Education and Fertility: Pension System and its Phase Out

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Abstract

This paper exploits a well accepted inefficiency that arises out of a Pay-As-You-Go (PAYG) pension system itself to phase it out in a Pareto way. The positive externalities of having children in a PAYG pension system is carefully utilized to phase the pension out. In a model with endogenous fertility and education choice of parents, it establishes the sub-optimality of parent's choices and recommends an intergenerationally balanced education and childcare subsidy package to correct for the externalities in a PAYG system. However, in a departure from the existing literature dealing with this externality, this paper proposes a phaseout plan which decreases PAYG pension tax in subsequent periods and pension is phased out in finite time, keeping the Pareto condition intact. Starting from an institutionalized pension system, a calibration exercise charts out the phaseout of PAYG pension under the proposed mechanism.

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1 Introduction

For a plethora of concerns, the relevance and sustainability of the age-old Pay-As-You-Go (PAYG) pension system in the present days is under serious doubt. Many countries with PAYG type pensions are pursuing reforms to downsize or eliminate this social security system. However, these efforts face the challenge of eliminating pensions in a Pareto way. There is a rich literature dealing with the transition from PAYG system to a funded system. The broad conclusion of this literature is that it is generally impossible to compensate the first generation of pensioners for the loss incurred without making at least one later generation worse off (unless there is some externality involved). This result is robust to the choice of fertility decisions being made exogenous (see, for example, Verbon (1988) and Breyer (1989)) or endogenous (see, for example, van Groezen et. al.(2003)). In this paper we present a novel mechanism that exploits an well accepted externality effect, spawning from having children in a contributory public pension system, to phase out pensions in a Pareto way.

Under the PAYG arrangement, each working generation funds pension for its parental generation and is provided with pension support by the children's generation. A severe budgetary pressure that PAYG type pensions are facing is mainly due to a demographic shift in the developed countries, typically the home of PAYG. A fall in fertility has coincided with the increase in size of the pension system (Boldrin et al. (2015)). Therefore, the tax base for providing pensions has fallen, raising concerns over feasibility of the pension system. Moreover, an increase in longevity implies that the number of old people dependent on pension support has increased, thus worsening the budgetary concerns further.[1] From a theoretical perspective too, the celebrated Aaron - Samuelson results confirm that the PAYG pension regime is welfare reducing if the economy is dynamically efficient (see Aaron (1966)).[2] There are some other concerns including behavioral ones that also go against this age-old instrument.[3] These create an impression that PAYG may not be desirable on its own unless it serves some other purposes. In an important contribution Boldrin and Montes (2005) show that if a perfect capital market to borrow funds for education is missing, an education subsidy - PAYG pension combination can

[1]How to adjust the features of the social security scheme to changes in longevity has been discussed by Andersen (2008), Mulligan and Sala-I-Martin (2003, 2004), among others.

[2] According to Abel et. al. (1989), the United States and other OECD countries are dynamically efficient.

[3]Admittedly pension programs have a lot of critiques, some of which are purely on philosophical grounds. Possibly the most important one is a myopia or present bias in consumption. Individuals differ in their tastes and the government may not be the best judge of what is in their best interest (see Friedman (1962), Feldstein (2005)). Surely there are things that an individual would enjoy more and would like to spend on when she is young rather than when she is old. Social security benefits thus infringe on an individual's liberty by changing her preferences. A myopic agent has an inherent preference towards consuming more when young than old. A paternalistic intervention like public pension based on the value judgement that the myopic agents save less for their old age than they ideally should, will admittedly increase the old age consumption of the agent (for a review of the literature dealing with the rationale for social security via its effect on savings see de la Croix and Michel (2002). Andersen and Bhattacharya (2011) revisits the role played by myopia in generating a rationale for PAYG social security in dynamically efficient economies. Also, provision of old age benefits distorts retirement behavior and the tax that is imposed on the working population may distort labour supply (see Feldstein (1985)).

well replicate the allocation that a complete market generates.[4] A two-armed oppositely directed inter-generational combination of transfers that may help sustain each other has been extensively discussed even earlier by Becker and Murphy (1988), Pogue and Sgontz (1977) and others.[5]

In an interesting recent contribution Andersen and Bhattacharya (2017) show that it is possible to demonstrate theoretically the empirically observed phenomenon of rise-and-fall of pensions starting from the complete market allocation of Boldrin and Montes (2005). They show that in a dynamically efficient economy a complete phaseout of pension is possible, but for that an externality in human capital, as is standard in the endogenous growth models (see, for example, Lucas (1988)), is necessary. The model however does not focus on the issue of fertility. In another exogenous fertility setup, under the assumption that the education loan market is imperfect, Bishnu et al. (2018) find that the expansion of one instrument along with the other emerges as the optimal response. However, once the complete market level of education is achieved, the optimal policy suggests phasing pensions out. Eventually the government leads the economy to an equilibrium with zero pension and the Golden Rule level of human capital accumulation. This is achieved in the analysis by exploiting only market opportunities without relying on any other factor including human capital externalities highlighted in the literature.

