# Climate Policy and Innovation in the Absence of Commitment*

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**Abstract**

We compare the effects of price and quantity instruments (an emissions tax and a quota with tradable permits) on the incentive to innovate to reduce the cost of an emission-free technology. We assume that the government cannot commit to the level of a policy instrument before R&D occurs, but sets the level to be socially optimal after the results of R&D are realized. The equivalence of price and quantity instruments in inducing innovation that is seen in end-of-pipe abatement models does not hold.

When the marginal cost of the dirty technology is constant, then a quota can induce R&D but a tax is completely ineffective. However, if the marginal cost function of the dirty technology is steep enough, then both a tax and a quota with tradable permits can induce R&D, and the tax will do so in a wider range of circumstances. Furthermore, in this case, an R&D subsidy may induce R&D and raise welfare whether a tax or a quota regime is in place.

*Keywords*: Climate, innovation, policy instruments, emissions tax, tradable permits, R&D, commitment.

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# 1 Introduction

The IPCC has made it clear that emission reduction is not enough to avoid dangerous climate change. Zero or negative GHG emissions will be required this century (Edenhofer et al. (2014), Chapter 6) to meet a 2-degree target. This puts the focus on climate policies that will induce R&D in *zero*-carbon technologies. Moreover, this has to be achieved in the absence of commitment by future governments to any given level of stringency in a policy. In this paper, we compare the effect of an emissions tax with that of an emissions quota with tradeable permits on a firm's incentive to conduct R&D in the absence of commitment by the government. We examine the conditions under which a subsidy to R&D can improve welfare when either of these instruments is in place.

While there is a considerable literature on the role of emission-reducing R&D, (for example, Kneese and Schulze (1975), Marin (1978), Downing and White (1986), Milliman and Prince (1989), Jung et al. (1996), Denicolo (1999), Innes and Bial (2002), Montero (2002), Amacher and Malik (2002), Fischer et al. (2003), Tarui and Polasky (2005), Kolstad (2010)), most of it concerns technologies that reduce the rate of emissions. This approach is suited to the study of end-of-pipe abatement technologies, or others where emissions rates can be reduced by changing the quality of fuel. But, as noted above, it is of limited applicability in studying carbon dioxide emissions, the most significant contributor to climate change. Of greater significance in the climate context are technologies that replace carbon-based fuels with an entirely different source of energy, such as solar, wind, or nuclear energy. In recent years, Montgomery and Smith (2007) studied the commitment problem in climate policy in a framework where innovation leads to development of zero-carbon technologies. They concluded that standard market-based environmental policy tools cannot create credible incentives for R&D. A crucial assumption in their paper was that the R&D sector is competitive. Thus, their negative result is a consequence of the non-appropriability of the returns from R&D. In our work, we assume a monopolistic R&D sector so that the returns from R&D are appropriable.[1] We obtain results that are much less pessimistic than Montgomery and Smith (2007).

An earlier paper by Laffont and Tirole (1996), also models a fall in the cost of an emission-free technology as a result of R&D. They assume that the marginal cost of the dirty technology is constant and point out that if the government can charge any price to pollute *ex-post*, then this undercuts the incentive to conduct R&D. (Our Proposition 1 is very close to their result.) They go on to consider the problems with committing to a pre-specified quota when the outcome of R&D is uncertain. They analyze the role of options to buy permits in this context. Our paper instead maintains the assumption of no pre-commitment to the level of

[1] This can serve as a benchmark for future models with more than one firm conducting R&D.

a policy, but compares tax and quota policies, and examines the role that R&D subsidies can play. We examine the increasing marginal cost case and show that this changes the first result – a pollution tax can induce R&D.

It is also instructive to compare our results with those of Denicolo (1999). Like us, Denicolo considers a monopolistic firm that decides how much to invest in R&D on the basis of its expectation about the level of an emissions tax or quota with tradeable permits. Denicolo assumes that the extent of emission reduction per unit of output is an increasing function of the amount invested in R&D but that the private marginal cost of producing a unit of output is unaffected by R&D. In contrast, we assume that R&D is used to reduce the marginal cost of zero-emission technologies. Our assumption is intended to model replacement technologies of the kind mentioned above, while his is better suited to modeling end-of-pipe abatement of a particular kind: one in which there is a sunk cost of abatement (the cost of R&D) but no variable cost of abatement. Denicolo shows that if the government sets the level of the emissions tax or aggregate quota to be optimal ex-post, that is, after the result of R&D is realized, then tax and quota policies are equivalent. They induce the same R&D. This result appears in end-of-pipe abatement models because the production technology is unchanged by R&D. In these models, R&D only shifts the marginal abatement cost curve. In contrast, we show that in our framework, taxes and quotas do not, in general, induce the same level of R&D. In fact, when the marginal cost of the dirty technology is constant, (as assumed by Denicolo (1999)), a tax can never induce R&D while a quota can do so. The underlying reason why our results are different is that in our model, *ex-post* there are two targets – emissions and total energy, or equivalently, dirty and clean energy, – but only one instrument available. In the standard model there is only one technology in use *ex-post* and one instrument is enough to deal with it.

An R&D subsidy in our model can be a direct transfer to a firm or any government expenditure that lowers the cost to the private sector of conducting R&D. For example, public-sector R&D that can be used by the private sector, or an increased supply of PhD's in relevant disciplines promoted by government funding. We ask when an R&D subsidy can improve welfare when either a tax or a quota with tradable permits is in place.

In Section 2 we lay out the structure of our model. In Section 3 we analyze the limiting special case of a constant marginal cost of dirty (emission-producing) energy. In Section 3.1, we show that an emissions tax is ineffective in inducing R&D. The reason for this is that a fall in the marginal cost of the emission-free technology as a result of R&D means that a lower tax is sufficient to allow the new technology to compete. Since a higher-than-necessary tax results in a welfare loss by giving the owner of the new technology monopoly power, the government reduces the emissions tax in response to successful R&D. This destroys the incentive to do R&D.

In Section 3.2 we examine the emissions quota with tradable permits. We show that the government will reduce the quota when the emission-free technology gets less expensive (as long as it remains more costly than the dirty alternative), because the cost of reducing emissions has fallen. This response induces R&D. Perhaps surprisingly, it is impossible for an R&D subsidy to improve welfare, and it may actually reduce it.

Since fossil fuels are subject to increasing marginal costs of production when harder to reach mineral deposits have to be extracted, it is, of course, more realistic to assume that the supply curve of dirty energy is upward-sloping. This is the case taken up in Section 4. We find that now both the tax and the quota can induce R&D, with the tax doing so in a wider range of circumstances. When the supply curve of dirty energy is sufficiently steep compared to the demand curve for energy, a subsidy to R&D can expand the range of parameter values under which R&D occurs and this can be welfare-improving. That is, a subsidy can induce R&D that would be too expensive to conduct with only the incentive of an emissions tax or a quota with tradable permits.

Thus, whether an R&D subsidy is welfare-improving depends both on the choice of instrument that is used *ex-post*, and on the shape of the cost curve of the dirty technology. In contrast, in an end-of-pipe abatement model, these considerations are irrelevant – an R&D subsidy is welfare-improving if the emissions price that yields the socially optimal level of emissions *ex-post* leads to an insufficient incentive to conduct R&D in the first place (Golombek et al. (2010)).

Section 5 concludes with some implications for further research.

## 2 The structure of the economy

There is a representative consumer who consumes two goods, energy ($e$) and the numeraire good ($y$). The consumer maximizes a quasi-linear utility function

$$U(e) + y = ae - \tfrac{b}{2}e^2 + y \tag{2.1}$$

subject to

$$Pe + y = Y, \tag{2.2}$$

where P is the price of energy and Y is the endowment with the consumer. Solving this problem gives the consumer's inverse demand function for energy

$$P = D^{-1}(e) = \begin{cases} a - be & \text{if} \quad e < \frac{a}{b} \\ 0 & \text{if} \quad e > \frac{a}{b} \end{cases} \tag{2.3}$$

So $b$ is the slope of the marginal social *benefit* of energy.

Energy in the economy can be produced in two ways. There is a competitive industry that produces dirty energy $e_d$, with a pollutant being emitted as a by-product.

$$\text{The marginal cost of producing } e_d = ce_d. \tag{2.4}$$

So $c$ denotes the slope of the dirty technology's marginal *cost*. In Section 3 we analyse the special case $c = 0$ when the private marginal cost of dirty energy is zero for all levels of production. The marginal cost of dirty energy when there is an emissions tax of $t$ is $t + ce_d$. When $c > 0$ the supply curve of dirty energy is

$$S_d(P) = \frac{P}{c}. \tag{2.5}$$

Energy can also be produced without any pollution emissions. The quantity of this green energy is denoted by $e_g$. The marginal cost of producing green energy depends on the research and development investment made by a monopolist in the period before production occurs. If $I$ is investment measured in units of the numeraire good, then the (constant) marginal cost of green energy that will be realized next period is $g = g(I)$ given by

$$g(I) = \begin{cases} \overline{g} - (\frac{I}{i})^{\frac{1}{2}} & \text{if } 0 \leq I < i\overline{g}^2 \\ 0 & \text{if } I \geq i\overline{g}^2 \quad \text{where} \quad i > 0. \end{cases} \tag{2.6}$$

Therefore,

$$g'(I) < 0, \quad g''(I) > 0, \quad g(0) = \overline{g} > 0 \tag{2.7}$$

Equation (2.6) can also be written as:

$$I : [0, \overline{g}] \to \mathbb{R}^+ \quad \text{where} \quad I(g) = i(\overline{g} - g)^2.$$

$\frac{1}{i}$ measures the impact of investment on the marginal cost of green energy. The lower the value of $i$, the more sensitive the marginal cost of green energy is to R&D investment.

Emissions produce an externality that is not internalized by the consumer. We choose units so that one unit of dirty energy produces one unit of emissions and we suppose that the damage from emissions is linear so that $e_d$ units of dirty energy result in an external damage of $\delta e_d$. Thus $\delta$ is the (constant) marginal *damage* of dirty energy.

The sequence of events in the model is as follows: The government inherits from the past the choice of policy instrument: tax or quota. It is assumed that it cannot change this. The government chooses a percentage subsidy for the firm's investment in research and development. Then the green firm chooses its investment in R&D. In the next period, as a result of the green firm's R&D, its marginal cost of production $g$ is realized. The government observes $g$ and then chooses the level of the quota or tax (as the case may be) with the objective of maximizing social welfare. We assume that in the first period the government cannot credibly commit to the level of the quota or to the tax rate it will impose in the second period. However, it *is* committed to the kind of instrument it has inherited, whether that is a tax or a quota. After observing the tax rate or the level of the quota, the green firm chooses its price and output.[2] [3]

In reality, we believe that the choice of quota or tax is made by governments on the basis of their usefulness in the current period. Governments are not looking half a decade, or even several decades ahead at the effects on the technologies that become available. Once this choice is made, an institutional infrastructure is locked in around it, so it is not easily reversible. On the other hand, the effective level of the tax or quota can be altered by future legislatures or governments that react to the prevailing conditions. This is the motivation for our assumptions above. Since we are interested in the effects of instruments on the incentive to innovate, we do not model production and emissions in the current period.

