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On confidence intervals for quantiles and tolerance intervals for subdistribution functions

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Abstract

We find distribution-free confidence intervals for quantiles of subdistribution functions and also tolerance intervals for subdistribution functions. Both these are based on order statistics. We tabulate these intervals for various choices of the parameters and compare them with those for distribution functions.

Key Words: Confidence intervals, improper distribution function, order statistics, quantiles, tolerance intervals.

1 Introduction

Let $F(.)$ be a continuous distribution function with q_p a quantile of order $p, q_p = \inf\{x : F(x) \geq p\}$. A technique for constructing distribution-free confidence intervals for q_p based on order statistics is well-known. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics corresponding to a random sample from F. Then $[X_{(r)}, X_{(s)}],\ r \leq s$, provides a $100(1-\alpha)\%$ confidence interval for q_p, where r and s are chosen such that

$$P(X_{(r)} \leq q_p \leq X_{(s)}) = \sum_{j=r}^{s-1} \binom{n}{j} p^j (1-p)^{n-j} = 1-\alpha,$$

see, for example, Gibbons and Chakraborti (2003), Hettmansperger (1984).

However, the results are not true if F is not a proper distribution function, that is $F(\infty) < 1$. In several situations we come across a pair of random variables (T, δ) - one of which is continuous and the other is discrete. For example, in reliability T denotes the lifetime of a series system and δ denotes the component which has failed, and in survival analysis T is the age at death and δ is the cause of death. For several other examples see Crowder (2001).

Suppose that there are two causes of death. The joint distribution of (T, δ) is given by

$$F(i,t) = P[T \leq t, \delta = i],\ i = 1, 2. \tag{1}$$

Note that $F(i,t)$ is a subdistribution function with $F(i,\infty) = P[\delta = i] < 1,\ i = 1, 2$. Assume that $F(i,t)$ is continuous. Let q_{ip} be the quantile of order p for the subdistribution function $F(i,t)$. In particular, the quantile q_{1p} satisfies the following

$$F(1, q_{1p}) = P[T \leq q_{1p}, \delta = 1] = p,\ \ 0 \leq p \leq F(1,\infty) = \theta\ \ (say). \tag{2}$$

We want to find distribution-free confidence intervals for q_{1p} based on ordered statistics corresponding to failures due to the cause 1. Such intervals are of interest in various fields, for example the policy makers for health care would like to have an idea of an interval in which the median age of failure due to cardiac problems or AIDS lies. Similarly car reliability engineers could be interested in a confidence interval of median failures due to electrical/mechanical problems.

One could also be interested in setting tolerance limits for a continuous subdistribution functions. A tolerance interval for a continuous distribution function with tolerance coefficient γ is a random interval such that the probability is γ that the area between the end points of the interval and under the probability density function is atleast a certain preassigned number p. That is, this random interval includes atleast 100 $p\%$ of the distribution with probability γ. Engineers giving guarantee for products (say refrigerators) need to ensure that the compressors of a high percentage of refrigerators survive certain reasonable time point with a high probability.

To the best of our knowledge this problem has not been looked into. Zhou (1997) obtained confidence intervals for quantiles of the distribution function in the presence of independent censoring. These were based on the quantiles of the Kaplan-Meier estimator. Zhou and Wu (2002) considered the sequential fixed-width interval estimation for quantiles with censored data. In each case the data used were of the form (T, δ) where T is the minimum of the failure time and the censoring time and δ the corresponding indicator function. However, we are interested in distribution-free confidence intervals for quantiles of the joint distribution function of (T, δ) and not in the confidence interval for quantiles of failure time distribution. Further there are not many known and documented parameteric or semiparameteric functional forms for subdistribution functions. We also extend the well studied procedures for distribution functions based on order statistics to the case of subdistribution functions.

In section 2, we state some simple but interesting results on order statistics coming from subdistribution functions and also the probability integral transformation for subdistribution functions. In section 3, we discuss distribution-free confidence intervals for quantiles of order p for $F(1,t)$. These are based on order statistics from a sample with failures due to cause 1. In section 4, we find distribution-free tolerance limits for $F(1,t)$, again based on order statistics. In section 5, we give tables for conditional and unconditional confidence intervals for the median (to be defined in section 2) and for $p = 0.5$ and compare them with the standard case for distribution functions and also give our conclusions.

2 Prelimnaries

First we look at some simple results on distribution of order statistics based on a random sample from improper distribution. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from a distribution function $F(x)$, where $F(x)$ is increasing and right continuous with $F(0) = 0, F(\infty) = \theta < 1$, that is, F is an improper distribution function. Note that the survival function is given by

$$\bar{F}(x) = F(\infty) - F(x) = \theta - F(x). \tag{3}$$

Let the density function be denoted by $f(x)$, where $\int_0^\infty f(x)dx = \theta < 1$. The median, m of the improper distribution $F(.)$ is such that $F(m) = \theta/2$. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denote the order statistics corresponding to the given random sample. The following theorem discusses the distribution of the minimum, the maximum, the rth order statistics as well as the joint distribution of rth and $sth, r < s$ order statistics. The results are straightforward but since they have not been reported elsewhere, we state them here for completeness.

Theorem 1 *Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from an improper distribution function $F(x)$. Then for $x < y,\ 1 \le r < s \le n$, the following are true*

$$\begin{aligned}
(i) \quad P[X_{(1)} > x] &= (\theta - F(x))^n, \\
(ii) \quad h_{X_{(1)}}(x) &= n(\theta - F(x))^{n-1}(x)f(x), \\
(iii) \quad P[X_{(n)} \le x] &= F^n(x), \\
(iv) \quad h_{X_{(n)}}(x) &= n(F(x))^{n-1}f(x),
\end{aligned}$$

$$
\begin{aligned}
(v) \qquad h_{X_{(r)}}(x) &= \frac{n!}{(r-1)!(n-r)!}F(x)^{r-1}(\theta - F(x))^{n-r}f(x), \;\; 1 \le r \le n, \\
(vi) \;\; h_{X_{(r)},X_{(s)}}(x) &= \frac{n!}{(r-1)!(s-r-1)!(n-s)!}F(x)^{r-1}(F(y) - F(x))^{s-r-1} \\
&\quad (\theta - F(y))^{n-s}f(x)f(y). \qquad (4)
\end{aligned}
$$

In particular, if we let $G(x) = F(x)/\theta$, then $G(x)$ is a proper distribution function. Let $g(x)$ be the corresponding density function. Then, the above results can be restated in terms of $G(x)$ as follows.

