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Parametric Estimation for Linear Stochastic Delay Differential Equations Driven by Fractional Brownian Motion

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Parametric Estimation for Linear Stochastic Delay Differential Equations Driven by Fractional Brownian Motion

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Abstract

Consider a linear stochastic differential equation

$$dX(t) = (aX(t) + bX(t-1))dt + dW_t^H, t \geq 0$$

with time delay driven by a fractional Brownian motion $\{W_t^H, t \geq 0\}$. We investigate the asymptotic properties of the maximum likelihood estimator of the parameter $\theta = (a, b)$.

Keywords and phrases: Linear stochastic differential equation; Time delay; fractional Ornstein-Uhlenbeck type process; fractional Brownian motion; Maximum likelihood estimation; Consistency; Local asymptotic normality; Local asymptotic mixed normality.

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1 Introduction

Statistical inference for diffusion type processes satisfying stochastic differential equations driven by Wiener processes has been studied earlier and a comprehensive survey of various methods is given in Prakasa Rao (1999a). There has been a recent interest to study similar problems for stochastic processes driven by a fractional Brownian motion. Le Breton (1998) studied parameter estimation and filtering in a simple linear model driven by a fractional Brownian motion. In a recent paper, Kleptsyna and Le Breton (2002) studied parameter estimation problems for fractional Ornstein-Uhlenbeck type process. This is a fractional analogue of the Ornstein-Uhlenbeck process, that is, a continuous time first order autoregressive process $X = \{X_t, t \geq 0\}$ which is the solution of a one-dimensional homogeneous linear stochastic differential equation driven by a fractional Brownian motion (fBm) $W^H = \{W_t^H, t \geq 0\}$ with Hurst parameter $H \in [1/2, 1)$. Such a process is the unique Gaussian process satisfying the linear integral equation

$$X_t = \theta \int_0^t X_s ds + \sigma W_t^H, t \geq 0. \tag{1. 1}$$

They investigate the problem of estimation of the parameters θ and σ^2 based on the observation $\{X_s, 0 \leq s \leq T\}$ and prove that the maximum likelihood estimator $\hat{\theta}_T$ is strongly consistent as $T \to \infty$.

We discussed more general classes of stochastic processes satisfying linear stochastic differential equations driven by a fractional Brownian motion (fBm) in Prakasa Rao (2003 a,b,c) and studied the asymptotic properties of the maximum likelihood and the Bayes estimators for parameters involved in such processes. Nonparametric inference problems were studied in Prakasa Rao (2003d). A comprehensive discussion of these results is given in Prakasa Rao (2003e).

In a recent paper, Gushchin and Kuchler (1999) investigated asymptotic inference for linear stochastic differential equations with time delay of the type

$$dX(t) = (aX(t) + bX(t-1))dt + dW_t, t \geq 0$$

driven by the standard Brownian motion $\{W_t, t \geq 0\}$ with the initial condition $X(t) = X_0(t), -1 \leq t \leq 0$ wher $X_0(t)$ is a continuous process independent of $W(.)$. They investigated the asymptotic properties of the maximum likelihood estimator (MLE) of the parameter $\theta = (a, b)$. They have shown that the asymptotic behaviour of the maximum likelihood estimator depends on the ranges of the values of a and b.

We now consider the linear stochastic differential equation

$$dX(t) = (aX(t) + bX(t-1))dt + dW_t^H, t \geq 0$$

with time delay driven by the fractional Brownian motion $\{W_t^H, t \geq 0\}$. We investigate the asymptotic properties of the maximum likelihood estimator of the parameter $\theta = (a, b)$.

2 Preliminaries

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$ be a stochastic basis satisfying the usual conditions.The natural fitration of a stochastic process is understood as the P-completion of the filtration generated by this process.