The green firm's profit net of investment in R&D is denoted by

$$\Pi = \pi - I$$

where $\pi$ denotes gross profit in the last stage of the game. Similarly, social welfare net of investment in R&D is denoted by

$$W = w - I$$

where $w$ is the gross social welfare that the government maximizes in the second stage of the game:

[2]Even with commitment, a single policy instrument will not be able to achieve the first best. The number of instruments required to achieve a vector of policy targets cannot be less than the number of elements in the vector. Since we have two targets: the level of abatement, given a marginal cost of abatement and the marginal cost of abatement itself, a single instrument is unable to achieve it (Tinbergen, 1964).
In Kolstad (2010), optimality is achieved as he assumes that policy targets abatement rather than the level of emissions, thus restricting the number of margins along which adjustment can take place.

[3]The owner of the patent for the green technology, could, of course, license it rather than engaging in production. This does not change the analysis in any way.

$$w = ae - \frac{b}{2}e^2 + Y - \delta e_d - \frac{c}{2}{e_d}^2 - ge_g. \tag{2.8}$$

We assume that in the absence of a green firm, it is socially optimal to produce a positive level of dirty energy, and that the initial marginal cost of the green firm is below the marginal social value of energy at $e = 0$. We also assume that the initial marginal cost of the green technology $\overline{g}$ is too high for it to be socially optimal to have any production of green energy.[4] These two assumptions can be written as:

**Assumption 2.1**

$$a > \overline{g} > P_d^*(\delta),$$

where

$$P_d^*(t) = \frac{ac + bt}{b + c} \tag{2.9}$$

denotes the equilibrium price of dirty energy when there is no green energy produced and there is an emissions tax of $t$.

It should be noted that in the special case when $c = 0$, that is, the supply curve of dirty energy is flat, then $P_d^*(\delta) = \delta$, so that Assumption 2.1 implies that $\overline{g} > \delta$. If $\overline{g} < \delta$, it would be optimal to use only the green technology. So the problem facing society would not be one of reducing emissions, but only that of making emission control less expensive.

The equilibrium quantity of dirty energy when there is no green sector and there is an emissions tax of $t$ is

$$e_d^*(t) \equiv S_d(P_d^*(t) - t) = \frac{a - t}{b + c}. \tag{2.10}$$

# 3 Horizontal Supply of Dirty Energy

Most papers in the literature make the assumption that $c = 0$, for example Denicolo (1999); **?**; Laffont and Tirole (1996). We start with this special case.

## 3.1 The Tax Regime

The government and the green firm play a sequential game with three stages,

1. The green firm chooses investment $I$ that results in a marginal cost $g$ of green energy.

[4] In Datta and Somanathan (2010) we show that the alternative assumption leads to qualitatively similar results.

2. The government chooses an emissions tax rate $t$.

3. The green firm chooses its price and output.

**Proposition 1** *If the supply curve of dirty energy is flat, then there will be no investment in research and development under the tax regime.*

*Proof*: Suppose the firm chooses $g > \delta$ (which can happen only if $\overline{g} > \delta$) in the first stage. Then the social marginal cost of green energy is greater than that of dirty energy. Thus the optimal tax is $\delta$, the difference between the social and private marginal costs of energy production. The green firm will not produce and so will incur a net loss with $\pi = -I(g) \leq 0$, where equality holds only when $g = \overline{g}$.

If $g \leq \delta$, then the optimal tax is infinitesimally greater than $g$. This is just sufficient to drive the dirty firms out of the market, but not enough to allow the green firm to exercise its monopoly power to restrict output. Now the green firm can only charge the tax, which is just infinitesimally greater than $g$. Thus the green firm incurs a loss of $-I(g) \leq 0$.

Therefore, the green firm must set $I = 0$ if it is to avoid a loss.

■

We remark that linearity of demand and of the damage from emissions are not required for this argument.

When dirty energy supply is flat, the optimal tax falls with $g$, wiping out the incentive to do R&D.[5] This is not the case in models of end-of-pipe abatement such as that of Denicolo (1999). *Ex-post* emissions taxation does not eliminate R&D in those models because they allow the innovating firm to choose the emission intensity of its technology from a continuum of possibilities. In equilibrium, the firm chooses an emission intensity far enough from zero so as to prevent the government from setting a very low emissions tax.

## 3.2 The Quota Regime

The government and the green firm play a sequential game with three stages,

1. The green firm chooses investment that results in a marginal cost $g$ of green energy.

[5] It can be shown that the decision of the green firm not to invest in R&D is the limiting case as $c$ gets close to zero. A proof is available from the authors on request.

2. The government choosing an emissions quota $q$.

3. The green firm chooses its price and output.

We begin with the third stage in which $g$ and $q$ have been chosen. Suppose $q \geq D(g)$ for some $g \geq 0$. Then the price of tradeable emissions permits will be $D^{-1}(q)$ and the dirty sector can supply energy at a price less than the green firm's cost. Thus the green energy firm will not produce. The price of energy will be $D^{-1}(q)$ and energy produced will be equal to the level of quota.

Now suppose $q < D(g)$ for $g \geq 0$. The green firm faces a residual demand curve of $D(P) - q$ for $P$ in the relevant range $D^{-1}(q) \geq P \geq 0$. It acts as a monopolist in this market and chooses $e_g$ to maximize

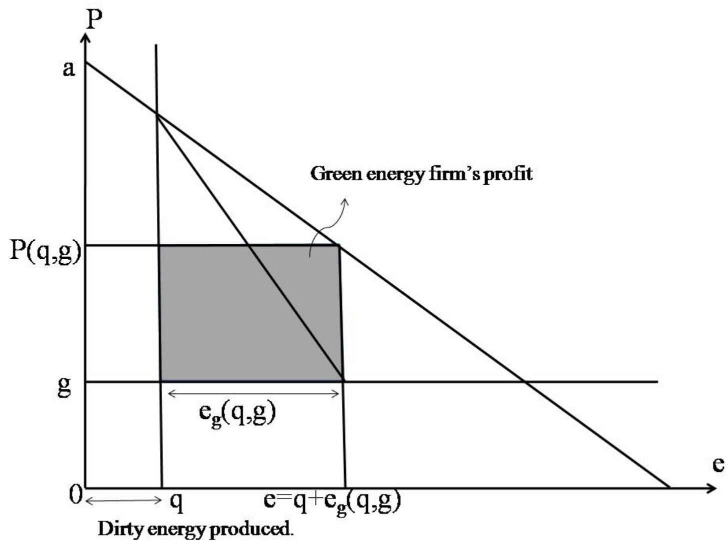


Figure 1: Quota Regime: $q < D(g)$

$$
\begin{aligned}
\pi &= e_g[D^{-1}(e_g + q) - g] \\
&= e_g[a - b(q + e_g) - g].
\end{aligned}
$$

$\pi$ is concave in $e_g$ and there is no corner solution. The monopoly price is the average of marginal cost and the highest point of the residual demand curve $D^{-1}(q)$. Thus the output of clean energy and total energy, the price of energy and the profit of the green firm are respectively:

$$
\begin{aligned}
e_g(g,q) &= \frac{1}{2}[D(g)-q] \\
&= \frac{a-bq-g}{2b}
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
e(g,q) &= \frac{1}{2}[D(g)+q] \\
&= \frac{a+bq-g}{2b}
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
P(g,q) &= \frac{1}{2}(D^{-1}(q)+g) \\
&= \frac{a-bq+g}{2}
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
\pi(g,q) &= \frac{1}{4}[D(g)-q][D^{-1}(q)-g] \\
&= \frac{(a-bq-g)^2}{4b}
\end{aligned}
\tag{3.4}
$$

Moving one step back in the game, we turn to the government's choice of $q$. Recall Assumption 2.1 which reduces to $\overline{g} > \delta$ since we are in the case $c = 0$. If $\overline{g} < \delta$, there would be no emissions problem, only a problem of making emission control less expensive.

If in Stage 1, the green firm had chosen $I$ so that $g > \delta$, the social marginal cost of green energy would be greater than that of dirty energy, and it would be inefficient to allow the green firm to operate. So the optimal $q \geq D(g)$. Optimality is attained at $q = D(\delta)$ where the marginal social cost of dirty energy equals the marginal social benefit from energy consumption.

Now consider the case $g < \delta$. Now the marginal social cost of green energy is lower than that of dirty energy. While a larger quota brings a welfare gain from from higher consumption of energy, it inflicts a welfare loss due to increased emissions. The government chooses the quota taking this tradeoff into account.

The net marginal social benefit of increasing the quota is

$$
\begin{aligned}
&\frac{\partial e}{\partial q}(P(q,g)-g)-(\delta-g) \\
&= \frac{1}{2}\left[\frac{1}{2}(D^{-1}(q)-g)\right]-(\delta-g).
\end{aligned}
\tag{3.5}
$$

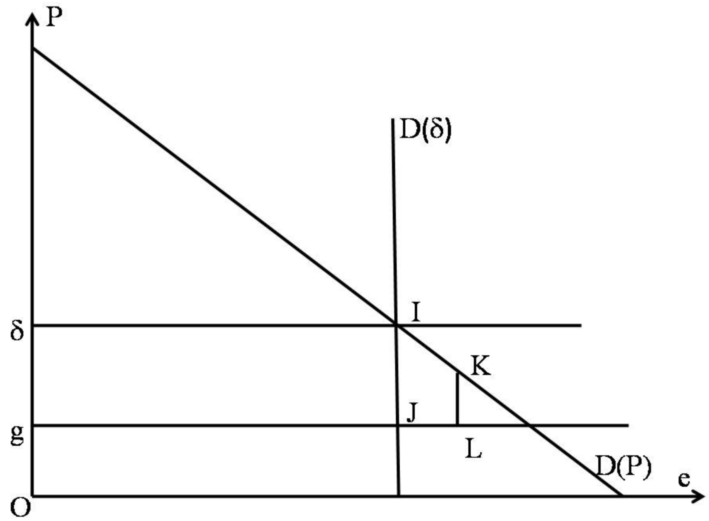


Figure 2: Quota Regime: Sub-optimality of $q = D(\delta)$ when $g^0 < g < \delta$

This is depicted in Figure 2 for $q = D(\delta)$. $\frac{1}{2}(D^{-1}(q) - g)$ (= KL in Figure 2) is the marginal gain in social surplus when energy consumption rises in response to the increase in $q$ while $\delta - g$ (= IJ in Figure 2) is the marginal increase in the social cost of energy as dirty energy replaces clean energy. The net marginal social benefit of increasing the quota is clearly negative at $q = D(\delta)$ and decreasing in $q$. Thus the optimal $q < D(\delta)$.

Setting the expression 3.5 equal to zero, we find that the optimal quota is given by

$$q(g) = \begin{cases} D(\delta), & \text{if} \quad g \geq \delta \\ \max\{\frac{a+3g-4\delta}{b}, 0\} & \text{if} \quad g < \delta \end{cases} \tag{3.6}$$

It is clear from this that if $g$ falls, then $q$ must also fall to restore equality (as long as $q$ remains positive). Thus, in contrast to the tax regime, a fall in $g$ induces a tightening of the emissions quota, thus reinforcing the incentive for the green firm to conduct R&D.