Theorem 2 *Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from the distribution function $G(x)$ defined above. Then for $x < y, \;\; 1 \le r < s \le n$, the following are true*

$$
\begin{aligned}
(i) \;\; P[X_{(1)} > x] &= \theta^n(1 - G(x))^n \\
(ii) \qquad h_{X_{(1)}}(x) &= n\theta^n(1 - G(x))^{n-1}g(x) \\
(iii) \;\; P[X_{(n)} \le x] &= \theta^n G^n(x) \\
(iv) \qquad h_{X_{(n)}}(x) &= n\theta^n(G(x))^{n-1}g(x) \\
(v) \qquad h_{X_{(r)}}(x) &= \theta^n \frac{n!}{(r-1)!(n-r)!}G(x)^{r-1}(1 - G(x))^{n-r}g(x), \;\; 1 \le r \le n, \\
(vi) \;\; h_{X_{(r)},X_{(s)}}(x) &= \theta^n \frac{n!}{(r-1)!(s-r-1)!(n-s)!}G(x)^{r-1}(G(y) - G(x))^{s-r-1} \\
&\quad (1 - G(y))^{n-s}g(x)g(y). \qquad (5)
\end{aligned}
$$

Probability integral transformation states that if X is a continuous random variable with distribution function $F(x)$, then the random variable $Y = F(X)$ has a uniform distribution on $(0, 1)$. It is a useful tool in nonparametric inference - in particular in the study of distribution-free confidence intervals and tolerance intervals. Next we extend this result to the case when F is not a proper distribution function. Let $F^{-1}(y) = \inf\{x | F(x) \ge y\}, \;\; 0 < y < \theta$.

Theorem 3 *Suppose X is a continuous random variable with an improper ditribution function $F(x)$, that is, $F(\infty) = \theta < 1$. Then the random variable $Y = F(X)$ has an improper uniform distribution given by*

$$
\begin{aligned}
P[Y \le y] &= 0, \;\; y \le 0, \\
&= y, \;\; 0 \le y \le \theta, \\
&= \theta, \;\; y \ge \theta. \qquad (6)
\end{aligned}
$$

Proof: Since $F(.)$ is continuous, $F(F^{-1}(y)) = y$ for $0 < y < \theta$. Since $F(.)$ is monotonic, we have $\{X \le F^{-1}(y)\}$ implies $\{F(X) \le F(F^{-1}(y)) = y\}$. Also

$$\{F(X) \le y\} = \{X \le F^{-1}(y)\}U\{X > F^{-1}(y)\text{and}F(X) = y\}.$$

Since X has a continuous distribution, we have $P[F(X) = y] = 0$. Hence,

$$P[Y \le y] = P[F(X) \le y] = P[X \le F^{-1}(y)] = y, \;\; 0 < y < \theta.$$

3 Confidence intervals for quantiles of subdistribution functions

Let $(T_1,\delta_1),(T_2,\delta_2),\ldots,(T_n,\delta_n)$ denote a random sample of failure times and the corresponding causes of failure. Suppose there are n_1 failures due to the first cause and n_2 failures due to the second cause, $n=n_1+n_2$. Let $T_{11},T_{21},\ldots,T_{n_1 1}$ denote the n_1 ordered lifetimes where the failure was due to risk 1.

Then

$$\begin{aligned}
P[T_{j1}\le q_{1p}] &= P[\text{at least } j\ T_{k1}\text{ 's are } \le q_{1p}] \\
&= \sum_{n_1=j}^{n}\sum_{k=j}^{n_1} P[k \text{ out of } n_1 T'_{i1}s \text{ are } \le q_{1p}]P[\text{there are } n_1 T_{j1}] \\
&= \sum_{n_1=j}^{n}\sum_{k=j}^{n_1}\binom{n_1}{k}p^k(\theta-p)^{n_1-k}\binom{n}{n_1}\theta^{n_1}(1-\theta)^{n-n_1}, \\
&= \sum_{n_1=j}^{n}\sum_{k=j}^{n_1}\binom{n_1}{k}p^k(\theta-p)^{n_1-k}\binom{n}{n_1}\theta^{n_1}(1-\theta)^{n-n_1}, \qquad (7)
\end{aligned}$$

and hence

$$P[T_{i1}\le q_{1p}\le T_{j1}] = \sum_{n_1=j}^{n}\sum_{k=i}^{j-1}\binom{n_1}{k}p^k(\theta-p)^{n_1-k}\binom{n}{n_1}\theta^{n_1}(1-\theta)^{n-n_1}, \qquad (8)$$

Note that if $n_1<j$, then, $P[T_{j1}\le q_{1p}]=0$. Hence we have the following theorem.

Theorem 4 $[T_{i1},T_{j1}]$, $i<j$ *is a* $100(1-\alpha)\%$ *confidence interval for* q_{1p} *if*

$$\sum_{n_1=j}^{n}\sum_{k=i}^{j-1}\binom{n_1}{k}p^k(\theta-p)^{n_1-k}\binom{n}{n_1}\theta^{n_1}(1-\theta)^{n-n_1}=(1-\alpha). \qquad (9)$$

Thus, we need to choose i and j so as to satisfy (9) for a given choice of n,p,θ,α.

Proof: Consider n i.i.d. $T_i's$. There are three mutually exclusive events. $X_1 : T_i\le q_{1p},\delta_i=1$ with probability p, $X_2 : T_i>q_{1p},\delta_i=1$ with probability $\theta-p$, and $X_3 : T_i,\delta_i=2$ with probability $1-\theta$. Hence,

$$P[X_1=x_1,X_2=x_2,X_3=x_3]=\frac{n!}{x_1!x_2!x_3!}p^{x_1}(\theta-p)^{x_2}(1-\theta)^{x_3}, \qquad (10)$$

where $x_1,x_2,x_3\ge 0, x_1+x_2+x_3=n$. This is a multinomial distribution. For given n_1, $x_1=0,1,\ldots,n_1;\ \ x_2=n_1-x_1; x_3=0,1,\ldots,n-n_1$. It is interesting to note that

$$\begin{aligned}
&\binom{n_1}{k}p^k(\theta-p)^{n_1-k}\binom{n}{n_1}\theta^{n_1}(1-\theta)^{n-n_1)} \\
=\ & \frac{n!}{k!(n_1-k)!(n-n_1)!}p^k(\theta-p)^{n_1-k}\theta^{n_1}(1-\theta)^{n-n_1} \\
=\ & \theta^{n_1}\frac{n!}{k!(n_1-k)!(n-n_1)!}p^k(\theta-p)^{n_1-k}(1-\theta)^{n-n_1}. \qquad (11)
\end{aligned}$$

Hence the terms in the summation in (11) are θ^{n_1} times multinomial probabilities.

