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Wavelet linear density estimation for associated stratified size-biased sample

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Abstract

Ramirez and Vidakovic (2010) considered an estimator of the density function based on wavelets for a random stratified sample from weighted distributions. We extend these results to both positively and negatively associated random variables within strata. An upper bound on L_p-loss for the estimator is given which extends such a result for the L_2-consistent given in Ramirez and Vidakovic(2010).

Key words and phrases: Negative and positive dependence; wavelets, random stratified sample, nonparametric estimation of a density, Rosenthal's inequality.

1 Introduction

Several methods of estimation of density and regression function are avilable in statistical literature. Recently, there has been a lot of interest in nonparametric estimation of such functions based on wavelets. The reader may be referred to Härdle *et al.*(1998) and Vidakovic (1999) for a detailed coverage of wavelet theory in statistics and to Prakasa Rao (1999b) for a comprehensive review and application of various methods of nonparametric functional estimation.

Antoniadis *et al.* (1994) and Masry (1994) among others discuss the estimation of regression and density function using the wavelets. Walter and Ghorai (1992) discuss the advantages and disadvantages of wavelet based methods of nonparametric estimation from *i.i.d.* sequences of random variables. Prakasa Rao (2003) Doosti *et al.* (2006) extended the results for negatively and positively associated sequences, respectively.

Definition 1.1 *A finite family of random variables (r.v.s) $\{X_i, 1 \leq i \leq n\}$ is said to be negatively associated (NA) if, for every pair of disjoint subsets A and B of $\{1, 2, ..., n\}$, we have*

$$Cov\{h_1(X_i, i \in A), h_2(X_j, j \in B)\} \leq 0,$$

whenever h_1 and h_2 are real-valued coordinate-wise increasing functions and the covariance exists. A random process $\{X_i\}_{i=-\infty}^{\infty}$ is NA if every finite sub-family is NA.

The dependence structure characterized by NA was first introduced by Alam and Saxena (1981) and later studied by Joag-Dev and Proschan (1983). Roussas (1996) provides an excellent review of the subject with a comprehensive list of references.

Definition 1.2 *A finite family of random variables $\{X_i, 1 \leq i \leq n\}$ is said to be positively associated (PA) if, we have*

$$Cov\{h_1(X_1, X_2, \ldots, X_n), h_2(X_1, X_2, \ldots, X_n)\} \geq 0,$$

whenever h_1 and h_2 are real-valued coordinate-wise increasing functions and the covariance exists. Similarly, a random process $\{X_i\}_{i=-\infty}^{\infty}$ is PA if every finite sub-family is PA.

PA random variables are of considerable interest in reliability studies, percolation theory and statistical mechanics. For a review of several probabilistic and statistical result for PA sequences, see Prakasa Rao and Dewan (2001).

Here, we adopt the method of estimation of a density function on the basis of a random stratified sample from weighted distributions, discussed in Ramirez and Vidakovic (2010) to the case of associated random variables within the strata. An upper bound on L_p-loss for the resulting estimator is given which extends such a result for the L_2-consistent given in Ramirez and Vidakovic (2010).

The organization of the paper is as follows. In section 2, we discuss the preliminaries of the wavelet based estimation of the density along with the necessary underlying setup considered in Ramirez and Vidakovic (2010). Then in section 3, we extend their result to in L_p-norm. This result is then generalized to associated cases (NA and PA) and finally we obtain upper bounds on the L_p-losses similar to the one obtained by Prakasa Rao (2003) and Doosti *et. al.* (2006) for density estimation for the case of positive and negative association, respectively.

2 Preliminaries

Our notation in the rest of paper follows Romirez and Vidakovic (2010). Let $\{Y_n, n \geq 1\}$ be a sequence of random variables on the probability space $(\Omega, \aleph, P)$. We suppose that Y_i has a stationery bounded and compactly supported marginal density $f^Y(.)$

Definition 2.1 *The density associated with a size-biased random variable Y, f^Y,is related to the underlying true density f^X by*

$$f^Y(y) = \frac{g(y)f^X(y)}{\mu},$$

where g is the so-called weighting or biasing function and μ is defined as the expected value of $g(X)$, $\mu = \mathbf{E}g(X) < \infty$.

As $f^X(y)$ is unknown, the parameter μ is also unknown. The problem is indirect since one observes Y and wants to estimate the density of an unobserved X.