From now on, we ignore corner solutions in $q$ for the sake of simplicity. In other words, we assume that the externality from emissions is not high enough to justify setting a zero quota. It follows from 3.6 that the required assumption is

**Assumption 3.1** *$a > 4\delta$.*

Substituting 3.6 into 3.2 yields

**Remark 1** *Total energy consumption must fall when $g$ falls below $\delta$.*

As will be seen shortly, this fact has important implications for the welfare effects of an R&D subsidy.

We now turn to the first stage of the game, the optimal choice of investment (and marginal cost) by the green firm given the reaction function 3.6 of the government.

The green firm's net profit function is

$$\begin{aligned}\Pi(g) &= \pi(g, q(g)) - I(g) \\ &= \frac{4}{b}(\delta - g)^2 - i(g - \overline{g})^2 \qquad \text{(using 3.4 and 3.6)}, \\ &< 0 \qquad \text{at } g = \delta.\end{aligned}$$

Now

$$\begin{aligned}\Pi'(g) &= -\frac{8}{b}(\delta - g) - 2i(g - \overline{g}) \\ &> 0 \qquad \text{at } g = \delta.\end{aligned}$$

Unless the positive slope of $\Pi$ at $g = \delta$ is reversed at a lower value of $g$, investment in R&D is ruled out. Now

$$\Pi''(g) = \frac{8}{b} - 2i.$$

It follows immediately that R&D can take place only if $\Pi$ is convex and, therefore, if and only if $\Pi > 0$ at $g = 0$. This argument is summarized in

**Proposition 2** *The quota regime induces R&D with $g = 0$ provided the marginal cost of green energy is sufficiently sensitive to R&D investment, that is, if (and only if) $i < \frac{4\delta^2}{b\overline{g}^2}$.*

This is illustrated in Figure 3. Thus, provided investment in R&D is not too costly, the quota regime will induce R&D while the tax regime will not. This last statement will be true even with a non-linear demand function and a convex damage function. To see this, note that for any $g$ below the intersection of the demand and marginal damage functions, the government will always do better to set a binding quota at this intersection than to set a non-binding quota. The reason is that, compared to no policy, this unambiguously reduces dirty energy when its marginal damage is greater than its marginal benefit. But such a quota guarantees the green firm a positive gross profit, and hence, also a positive net profit if $i$ is sufficiently small.

## 3.3 R&D Subsidies and Social Welfare

We now ask whether a government subsidy for R&D would improve social welfare under each regime. When the rate of subsidy is $s$, the amount the green firm has to spend on R&D

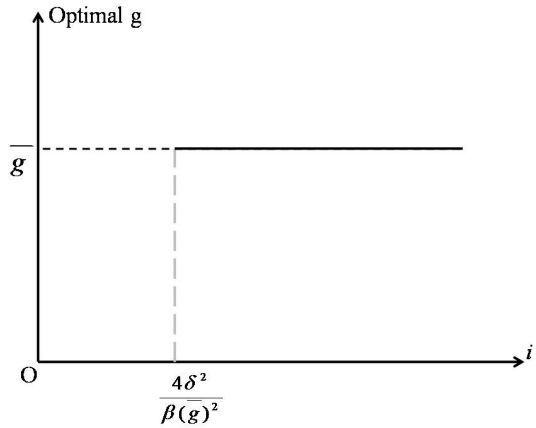


Figure 3: Optimal Choice of $g$ in the quota regime.

in order to achieve a marginal cost $g$ becomes $(1-s)i(g-\overline{g})^2$. Thus, a subsidy reduces the effective $i$ for the green firm. It can have no effect in a tax regime (as long as $s < 1$ which we assume), since any expenditure at all is sufficient to deter the firm from conducting R&D. In a quota regime, it is clear from Proposition 2 that it will have an effect if and only if it moves the effective $i$ for the firm below the threshold $\frac{4\delta^2}{b\overline{g}^2}$. At this threshold value of $i$, the firm is indifferent between conducting R&D and not doing so. That is $\Pi(0) = \Pi(\overline{g}) = 0$. Therefore, its gross profit if it conducts R&D, $\pi(0)$, must equal $i\overline{g}^2$, the social cost of R&D at the threshold level of $i$.

Hence, welfare will be raised by inducing the firm to conduct R&D if and only if the social return to R&D, $w(0) - w(\overline{g})$, exceeds $\pi(0)$, the private return to R&D.

These two quantities are easily compared in Figure 4. In drawing this figure with $e(0)$, the total energy supplied when $g = 0$, being less than the total energy supplied when $g = \overline{g}$, we make use of Remark 1. In Figure 4, $\pi(0)$ is the area of the rectangle $ADIH$. $w(0) - w(\overline{g})$ is the social surplus from dirty energy (area $aBE\delta$) plus the social surplus from green energy (area $BDIH$) less the social surplus from dirty energy when there is no R&D (area of $\triangle aG\delta$). This equals area $EFIH$ - the area of $\triangle DFG$ which is clearly less than $\pi(0)$. We conclude that

**Proposition 3** *In a tax regime, an R&D subsidy is ineffective (has no impact on R&D). Under a quota regime, an R&D subsidy is either ineffective, or, if effective, reduces welfare.*

It is clear from this argument that if $i$ is only slightly less than the threshold value given in Proposition 2, then R&D will occur under a quota regime despite it being welfare-reducing. This is a consequence of the government's inability to commit itself to not imposing a quota

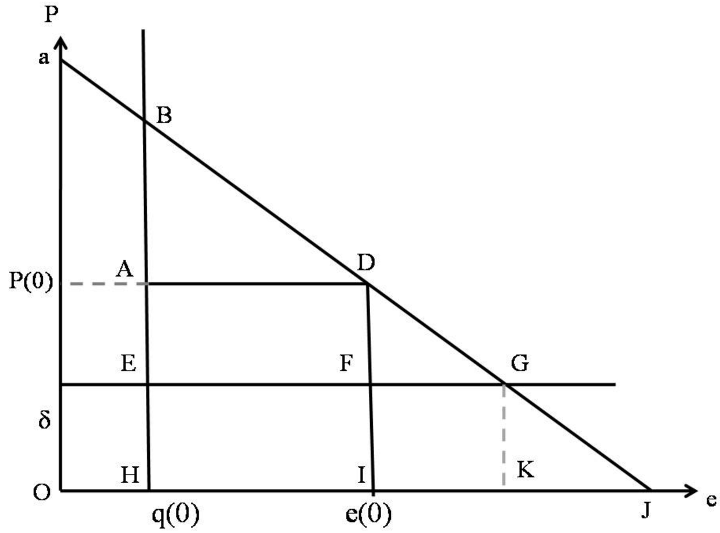


Figure 4: Welfare Analysis of Tax and Quota Regimes

once $g$ has been chosen. As discussed in the introduction, we believe this is realistic. If the government could commit to a particular $q(g)$ for each possible value of $g$, then many more outcomes become implementable. Exploring this further is beyond the scope of this paper.

Finally, one may ask whether social welfare is indeed higher under the quota regime than under the tax regime when R&D is sufficiently cheap (that is, $i$ is sufficiently small). The answer is yes and we record this as

**Proposition 4** *A quota regime that induces R&D results in higher welfare than a tax regime (that never induces R&D) provided the marginal cost of green energy is sufficiently sensitive to R&D investment, that is, if* $i < \frac{3}{2}\frac{\delta^2}{b\overline{g}^2}$. *If* $\frac{3}{2}\frac{\delta^2}{b\overline{g}^2} < i < \frac{4\delta^2}{b\overline{g}^2}$, *then welfare is lower in the quota regime than in the tax regime.*

*Proof*: Welfare is higher under the quota regime than under the tax regime iff the area $EFIH$ - the area of $\triangle DFG$ in Figure 4 is positive. It is easily checked using 3.1-3.3 and 3.6, that this is the case iff $i < \frac{3}{2}\frac{\delta^2}{b\overline{g}^2}$. ■

As an aside, if we supposed that the government is not committed even to the choice of instrument, then it is easy to see, looking at Figure 4, that it will always choose a tax if R&D occurs. But this, of course, would guarantee that R&D would not occur.

# 4 Steep Marginal Cost of Dirty Energy

We turn to the more realistic case when $c > 0$.

## 4.1 Tax Regime

In the previous section, we saw that if the green firm's marginal cost $g$ ever falls below the marginal social damage of emissions $\delta$, then it is optimal for the government to set a tax that eliminates the dirty sector. Whether or not to eliminate the dirty sector is an all-or-nothing choice in the tax regime when $c = 0$. As we will see in this section, with $c > 0$, the government can, by its choice of the tax rate, determine *how much* of the dirty sector will survive. In fact, for $c$ large enough, it is never optimal for the government to set a tax high enough to eliminate the dirty sector entirely. This is the case we now discuss. It is shown in Appendix A that the required assumption is

**Assumption 4.1** $c > \frac{b(2\delta+\sqrt{a\delta})}{(a-4\delta)}$.

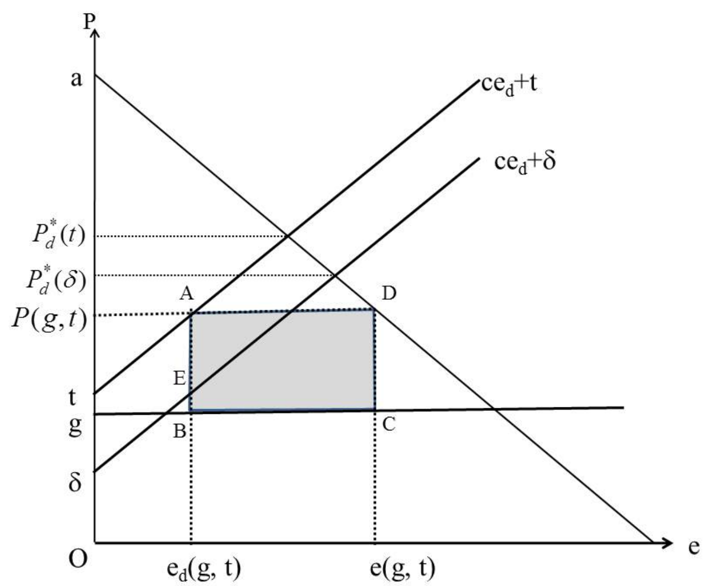


Figure 5: Price determination in a tax regime.

Referring to Figure 5, $P_d^*(\delta)$ is the price of energy that would prevail if there were no green firm and emissions were optimally taxed by setting $t = \delta$. Clearly, for any $g > P_d^*(\delta)$ the optimal tax is just $\delta$, and the green firm will not produce. So let us consider a value of $g < P_d^*(\delta)$ as in the figure. Suppose the tax is then set at some $t$ as in the figure.

The green firm can get a positive market share by pricing its energy anywhere between $g$ and $P_d^*(t)$. For any given price $P$, the dirty sector produces $\frac{P-t}{c}$ denoted by $e_d(g,t)$ in Figure 5, while the remaining demand is served by the green firm. While a higher price ensures a higher profit per unit of green energy produced, it reduces the green energy produced. The green firm balances these two effects and chooses the profit-maximizing price $P(g,t)$, which (it is easy to show) is the average of $g$ and $P_d^*(t)$. Green energy produced is $e(g,t) - e_d(g,t)$. The green firm's gross profit $\pi$ is shown by the rectangle shaded in grey.