Thus, given θ, the probability of failing due to cause 1, we can easily find appropriate choices of i, j. However, if θ is unknown, we can estimate it and then use multinomial tables to find i and j.

The case when T and δ are independent is of special interest as it simplifies the problem. Dewan *et al.* (2004) have proposed distribution-free tests for testing independence of T and δ against various dependence alternatives. In this case $p = \theta F(q_{1p})$, where F is the cdf of T. Then we have the following result.

Theorem 5 *If T and δ are independent, then $[T_{i1}, T_{j1}]$, $i < j$ is a $100(1-\alpha)\%$ confidence interval for q_{1p} such that*

$$\sum_{n_1=j}^{n} \binom{n}{n_1} \theta^{2n_1}(1-\theta)^{n-n_1} \sum_{k=i}^{j-1} \binom{n_1}{k} (F(q_{1p}))^k (1-F(q_{1p}))^{n_1-k} = (1-\alpha). \tag{12}$$

The following theorem gives conditional confidence intervals for quantiles, conditional on the number of failures due to cause 1.

Theorem 6 *Given n_1, the number of failures due to cause 1, $[T_{i1}, T_{j1}]$, $i < j$ is a conditional $100(1-\alpha)\%$ confidence interval for q_{1p} such that*

$$\sum_{k=i}^{j-1} \binom{n_1}{k} p^k (\theta - p)^{n_1-k} = (1-\alpha). \tag{13}$$

Thus, we need to choose i and j so as to satisfy (13) for a given choice of n, n_1, p, θ, and α.

Note that confidence interval in (13) is same as that for confidence interval for quantiles of distribution functions, with $(1-p)$ replaced by $(\theta - p)$ (see Gibbons and Chakraborti, 2003).

4 Tolerance intervals for subdistribution functions

Let the random variable T_1 denote the failure time due to cause 1 having subdistribution function $F(1,t)$. Let T_{1r} and $T_{1s}, r < s$ denote the rth and sth ordered statistic from a sample of size n_1 from T_1. We have to find r, s such that for given n, γ, β

$$\gamma = P[P[T_{1r} < T_1 < T_{1s}] \geq \beta]. \tag{14}$$

Theorem 7 *Given n_1, $[T_{1r}, T_{1s}]$ is a conditional tolerance interval for subdistribution function $F(1,t)$, with tolerance coefficient*

$$\gamma = 1 - \sum_{j=s-r}^{n_1} \theta^{n_1} \binom{n_1}{j} \beta^j (1-\beta)^{n_1-j}. \tag{15}$$

Proof: For $1 \le r < s \le n_1$, we have

$$\begin{aligned}\gamma &= P[P[T_{1r} < T_1 < T_{1s}] \ge \beta] \\ &= P[F(1,s) - F(1,r) \ge \beta]. \qquad (16)\end{aligned}$$

Using Theorems 1-3, the joint density of the rth and the sth order statistics from distribution given by

$$\frac{n_1!}{(r-1)!(s-r-1)!(n_1-s)!} x^{r-1}(y-x)^{s-r-1}(\theta-y)^{n_1-s},\ 0 < x < y < \theta. \qquad (17)$$

Hence the distribution of $F(1,s) - F(1,r)$, is given by

$$g_1(z) = \frac{z^{s-r-1}(\theta-z)^{n_1-s+r}}{B(s-r, n_1-s+r+1)},\ 0 < z < \theta, \qquad (18)$$

which is an improper beta density function. Hence

$$\begin{aligned}\gamma &= \int_\beta^\theta g_1(z)dz \\ &= 1 - \sum_{j=s-r}^{n_1} \theta^{n_1} \binom{n_1}{j} \beta^j (1-\beta)^{n_1-j}, \qquad (19)\end{aligned}$$

The following theorem gives the unconditional tolerance intervals.

Theorem 8 $[T_{1r}, T_{1s}]$ *is an unconditional tolerance interval for subdistribution function* $F(1,t)$, *with tolerance coefficient*

$$\gamma = \sum_{n_1=s}^{n} [1 - \sum_{j=s-r}^{n_1} \theta^{n_1} \binom{n_1}{j} \beta^j (1-\beta)^{n_1-j}] \binom{n}{n_1} \theta^{n_1} (1-\theta)^{n-n_1}. \qquad (20)$$

5 Computation of confidence intervals

Here we illustrate the confidence intervals and corresponding confidence coefficients for the quantiles of order $p = \theta/2$ and $p = 0.5$ when the order statistics are from the proper distribution function and from the subdistribution functions and for given values of n, n_1, θ and two choices of $(1-\alpha) = 0.99$ and 0.95. The tables give the confidence coefficients corresponding to the specific values of i and j of order statistics for the following three cases:

(i) the standard case when i and j are order statistics based on a sample of size n from a proper distribution function and the confidence coefficient is given by

$$\sum_{k=i}^{j-1} \binom{n}{k} p^k (1-p)^{n-k} = \text{F}\textit{ull Probability},$$

(ii) given $n_1(< n)$ number of failures due to cause 1, i and j are order statistics based on failures due to the first cause and the confidence coefficient is given by

$$\sum_{k=i}^{j-1} \binom{n_1}{k} p^k (\theta-p)^{n_1-k} = \textit{Conditional Probability},$$

and

(iii) unconditional confidence coefficient by summing over all possible values of n_1 in (ii) and the confidence coefficient is given by

$$\sum_{n_1=j}^{n}\sum_{k=i}^{j-1}\binom{n_1}{k}p^k(\theta-p)^{n_1-k}\binom{n}{n_1}\theta^{n_1}(1-\theta)^{n-n_1} = \text{U}\mathit{nconditional\ Probability}.$$

Note that for a given choice of p, the full probability depends only on n, the unconditional probability varies with θ and n. Both these are not affected by the choice of n_1. The conditional probability varies with n , θ and n_1.