To the best of our knowledge, this paper is the first attempt to show a complete phase-out of pensions when both education and fertility choices are endogenous and agents care about the quality - quantity trade-off for their offsprings. More importantly, to phase out pensions, the model exploits an externality that is associated with the PAYG pension system itself, arising through endogenous fertility decisions; it does not need to exploit market failure appearing outside, for example, in education. Let us explain this externality first. Benefits of having more children are enjoyed by the society if there is a publicly funded contributory system such as the PAYG since having a child and educating her

[4]Research along this line has gained momentum and a framework where two differently directed intergenerational transfers are present has been used extensively thereafter. For example, Docquier et al. (2007) show that because of the externalities in human capital accumulation, allocations of human and physical capital in a competitive equilibrium differ from the planner's allocations and a possibility arises where the laissez-faire equilibrium experiences higher physical capital accumulation but lower human capital accumulation compared to the planner's allocations. Bishnu (2013) uses education subsidy to correct for consumption externalities that may result in different education levels of the agents in a society and shows that the overaccumulation (underaccumulation) of human capital will always be accompanied by overaccumulation (underaccumulation) in physical capital. Wang (2014) extends Boldrin and Montes (2005) by endogenizing the imperfection of the credit market and shows that the result could hold for a wide range of parameters even when borrowing constraints for education loan arise endogenously as the result of limited commitment.

[5] Becker and Murphy (1988) provide a rationale for pensions when parents invest insufficiently in their children's education. A welfare state can correct this by financing public education. Parents pay for children's education and get pension in return when they are old. In this strand of literature, pension is one arm of inter-generational transfers, payments for children's education being the other arm. There exists some inefficiency in education of children. Working population is taxed to correct for this inefficiency and they are compensated in old age. However, in the absence of any externalities such as those associated with human capital, the standard Aaron-Samuelson result is that pensions are inefficient in steady state. Even in case of inefficiencies, when the original inefficiency has been corrected, the existence of pensions is not desirable as PAYG pension system gives a lower return than capital market in a dynamically efficient economy.

broadens and deepens the pension tax base. An individual agent however does not take this benefit into account fully. Since this increase in the tax base will be shared with all individuals, an individual agent's own share of this increase in old-age pension is very small and imperceptible to her. That is, an individual's decision to have children and educate them is driven solely by the direct utility she gets from her children and their human capital, whereas they have a positive externality on the society. This particular form of externality was recognized quite some time ago and is now well-established in the pension-related literature.[6] While Cigno (1993) termed this as positive externality of children, Folbre (1994) called children as public capital good. This social benefit of children can lead to a free-riding behavior where agents no longer need children for old-age support: they do not need to spend on children as they can avail the benefits of having children through the pension system. Ehrlich and Kim (2007) show that when children are both investment as well as consumption goods, an increase in social security leads to a fall in fertility as the investment good role of children is fulfilled by the public pension system. We, in particular, take advantage of this positive social externality of having children and show that, in its presence, a complete phase out of pensions as desired in a dynamically efficient economy is possible. Interestingly, throughout this phaseout period, the Pareto condition is never compromised.

Several solutions have been proposed to handle this well accepted positive externality of children and their human capital. Sinn (1997, 1998) proposes a hybrid pension system where parents with insufficient number of children must also contribute to a funded system. Razin and Sadka (1999) proposes to allow immigration as an alternative way to broaden the pension tax base. However, the most intuitive way to correct for this externality is by providing child care and education subsidies. A pioneering work by van Groezen et al. (2003) suggests a child allowance scheme to correct the fertility choice of children. Cremer et al. (2011) suggest an education subsidy and possibly a child subsidy to handle this externality. van Groezen et al. (2003) have also pointed out that in the presence of such externalities, a pension should be accompanied by a child care subsidy as "Siamese twins". From that angle, one particular result in our paper can be seen as a generalization of their "Siamese twins" arguments. In particular, we show that in the presence of this externality, a pension scheme should be accompanied by *both* child care and education subsidies, only one of these two instruments is not enough. That is, the transfers are intergenerationally balanced. We infer that the concept of "Siamese twins" can be generalized to all the differently directed intergenerational transfers. From this point of view our findings broaden the reach of Boldrin and Montes (2005) too that explicitly models the dependency between two welfare state instruments, namely education subsidy and PAYG pension. In our framework, at the optimum, the backward intergenerational transfer, that is, pension, should be accompanied by subsidies to all the forward intergenerational transfers, that is, to both education as well as child care

[6] For a nice and up-to-date discussion of the fiscal externality due to children see Barnett et al. (2018).

subsidies.[7] This overall interplay between the forward and backward intergeneratonal goods in a game theoretic framework has also been reflected in Rangel (2003).