We estimate this density from n observations $Y_i, i = 1, ..., n$. For any function $f \in \mathbf{L}_2(\mathbf{R})$, we can write a formal expansion (see Daubechies (1992)):

$$f^X(y) = \sum_{k \in Z} c_{J,k}\phi_{J,k}(y) + \sum_{j \geq J}\sum_{k \in Z} d_{j,k}\psi_{j,k}(y) = P_J f^X + \sum_{j \geq J} D_j f^X$$

where J is resolution level and the functions

$$\phi_{J,k}(y) = 2^{J/2}\phi(2^J y - k)$$

and

$$\psi_{j,k}(y) = 2^{j/2}\psi(2^j y - k)$$

constitute an (inhomogeneous) orthonormal basis of $\mathbf{L}_2(\mathbf{R})$. Here $\phi(y)$ and $\psi(y)$ are the scale function and the orthogonal wavelet, respectively. Wavelet coefficients are given by the integrals

$$c_{J,k} = \int f^X(y)\phi_{J,k}(x)dy, \quad d_{j,k} = \int f^X(y)\psi_{j,k}(y)dy$$

We suppose that both ϕ and $\psi \in \mathbf{C}^r$, (space of functions with r continuous derivatives), r being a positive integer and have compact supports included in $[-\delta, \delta]$, for some $\delta > 0$. Note that, by corollary 5.5.2 in Daubechies (1988), ψ is orthogonal to polynomials of degree $\leq r$, *i.e.*

$$\int \psi(y)y^l dy = 0, \forall \;\; l = 0, 1, ..., r$$

We suppose that f^X belongs to the Besov class (see Meyer (1992), p.50), $F_{s,p,q} = \{f^X \in B^s_{p,q}, \|f^X\|_{B^s_{p,q}} \leq M\}$ for some $0 \leq s \leq r+1, p \geq 1$ and $q \geq 1$, where

$$\|f^X\|_{B^s_{p,q}} = \|P_J f^X\|_p + (\sum_{j \geq J}(\|D_j f^X\|_p 2^{js})^q)^{1/q}$$

Ramirez and Vidakovic (2010) proposed the following estimator.

$$\hat{f}^X_J(y) = \sum_{k \in K_n} \hat{c}_{J,k}\phi_{J,k}(y), \qquad \text{with} \quad \hat{c}_{J,k} = \frac{\mu}{n}\sum_{i=1}^{n}\frac{\phi_{J,k}(Y_i)}{g(Y_i)}, \tag{2.1}$$

where K_n is the set of k such that $supp(f^X) \cap supp(\phi_{J,k}) \neq \emptyset$. The fact that ϕ has a compact support implies that K_n is finite and $card(K_n) = O(2^J)$. The above estimator can be represented as (3) in Ramirez and Vidakovic(2010):

$$\hat{f}_J^X(y) = \sum_{k \in K_n} \hat{c}_{j_0,k}\phi_{j_0,k}(y) + \sum_{j_0 \leq j \leq J} \sum_{k \in K_n} \hat{d}_{j,k}\psi_{j,k}(y).$$

We observe the stratified sample from M strata, $y_{11}, ..., y_{1n_1}; ...; y_{M1}, ..., y_{Mn_M}$ with common underlying density f^X. Let $N = \sum_{i=1}^{M} n_i$ be the total sample size. Suppose $g_m(y)$ is the strata dependent biasing function. We modify the estimator in (2.1) to account for strata based biasing .

Then a linear estimator of f^X based on all observations, can be defined as

$$\hat{f}_J^X(y) = \sum_{m=1}^{M} \alpha_m \hat{f}_{m,J(m)}^X(y), \tag{2.2}$$

where $\alpha_m = \frac{n_m}{N}$, $J(m)$ is the projection level in the mth stratum, and J is defined as $J := \min J(1), ..., J(M)$

In(2.2), $\hat{f}_J^X(y)$ represents the projection estimate of f^X defined in (2.1), based on the mth stratum, depending on

$$\hat{c}_{J(m),k}^m = \frac{\mu_m}{n_m} \sum_{i=1}^{n_m} \frac{\phi_{J(m),k}(Y_{mi})}{g_m(Y_m i)}, \tag{2.3}$$

the estimate of $c_{J(m),k}$ withing stratum m. $\mu_m, m = 1, ..., M$ are unknown parameters which are related to unknown density function f^X. Ramirez and Vidakovic (2010) proposed an unbiased estimators for inverse of these parameters, but in the proof of their main theorem they considered them as a known parameters which we follow this in next section.
The following two lemmas are needed for the proofs in the next section.