For Table 1, the values of i and j were chosen such that the Full Probability is at least 99% and $p = 0.5$ for various values of n. The Conditional Probability and Unconditional Probability are reported for these choices of i and j and for $\theta = 0.6, 0.7, 0.8, 0.9$. For $n = 16$ and $\theta = 0.7$, the standard confidence interval for a quantile of order $p = 0.5$ of the distribution function is based on the third and the fourteenth order statistics and the confidence coefficient is given by 0.9958. The unconditional confidence coefficient for this choice is 0.0006. However, given $n_1 = 14, 15, 16$, the third and the fourteenth order statistics from failures due to cause 1 give the confidence intervals for quantile of order $p = 0.5$ of the subdistribution function with confidence coefficients $0.0067, 0.0045, 0.0029$, respectively. Similarly, Table 2 is created for those i and j such that the Full Probability is at least 95%. We find that the confidence coefficient significantly drops in each of the cases - being slightly higher for relatively large values of $(\theta - p)$. The reason for the drop is the small value for θ^{n_1}. The multiplicative effect of small numbers is extremely high, leading to extremely small values of the confidence coefficients.

Tables 3 and 4 give 99% and 95% confidence intervals for the median of the subdistribution function ($p = \theta/2$) for $\theta = 0.6, 0.7, 0.8, 0.9$. As an illustration let us take $\theta = 0.6$. For $n = 16$, the standard confidence interval for a quantile of order $p = 0.3$ of the distribution function is based on the first and the eleventh order statistics and the confidence coefficient is given by 0.9951. The unconditional confidence coefficient for this choice is 0.0009. However, given $n_1 = 11, 12, \ldots, 16$, the first and the eleventh order statistics from failures due to cause 1 give the confidence intervals for quantile of order $p = 0.3$ of the subdistribution function with confidence coefficients $0.0036, 0.0022, 0.0013, 0.0007$, respectively. Note that all other parameters remaining fixed, the confidence coefficient decreases with increase in n_1. Here again the confidence coefficient drops significantly when one moves from confidence intervals for distribution functions to confidence intervals for subdistribution functions .

One could similarly work out distribution-free tolerance intervals and the tolerance coefficients continue to be small as in the case of confidence intervals.

6 Conclusions

The mathematical theory for finding confidence intervals for quantiles of subdistribition functions and tolerance intervals is simple and is an elegant extension of the similar theory for distribution functions. As seen from the tables, the confidence coefficients corresponding to the proper distribution function are very high compared to the conditional as well as unconditional confidence coefficients. The tables are just illustrative - one could work them out for other choices of n, p, θ.

For large sample sizes, one could approximate binomial probabilities by normal probabilities, but the confidence coefficients are extremely small. There is a need to explore alternative distribution-free procedures for finding confidence intervals for quantiles and tolerance intervals for subdistribution functions for small as well as large sample sizes.

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Table 1: 99% Confidence intervals for $p = 0.5$

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
8	0.6	8	1	8	0.0129	0.0002	0.9922
8	0.7	8	1	8	0.0537	0.0031	0.9922
8	0.8	8	1	8	0.1638	0.0275	0.9922
8	0.9	8	1	8	0.4259	0.1833	0.9922
10	0.6	10	1	10	0.0051	0.00003	0.9980
10	0.7	10	1	10	0.0273	0.0007	0.9980
10	0.8	10	1	10	0.1064	0.01142	0.9980
10	0.9	10	1	10	0.3476	0.1212	0.9980
12	0.7	11	2	11	0.0193	0.0015	0.9936
12	0.7	12	2	11	0.0124	0.0015	0.9936
12	0.8	11	2	11	0.0854	0.0222	0.9936
12	0.8	12	2	11	0.0667	0.0222	0.9936
12	0.9	11	2	11	0.3127	0.1967	0.9936
12	0.9	12	2	11	0.2796	0.1967	0.9936
14	0.7	13	2	12	0.0089	0.0019	0.9926
14	0.7	14	2	12	0.0055	0.0019	0.9926
14	0.8	12	2	12	0.0685	0.0272	0.9926
14	0.8	13	2	12	0.0539	0.0272	0.9926
14	0.8	14	2	12	0.0414	0.0272	0.9926
14	0.9	12	2	12	0.2819	0.2137	0.9926
14	0.9	13	2	12	0.2527	0.2137	0.9926
14	0.9	14	2	12	0.2244	0.2137	0.9926
16	0.7	14	3	14	0.0067	0.0006	0.9958
16	0.7	15	3	14	0.0045	0.0006	0.9958
16	0.7	16	3	14	0.0029	0.0006	0.9958
16	0.8	14	3	14	0.0439	0.0139	0.9958
16	0.8	15	3	14	0.0349	0.0139	0.9958
16	0.8	16	3	14	0.0273	0.0139	0.9958
16	0.9	14	3	14	0.2283	0.1644	0.9958
16	0.9	15	3	14	0.2053	0.1644	0.9958
16	0.9	16	3	14	0.1838	0.1644	0.9958

Table 1 continued ...

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
18	0.7	15	4	15	0.0047	0.0007	0.9925
18	0.7	16	4	15	0.0032	0.0007	0.9925
18	0.7	17	4	15	0.0021	0.0007	0.9925
18	0.7	18	4	15	0.0013	0.0007	0.9925
18	0.8	15	4	15	0.0351	0.0149	0.9925
18	0.8	16	4	15	0.0279	0.0149	0.9925
18	0.8	17	4	15	0.0221	0.0149	0.9925
18	0.8	18	4	15	0.0171	0.0149	0.9925
18	0.9	15	4	15	0.2047	0.1587	0.9925
18	0.9	16	4	15	0.1845	0.1587	0.9925
18	0.9	17	4	15	0.1657	0.1587	0.9925
18	0.9	18	4	15	0.1479	0.1587	0.9925
20	0.7	16	4	16	0.0033	0.0006	0.9928
20	0.7	17	4	16	0.0023	0.0006	0.9928
20	0.7	18	4	16	0.0015	0.0006	0.9928
20	0.7	19	4	16	0.0009	0.0006	0.9928
20	0.7	20	4	16	0.0005	0.0006	0.9928
20	0.8	16	4	16	0.0281	0.0141	0.9928
20	0.8	17	4	16	0.0224	0.0141	0.9928
20	0.8	18	4	16	0.0177	0.0141	0.9928
20	0.8	19	4	16	0.0139	0.0141	0.9928
20	0.8	20	4	16	0.0106	0.0141	0.9928
20	0.9	16	4	16	0.1847	0.1415	0.9928
20	0.9	17	4	16	0.1664	0.1415	0.9928
20	0.9	18	4	16	0.1495	0.1415	0.9928
20	0.9	19	4	16	0.1338	0.1415	0.9928
20	0.9	20	4	16	0.1189	0.1415	0.9928