Let $W^H = \{W_t^H, t \geq 0\}$ be a normalized fractional Brownian motion with Hurst parameter $H \in (0, 1)$, that is, a Gaussian process with continuous sample paths such that $W_0^H = 0, E(W_t^H) = 0$ and

$$E(W_s^H W_t^H) = \frac{1}{2}[s^{2H} + t^{2H} - |s-t|^{2H}], t \geq 0, s \geq 0. \tag{2. 1}$$

Let us consider a stochastic process $Y = \{Y_t, t \geq 0\}$ defined by the stochastic integral equation

$$Y_t = \int_0^t C(s)ds + \int_0^t B(s)dW_s^H, t \geq 0 \tag{2. 2}$$

where $C = \{C(t), t \geq 0\}$ is an $(\mathcal{F}_t)$-adapted process and $B(t)$ is a nonvanishing nonrandom function. For convenience we write the above integral equation in the form of a stochastic differential equation

$$dY_t = C(t)dt + B(t)dW_t^H, t \geq 0 \tag{2. 3}$$

driven by the fractional Brownian motion W^H. The integral

$$\int_0^t B(s)dW_s^H \tag{2. 4}$$

is not a stochastic integral in the Ito sense but one can define the integral of a deterministic function with respect to the fbM in a natural sense (cf. Norros et al. (1999)). Even though the process Y is not a semimartingale, one can associate a semimartingale $Z = \{Z_t, t \geq 0\}$ which is called a *fundamental semimartingale* such that the natural filtration $(\mathcal{Z}_t)$ of the process Z coincides with the natural filtration $(\mathcal{Y}_t)$ of the process Y (Kleptsyna et al. (2000)). Define, for $0 < s < t$,

$$k_H = 2H\,\Gamma(\frac{3}{2} - H)\Gamma(H + \frac{1}{2}), \tag{2. 5}$$

$$k_H(t,s) = k_H^{-1} s^{\frac{1}{2}-H}(t-s)^{\frac{1}{2}-H}, \tag{2. 6}$$

$$\lambda_H = \frac{2H\,\Gamma(3-2H)\Gamma(H+\frac{1}{2})}{\Gamma(\frac{3}{2}-H)}, \tag{2. 7}$$

$$w_t^H = \lambda_H^{-1} t^{2-2H}, \tag{2. 8}$$

and

$$M_t^H = \int_0^t k_H(t,s)dW_s^H, t \geq 0. \tag{2. 9}$$

The process M^H is a Gaussian martingale, called the *fundamental martingale* (cf. Norros et al. (1999)) and its quadratic variation $< M_t^H >= w_t^H$. Further more the natural filtration of the martingale M^H coincides with the natural fitration of the fbM W^H. In fact the stochastic integral

$$\int_0^t B(s)dW_s^H \tag{2. 10}$$

can be represented in terms of the stochastic integral with respect to the martingale M^H. For a measurable function f on $[0,T]$, let

$$K_H^f(t,s) = -2H\frac{d}{ds}\int_s^t f(r)r^{H-\frac{1}{2}}(r-s)^{H-\frac{1}{2}}dr, 0 \leq s \leq t \tag{2. 11}$$

when the derivative exists in the sense of absolute continuity with respect to the Lebesgue measure (see Samko et al. (1993) for sufficient conditions). The following result is due to Kleptsyna et al. (2000).

Therorem 2.1: Let M^H be the fundamental martingale associated with the fbM W^H defined by (2.9). Then

$$\int_0^t f(s)dW_s^H = \int_0^t K_H^f(t,s)dM_s^H, t \in [0,T] \tag{2. 12}$$

a.s $[P]$ whenever both sides are well defined.

Suppose the sample paths of the process $\{\frac{C(t)}{B(t)}, t \geq 0\}$ are smooth enough (see Samko et al. (1993)) so that the process

$$Q_H(t) = \frac{d}{dw_t^H}\int_0^t k_H(t,s)\frac{C(s)}{B(s)}ds, t \in [0,T] \tag{2. 13}$$

is well-defined where w^H and k_H are as defined in (2.8) and (2.6) respectively and the derivative is understood in the sense of absolute continuity. The following theorem due to Kleptsyna et al. (2000) associates a *fundamental semimartingale* Z associated with the process Y such that the natural filtration $(\mathcal{Z}_t)$ coincides with the natural filtration $(\mathcal{Y}_t)$ of Y.

Theorem 2.2: Suppose the sample paths of the process Q_H defined by (2.13) belong P-a.s to $L^2([0,T], dw^H)$ where w^H is as defined by (2.8). Let the process $Z = (Z_t, t \in [0,T])$ be defined by

$$Z_t = \int_0^t k_H(t,s) B^{-1}(s) dY_s \tag{2. 14}$$

where the function $k_H(t,s)$ is as defined in (2.6). Then the following results hold:
(i) The process Z is an $(\mathcal{F}_t)$ -semimartingale with the decomposition

$$Z_t = \int_0^t Q_H(s) dw_s^H + M_t^H \tag{2. 15}$$

where M^H is the fundamental martingale defined by (2.9),
(ii) the process Y admits the representation

$$Y_t = \int_0^t K_H^B(t,s) dZ_s \tag{2. 16}$$

where the function K_H^B is as defined in (2.11), and
(iii) the natural fitrations of $(\mathcal{Z}_t)$ and $(\mathcal{Y}_t)$ coincide.