Lemma 2.1 *Let $\{\xi_i, 1 \leq i \leq n\}$ be a sequence of NA identically distributed random variables such that $\mathbf{E}(\xi_i) = 0$, and $\|\xi_i\|_\infty < M < \infty$. Then there exist positive constant $C(p)$ such that*

$$\mathbf{E}(|\sum_{i=1}^{n} \xi_i|^p) \leq C_1(p)\{M^{p-2} \sum_{i=1}^{n} \mathbf{E}(\xi_i^2) + (\sum_{i=1}^{n} \mathbf{E}(\xi_i^2))^{p/2}\}, p > 2 \tag{2.4}$$

Proof: This is readily obtained by using the results (1.6) and (1.7) of Theorem 2 of Shao (2000).

Lemma 2.2 *Let $\{\xi_i, 1 \leq i \leq n\}$ be a sequence of PA identically distributed random variables satisfifying $\mathbf{E}(\xi_i) = 0$, and $\mathbf{E}|\xi_i|^{p+\delta} \leq M$ for some $p > 2$ and $\delta > 0$. Assume*

$$u(n) = \sup_{k \geq 1} \sum_{j:|j-k| \geq n} Cov(\xi_j, \xi_k) = O(n^{-(p-2)(p+\delta)/(2\delta)}) \tag{2.5}$$

then there is a constant B not depending on n such that for all n

$$\mathbf{E}(|\sum_{i=1}^{n}\xi_i|^p) \leq B\{M^{(p-2)/z}n^{p/2}\}, \tag{2.6}$$

where $z = \delta + (p-2)(p+\delta)$.

Proof: This is a result from Theorem 1 Birkel (1988) p.1185. The only difference is that we drop the assumption (2.1) in Theorem 1, Birkel (1988) and instead we assume upper bound for $\mathbf{E}|\xi_i|^{r+\delta} \leq M$. Conclusions of the theorem hold even if M is not finite.

3 Main Results

First, we consider the i.i.d. sequence $\{Y_{ij}, j = 1, ..., n_i,\ i = 1, ..., M;\}$ of random variables and extend the result of Ramirez and Vidakovic (2010) when the error is measured in L_p-norm. Next, we obtain similar results for sequences are NA and PA within the strata in Theorems 3.2 and 3.3. Here an additional condition on the scale function, namely bounded variation, is imposed. We note that C is a generic positive number in the rest of paper.

Theorem 3.1 *Let $f^X(x) \in F_{s,p,q}$ with $0 < B_1 \leq g_m(y), s \geq 1/p, p \geq 1,$ and $q \geq 1$. The density function f^X is uniformly bounded and f_J^X is the linear wavelet density estimator in Eq. (2.2) for the i.i.d. sequence of random variables $\{Y_{ij}, j = 1, ..., n_i,\ i = 1, ..., M;\}$, Then for $p' \geq max(2, p)$, there exists a constant C such that*

$$\mathbf{E}\|\hat{f}_J^X(y) - f^X(y)\|_{p'}^2 \leq C\ N^{-\frac{2s'}{1+2s'}}$$

where $s' = s + 1/p' - 1/p$ and $2^{J(m)} = n_m^{\frac{1}{1+2s'}}$.

Proof: We have

$$\mathbf{E}\|\hat{f}_J^X(y) - f^X(y)\|_{p'}^2 \leq \sum_{m=1}^{m} \alpha_m^2 \mathbf{E}\|\hat{f}_{m,J(m)}^X(y) - f^X(y)\|_{p'}^2 \tag{3.1}$$

It is sufficient to find an upper bound for $\mathbf{E}\|\hat{f}_{m,J(m)}^X(y) - f^X(y)\|_{p'}^2$ for some m. First, we decompose $\mathbf{E}\|\hat{f}_{m,J(m)}^X - f^X\|_{p'}^2$ into a bias term and stochastic term

$$\mathbf{E}\|\hat{f}_{m,J(m)}^X - f^X\|_{p'}^2 \leq 2(\|f^X - P_{J(m)}f^X\|_{p'}^2 + \mathbf{E}\|\hat{f}_{m,J(m)}^X - P_{J(m)}f^X\|_{p'}^2) = 2(T_1 + T_2) \tag{3.2}$$

Now, we want to find upper bounds for T_1 and T_2 using techniques of Leblanc (1996), p.83 (see also Prakasa Rao (2003), p.373).

$$T_1 \leq C2^{-2s'J(m)}, \tag{3.3}$$

where $s' = s + 1/p' - 1/p$.