Turning to the government's choice of $t$, we note that raising the tax results in a higher energy price. This shrinks total energy consumption and so also consumer surplus from energy consumption. However, a higher tax also results in less dirty energy produced.[6] The government sets the optimal tax by trading off these two effects. Unlike in Section 3 in which the optimal tax chases $g$ downwards, it can be shown that here the government's optimal tax actually increases as $g$ falls.

One can now see why R&D in the tax regime can occur in equilibrium. Suppose the green firm chooses $g < P_d^*(0)$. The worst possible tax from the green firm's point of view is $t = 0$. But even this will result in a positive gross profit. So if $i$ is low enough, this will mean a positive profit net of the cost of R&D. Of course, when the tax is set to maximize social welfare, R&D will occur for a larger range of $i$. Clearly, this conclusion does not depend on the linearity of marginal benefit, marginal cost, and damage functions.

**Proposition 5** *In the tax regime with c large enough (Assumption 4.1 holds), the marginal cost chosen by the green firm is*

$$g = \begin{cases} 0, & \text{if} \quad i \leq i_T, \\ \overline{g}, & \text{if} \quad i \geq i_T, \end{cases} \tag{4.1}$$

*where* $i_T \equiv \frac{(b+2c)^2(ac+b\delta)^2}{bc\overline{g}^2(b+c)(b+4c)^2}$.

*Proof*: In Appendix A. ■

Unlike in the case $c = 0$, the green firm will undertake R&D if its cost of doing so is not too large. The steepness of the dirty sector supply curve generates monopoly power for the green firm, thus creating the incentive for R&D when it is inexpensive to conduct ($i$ is low enough).[7] As explained above, the policy response re-inforces this incentive. Figure 6 shows

[6] $e_d(g,t) = \frac{P-t}{c}$ and $P$ rises at a slower rate than $t$. This can be seen from Figure 5. Consider a tax increase from $\delta$ to $t$ as shown. Then $P_d^*(t) - P_d^*(\delta)$ is less than the tax increase, and since $P$ is the average of $P_d^*(t)$ and $g$, it rises at half the rate that $P_d^*(t)$ does.

[7] Reversing Assumption 2.1 that $\overline{g} > P_d^*(\delta)$ implies a smoothly increasing $g(i)$ function for $i$ large enough (see Proposition 9 in Datta and Somanathan (2010)).

the optimal choice of $g$ by the green firm.

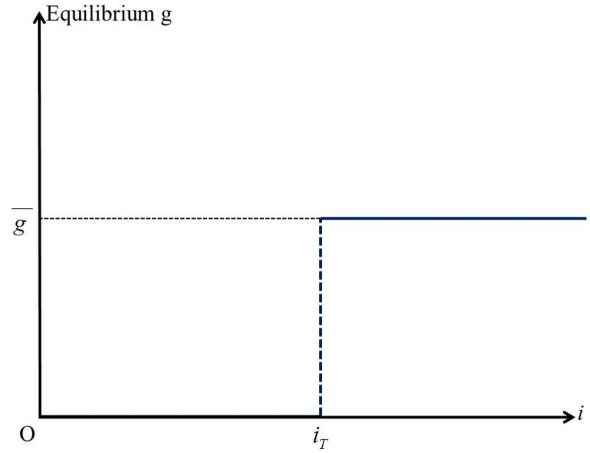


Figure 6: Equilibrium green marginal cost $g$ as a function of $i$ in the tax regime.

Note that in the equilibrium, emissions and energy output are not at their socially optimal levels *ex-post*, that is, after the result of R&D is realized. The reason is that the government has only a single instrument at its disposal to meet two targets, emissions and energy output, or equivalently, dirty and green energy. This is in contrast to the many end-of-pipe abatement models from Downing and White (1986) onwards in which emissions are at their first-best level when the government sets the emissions tax *ex-post*. This is because the marginal cost curve of the lower-emission technology in the end-of-pipe abatement model is simply that of the old technology minus a (tax-dependent) constant. So only the low-emission technology is used in equilibrium, and hence one instrument is sufficient to achieve the first-best. In the end-of-pipe model, the new technology strictly dominates the old one; given any positive emissions price, it costs less than the old technology at every level of output. It is, of course, the same technology with a lower emissions intensity. It is this fact that also generates the equivalence of tax and quota regimes in these models since either can be used to generate the emissions price that delivers the desired level of output. In our model, the green technology has a different marginal cost curve since it is not dependent on fossil resources.

We now turn to the role of a subsidy to R&D. As seen in Section 3.3, it can have no effect when the marginal cost of the dirty sector is flat ($c = 0$), since any expenditure at all is sufficient to deter the firm from conducting R&D. However, under Assumption 4.1 ($c > \frac{b(2\delta+\sqrt{a\delta})}{(a-4\delta)}$), when $i > i_T$, a large enough subsidy can reduce the effective $i$, i.e. $(1-s)i$ below $i_T$. In this case, a subsidy to R&D is effective in inducing R&D. Under what conditions will it improve welfare? The next proposition shows that it will do so when the supply curve of dirty energy is steep enough and R&D is not too expensive.

**Proposition 6** *In the tax regime, when Assumption 4.1 is satisfied, then an R&D subsidy will induce investment* and *increase welfare if and only if* $c > \frac{1+\sqrt{17}}{8} b$ *and* $i \in [i_T, \frac{(b+3c)(ac+b\delta)^2}{2bc\bar{g}^2(b+c)(b+4c)}]$.

*Proof*: See Appendix B. ■

We have assumed all along that the government cannot commit to a tax rate before the firm conducts R&D. It is now easy to see that if such a commitment were possible, then it would be welfare-improving under conditions similar to those required for an R&D subsidy to be welfare-improving under no commitment. Suppose Assumption 4.1 is satisfied, and that $c > \frac{1+\sqrt{17}}{8} b$. When $i$ is only slightly greater than $i_T$, the firm needs only a small increase in prospective profit after R&D to push it over the threshold and induce it to conduct R&D. Thus, by committing to a tax just a little higher than the optimal tax under no commitment, the government could push the firm over the threshold, induce it to conduct R&D, and improve welfare. Note, however, that this option is dominated by the no commitment case if the government avails of the option of subsidizing R&D. This is, of course, because changing the tax from its *ex-post* optimal level inflicts a welfare loss that is avoided by using the R&D subsidy instead.[8]

## 4.2 Quota Regime

The analysis of the quota regime when $c > 0$ is qualitatively similar to that in the case $c = 0$ except in two respects. First, when the supply curve of dirty energy is upward-sloping, it is possible for the green firm to make a positive gross profit by choosing $g < P_d^*(0)$ even if there is no quota, or equivalently, if the quota is set high enough to be non-binding. Hence, it can make a profit net of the cost of R&D if $i$ is small enough.

Second, it is now possible for an R&D subsidy to *increase* welfare by inducing R&D when it would otherwise have not occurred. This is true when $c$ is sufficiently large.

**Proposition 7** *If* $c > \frac{5}{4} b$, *then there exists a range of* $i$ *for which an R&D subsidy is welfare-improving in the quota regime. The maximum value of* $i$ *for which R&D takes place in the quota regime is less than* $i_T$. *For large enough* $c$ *(that is, when Assumption 4.1 holds), the tax regime induces at least as much R&D as the quota regime for every value of* $i$,.

[8]If the government could commit to any tax rate for each possible $g$, then it could induce the optimal $g$ and *ex-post* optimal $t$ by the use of a trigger strategy. If the firm were to choose any $g$ other than the one desired by the government, it would face a zero tax. We do not believe this is an interesting model.

*Proof*: See Appendix C. ■

Figure 7 illustrates the set of $(c, i)$ pairs (the green region) for which the tax regime delivers higher welfare than the quota regime, when there is no R&D subsidy. In this simulation, the other parameters have been set at the following values: $a = 100, b = 1, \overline{g} = 75, \delta = 10$. For these parameter values, the tax regime not only induces R&D for a larger set of $(c, i)$ combinations, it also delivers higher welfare for a larger set of $(c, i)$ pairs than the quota regime. The threshold value of $c$ above which Assumption 4.1 holds is about 0.86.

In the white region, no R&D investment is made in either of the two regimes, so that welfare is equal in the two. Below the black curve, R&D occurs in the tax regime, and below the red curve, it occurs in the quota regime. The black curve passes through the origin illustrating Proposition 1 that there can be no R & D in the tax regime when $c = 0$. The red line has a positive vertical intercept in accordance with Proposition 2 which states that R&D occurs in the quota regime in the flat supply case if $i$ is small enough.

The region shaded in blue refers to $(c, i)$ pairs for which welfare is higher in the quota regime than in the tax regime. It should be noted that the vertical axis is shaded in blue close to the origin, but not up to the cutoff value above which there is no R&D in the quota regime. This illustrates Proposition 4 — in the flat supply case, R&D in the quota regime lowers welfare unless the marginal cost of green energy is sufficiently sensitive to R&D investment.

## 5 Conclusion

Technological innovation in the energy sector is clearly of central importance in any strategy to avoid too much climatic change. In this respect, the climate problem is distinct from many environmental problems in that it is probably more feasible to replace existing technologies entirely than to reduce their emission intensity. Accordingly, we have departed from most of the literature on innovation in environmental economics and modeled the incentive to conduct R&D to lower the cost of such replacements. We have done this in a context in which the government is unable to commit to the future level of any policy instrument (although it is committed to the choice of instrument). This is quite a realistic assumption, given the fairly long delay to be expected between the decision to conduct R&D and the arrival of the resulting technology in the market. We consider a single innovator. This model can be thought of as a benchmark from which various extensions with more than one innovator can be explored in future research.

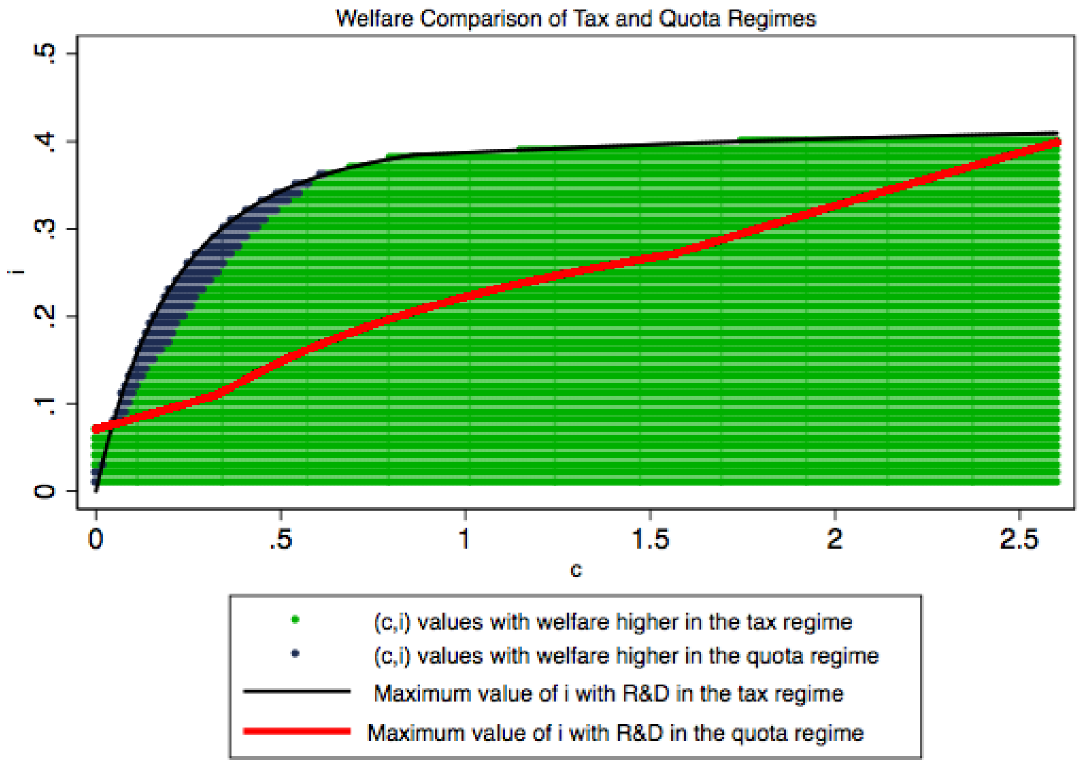


Figure 7: Tax and quota regimes compared.