Table 2: 95% Confidence intervals for $p = 0.5$

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
8	0.6	7	1	7	0.0202	0.0019	0.9609
8	0.6	8	1	7	0.0066	0.0019	0.9609
8	0.7	7	1	7	0.0745	0.0171	0.9609
8	0.7	8	1	7	0.0412	0.0171	0.9609
8	0.8	7	1	7	0.2017	0.0921	0.9609
8	0.8	8	1	7	0.1451	0.0921	0.9609
8	0.9	7	1	7	0.4688	0.3519	0.9609
8	0.9	8	1	7	0.4009	0.3519	0.9609
10	0.6	9	2	9	0.0081	0.0003	0.9785
10	0.6	10	2	9	0.0031	0.0003	0.9785
10	0.7	9	2	9	0.0384	0.0053	0.9785
10	0.7	10	2	9	0.0234	0.0053	0.9785
10	0.8	9	2	9	0.1319	0.0462	0.9785
10	0.8	10	2	9	0.1004	0.0462	0.9785
10	0.9	9	2	9	0.3823	0.2661	0.9785
10	0.9	10	2	9	0.3385	0.2661	0.9785
12	0.6	10	3	10	0.0051	0.0004	0.9614
12	0.6	11	3	10	0.0021	0.0004	0.9614
12	0.6	12	3	10	0.0007	0.0004	0.9614
12	0.7	10	3	10	0.0272	0.0059	0.9614
12	0.7	11	3	10	0.0171	0.0059	0.9614
12	0.7	12	3	10	0.0098	0.0059	0.9614
12	0.8	10	3	10	0.1055	0.0509	0.9614
12	0.8	11	3	10	0.0819	0.0509	0.9614
12	0.8	12	3	10	0.0608	0.0509	0.9614
12	0.9	10	3	10	0.3389	0.2683	0.9614
12	0.9	11	3	10	0.3048	0.2683	0.9614
12	0.9	12	3	10	0.2675	0.2683	0.9614
14	0.6	11	3	11	0.0031	0.0003	0.9648
14	0.6	12	3	11	0.0013	0.0003	0.9648
14	0.6	13	3	11	0.0005	0.0003	0.9648
14	0.6	14	3	11	0.0001	0.0003	0.9648
14	0.7	11	3	11	0.0193	0.0055	0.9648
14	0.7	12	3	11	0.0124	0.0055	0.9648
14	0.7	13	3	11	0.0074	0.0055	0.9648
14	0.7	14	3	11	0.0041	0.0055	0.9648
14	0.8	11	3	11	0.0851	0.0473	0.9648
14	0.8	12	3	11	0.0666	0.0473	0.9648
14	0.8	13	3	11	0.0504	0.0473	0.9648
14	0.8	14	3	11	0.0366	0.0473	0.9648
14	0.9	11	3	11	0.3091	0.2428	0.9648
14	0.9	12	3	11	0.2778	0.2428	0.9648
14	0.9	13	3	11	0.2458	0.2428	0.9648
14	0.9	14	3	11	0.2127	0.2428	0.9648
16	0.6	12	4	12	0.0019	0.0002	0.9509
16	0.6	13	4	12	0.0009	0.0002	0.9509
16	0.6	14	4	12	0.0003	0.0002	0.9509
16	0.6	15	4	12	0.0001	0.0002	0.9509
16	0.7	12	4	12	0.0136	0.0045	0.9509
16	0.7	13	4	12	0.0089	0.0045	0.9509
16	0.7	14	4	12	0.0055	0.0045	0.9509
16	0.7	15	4	12	0.0031	0.0045	0.9509
16	0.7	16	4	12	0.0016	0.0045	0.9509

Table 2 continued ...

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
16	0.8	12	4	12	0.0678	0.0396	0.9509
16	0.8	13	4	12	0.0536	0.0396	0.9509
16	0.8	14	4	12	0.0413	0.0396	0.9509
16	0.8	15	4	12	0.0307	0.0396	0.9509
16	0.8	16	4	12	0.0219	0.0396	0.9509
16	0.9	12	4	12	0.2729	0.2057	0.9509
16	0.9	13	4	12	0.2481	0.2057	0.9509
16	0.9	14	4	12	0.2221	0.2057	0.9509
16	0.9	15	4	12	0.1952	0.2057	0.9509
16	0.9	16	4	12	0.1676	0.2057	0.9509
18	0.6	13	3	13	0.0012	0.0002	0.9512
18	0.6	14	3	13	0.0005	0.0002	0.9512
18	0.6	15	3	13	0.0002	0.0002	0.9512
18	0.7	13	3	13	0.0096	0.0035	0.9512
18	0.7	14	3	13	0.0064	0.0035	0.95121
18	0.7	15	3	13	0.0040	0.0035	0.9512
18	0.7	16	3	13	0.0024	0.0035	0.9512
18	0.7	17	3	13	0.0013	0.0035	0.9512
18	0.7	18	3	13	0.0007	0.0035	0.9512
18	0.8	13	3	13	0.0548	0.0315	0.9512
18	0.8	14	3	13	0.0434	0.0315	0.9512
18	0.8	15	3	13	0.0337	0.0315	0.9512
18	0.8	16	3	13	0.0255	0.0315	0.9512
18	0.8	17	3	13	0.0186	0.0315	0.9512
18	0.8	18	3	13	0.0130	0.0315	0.9512
18	0.9	13	3	13	0.2531	0.1731	0.9512
18	0.9	14	3	13	0.2276	0.1731	0.9512
18	0.9	15	3	13	0.2032	0.1731	0.9512
18	0.9	16	3	13	0.1794	0.1731	0.9512
18	0.9	17	3	13	0.1559	0.1731	0.9512
18	0.9	18	3	13	0.1325	0.1731	0.9511
20	0.7	15	6	15	0.0047	0.0014	0.9586
20	0.7	16	6	15	0.0032	0.0014	0.9586
20	0.7	17	6	15	0.0021	0.0014	0.9586
20	0.7	18	6	15	0.0013	0.0014	0.9586
20	0.7	19	6	15	0.0007	0.0014	0.9586
20	0.7	20	6	15	0.0004	0.0014	0.9586
20	0.8	15	6	15	0.0344	0.0197	0.9586
20	0.8	16	6	15	0.0277	0.0197	0.9586
20	0.8	17	6	15	0.0219	0.0197	0.9586
20	0.8	18	6	15	0.0170	0.0197	0.9586
20	0.8	19	6	15	0.0129	0.0197	0.9586
20	0.8	20	6	15	0.0094	0.0197	0.9586
20	0.9	15	6	15	0.1913	0.1430	0.9586
20	0.9	16	6	15	0.1769	0.1430	0.9586
20	0.9	17	6	15	0.1615	0.1430	0.9586
20	0.9	18	6	15	0.1456	0.1430	0.9586
20	0.9	19	6	15	0.1296	0.1430	0.9586
20	0.9	20	6	15	0.1135	0.1430	0.9586