By using Lemma 1 in Leblanc (1996), p. 82 (using Meyer (1992), lemma 8, p.30),

$$T_2 \leq C\mathbf{E}\{\|\hat{c}^m_{J(m),k} - c_{J(m),k}\|^2_{l_{p'}}\}2^{2J(m)(1/2-1/p')}.$$

Further, by using Jensen's inequality the above equation implies,

$$T_2 \leq C2^{2J(m)(1/2-1/p')}\{\sum_{k\in K_n} \mathbf{E}|\hat{c}^m_{J(m),k} - c_{J(m),k}|^{p'}\}^{2/p'}. \tag{3.4}$$

To complete the proof, it is sufficies to estimate $\mathbf{E}|\hat{c}^m_{J(m),k} - c_{J(m),k}|^{p'}$. We know that

$$\hat{c}^m_{J(m),k} - c_{J(m),k} = \frac{1}{n_m}\sum_{i=1}^{n_m}\{[\frac{\phi_{J(m),k}(Y_{m_i})}{g_m(Y_{m_i})} - c_{J(m),k}]\}.$$

Denote $\xi_i = [\frac{\phi_{J(m),k}(Y_{mi})}{g_m(Y_{mi})} - c_{J(m),k}]$. Note that $\|\xi_i\|_\infty \leq K.2^{1/2J(m)}\|\phi\|_\infty, \mathbf{E}\xi_i = 0$, $\mathbf{E}\xi_i^2 < \infty$ and $|\hat{c}^m_{J(m),k} - c_{J(m),k}| = \frac{1}{n_m}|\sum_{i=1}^{n_m}\xi_i|$. Hence applying the Rosenthal's inequality for *i.i.d* random variables, e.g. in Härdle *et al.* (1998) p. 244, and using $card(K_n) = O(2^{J(m)})$ we have

$$\begin{aligned}
\{\sum_{k\in K_n} \mathbf{E}|\hat{c}^m_{J(m),k} - c_{J(m),k}|^{p'}\}^{2/p'} &\leq \{C2^{J(m)}\frac{1}{n^{p'}}(n_m 2^{(J(m)/2)(p'-2)} + n_m^{p'/2})\}^{2/p'} \\
&\leq C\{\frac{2^{J(m)(1)}}{n_m^{2(1-1/p')}} + \frac{2^{2(J(m)/p\prime)}}{n_m}\}. \qquad (3.5)
\end{aligned}$$

Now by substituting the above bound in (3.4), we get

$$T_2 \leq C\{\frac{2^{J(m)}}{n_m}(\frac{2^{J(m)}}{n_m})^{1-2/p'} + \frac{2^{J(m)}}{n_m}\}. \tag{3.6}$$

Since $n_m \geq 2^{J(m)}$ and $1 - 2/p' \geq 0$ imply $(\frac{2^{J(m)}}{n_m})^{1-2/p'} \leq 1$, we have the inequality

$$T_2 \leq \frac{C2^{J(m)}}{n_m}. \tag{3.7}$$

By using the bounds obtained in (3.3) and (3.7),$n_m = O(N)$ for aome m, and choosing $J(m)$ such that $2^{J(m)} = n_m^{\frac{1}{1+2s'}}$ in (3.1), the theorem is proved. □

Now, in the rest of paper we consider $\{Y_{ij}\}$ as a negatively(positively) associated sequence of random variables within strata and independent accross strata. We also assume $\phi_{J(m),k}/g_m$, to be a function of bounded variation (BV) .

Theorem 3.2 *Let $\phi_{J(m),k}/g_m$ be BV, $f^X \in F_{s,p,q}$ with $s \geq \max(1/p, d), p \geq 1,$ and $q \geq 1$. Consider the linear wavelet based estimator in Eq. (2.2) for NA sequence of random variables $\{Y_{ij}\}$ within strata. Then, for $p' > max(2,p)$, there exists a constant C such that*

$$\mathbf{E}\|\hat{f}_J^X(y) - f^X(y)\|_{p'}^2 \leq C\ N^{-\frac{2s}{1+2s'}}$$

where $s' = s + 1/p' - 1/p$, and $2^{J(m)} = n_m^{\frac{1}{1+2s'}}$, for some m.