Kleptsyna et al. (2000) derived the following Girsanov type formula as a consequence of the Theorem 2.2.

Theorem 2.3: Suppose the assumptions of Theorem 2.2 hold. Define

$$\Lambda_H(T) = \exp\{-\int_0^T Q_H(t) dM_t^H - \frac{1}{2}\int_0^t Q_H^2(t) dw_t^H\}. \tag{2. 17}$$

Suppose that $E(\Lambda_H(T)) = 1$. Then the measure $P^* = \Lambda_H(T)P$ is a probability measure and the probability measure of the process Y under P^* is the same as that of the process V defined by

$$V_t = \int_0^t B(s) dW_s^H, 0 \le t \le T. \tag{2. 18}$$

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3 Maximum Likelihood Estimation

Let us consider the stochastic differential equation

$$dX(t) = (aX(t) + bX(t-1))dt + dW_t^H, t \ge 0 \tag{3. 1}$$

where $\theta = (a,b) \in \Theta \subset R^2, W = \{W_t^H, t \geq 0\}$ is a fractional Brownian motion with a *known* Hurst parameter H with the initial condition $X(t) = X_0(t), t \in [-1,0]$ where $X_0(.)$ is a continuous Gaussian stochastic process independent of W^H. In other words $X = \{X_t, t \geq 0\}$ is a stochastic process satisfying the stochastic integral equation

(3. 2)
$$\begin{aligned} X(t) &= X(0) + \int_0^t [aX(s) + bX(s-1)]ds + W_t^H, t \geq 0, \\ X(t) &= X_0(t), -1 \leq t \leq 0. \end{aligned}$$

Let

(3. 3)
$$C(\theta, t) = aX(t) + bX(t-1), t \geq 0$$

and assume that the sample paths of the process $\{C(\theta,t)\}, t \geq 0$ are smooth enough so that the process

(3. 4)
$$Q_{H,\theta}(t) = \frac{d}{dw_t^H} \int_0^t k_H(t,s) C(\theta, s) ds, t \geq 0$$

is well-defined where w_t^H and $k_H(t,s)$ are as defined in (2.8) and (2.6) respectively. Suppose the sample paths of the process $\{Q_{H,\theta}, 0 \leq t \leq T\}$ belong almost surely to $L^2([0,T], dw_t^H)$. Define

(3. 5)
$$Z_t = \int_0^t k_H(t,s) dX_s, t \geq 0.$$

Then the process $Z = \{Z_t, t \geq 0\}$ is an $(\mathcal{F}_t)$-semimartingale with the decomposition

(3. 6)
$$Z_t = \int_0^t Q_{H,\theta}(s) dw_s^H + M_t^H$$

where M^H is the fundamental martingale defined by (2.9) and the process X admits the representation

(3. 7)
$$\begin{aligned} X(t) &= X(0) + \int_0^t K_H(t,s) dZ_s, t \geq 0, \\ X(t) &= X_0(t), -1 \leq t \leq 0 \end{aligned}$$

where the function K_H is as defined by (2.11) with $f \equiv 1$. Let P_T^θ be the measure induced by the process $\{X_t, -1 \leq t \leq T\}$ on $C[-1,T]$ when θ is the true parameter conditional on $X(t) = X_0(t), -1 \leq t \leq 0$. Following Theorem 2.3, we get that the Radon-Nikodym derivative of P_T^θ with respect to $P_T^{(0,0)}$ is given by

(3. 8)
$$\frac{dP_T^\theta}{dP_T^{(0,0)}} = \exp[\int_0^T Q_{H,\theta}(s) dZ_s - \frac{1}{2} \int_0^T Q_{H,\theta}^2(s) dw_s^H].$$

We now consider the problem of estimation of the parameter $\theta = (a,b)$ based on the observation of the process $X = \{X_t, 0 \leq t \leq T\}$ conditional on $X(t) = X_0(t), -1 \leq t \leq 0.$ and study its asymptotic properties as $T \to \infty$.

