system. We use analytical hierarchy process (AHP) technique for evaluating the fitness of COTS components based upon various criteria and sub-criteria thereby providing overall score of each COTS component. We develop optimization models integrating AHP and multi-criteria decision making, which aim at: (i) maximize the total value of purchasing (TVP) subject to budget, compatibility, and reliability constraints, and (ii) maximize TVP and minimize the total cost of purchase simultaneously subject to compatibility and reliability constraints. The efficiency of the models is illustrated by means of numerical illustrations.

**Keywords:** Multi-criteria decision making; AHP; COTS component selection

**Railways Decision Making Problem on Special Trains in a Bilevel Programming Framework**

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The railways being mass service provider for domestic travelling, plan to run trains through places such that maximum number of travellers can avail its services. In some seasons or special occasions, like festivals or holidays, a comparatively much higher demand for travelling services are observed on some specific routes. To cater to this demand, railways introduce some special trains using some of its available unused resources. The railways also have to decide on the fares for such trains, which are generally higher than the regular train fares, so as to get higher profits which can further be utilized as funds to develop and improve the widespread infrastructure of railways network.

At the same time, other travelling service providers like, airlines, tourist buses, and taxis etc. also try to get a profitable share of the arisen additional demand of travellers. They also attempt to compete through prices with the railways and among themselves. This price competition between various competitors is settled as the Nash-equilibrium prices.

Consequently, the railways decide on the train routes, type of trains, number of coaches of different types within the train, to fulfil the demand share which it has received as a result of the price competition. This leaves the railways on a receiving end of its competitors.

We propose an optimization model for the decision making problem of railways on types of special trains on various routes, number of coaches in these trains and corresponding fare prices. We cater the reaction of the competitors in terms of their prices and the demand share. The model is developed as a bilevel mixed integer programming problem, placing the railways at leading end. We propose to solve the model using the generalized genetic algorithm.

*Keywords*: railways, special trains, Nash-equilibrium, bilevel programming, genetic algorithm

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## Higher order symmetric duality in multiobjective programming over cone-invex function

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In this paper we established weak, strong and converse duality results for a pair of multiobjective higher-order symmetric dual models with cone constraints under the assumption of higher-order cone-invexity. A nontrivial example has also been exemplified to show the existence of higher-order cone-invexity functions. Several known results are obtained as special cases.

Keywords : Higher-order symmetric duality; Duality theorems; Higher-order invexity/generalized invexity.

**Duality for multiobjective mathematical programming problems with equilibrium constraints**

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In this paper, we consider a multiobjective mathematical programming problem with equilibrium constraints (MMPEC). We formulate Mond-Weir type dual and Wolfe type dual models for the multiobjective mathematical programming problem with equilibrium constraints . We establish weak duality and strong duality results relating to the multiobjective mathematical programming problem with equilibrium constraints and the two dual models under convexity and generalized convexity assumptions.

**Inventory model for substitutable deteriorating items with stock out based substitution under joint replenishment**

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In this paper, we develop an inventory model to determine the optimal ordering quantities for a set of two substitutable deteriorating items. The inventory level of both items depleted due to demands and deterioration and when an item is out of stock, its demands are fulfilled by the other item and all unsatisfied demand is lost. Each substituted item incurs a cost of substitution and the demand and deterioration is considered as deterministic and constant. Both items are order jointly in each ordering cycle in order to take the advantages of joint replenishment for minimizing the total inventory cost. We determine the optimal ordering quantities of both product and derived a solution procedure that minimizes the total inventory cost. We also provide an extensive numerical and graphical analysis to illustrate the different parameter of the model. The key observation on the basis of numerical analysis,

there is substantial improvement in the total optimal cost of the inventory model with substitution over without substitution.

***Keywords*****:** Inventory control; Substitutable items; Cost of Substitution; Deterioration; Optimal ordering quantities; Joint replenishment.

## Multistage Stochastic Decision Making in Dynamic Multi-level

## Bi-criteria multi-choice assignment problem

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Owing to different capacities or efficiencies of machines there might be multiple cost and time parameters available for a single job-machine pair. The multi-choice cost and time coefficients help in incorporating marketing concepts such as differential pricing strategy to the assignment model. The present paper addresses this problem and models it as a bi-criteria multi-choice assignment problem, the two objectives being assignment cost and bottleneck time. The proposed methodology iteratively solves a multi-choice cost minimising assignment problem to obtain all Pareto-optimal time-cost pairs. Finally based on a multi-criteria decision making technique, namely, TOPSIS a single efficient pair is obtained.

Keywords Multi-choice programming, assignment problem, time-cost trade-off.

**Trust Region Levenberg-Marquardt Method for Support Vector Machines**

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Support Vector Machines (SVM) is an optimal margin based classification technique in Machine Learning. In this paper, we have proposed Trust Region Levenberg-Marquardt (TRLM) method as a novel problem solver for L2 regularized L2 loss (L2RL2) primal SVM classification problem. Levenberg-Marquardt (LM) method is an extension of Gauss-Newton method for solving least squares non-linear optimization problems and Trust-Region (TR) method is used for checking the acceptability of solution, if the solution lies in the trust region then the solution is accepted else solution is rejected. In LM method, LM parameter $\lambda$ is changed by an arbitrary factor but in TRLM instead of changing $\lambda$, the trust region radius $\triangle$ is changed. The proposed solver for L2RL2 primal SVM, performs well with medium sized problems. Experimental results establish TRLM as a solver for linear SVM as it performs at par and better in selective cases than existing state of the art solvers in terms of test accuracy.

**Keywords** Support Vector Machines · SVM · Trust Region Levenberg-Marquardt method · classification · optimization · least squares