We find that when the slope of marginal cost $c$ of the dirty technology is zero, then an emissions tax can never induce R&D because the innovator's profit is wiped out by the tax being reduced to the level of the innovator's marginal cost. A tax can be effective in inducing R&D only if $c$ is positive so that the innovator has some monopoly power *ex-post*. Since an emissions quota with tradeable permits does give the innovator monopoly power, it can induce R&D even for $c = 0$. However, for large enough $c$, a tax may induce R&D in circumstances in which a quota will not. This can happen when it is somewhat costly to use R&D expenditure to lower the marginal cost $g$ of the green technology.

Our results differ dramatically from those in end-of-pipe abatement models. In those models, the innovator chooses the reduction in emissions intensity of the dirty technology by investing in R&D. In the flat supply case, this means that the innovator will always choose an emissions intensity far enough from zero to ensure that the socially optimal tax is bounded away from zero. Thus, R&D will occur in the tax regime. Further, the fact that the new technology strictly dominates the old technology means that only the new technology will be used. Therefore, *ex-post*, the government has only one target to meet with one instrument.

It can, therefore, implement the first-best conditional on the existence of the new technology, and either a price or a quantity instrument will do. In our framework, since there are two technologies with different cost curves, there are two targets – emissions and energy output, and only one instrument available *ex-post*. Thus, the first-best cannot be implemented and the instruments are not equivalent.

We have shown that when the slope of marginal cost $c$ of the dirty technology is large enough relative to the slope of inverse demand $b$, subsidies to R&D can be welfare-improving in conjunction with either a tax or a quota.

There are two factors not considered in this paper that strengthen the case for R&D subsidies. First, since increasing marginal extraction costs in fossil fuel industries give rise to rents, it is to be expected that rentiers will lobby to protect their rents. This introduces uncertainty about whether there will be any climate policy when the results of R&D are realized. Datta and Somanathan (2010) show that in the presence of such uncertainty, a subsidy to R&D, because it takes effect in the present rather than the future, becomes a more attractive policy instrument. Second, since this paper was focusing on the environmental externality, the externalities from research and development were not modeled. Standard theory suggests that taking this public-good nature of R&D into account makes R&D subsidies more attractive, for example, in the form of basic research that lowers the innovator's cost of research.

The paper has considered only a single final good - energy. There are, of course, several forms of energy services that are sometimes complementary and sometimes substitutes. In this context, the game between innovators during the R&D stage and market structure in the output markets remains to be studied. The role of R&D spillovers between innovators in green technologies is another area for further research. Since the welfare implications of the choice between a tax and a quota regime is related to the extent of market power the innovator gets, allowing for oligopoly may have implications for this choice. Finally, careful modeling of political economy and lobbying in the context of rents in both dirty and green industries would be interesting.

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# APPENDICES

## A Proof of Proposition 5.

Suppose the choice of R & D investment has been made by the green firm. If $g > P_d^*(\delta)$, the government chooses a tax equal to the marginal damage from emissions ($t = \delta$) and allows only the dirty sector to operate. Green energy is too costly to be produced.

Now suppose $g < P_d^*(\delta)$. Let $t_L(g)$ be the lowest (non-negative) tax that keeps the green firm viable. The government never chooses a tax less than $t_L(g)$ for the following reason. A reduction in the tax from $t_L(g)$ increases the production of energy (all of which is dirty energy) which is already higher than what is optimal.[9] Let $t_H(g)$ be the highest tax rate that lets the dirty sector survive. It is never optimal for the government to choose a tax above $t_H(g)$. This is because choosing $t > t_H(g)$ raises the price of energy, and so reduces energy consumption and consumer surplus, but is unable to achieve any welfare gain through reduced emissions since emissions are already zero. Thus when $g < P_d^*(\delta)$, the government's optimal tax must lie in the interval $[t_L, t_H]$.

It can be shown that for $g \in [0, P_d^*(\delta))$ and $t \in [t_L(g), t_H(g))$, the optimal choice of price, energy consumption and gross profit by the firm is given by the following equations:

$$P(g,t) = \frac{1}{2}[g + P_d^*(t)], \tag{A.1}$$

$$e(g,t) = \frac{1}{2}[e_d^*(t) + D(g)], \tag{A.2}$$

$$e_d(g,t) = \frac{1}{2}[e_d^*(t) + S_d(g-t)], \tag{A.3}$$

$$e_g(g,t) = \frac{1}{2}[D(g) - S_d(g-t)], \tag{A.4}$$

$$\pi(g,t) = \frac{b+c}{4bc}[P_d^*(t) - g]^2, \tag{A.5}$$

where $S_d(P)$ denotes the supply curve of the dirty sector in the absence of a tax.

[9] This is because, by definition, $t_L(g)$ must be less than $\delta$.

Substituting A.3 and A.4 in 2.8, we obtain an expression for gross welfare in terms of $t$. It is not optimal for the government to choose $t = t_H$ if

$$\begin{aligned} \left.\frac{\partial w}{\partial t}\right|_{t=t_H} &< 0 \\ \text{or} \quad g &< \frac{\delta(b+2c)^2 - ac^2}{(b+c)(b+3c)}. \end{aligned} \tag{A.6}$$

We define $g_H \equiv \frac{\delta(b+2c)^2-ac^2}{(b+c)(b+3c)}$. Assumption 4.1 ensures that $g_H < 0$. Thus $t_H$ cannot be an optimal tax.

The government chooses the optimal $t$ by equating the social marginal benefit (SMB) and social marginal cost (SMC) of changing the tax by a unit.

$$\begin{aligned} \text{The SMB of increasing the tax} &= |\frac{\partial e_d}{\partial t}| * \text{SMB of replacing dirty with clean energy} \\ &= |\frac{\partial e_d}{\partial t}| * (\delta + ce_d - g) \\ &= |\frac{\partial e_d}{\partial t}| * EB \quad \text{(refer to Figure 5)}, \\ \text{and the SMC of increasing the tax} &= |\frac{\partial e}{\partial t}| * \text{SMC of reducing energy consumption} \\ &= |\frac{\partial e}{\partial t}| * (a - be - g) \\ &= |\frac{\partial e}{\partial t}| * DC \quad \text{(refer to Figure 5)}. \end{aligned}$$

From A.2 and A.3, we see that $|\frac{\partial e_d}{\partial t}| > |\frac{\partial e}{\partial t}|$. Thus, to equate the marginal benefit and cost of the tax, the marginal social benefit from replacing dirty with clean energy must be less than the marginal social cost of reducing energy consumption. Since $t_L < \delta$, this shows, looking at Figure 5, that $t_L$ cannot be an optimal tax. Hence, the optimal tax is in $(t_L, t_H)$ and satisfies:

$$(a - be)\frac{\partial e}{\partial t} = (\delta + ce_d)\frac{\partial e_d}{\partial t} + g\frac{\partial e_g}{\partial t}, \tag{A.7}$$

where

$$\begin{aligned} \frac{\partial e_d}{\partial t} &= -(\frac{1}{2(b+c)} + \frac{1}{2c}), \\ \frac{\partial e_g}{\partial t} &= \frac{1}{2c}, \\ \frac{\partial e_d}{\partial t} + \frac{\partial e_g}{\partial t} &= \frac{\partial e}{\partial t}. \end{aligned}$$

From these expressions we obtain the government's reaction function:

$$t(g) = \begin{cases} \frac{ac-(b+c)g+2\delta(b+2c)}{b+4c}, & \text{if} \quad g \in [0, P_d^*(\delta)], \\ \delta, & \text{if} \quad g > P_d^*(\delta). \end{cases} \tag{A.8}$$

The green firm chooses its level of R & D by taking the government's reaction function as given when maximizing net profit. The firm never chooses $g \in [P_d^*(\delta), \overline{g})$ as this choice would not allow it to operate in the market in the second period. In the range $[0, P_d^*(\delta))$, the net profit function is obtained by sequentially substituting the firm's second-period reaction function and the government's reaction function into the profit function A.5 and subtracting the cost of investment in R & D:

$$\Pi(g) = \frac{b+c}{4bc}\left[\frac{ac+bt(g)}{b+c} - g\right]^2 - i(\overline{g}-g)^2. \tag{A.9}$$

Differentiating A.9 with respect to $g$,

$$\frac{d\Pi(g)}{dg} = \frac{(b+c)}{2bc}\left[\frac{ac+bt}{b+c} - g\right]\left[\frac{b}{b+c}\frac{dt(g)}{dg} - 1\right] + 2i(\overline{g}-g), \tag{A.10}$$

$$\text{where} \quad \frac{dt(g)}{dg} = -\frac{b+c}{b+4c}.$$

Substituting, we get:

$$\frac{d\Pi(g)}{dg} = -\frac{2(b+c)(b+2c)^2}{bc(b+4c)^2}[P_d^*(\delta) - g] + 2i(\overline{g}-g), \tag{A.11}$$

$$\text{and} \quad \frac{d^2\Pi(g)}{dg^2} = \frac{2(b+c)(b+2c)^2}{bc(b+4c)^2} - 2i. \tag{A.12}$$

Note that:

$$\left.\frac{d\Pi(g)}{dg}\right|_{g=P_d^*(\delta)} > 0. \tag{A.13}$$

If the net profit curve is concave, inequality (A.13) ensures that in the range $[0, P_d^*(\delta)]$, $g = P_d^*(\delta)$ gives the lowest net loss. Thus, when $\Pi(g)$ is concave, the only choice of $g$ that can give non-negative net profits is $\overline{g}$. Thus $\overline{g}$ is the global optimum.

If the net profit curve is convex, then in the range $[0, P_d^*(\delta)]$, there are two candidates for a maximum: 0 and $P_d^*(\delta)$. We know that at $g = P_d^*(\delta)$, net profit is negative. Thus, we are left with just two candidates for maximum: 0 and $\overline{g}$.