Let $L_T(\theta)$ denote the Radon-Nikodym derivative $\frac{dP_T^\theta}{dP_T^{(0,0)}}$. The maximum likelihood estimator (MLE) is defined by the relation

(3. 9) $$L_T(\hat{\theta}_T) = \sup_{\theta\in\Theta} L_T(\theta).$$

We assume that there exists a measurable maximum likelihood estimator. Sufficient conditions can be given for the existence of such an estimator (cf. Lemma 3.1.2, Prakasa Rao (1987)). Note that

(3. 10)
$$\begin{aligned} Q_{H,\theta}(t) &= \frac{d}{dw_t^H}\int_0^t k_H(t,s)C(\theta,s)ds \\ &= \frac{d}{dw_t^H}\int_0^t k_H(t,s)aX(s)ds + \frac{d}{dw_t^H}\int_0^t k_H(t,s)bX(s-1)ds \\ &= aJ_1(t)+bJ_2(t).\ \text{(say)} \end{aligned}$$

Then

(3. 11) $$\log L_T(\theta) = \int_0^T (aJ_1(t)+bJ_2(t))dZ_t - \frac{1}{2}\int_0^T (aJ_1(t)+bJ_2(t))^2 dw_t^H$$

and the likelihood equations are given by

(3. 12)
$$\begin{aligned} \int_0^T J_1(t)dZ_t &= a\int_0^T J_1^2(t)dw_t^H + b\int_0^T J_1(t)J_2(t)dw_t^H, \\ \int_0^T J_2(t)dZ_t &= b\int_0^T J_2^2(t)dw_t^H + a\int_0^T J_1(t)J_2(t)dw_t^H. \end{aligned}$$

(3. 13)

Solving the above equations, we obtain that the maximum likelihood estimator $\hat{\theta}_T$ of $\theta=(a,b)'$ is given by

(3. 14) $$\hat{\theta}_T = (I_T^0)^{-1}V_T^0$$

where

(3. 15) $$V_T^0 = (\int_0^T J_1(t)dZ_t, \int_0^T J_2(t)dZ_t)$$

and

(3. 16) $$I_T^0 = ((I_{ij}))$$

is the observed Fisher information matrix with

(3. 17) $$I_{ii} = \int_0^T J_i^2(t)dw_t^H, i=1,2$$

and

(3. 18) $$I_{12} = I_{21} = \int_0^T J_1(t)J_2(t)dw_t^H.$$

We can write the log-likelihood function in the form

(3. 19) $$\log\frac{dP_T^\theta}{dP_T^{(0,0)}} = \theta' V_T^0 - \frac{1}{2}\theta' I_T^0\theta, \theta\in R^2.$$

Let $\theta_0 = (a,b)' \in R^2$ be arbitrary but fixed. Let $\theta_0 + \phi_T\gamma$ where $\gamma = (\alpha,\beta)' \in R^2$ and $\phi_T = \phi_T(\theta_0)$ is a normalizing matrix with $||\phi_T|| \to 0$ as $T \to \infty$. It is easy to see that

$$\log \frac{dP_T^{\theta}}{dP_T^{(0,0)}} = \gamma' V_T - \frac{1}{2}\gamma' I_T \gamma \tag{3. 20}$$

where

$$V_T' = (\int_0^T J_1(t)dM_t^H, \int_0^T J_2(t)dM_t^H)\phi_T \tag{3. 21}$$

and

$$I_T = \phi_T' I_T^0 \phi_T. \tag{3. 22}$$

For linear stochastic differential equations with time delay driven by a standard Wiener process, Gushchin and Kuchler (1999) discussed different conditions under which the family of measures $\{P_T^\theta\}$ is locally asymptotically normal (LAN) or locally asymptotically mixed normal (LAMN) or in general locally asymptotically quadratic (LAQ). For a discussion of these concepts, see Prakasa Rao (1999b), Chapter 6.