Table 3: 99% Confidence intervals for median $p = \theta/2$

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
10	0.8	9	1	9	0.1337	0.0473	0.9929
10	0.8	10	1	9	0.1061	0.0473	0.9929
10	0.9	9	1	9	0.3859	0.2697	0.9929
10	0.9	10	1	9	0.3446	0.2697	0.9929
12	0.7	10	1	10	0.0282	0.0063	0.9935
12	0.7	11	1	10	0.0196	0.0063	0.9935
12	0.7	12	1	10	0.0136	0.0063	0.9935
12	0.8	10	1	10	0.1072	0.0526	0.9935
12	0.8	11	1	10	0.0853	0.0526	0.9935
12	0.8	12	1	10	0.0674	0.0526	0.9935
12	0.9	10	1	10	0.3479	0.2757	0.9935
12	0.9	11	1	10	0.3118	0.2757	0.9935
12	0.9	12	1	10	0.2769	0.2757	0.9935
14	0.6	10	1	10	0.0060	0.0013	0.9915
14	0.6	11	1	10	0.0036	0.0013	0.9915
14	0.6	12	1	10	0.0021	0.0013	0.9915
14	0.6	13	1	10	0.0012	0.0013	0.9915
14	0.6	14	1	10	0.0007	0.0013	0.9915
14	0.7	10	1	10	0.0282	0.0122	0.9915
14	0.7	11	1	10	0.0196	0.0122	0.9915
14	0.7	12	1	10	0.0136	0.0122	0.9915
14	0.7	13	1	10	0.0092	0.0122	0.9915
14	0.7	14	1	10	0.0062	0.0122	0.9915
14	0.8	11	1	11	0.0858	0.0488	0.9953
14	0.8	12	1	11	0.0685	0.0488	0.9953
14	0.8	13	1	11	0.0543	0.0488	0.9953
14	0.8	14	1	11	0.0427	0.0488	0.9953
14	0.9	12	1	12	0.2823	0.2148	0.9976
14	0.9	13	1	12	0.2537	0.2148	0.9976
14	0.9	14	1	12	0.2273	0.2148	0.9976
16	0.6	11	1	11	0.0036	0.0009	0.9951
16	0.6	12	1	11	0.0022	0.0009	0.9951
16	0.6	13	1	11	0.0013	0.0009	0.9951
16	0.6	14	1	11	0.0008	0.0009	0.9951
16	0.6	15	1	11	0.0004	0.0009	0.9951
16	0.6	16	1	11	0.0002	0.0009	0.9951
16	0.7	11	1	11	0.0197	0.0089	0.9928
16	0.7	12	1	11	0.0137	0.0089	0.9928
16	0.7	13	1	11	0.0096	0.0089	0.9928
16	0.7	14	1	11	0.0066	0.0089	0.9928
16	0.7	15	1	11	0.0045	0.0089	0.9928
16	0.7	16	1	11	0.0029	0.0089	0.9928
16	0.8	12	1	12	0.0687	0.0411	0.9948
16	0.8	13	1	12	0.0549	0.0411	0.9948
16	0.8	14	1	12	0.0437	0.0411	0.9948
16	0.8	15	1	12	0.0346	0.0411	0.9948
16	0.8	16	1	12	0.0271	0.0411	0.9948
16	0.9	13	1	13	0.2541	0.2004	0.9965
16	0.9	14	1	13	0.2285	0.2004	0.9965
16	0.9	15	1	13	0.2051	0.2004	0.9965
16	0.9	16	1	13	0.1833	0.2004	0.9965

Table 3 continued ...