$\overline{g}$ is preferred iff $i$ is greater than the level of $i$ at which

$$\begin{aligned} \Pi(0) &= 0, \\ \text{or,} \quad i &= \frac{(b+2c)^2(ac+b\delta)^2}{bc\overline{g}^2(b+c)(b+4c)^2} && \text{(A.14)} \\ &\equiv i_T \quad \text{(say)}. && \text{(A.15)} \end{aligned}$$

Hence, in equilibrium,

$$g = \begin{cases} 0, & \text{if} \quad i \leq i_T \\ \overline{g}, & \text{if} \quad i \geq i_T. \end{cases} \tag{A.16}$$

# B Proof of Proposition 6.

By Proposition 5, at $i = i_T$, $\Pi(\overline{g}) = \Pi(0)$, so $\pi(0) = i\overline{g}^2$. Therefore, a welfare-improving R&D subsidy will exist for $i$ greater than and sufficiently close to $i_T$ if $W(0) - W(\overline{g}) > 0$ at $i = i_T$. The last condition will be met if, at $i = i_T$,

$$\begin{aligned} w(0) - i\overline{g}^2 - w(\overline{g}) &> 0, \\ \text{or} \quad w(0) - w(\overline{g}) &> \pi(0). \end{aligned}$$

To find the conditions under which this will be true, first note that by A.8,

$$t(0) = \frac{ac + 2\delta(b+2c)}{(b+4c)} > \delta,$$

and by A.1 and 2.9,

$$\begin{aligned} P(0, t(0)) &= \frac{(ac+b\delta)(b+2c)}{(b+c)(b+4c)} \\ &= \frac{(b+2c)}{(b+4c)} P_d^*(\delta) \\ &< P_d^*(\delta). \end{aligned}$$

This situation is depicted in Figure 8. $\pi(0)$ is given by the rectangle $EGLK$. Gross welfare when there is R & D and $g = 0$ is

$$w(0) = ADJC + DKLG$$

If there is no R & D, $w(\overline{g}) = \triangle ABC$. Therefore,

$$w(0) - w(\overline{g}) = JKLGB \quad \text{(shaded in grey)}.$$

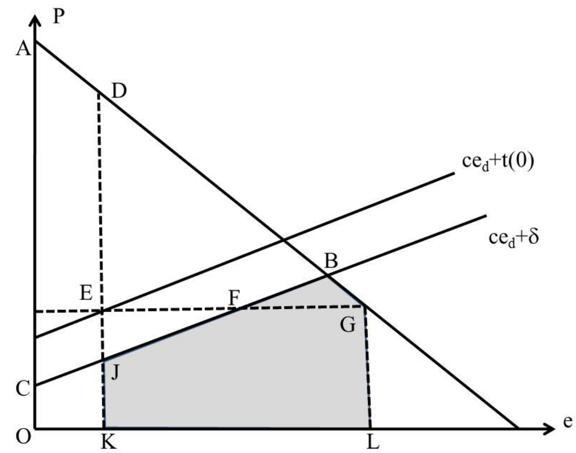


Figure 8: The welfare consequences of an R & D subsidy.

Therefore,

$$w(0) - w(\overline{g}) - \pi(0) = \triangle FGB - \triangle EFJ.$$

Hence, a welfare-improving R&D subsidy will exist for $i$ greater than and sufficiently close to $i_T$ iff $\triangle FGB - \triangle EFJ > 0$. We now show that $\triangle FGB > \triangle EFJ$ in Figure 8 iff $4c^2 - bc - b^2 > 0$. Consider Figure 8.

$$\begin{aligned}
\triangle EFJ &= \frac{1}{2}.(EJ).(EF) \\
&= \frac{1}{2}.[P(0,t(0)) - (\delta + ce_d(0,t(0)))].\left[\frac{P(0,t(0)) - \delta}{c} - e_d(0,t(0))\right] \\
&= \frac{1}{2}.\frac{ac + b\delta}{b + 4c}.\frac{ac + b\delta}{c(b + 4c)} \\
&= \frac{1}{2c}\left(\frac{ac + b\delta}{b + 4c}\right)^2. \qquad \text{(B.1)}
\end{aligned}$$

$$\begin{aligned}
\triangle FGB &= \frac{1}{2}.FG.[\text{Perpendicular distance of edge } FG \text{ from } B] \\
&= \frac{1}{2}.\left[D(P(0,t(0))) - \frac{P(0,t(0)) - \delta}{c}\right].[P_d^*(\delta) - P(0,t(0))] \\
&= \frac{1}{2}.\frac{2(ac + b\delta)}{b(b + 4c)}.\frac{2c(ac + b\delta)}{(b + c)(b + 4c)} \\
&= \frac{2c}{b(b + c)}\left(\frac{ac + b\delta}{b + 4c}\right)^2. \qquad \text{(B.2)}
\end{aligned}$$

$$\text{Therefore,} \quad \triangle FGB - \triangle EFJ = \left(\frac{ac+b\delta}{b+4c}\right)^2 . \left[\frac{2c}{b(b+c)} - \frac{1}{2c}\right]. \tag{B.3}$$

Thus, $\triangle FGB - \triangle EFJ$ is positive iff $4c^2 - bc - b^2 > 0$, or, $c > \frac{1+\sqrt{17}}{8}b$.

Therefore, a welfare-improving subsidy to R&D exists for $i$ greater than and sufficiently close to $i_T$ iff $4c^2 - bc - b^2 > 0$. Now suppose, $4c^2 - bc - b^2 > 0$. In this case, $w(0) - w(\overline{g}) > \pi(0) = \frac{(b+2c)^2(ac+b\delta)^2}{bc\overline{g}^2(b+c)(b+4c)^2}$.

Now, an R & D subsidy can be welfare improving if $i \geq i_T$ and $w(0) - i\overline{g}^2 \geq w(\overline{g})$. The last inequality can be re-written as $i \leq \frac{w(0)-w(\overline{g})}{\overline{g}^2}$.

Now,

$$\begin{aligned} w(0) - w(\overline{g}) &= \text{Area of } \quad EKLG + \text{Area of } \triangle FGB - \text{Area of } \triangle EFJ \\ &= P(t(0),0).e_g(t(0),0) + \text{Area of } \triangle FGB - \text{Area of } \triangle EFJ \\ &= \frac{(ac+b\delta)^2(b+2c)^2}{bc(b+c)(b+4c)^2} + \left[\left(\frac{ac+b\delta}{b+4c}\right)^2 . \left[\frac{2c}{b(b+c)} - \frac{1}{2c}\right]\right] \\ &= \frac{(b+3c)(ac+b\delta)^2}{2bc(b+c)(b+4c)}. \end{aligned}$$

Thus, $\frac{(b+3c)(ac+b\delta)^2}{2bc\overline{g}^2(b+c)(b+4c)}$ is the highest level of $i$ for which a subsidy that encourages R & D can be welfare-improving.

# C Proof of Proposition 7

If $g > P_d^*(\delta)$, it is clear that the optimal quota is $e_d^*(\delta)$ and the green firm is shut out of the market. Energy consumption is $e_d^*(\delta)$ and its price is $P_d^*(\delta)$.

Now consider the case $P_d^*(0) \leq g \leq P_d^*(\delta)$, depicted in Figure 9. The green firm chooses its output to maximize its profit given the residual demand curve for energy after the dirty sector has produced $q$.[10]

The profit function of the green firm is :

$$\begin{aligned} \pi &= e_g[D^{-1}(e_g+q) - g] \\ &= e_g[a - b(q+e_g) - g]. \end{aligned}$$

[10]It is clear that the quota must be less than $D(g)$. If not, then the green firm would be shut out of the market. So $q = e_d^*(\delta)$ would yield higher welfare.

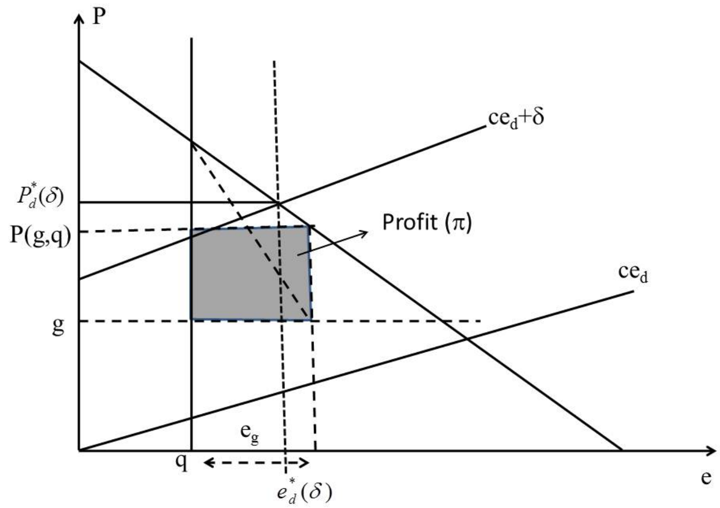


Figure 9: Choice of green energy output when $P_d^*(0) \leq g \leq P_d^*(\delta)$.

$\pi$ is concave in $e_g$ and there is no corner solution. The monopoly price is the average of marginal cost and the highest point of the residual demand curve $D^{-1}(q)$. Thus the output of clean energy and total energy, the price of energy and the profit of the green firm are given by 3.1, 3.2, 3.3 and 3.4 respectively (Refer to Figure 9).

Next, consider the case $g < P_d^*(0)$. It is straightforward to show that in the absence of government intervention, that is, if there were no quota set for dirty energy, then profit maximization by the green firm would lead to an energy price $P = \frac{1}{2}[P_d^*(0) + g]$ and dirty energy production $e_d = \frac{1}{2c}[P_d^*(0) + g]$. The government can always achieve this outcome by setting a large enough quota; any $q \geq e_d^*(0)$ will do. We note that the government may, under some parameter configurations and values of $g$, actually choose to set such a non-binding quota. The reason is that setting a quota slightly less than $\frac{1}{2c}[P_d^*(0)+g]$ gives the green firm monopoly power. The resulting high energy price restricts total energy output so much that it is preferable to set a non-binding quota and allow $e_d = \frac{1}{2c}[P_d^*(0) + g]$.

If $g < P_d^*(0)$ and the government's choice of $q$ is from the interior of the interval $\left[0, \frac{1}{2c}(P_d^*(0) + g)\right]$, then the quota is binding and optimal green energy production, total energy production, the price of energy, and the gross profit of the green firm are given by (3.1), (3.2), (3.3) and (3.4) respectively.

When $g \leq P_d^*(\delta)$ and the government is choosing a binding quota the gross welfare function is

$$w(g,q) = a(\frac{a+bq-g}{2b}) - \frac{b}{2}(\frac{a+bq-g}{2b})^2 - \delta q - \frac{cq^2}{2} - g(\frac{a-bq-g}{2b}) \tag{C.1}$$

The marginal social benefit from tightening the quota is, as in Section 3.1, the reduction in social cost when dirty energy is replaced by green energy, while the marginal welfare loss arises from the reduction in net welfare from energy consumption. The first-order condition from maximizing C.1 with respect to $q$ can, therefore, be written as

$$cq + \delta - g = \frac{1}{2}[\frac{a-bq-g}{2}] \tag{C.2}$$

provided the solution is interior, which we assume. It is easy to see that the assumption needed to rule out a zero quota is $a > 4\delta$.[11]

If the government is restricted to choose a quota from the interval $\big(0, \frac{1}{2c}(P_d^*(0)+g)\big)$, it follows easily from C.2 that the government's reaction function is:

$$q(g) = \begin{cases} e_d^*(\delta) & \text{if} \quad g > P_d^*(\delta) \\ \frac{a+3g-4\delta}{b+4c} & \text{if} \quad g \leq P_d^*(\delta) \end{cases} \tag{C.3}$$

However, if the government is not committed to a binding quota and $g$ is low enough, it could flood the markets with permits, so that the green firm is forced to choose

$$P = \frac{1}{2}[P_d^*(0) + g]. \tag{C.4}$$

Emissions would then equal the business-as-usual emissions

$$e_d = \frac{1}{2c}[P_d^*(0) + g], \tag{C.5}$$

and energy consumption would be

$$e = \frac{a - \frac{1}{2}[P_d^*(0) + g]}{b}. \tag{C.6}$$

By substituting Equation C.5 and Equation C.6 in the gross welfare function

$$w = ae - \frac{b}{2}e^2 - \delta e_d - \frac{c}{2}{e_d}^2 - g(e - e_d),$$

we obtain the gross welfare when a non-binding quota is chosen. Let us denote this welfare level by $w^{nb}$.