Proof: The proof is similar to the proof of Theorem 3.1 , see Doosti *et al.*(2006). They assumed the monotonicity of the scale function, which is a rather restrictive condition. Here the bounded variation property of the $\phi_{J(m),k}/g_m$ is assumed which is more general in contrast. The method of proof follows in theorem 3.2 in Chaubey *et al.* (2008) p.460. We shall prove that (3.7) still remains true under NA case. Since $\phi_{J(m),k}/g_m$ is function of BV, so it is the difference of two monotone increasing function, say ϕ_1, ϕ_2, on $[-\delta, \delta]$, i.e., $\phi_{J(m),k}/g_m = \phi_{1(J(m),k)} - \phi_{2(J(m),k)}$. Furthermore If we define:

$$\begin{aligned}
a_1 &= \int \phi_{1(J(m),k)}(y) f^X(y) dy, \\
a_2 &= \int \phi_{2(J(m),k)}(y) f^X(y) dy, \\
\xi_{1(i)} &= \phi_{1(J(m),k)}(Y_i) - a_1, \\
\xi_{2(i)} &= \phi_{2(J(m),k)}(Y_i) - a_2,
\end{aligned}$$

then it is easy to see:

$$\begin{aligned}
\mathbf{E}\xi_{1(i)} &= \mathbf{E}\xi_{2(i)} = 0 \\
\|\xi_{l(i)}\|_\infty &\leq C\|\xi_i\|_\infty < C2^{J(m)/2}\|\phi\|_\infty, l = 1, 2, \\
\mathbf{E}\xi_{l(i)}^2 &\leq C\mathbf{E}\xi_i^2 < \infty, l = 1, 2,
\end{aligned}$$

, where ξ_i defined in Theorem 3.1.

Since $\{Y_{ij}; 1 \leq j \leq n_i\}$ is NA and the monotonicity of the functions $\phi_{1(J(m),k)}$ and $\phi_{2(J(m),k)}$, it follows that the sequences $\{\xi_{1(i)}, i \geq 1\}$ and $\{\xi_{2(i)}, i \geq 1\}$ are also a sequences of NA random variables. Now by considering lemma 2.1 and using following inequality, we see the Eq. (3.7) remains true.

$$|\sum_{i=1}^{n_m} \xi_i|^{p'} \leq 2^{p'}(|\sum_{i=1}^{n_m} \xi_{1(i)}|^{p'} + |\sum_{i=1}^{n_m} \xi_{2(i)}|^{p'}). \tag{3.8}$$

The rest of proof is similar to the proof of Theorem 3.1. □

Theorem 3.3 *Let $\phi_{J(m),k}/g_m$ be BV, $\{\xi_{l(i)}\}, l = 1, 2$ satistiefied in (2.5), $f^X \in F_{s,p,q}$ with $s \geq \max(1/p, d), p \geq 1,$ and $q \geq 1$. Consider the linear wavelet based estimator in Eq. (2.2)* *for PA sequence of random variables* $\{Y_{ij}\}$ *within strata. Then, for* $p' > max(2,p)$*, there exists a constant* C *such that*

$$\mathbf{E}\|\hat{f}_J^X(y) - f^X(y)\|_{p'}^2 \leq C\ \frac{2^{\frac{J(m)}{p'}[(p'+\delta-2)(p'-2)/z+2]}}{N},$$

where $z = \delta + (p'-2)(p'+\delta)$*.*

Proof: The proof follows the proof of last theorem which we use Lemma 2.2 instead of Lemma 2.1.

Remark 1: One can adopt the method in Prakasa Rao (2003) in p.377 and in Theorem 3.3 Chaubey *et al.* (2008) to get upper bound similar to those as in last three theorems for the expected loss $\mathbf{E}\|\hat{f}_{m,J(m)}^X - f^X\|_{p'}^{p'}$ when $1 \leq p' \leq 2$. Observe that

$$\mathbf{E}\|\hat{f}_{m,J(m)}^X - f^X\|_{p'}^{p'} \leq 2^{p'-1}(\|f^X - P_{J(m)}f^X\|_{p'}^{p'} + \mathbf{E}\|\hat{f}_{m,J(m)}^X - P_{J(m)}f^X\|_{p'}^{p'}) \tag{3.9}$$

and

$$\|f^X - P_{J(m)}f^X\|_{p'}^{p'} \leq C2^{-p's'J(m)} \tag{3.10}$$

we have

$$\mathbf{E}\|\hat{f}_{m,J(m)}^X - P_{J(m)}f^X|_{p'}^{p'} \leq C2^{2J(m)(p'/2-1)}\{\sum_{k\in K_n}\mathbf{E}|\hat{c}_{J(m),k} - c_{J(m),k}|^{p'}\}. \tag{3.11}$$

Remark 2: We may use other version of Rosenthal's inequality as in Prakasa Rao (2003) to find another upper bound .

Remark 3: By using similar techniques and the results in Chaubey *et al.* (2006, 2008) and Prakasa Rao (1996, 1999a, 1999b) one can find upper boundes for derivatives of density function for positive and negative associated sequences of random variables within strata.

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