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
18	0.6	11	1	11	0.0036	0.0012	0.9923
18	0.6	12	1	11	0.0022	0.0012	0.9923
18	0.6	13	1	11	0.0013	0.0012	0.9923
18	0.6	14	1	11	0.0008	0.0012	0.9923
18	0.6	15	1	11	0.0004	0.0012	0.9923
18	0.6	16	1	11	0.0002	0.0012	0.9923
18	0.6	17	1	11	0.0001	0.0012	0.9923
18	0.7	12	1	12	0.0138	0.0063	0.9934
18	0.7	13	1	12	0.0097	0.0063	0.9934
18	0.7	14	1	12	0.0067	0.0063	0.9934
18	0.7	15	1	12	0.0047	0.0063	0.9934
18	0.7	16	1	12	0.0032	0.0063	0.9934
18	0.7	17	1	12	0.0021	0.0063	0.9934
18	0.7	18	1	12	0.0014	0.0063	0.9934
18	0.8	13	1	13	0.0549	0.0327	0.9941
18	0.8	14	1	13	0.0439	0.0327	0.9941
18	0.8	15	1	13	0.0350	0.0327	0.9941
18	0.8	16	1	13	0.0278	0.0327	0.9941
18	0.8	17	1	13	0.0219	0.0327	0.9941
18	0.8	18	1	13	0.0171	0.0327	0.9941
18	0.9	14	1	14	0.2287	0.1749	0.9951
18	0.9	15	1	14	0.2058	0.1749	0.9951
18	0.9	16	1	14	0.1849	0.1749	0.9951
18	0.9	17	1	14	0.1657	0.1749	0.9951
18	0.9	18	1	14	0.1478	0.1749	0.9951
20	0.6	12	1	12	0.0022	0.0007	0.9941
20	0.6	13	1	12	0.0013	0.0007	0.9941
20	0.6	14	1	12	0.0008	0.0007	0.9941
20	0.6	15	1	12	0.00056	0.0007	0.9941
20	0.6	16	1	12	0.0003	0.0007	0.9941
20	0.6	17	1	12	0.0001	0.0007	0.9941
20	0.7	13	1	13	0.0097	0.0044	0.9938
20	0.7	14	1	13	0.0068	0.0044	0.9938
20	0.7	15	1	13	0.0047	0.0044	0.9938
20	0.7	16	1	13	0.0033	0.0044	0.9938
20	0.7	17	1	13	0.0023	0.0044	0.9938
20	0.7	18	1	13	0.0015	0.0044	0.9938
20	0.7	19	1	13	0.0010	0.0044	0.9938
20	0.7	20	1	13	0.0007	0.0044	0.9938
20	0.8	14	1	14	0.0439	0.0250	0.9935
20	0.8	15	1	14	0.0352	0.0250	0.9935
20	0.8	16	1	14	0.0281	0.0250	0.9935
20	0.8	17	1	14	0.0224	0.0250	0.9935
20	0.8	18	1	14	0.0177	0.0250	0.9935
20	0.8	19	1	14	0.0139	0.0250	0.9935
20	0.8	20	1	14	0.0109	0.0250	0.9935
20	0.9	15	1	15	0.2059	0.1481	0.9936
20	0.9	16	1	15	0.1852	0.1481	0.9936
20	0.9	17	1	15	0.1666	0.1481	0.9936
20	0.9	18	1	15	0.1495	0.1481	0.9936
20	0.9	19	1	15	0.1338	0.1481	0.9936
20	0.9	20	1	15	0.1191	0.1481	0.9936

Table 4: 95% Confidence intervals for median $p = \theta/2$

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
8	0.7	7	1	7	0.0811	0.0192	0.9646
8	0.7	8	1	7	0.0554	0.0192	0.9646
8	0.8	7	1	7	0.2064	0.0963	0.9747
8	0.8	8	1	7	0.1612	0.0963	0.9747
8	0.9	7	1	7	0.4708	0.3582	0.9735
8	0.9	8	1	7	0.4136	0.3582	0.9735
10	0.6	7	1	7	0.0275	0.0083	0.9612
10	0.6	8	1	7	0.0161	0.0083	0.9612
10	0.6	9	1	7	0.0091	0.0083	0.9612
10	0.6	10	1	7	0.0050	0.0083	0.9612
10	0.7	7	1	7	0.0811	0.0397	0.9605
10	0.7	8	1	7	0.0554	0.0397	0.9605
10	0.7	9	1	7	0.0366	0.0397	0.9605
10	0.7	10	1	7	0.0234	0.0397	0.9605
10	0.8	8	1	8	0.1665	0.0964	0.9816
10	0.8	9	1	8	0.1313	0.0964	0.9816
10	0.8	10	1	8	0.1014	0.0964	0.9816
10	0.9	8	1	8	0.4271	0.3444	0.9701
10	0.9	9	1	8	0.3791	0.3444	0.9701
10	0.9	10	1	8	0.3293	0.3444	0.9701
12	0.6	8	1	8	0.0167	0.0054	0.9767
12	0.6	9	1	8	0.0099	0.0054	0.9767
12	0.6	10	1	8	0.0057	0.0054	0.9767
12	0.6	11	1	8	0.0032	0.0054	0.9767
12	0.6	12	1	8	0.0017	0.0054	0.9767
12	0.7	8	1	8	0.0572	0.0286	0.9688
12	0.7	9	1	8	0.0395	0.0286	0.9688
12	0.7	10	1	8	0.0267	0.0286	0.9688
12	0.7	11	1	8	0.0175	0.0286	0.9688
12	0.7	12	1	8	0.0111	0.0286	0.9688
12	0.8	9	1	9	0.1337	0.0831	0.9825
12	0.8	10	1	9	0.10617	0.0831	0.9825
12	0.8	11	1	9	0.08307	0.0831	0.9825
12	0.8	12	1	9	0.0637	0.0831	0.9825
12	0.9	9	1	9	0.3859	0.3004	0.9636
12	0.9	10	1	9	0.3446	0.3004	0.9636
12	0.9	11	1	9	0.3034	0.3004	0.9636
12	0.9	12	1	9	0.2617	0.3004	0.9636
14	0.6	8	1	8	0.0167	0.0067	0.9617
14	0.6	9	1	8	0.0099	0.0067	0.9617
14	0.6	10	1	8	0.00579	0.0067	0.9617
14	0.6	11	1	8	0.0032	0.0067	0.9617
14	0.6	12	1	8	0.0017	0.0067	0.9617
14	0.6	13	1	8	0.0009	0.0067	0.9617
14	0.6	14	1	8	0.0005	0.0067	0.9617
14	0.7	9	1	9	0.0402	0.0198	0.9732
14	0.7	10	1	9	0.0279	0.0198	0.9732
14	0.7	11	1	9	0.0191	0.0198	0.9732
14	0.7	12	1	9	0.0128	0.0198	0.9732
14	0.7	13	1	9	0.0084	0.0198	0.9732
14	0.7	14	1	9	0.0053	0.0198	0.9732

Table 4 continued ...