In view of the representation (3.20) for the log-likelihood ratio process, the family of measures $\{P_T^\theta\}$ is LAQ at θ_0 if we can choose the normalizing matrix $\phi_T(\theta_0)$ in such a way that (i) the vectors V_T and I_T are bounded in probability as $T \to \infty$, (ii) if (V_{T_n}, I_{T_n}) converges in distribution to a limit (V_∞, I_∞) for a subsequence $T_n \to \infty$, then

$$E(\exp(\gamma' V_\infty - \frac{1}{2} I_\infty \gamma)) = 1$$

for every $\gamma \in R^2$, and (iii) if I_{T_n} converges in distribution to a limit I_∞ for a subsequence $T_n \to \infty$, then I_∞ is almost surely positive definite. The family of measures is LAMN at θ_0 if (V_T, I_T) converges in distribution to $(I_\infty^{1/2} Z, I_\infty)$ as $T \to \infty$ where the matrix I_∞ is almost surely positive definite and Z is a standard gaussian vector independent of I_∞. If, in addition, I_∞ is nonrandom, then the family of measures is LMN at θ_0. For the case $b = 0$, the process $X(t)$ reduces to the fractional Ornstein-Uhlenbeck type process. Strong consistency of the maximum likelihood estimator was proved for such a process in Kleptsyna and Lebreton (2002). Properties such as the strong consistency and the existence of the limiting distribution of the MLE for this process as well as for more general processes governed by linear stochastic differential equations driven by a fBm were studied in Prakasa Rao (2003a,b).

Suppose we can are able to obtain a normalizing matrix ϕ_T such that $||\phi_T|| \to 0$ as $T \to \infty$ and

$$(V_T, I_T) \xrightarrow{\mathcal{L}} (V_\infty, I_\infty)$$

as $T \to \infty$. Then we have

$$\phi_T^{-1}(\hat{\theta}_T - \theta_0) \xrightarrow{\mathcal{L}} I_\infty^{-1} V_\infty$$

which shows the asymptotic behaviour of the MLE $\hat{\theta}_T$ as $T \to \infty$. If the family of measures $\{P_T^\theta\}$ is LAMN, then the local asymptotic minimax bound holds for any arbitrary estimator $\tilde{\theta}_T$ of θ and it is given by

$$\text{(3. 23)} \qquad \begin{aligned} \lim_{r\to\infty} \liminf_{T\to\infty} \sup_{\|\phi_T^{-1}(\theta-\theta_0)\|<r} E_\theta[w(\phi_T^{-1}(\tilde{\theta}_T - \theta))] &\geq E[w(I_\infty^{-1}V_\infty)] \\ &= E[w(I_\infty^{-1/2}\mathbf{Z})] \end{aligned}$$

where $\mathbf{Z}$ is a bivariate vector with independent standard normal distributions and $w : R^2 \to [0, \infty)$ is bowl-shaped loss function. The maximum likelihood estimator is asymptotically efficient in the sense that the Hajek-Le Cam lower bound obtained above is achieved by the MLE $\hat{\theta}_T$. These results are consequences of the LAMN property for the family of measures $\{P_T^\theta\}$.

We will discuss sufficient conditions for LAMN later in this paper.

4 A Representation for the Solution of (3.1)

Let us consider again the stochastic differential equation

$$\text{(4. 1)} \qquad dX(t) = (aX(t) + bX(t-1))dt + dW_t^H, t \geq 0$$

where $\theta = (a, b) \in \Theta \subset R^2, W = \{W_t^H, t \geq 0\}$ is a fractional Brownian motion with the Hurst parameter H with the initial condition $X(t) = X_0(t), t \in [-1, 0]$ where $X_0(.)$ is a continuous Gaussian stochastic process independent of W^H. Observe that the process $\{W_t^H, t \geq 0\}$ is a process with stationary increments. Applying the results in Mohammed and Scheutzow (1990), we obtain that there exists a unique solution $X = \{X(t), t \geq -1\}$ of the equation (4.1) and it can be represented in the form

$$\text{(4. 2)} \qquad X(t) = x_0(t)X_0(0) + b\int_{-1}^{0} x_0(t-s-1)X_0(s)ds + \int_0^t x_0(t-s)dW_s^H, t \geq 0.$$

This process has continuous sample paths for $t \geq 0$ almost surely and conditionally on X_0, the process X is a Gaussian process. Further more the function $x_0(.)$, defined for $t \geq -1$, is the *fundamental solution* of the differential equation

$$\text{(4. 3)} \qquad \frac{dx(t)}{dt} = ax(t) + bx(t-1), t > 0$$

subject to the conditions $x(0) = 1, x(t) = 0, t \in [-1, 0)$.