[11] At $q = 0$,

$$\frac{\partial w}{\partial q} = \frac{a}{4} + \frac{3g}{4} - \delta.$$

To rule out the corner solution, we need $a + 3g - 4\delta > 0, \quad \forall g \geq 0$. This requires $a - 4\delta > 0$.

On the other hand if the government chooses the binding quota given by Equation C.3, gross welfare is obtained by substituting Equation C.3 in Equation C.1. Let us denote this welfare level by $w^b$.

When is it optimal for the government to flood the market with permits ($q > P_d^*(0)$) rather than to choose a binding quota? Comparing the welfare levels for optimal binding and non-binding quotas shows that the government chooses a large non-binding quota if and only if

$$\begin{aligned} w^{nb} &> w^b, \\ \text{or,} \quad g &< \frac{c(a-4\delta)}{b+c} \equiv g_b. \end{aligned} \tag{C.7}$$

When $g = g_b$, the welfare level from choosing an quota given by Equation C.3 is equal to the welfare level from the government choosing a non-binding quota. We assume that the government chooses a binding quota (given by Equation C.3) when $g = g_b$.[12]

We now turn to the green firm's choice of R & D investment. The firm never chooses $g \in [P_d^*(\delta), \overline{g})$. This is because while such cost reduction from $\overline{g}$ is costly, it does not allow the firm to operate in the market in the second period. In the range $[g_b, P_d^*(\delta))$, the net profit function is obtained by substituting the government's reaction function (C.3) into the profit function 3.4 and subtracting the investment cost. For $g < g_b$, the net profit as a function of $g$ is obtained by substituting Equation C.4, Equation C.5 and Equation C.6 in the net profit function

$$\pi = (P-g)(e-e_d) - i(\overline{g}-g)^2.$$

Thus the net profit function is

$$\Pi(g) = \begin{cases} -i(\overline{g}-g)^2 \quad \text{if} \quad P_d^*(\delta) < g \leq \overline{g}, \\ \frac{4}{b(b+4c)^2}[ac+b\delta-(b+c)g]^2 - i(\overline{g}-g)^2 \quad \text{if} \quad g_b \leq g \leq P_d^*(\delta), \\ \frac{b+c}{4bc}\left[\frac{ac}{b+c}-g\right]^2 - i(\overline{g}-g)^2 \quad \text{if} \quad g < g_b. \end{cases} \tag{C.8}$$

Consider values of $g$ in the range $[g_b, P_d^*(\delta)]$. Differentiating $\Pi(g)$ with respect to $g$,

$$\frac{d\Pi}{dg} = -\frac{8(b+c)}{b(b+4c)^2}[P_d^*(\delta)-g] + 2i(\overline{g}-g),$$

[12] The reason for this assumption is the following. Suppose it did not hold. Let the government choose the best binding quota with probability $p$ and the non-binding quota with probability $(1-p)$, $0 \leq p < 1$. Since choosing a non-binding quota reduces the profits of the green firm compared to the binding quota scenario, the net profit function experiences a discontinuous jump at $g = g_b$. As a result, the green firm never chooses $g = g_b$, since choosing $g$ higher than $g_b$ by an infinitesimal amount leads to a strictly higher profit.

$$\text{So} \quad \frac{d\Pi}{dg}\Big|_{P_d^*(\delta)} = 2i(\overline{g} - P_d^*(\delta)) > 0, \tag{C.9}$$

$$\frac{d^2\Pi}{dg^2} = \frac{8(b+c)}{b(b+4c)^2} - 2i.$$

Now suppose the net profit curve is concave in the range $[g_b, P_d^*(\delta))$. (C.9) implies that $P_d^*(\delta)$ maximizes net profit in this range. We know that net profit is negative at $P_d^*(\delta)$. Thus $\overline{g}$ is the profit-maximizing level of $g$ in the range $[g_b, P_d^*(\delta))$ when the profit function is concave.

If the net profit curve is convex in the range $[g_b, P_d^*(\delta))$, there are two candidates for a maximum in the range $[g_b, P_d^*(\delta))$: $g_b$ and $P_d^*(\delta)$. We know that net profit is negative at $P_d^*(\delta)$. Thus $\overline{g}$ and 0 are the two candidates for a for profit-maximizing $g$ in the range $[g_b, \overline{g}]$.

In the range $[0, g_b)$, differentiating the net profit function, we get:

$$\frac{d\Pi}{dg} = -\frac{b+c}{2bc}\left[\frac{ac}{b+c} - g\right] + 2i(\overline{g} - g),$$

$$\frac{d^2\Pi}{dg^2} = \frac{b+c}{2bc} - 2i.$$

When $i < \frac{b+c}{4bc}$, the profit function is convex. In this case, the maximum can either be 0 or $g_b$. When $i > \frac{b+c}{4bc}$, the profit function is concave. Note that $\lim_{g \to g_b+} \frac{d\Pi g}{dg} > 0$ if $i > \frac{\delta(b+c)}{b((b+c)\overline{g} - c(a-4\delta))}$. Now $\frac{\delta(b+c)}{b((b+c)\overline{g} - c(a-4\delta))} < \frac{b+c}{4bc}$ because $\overline{g} > P_d^*(\delta) = \frac{ac+b\delta}{b+c}$. Thus, the net profit curve is positively sloped in the neighbourhood of $g_b$ whenever it is concave. Hence, an interior solution is ruled out in the concave case as well.

Thus in the range $[0, g_b]$, there are two candidates for the equilibrium level of $g$: 0 & $g_b$. Globally, there are three candidates for an equilibrium: 0, $g_b$ and $\overline{g}$.

**The choice among $g = 0$, $g = g_b$, and $g = \overline{g}$ :**

If the gross profit of the green firm at $g = g_b$ is higher than that at $g = 0$, the same holds true for the net profit as well. Thus, if $\pi(0) < \pi(g_b)$, then $\Pi(0) < \Pi(g_b)$. Using (C.8),

$$\begin{aligned} \pi(0) &< \pi(g_b) \\ \text{if and only if} \quad \delta^2 &> \frac{a^2c}{16(b+c)}. \end{aligned} \tag{C.10}$$

Thus, when $\delta^2 > \frac{a^2c}{16(b+c)}$, then $\Pi(0) < \Pi(g_b) \forall i$.

Now we have two cases:

- **Case 1:** $\delta^2 > \frac{a^2 c}{16(b+c)}$.
  The two candidates for a maximum are: $g_b$ and $\overline{g}$. Substituting these two values of $g$ in the profit function (C.8), we identify the cut-off value of $i$ at which the firm is indifferent between $g = g_b$ and $g = \overline{g}$.

$$\begin{aligned} \Pi(\overline{g}) &< \Pi(g_b) \\ \iff 0 &< \frac{4\delta^2}{b} - i\frac{((b+c)\overline{g} - c(a-4\delta))^2}{(b+c)^2} \\ \iff i &< \frac{4\delta^2(b+c)^2}{b((b+c)\overline{g} - c(a-4\delta))^2} \equiv i_Q^1. \end{aligned} \tag{C.11}$$

  Thus,

$$g(i) = \begin{cases} g_b, & \text{if} \quad i \leq i_Q^1, \\ \overline{g}, & \text{if} \quad i \geq i_Q^1. \end{cases} \tag{C.12}$$

  Let

$$i_Q = \frac{4(ac+b\delta)^2}{b(b+4c)^2\overline{g}^2}$$

  Note that

$$\frac{i^T}{i^Q} = \frac{(b+2c)^2}{4c(b+c)} > 1$$

  Now,

$$\begin{aligned} i_Q^1 &< i_Q \\ \iff \frac{4\delta^2(b+c)^2}{b((b+c)\overline{g} - c(a-4\delta))^2} &< \frac{4(ac+b\delta)^2}{b(b+4c)^2\overline{g}^2} \\ \iff \frac{\delta(b+c)}{(b+c)\overline{g} - c(a-4\delta)} &< \frac{ac+b\delta}{(b+4c)\overline{g}} \text{ because } \overline{g} > \frac{ac+b\delta}{b+c} > \frac{ac-4\delta c}{b+c} \\ \iff c(a-4\delta)(ac+b\delta) &< c(b+c)(a-4\delta)\overline{g} \\ \iff \frac{ac+b\delta}{b+c} &< \overline{g}. \end{aligned} \tag{C.13}$$

  Thus, $i_Q^1 < i_Q$ because $\overline{g} > P_d^*(\delta)$. So $i_Q^1 < i_T$ because $i_Q < i_T$. The equilibrium $g(i)$ curve for Case 1 is depicted in Figure 10.

- **Case 2:** $\delta^2 < \frac{a^2 c}{16(b+c)}$.
  Since Inequality C.10 is not satisfied, $g = 0$ maximizes profit for values of $i$ very close to zero. We know that at $i = i_Q^1$,

$$\Pi(g_b) = \Pi(\overline{g}) = 0.$$

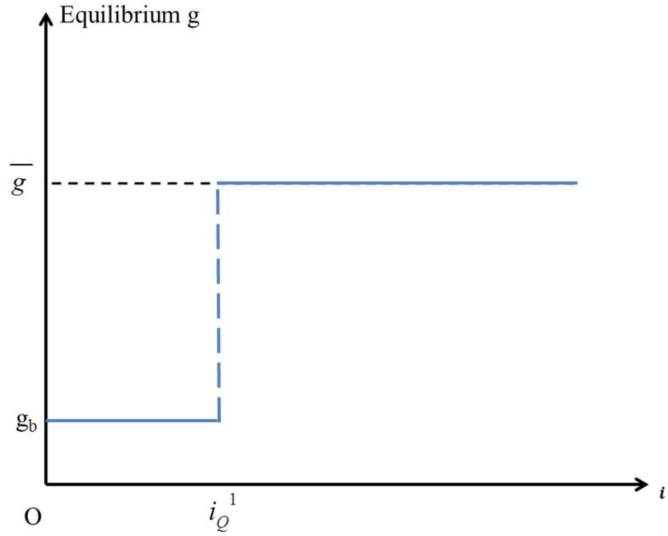


Figure 10: Equilibrium green marginal cost $g$ as a function of $i$

– **Case 2a:** If, at $i = i_Q^1$,

$$\begin{aligned} \Pi(0) &> \Pi(g_b) \\ \text{so that} \quad \delta^2 &< \frac{a^2c}{16(b+c)}\left[\frac{(b+c)\overline{g} - c(a-4\delta)}{(b+c)\overline{g}}\right]^2, \end{aligned} \tag{C.14}$$

then there are just two candidates for the equilibrium $g$: 0 and $\overline{g}$. Solving the equation $\Pi(0) = \Pi(\overline{g})$, we get $i = \frac{a^2c}{4b\overline{g}^2(b+c)} \equiv i_Q^3$. Thus if the inequality C.14 holds, the equilibrium $g$ is given by:

$$g(i) = \begin{cases} 0, & \text{if} \quad i \leq i_Q^3, \\ \overline{g}, & \text{if} \quad i \geq i_Q^3. \end{cases} \tag{C.15}$$

