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Maths 271: Mathematical Methods
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Practice Problems
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1. Find a basis for the nullspace of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix},$$

and verify that it is orthogonal to the row space of A. Given $x = (3, 3, 3)$, split it into a row space component x_r and a nullspace component x_n. That is, find $x_r \in \mathcal{R}(A')$ and $x_n \in \mathcal{N}(A)$ with $x = x_r + x_n$.

2. Find a matrix whose row space contains (1,2,1) and whose nullspace contains (1,-2,1), or prove that there is no such matrix.

3. Find all vectors which are orthogonal to (1,4,4,1) and (2,9,8,2).

4. True or False:

(a) $V \perp W \Rightarrow V^{\perp} \perp W^{\perp}$.

(b)$V \perp W$ and $W \perp Z \Rightarrow V \perp Z$.

5. What multiple of $a = (1, 1, 1)$ is closest to the point $b = (2, 4, 4)$?

6. The following system has no solution:

$$Xb = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix} = y.$$

Sketch and solve a straight line line fit that leads to the minimization of the quadratic $(b_1 - b_2 - 4)^2 + (b_1 - 5)^2 + (b_1 + b_2 - 9)^2$. What is the projection of y onto the column space of X?

7. If $P = P'P$, show that P is a projection matrix.

8. What subspace does the matrix $P = 0$ project onto?