Consider the characteristic equation

$$\text{(4. 4)} \qquad \lambda - a - be^{-\lambda} = 0$$

of the above differential equation. A complex number λ is a solution of (4.4) if and only if the function $e^{\lambda t}$ is a solution of the differential equation

$$\frac{dx(t)}{dt} = ax(t) + bx(t-1), t \geq 0.$$

Let Λ be the set of solutions of the equation (4.4). Define

$$v_0 = \max\{Re\lambda | \lambda \in \Lambda\}$$

and

$$v_1 = \max\{Re\lambda | \lambda \in \Lambda, Re\lambda < v_0\}.$$

A complete discussion of the existence and representation of the fundamental solution $x_0(t)$ of (4.3) is given in the Lemma 1.1 and the following discussion in Gushchin and Kuchler (1999). For the class of linear stochastic differential equations driven by the standard Wiener process, Gushchin and Kuchler (1999) have proved that the corresponding family of measures $\{P_T^\theta\}$ form (i) a LAN family if $v_0 < 0$, (ii) a LAQ family if $v_0 = 0$, and (iii) a LAMN family if $v_0 > 0$, and $v_1 < 0$ or $v_1 > 0$ and $v_1 \in \Lambda$.

5 Local Asymptotic Mixed Normality

Observe that the processes

(5. 1) $$R_i(T) = \int_0^T J_i(t) dM_t^H, i = 1, 2$$

are zero mean local martingales with the quadratic covariation processes

(5. 2) $$< R_m, R_n >_T = \int_0^T J_m(t) J_n(t) dw_t^H, 1 \leq m, n \leq 2.$$

Let

(5. 3) $$R'_T = (R_1(T), R_2(T)).$$

Let $\{< R, R >_t, t \geq 0\}$ be the matrix of covariate processes defined above. Suppose that we can choose a norming function $Q_t \to 0$ as $t \to \infty$ such that

$$Q_t^2 < R, R >_t \xrightarrow{\mathcal{L}} \boldsymbol{\eta^2}$$

as $t \to \infty$ where η is a symmetric positive definite random matrix with probability one. Applying the multidimensional version of the central limit theorem for continuous local martingales (cf. Theorem 1.49,; Remark 1.47, Prakasa Rao (1999b)), it follows that

$$(Q_T R_T, Q_T^2 < R, R >_T) \xrightarrow{\mathcal{L}} (\eta \mathbf{Z}, \boldsymbol{\eta^2})$$

as $T \to \infty$ where $\mathbf{Z}' = (Z_1, Z_2)$ is a bivariate random vector with independent N(0,1) components independent of the random matrix η. Hence we have the following result leading to sufficient conditions for LAMN property of the family of mesures $\{P_T^\theta\}$.

Theorem 5.1: Suppose the parameters a and b are such that there exists a norming function $Q_t \to 0$ as $t \to \infty$ with the property

$$Q_t^2 < R, R >_t \xrightarrow{\mathcal{L}} \boldsymbol{\eta^2}$$

as $t \to \infty$ where η is a symmetric positive definite random matrix with probability one. Then the family of measures $\{P_T^\theta\}$ form a LAMN family.

Remarks: If the matrix η is nonrandom, then the family of measures $\{P_T^\theta\}$ form a LAN family. We conjecture that the family is (i) LAN if $v_0 < 0$ with the norming diagonal matrix with the diagonal elements $(T^{-1/2}, T^{-1/2})$, (ii) LAMN if $v_0 > 0$ and $v_1 < 0$ with the norming diagonal matrix with the diagonal elements $(e^{-v_0 T}, T^{-1/2})$ and (iii) LAMN if $v_0 > 0, v_1 > 0$ and $v_1 \in \Lambda$ with the norming diagonal matrix with the diagonal elements $(e^{-v_0 T}, e^{-v_1 T})$. This conjecture is supported by the results obtained by Gushchin and Kuchler (1999) for linear stochastic differential equations with time delay driven by a Wiener process and by the results in Kleptsyna and Leadbetter (2002) for the fractional Ornstein-Uhlenbeck type process (the case $b = 0$) which implies that $v_0 = a$. In order to check this conjecture, one method is obtain the moment generating function of the matrix R_T explicitly using the methods developed in Kleptsyna and Le Breton (2002) and then study the asymptotic behaviour of the matrix $< R, R >_T$ under different conditions on the parameters a and b.

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