We now show that $i_Q^3 < i_Q$. It is easily checked graphically that when $g = 0$, gross profits are higher if the government chooses a binding quota (given by the second part of Equation C.3) rather than choosing an large non-binding quota. When $g = 0$, let $\pi_0^B$ and $\pi_0^{NB}$ denote the gross profits attained when a binding quota (given by substituing $g = 0$ in the second part of Equation C.3) and a large non binding quota are chosen respectively. Thus, $\pi_0^B > \pi_0^{NB}$. By definition of $i_Q^3$ and $i_Q$,

$$\begin{aligned} \pi_0^{NB} - i_Q^3\overline{g}^2 &= \Pi(\overline{g}) = 0 \\ \text{or,} \quad i_Q^3 &= \frac{\pi_0^{NB}}{\overline{g}^2} \end{aligned} \tag{C.16}$$

$$\begin{aligned}\pi_0^B - i_Q\overline{g}^2 &= \Pi(\overline{g}) = 0 \\ \text{or,}\quad i_Q &= \frac{\pi_0^B}{\overline{g}^2}\end{aligned} \tag{C.17}$$

Since $\pi_0^B > \pi_0^{NB}$, therefore $i_Q^3 < i_Q$.

– **Case 2b:** Inequality C.14 does not hold. In this case,

$$\frac{a^2c}{16(b+c)}\left[\frac{(b+c)\overline{g} - c(a-4\delta)}{(b+c)\overline{g}}\right]^2 < \delta^2 < \frac{a^2c}{16(b+c)}.$$

By (C.11), we know that $g = \overline{g}$ is not an equilibrium for any $i < i_Q^1$. Solving

$$\Pi(0) = \Pi(g_b),$$

we get the cut-off level of $i$ below which $g = 0$ is the profit-maximizing choice for the green firm. The cut-off level is denoted by

$$i_Q^2 = \frac{(b+c)(a^2c - 16\delta^2(b+c)}{4bc(a-4\delta)(2(b+c)\overline{g} - c(a-4\delta))}$$

Thus, in Case 2b the equilibrium value of $g$ is:

$$g(i) = \begin{cases} 0, & \text{if} \quad i \leq i_Q^2, \\ g_b, & \text{if} \quad i_Q^2 \leq i \leq i_Q^1, \\ \overline{g}, & \text{if} \quad i \geq i_Q^1. \end{cases} \tag{C.18}$$

Moreover, $i_Q^1 < i_Q$ because $\overline{g} > P_d^*(\delta)$. Therefore, $i_Q^1 < i_T$. The equilibrium $g(i)$ curve in Case 2b is depicted in Figure 11.

*This discussion has shown that in all the three cases 1, 2a, and 2b, the tax regime ensures more R & D than the quota regime. For the remaining values of i, the amount of R & D is equal in the two regimes.*

### *C.0.1 Conditions under which a subsidy to R & D is welfare-improving.*

Consider Case 1 and Case 2(b). In these two cases, if $i > i_Q^1$, then there is no R & D in the absence of a subsidy. As in the paper, a subsidy can ensure R & D only if it takes the effective $i$ , that is, $(1-s)i$ below $i_Q^1$. Suppose $i$ is infinitesimally higher than $i_Q^1$. A small subsidy then pulls the effective $i$ below the cut-off $i_Q^1$. The equilibrium $g$ is then $g_b$.

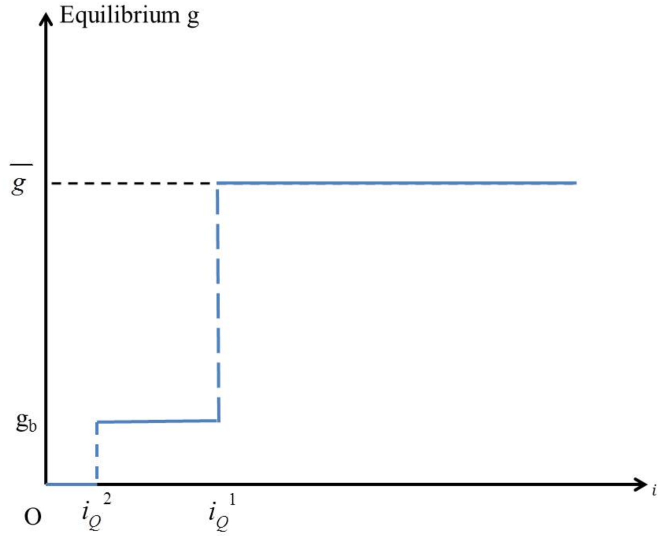


Figure 11: Equilibrium green marginal cost $g$ as a function of $i$

By substituting the value of $g_b$ from Equation C.7 into the second part of the optimal quota function C.3,

$$\begin{aligned}
q(g_b) &= \frac{a + 3g_b - 4\delta}{b + 4c} \\
&= \frac{a + 3(\frac{c(a-4\delta)}{b+c}) - 4\delta}{b + 4c} \\
&= \frac{a(b+c) + 3c(a - 4\delta) - 4\delta(b + c)}{(b + c)(b + 4c)} \\
&= \frac{a(b + 4c) - 4\delta(b + 4c)}{(b + c)(b + 4c)} \\
&= \frac{a - 4\delta}{b + c} \\
&= \frac{g_b}{c}.
\end{aligned} \tag{C.19}$$

$$P(g_b, q(g_b)) = \frac{(2(b + c)(ac + b\delta) - c(b - 2c)(a - 4\delta))}{(b + c)(b + 4c)}. \tag{C.20}$$

Now, it can be shown that $P(g_b, q(g_b)) > P_d^*(\delta)$ iff $c < \frac{b}{2}$. Thus, we consider two cases:

- Case (a): $c < \frac{b}{2}$.

  Refer to Figure 12. When $i = i_Q^1$,

$$\Pi(g_b) = \Pi(\overline{g}).$$

$$\text{So} \quad \pi(g_b) = i(\overline{g} - g_b)^2.$$

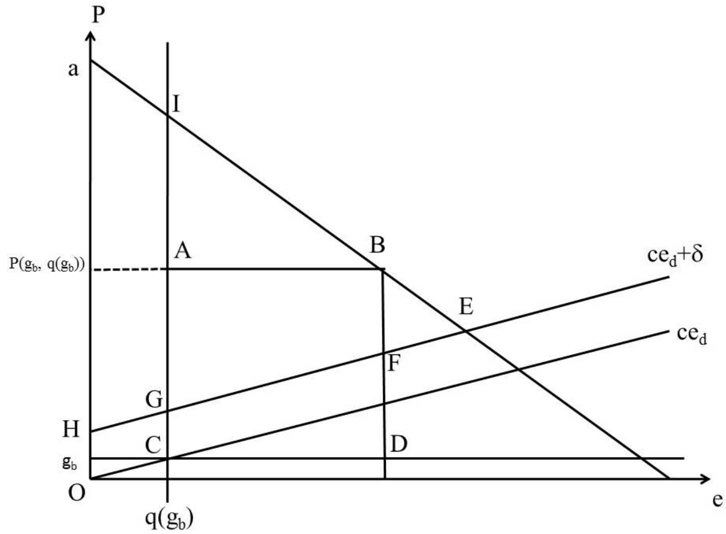


Figure 12: Case (a): Role of Subsidy

In Figure 12,

$$\pi(g_b) = ABCD.$$

$$\begin{aligned}
w(g_b) - w(\overline{g}) &= [IBDC + aIGH] - \triangle aEH \\
&= GFDC - \triangle BEF \\
&= ABCD - (ABFG + \triangle BEF) \\
&< ABCD = \pi(g_b) = i(\overline{g} - g_b)^2. \\
\text{So} \quad w(g_b) - i(\overline{g} - g_b)^2 &< w(\overline{g}), \\
\text{or} \quad W(g_b) &< W(\overline{g}).
\end{aligned}$$

Thus, when $c < \frac{b}{2}$, R & D subsidies are not welfare-improving.

- Case (b): $c > \frac{b}{2}$.

  When $c > \frac{b}{2}$, $P_d^*(\delta) > P(g_b, q(g_b)) >$ the social marginal cost of dirty energy at $q(g_b)$. In Figure 13,

$$\pi(g_b) = AFEC.$$

$$\begin{aligned}
w(g_b) - w(\overline{g}) - \pi(g_b) &= BCEFI - AFEC \\
&= \triangle IFL - \triangle ALB \\
&= \frac{1}{2}\frac{(2c-b)^2\delta^2}{bc(b+c)} - \frac{\delta^2}{2c}.
\end{aligned}$$

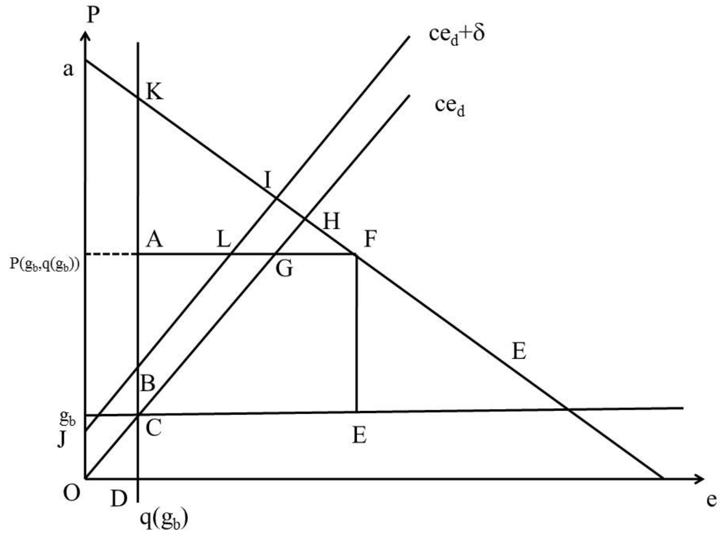


Figure 13: Case (b): Role of Subsidy

Now $w(g_b) - w(\bar{g}) - \pi(g_b) > 0$ requires $c > \frac{5}{4}b$. Thus, for the subsidy to be welfare-improving, we require $c > \frac{5}{4}b$.

Note that in *Case 2a* (refer to equation C.15), the green firm is indifferent between choosing $g = \bar{g}$ and $g = 0$ at $i = i_Q^3$. If for some value of $i$, the firm chooses $g = 0$, the government will not choose a binding quota. Thus in this case the firms choice of price and energy is given by equation C.4 and C.6 respectively. The dirty energy produced (when $g = 0$) is less than $e_d^*(\delta)$ due to Assumption 3.1. Thus it is simple to see that a subsidy is welfare improving provided $i \geq i_Q^3$ but $i$ is not much higher than $i_Q^3$.