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
14	0.8	10	1	10	0.1072	0.0665	0.9817
14	0.8	11	1	10	0.0853	0.0665	0.9817
14	0.8	12	1	10	0.0674	0.0665	0.9817
14	0.8	13	1	10	0.0524	0.0665	0.9817
14	0.8	14	1	10	0.0400	0.0665	0.9817
14	0.9	10	1	10	0.3479	0.2528	0.9571
14	0.9	11	1	10	0.3118	0.2528	0.9571
14	0.9	12	1	10	0.2769	0.2528	0.9571
14	0.9	13	1	10	0.2424	0.2528	0.9571
14	0.9	14	1	10	0.2082	0.2528	0.9571
16	0.6	9	1	9	0.0100	0.0039	0.9710
16	0.6	10	1	9	0.0059	0.0039	0.9710
16	0.6	11	1	9	0.0035	0.0039	0.9710
16	0.6	12	1	9	0.0020	0.0039	0.9710
16	0.6	13	1	9	0.0011	0.0039	0.9710
16	0.6	14	1	9	0.0006	0.0039	0.9710
16	0.6	15	1	9	0.0003	0.0039	0.9710
16	0.6	16	1	9	0.0002	0.0039	0.9710
16	0.7	10	1	10	0.0282	0.0134	0.9761
16	0.7	11	1	10	0.0196	0.0134	0.9761
16	0.7	12	1	10	0.0136	0.0134	0.9761
16	0.7	13	1	10	0.0092	0.0134	0.9761
16	0.7	14	1	10	0.0062	0.0134	0.9761
16	0.7	15	1	10	0.0040	0.0134	0.9761
16	0.7	16	1	10	0.0026	0.0134	0.9761
16	0.8	11	1	11	0.0858	0.0508	0.9806
16	0.8	12	1	11	0.0685	0.0508	0.9806
16	0.8	13	1	11	0.0543	0.0508	0.9806
16	0.8	14	1	11	0.0427	0.0508	0.9806
16	0.8	15	1	11	0.0331	0.0508	0.9806
16	0.8	16	1	11	0.0252	0.0508	0.9806
16	0.9	11	1	11	0.3135	0.2101	0.9513
16	0.9	12	1	11	0.2815	0.2101	0.9513
16	0.9	13	1	11	0.2513	0.2101	0.9513
16	0.9	14	1	11	0.2222	0.2101	0.9513
16	0.9	15	1	11	0.1937	0.2101	0.9513
16	0.9	16	1	11	0.1658	0.2101	0.9513
18	0.6	10	1	10	0.0060	0.0023	0.9774
18	0.6	11	1	10	0.0036	0.0023	0.9774
18	0.6	12	1	10	0.0021	0.0023	0.9774
18	0.6	13	1	10	0.0012	0.0023	0.9774
18	0.6	14	1	10	0.0007	0.0023	0.9774
18	0.6	15	1	10	0.0004	0.0023	0.9774
18	0.6	16	1	10	0.0002	0.0023	0.9774
18	0.6	17	1	10	0.0001	0.0023	0.9774
18	0.7	11	1	11	0.0197	0.0089	0.9783
18	0.7	12	1	11	0.0138	0.0089	0.9783
18	0.7	13	1	11	0.0096	0.0089	0.9783

Table 4 continued ...

n	θ	n_1	i	j	Cond. Prob.	Uncond. Prob.	Full Prob.
18	0.7	14	1	11	0.0066	0.0089	0.9783
18	0.7	15	1	11	0.0045	0.0089	0.9783
18	0.7	16	1	11	0.0029	0.0089	0.9783
18	0.7	17	1	11	0.0019	0.0089	0.9783
18	0.7	18	1	11	0.0012	0.0089	0.9783
18	0.8	12	1	12	0.0687	0.0379	0.9796
18	0.8	13	1	12	0.0549	0.0379	0.9796
18	0.8	14	1	12	0.0437	0.0379	0.9796
18	0.8	15	1	12	0.0346	0.0379	0.9796
18	0.8	16	1	12	0.0271	0.0379	0.9796
18	0.8	17	1	12	0.0209	0.0379	0.9796
18	0.8	18	1	12	0.0159	0.0379	0.9796
18	0.9	13	1	13	0.2541	0.1782	0.9817
18	0.9	14	1	13	0.2285	0.1782	0.9817
18	0.9	15	1	13	0.2051	0.1782	0.9817
18	0.9	16	1	13	0.1833	0.1782	0.9817
18	0.9	17	1	13	0.1627	0.1782	0.9817
18	0.9	18	1	13	0.1429	0.1782	0.9817
20	0.6	10	1	10	0.0060	0.0020	0.9512
20	0.6	11	1	10	0.0036	0.0020	0.9512
20	0.6	12	1	10	0.0021	0.0020	0.9512
20	0.6	13	1	10	0.0012	0.0020	0.9512
20	0.6	14	1	10	0.0007	0.0020	0.9512
20	0.6	15	1	10	0.0004	0.0020	0.9512
20	0.6	16	1	10	0.0002	0.0020	0.9512
20	0.6	17	1	10	0.0001	0.0020	0.9512
20	0.7	12	1	12	0.0138	0.0059	0.9802
20	0.7	13	1	12	0.0097	0.0059	0.9802
20	0.7	14	1	12	0.0067	0.0059	0.9802
20	0.7	15	1	12	0.0047	0.0059	0.9802
20	0.7	16	1	12	0.0032	0.0059	0.9802
20	0.7	17	1	12	0.0021	0.0059	0.9802
20	0.7	18	1	12	0.0014	0.0059	0.9802
20	0.7	19	1	12	0.0009	0.0059	0.9802
20	0.7	20	1	12	0.0006	0.0059	0.9802
20	0.8	13	1	13	0.0549	0.0277	0.9789
20	0.8	14	1	13	0.0439	0.0277	0.9789
20	0.8	15	1	13	0.0350	0.0277	0.9789
20	0.8	16	1	13	0.0278	0.0277	0.9789
20	0.8	17	1	13	0.0219	0.0277	0.9789
20	0.8	18	1	13	0.0171	0.0277	0.9789
20	0.8	19	1	13	0.0132	0.0277	0.9789
20	0.8	20	1	13	0.0100	0.0277	0.9789
20	0.9	14	1	14	0.2287	0.1481	0.9786
20	0.9	15	1	14	0.2058	0.1481	0.9786
20	0.9	16	1	14	0.1849	0.1481	0.9786
20	0.9	17	1	14	0.1657	0.1481	0.9786
20	0.9	18	1	14	0.1478	0.1481	0.9786
20	0.9	19	1	14	0.1308	0.1481	0.9786
20	0.9	20	1	14	0.1146	0.1481	0.9786