More importantly, this literature suddenly stops after correcting the suboptimal choices of agents through government intervention, completely overlooking the critical existence problem that PAYG is facing these days. Theoretically too, even after correcting for pension-related externalities, the standard inefficiency of pensions remains if the economy is dynamically efficient. Then it is wise to phase out pensions instead of continuing with it. We demonstrate how to phase pensions out in finite time and, more importantly, achieve this without violating the Pareto condition.[8] Since our focus is on phasing pensions out, our Pareto criterion is very simple and precise: during the policy implementation period, no generation is worse off than it was under the PAYG pension system. Thus, throughout the pension phase out period, we guarantee that the welfare never goes below the level that is ensured under the PAYG pensions before the policy was undertaken. Needless to mention that without the positive externality of children and their human capital, phasing out pensions in a Pareto way is not possible. This is similar to Andersen and Bhattacharya (2017) who also had to exploit an externality, namely the human capital externality, to phase out the pensions.

Let us briefly explain the mechanism that is at work in our paper. From the literature discussing the positive externality of children and their human capital in a PAYG pension regime we know that any generation can be made better off by implementing a government subsidy scheme. Government subsidies on education and children lower the costs of raising children and their education. This provides incentive to individuals to have more children and educate them, thus broadening the tax base for their pension benefits. Increase in tax base for pension benefits was the positive externality of children which was ignored by individual agents and is now taken care of by the subsidies for children and their education. We first show that a government subsidies scheme financed by taxing parents is necessary to handle the externalities of children and their education generated due to the PAYG pension system. However, as mentioned above, our scheme does not stop at just implementing the child care and education subsidies. The subsidies scheme makes the current generation better off as it has more children and hence higher pensions. Suppose that introducing the subsidies scheme increases the current generation's lifetime utility from V^* to $V_1 > V^*$. This increase in current generation's utility allows the gov-

[7] Of course the focus is different; while in Boldrin and Montes (2005) a balanced package is required to implement the complete market allocations, in our setup a balanced package is used to phase pensions out.

[8] When fertility is endogenous, the usual notion of Pareto efficiency is not well-defined. There is a literature that redefines the concept of Pareto efficiency in environments with endogenous fertility. Some of the prominent studies are Golosov et. al. (2007) which proposes the notion of $\mathcal{A}$- efficiency and $\mathcal{P}$- efficiency, Michel and Wigniolle (2007) which proposes RC-efficiency, and Conde-Ruiz et. al. (2010) which introduces the notion of Millian efficiency. This literature on efficiency under endogenous fertility has gained momentum lately. Very recently Cordoba and Liu (2018) has investigated the properties of socially optimal allocations in an environment characterized by endogenous fertility along with fixed resources under the Malthusian regime. Later in the paper we have shown that our use of Pareto obeys all the standard efficiency criteria defined in the context of endogenous fertility.

ernment to reduce its pension benefits while ensuring that the utility does not fall below V^*. Therefore, while satisfying the Pareto criterion of not making any generation worse off than under the PAYG pension system, the government can decrease this generation's pension benefits and hence the next generation's pension tax. Now the next generation faces a lower pension tax and there is a subsidy scheme in place to take care of the externalities of children in a PAYG pension system. Therefore, its utility is greater than the benchmark utility level V^* of the PAYG system. So the government can again reduce this generation's pension benefits and hence the following generation's pension tax. As the next generation faces a lower pension tax, the government can keep on decreasing subsequent generations' pension taxes. We show that this process of reducing pensions in conjunction with subsidies for correcting the PAYG pension related externalities will lead to a zero pension tax in finite time. Hence, starting from an institutionalized PAYG pension system, we show the existence of a sequence of government subsidies for children and their education and pension taxes which culminates in the complete phaseout of the PAYG pension system. At the end, we present a calibration exercise and provide the evolution of pension, fertility, income and consumption during the policy implementation period till the pension is completely phased out.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 explains the suboptimal individualistic choices in a PAYG pension regime. Section 4 introduces the government subsidies scheme to achieve the optimal choices. Section 5 presents the pension phaseout scheme. In section 6 we calibrate the model and chart out the path of phasing pensions out. Finally, we conclude in section 7. All the proofs are presented in the appendix.

2 The model

We consider an overlapping generations economy where agents live for three periods. They are young in the first period, middle-aged in the second and old in the third. An agent is born in the first period and gets educated. Her education is financed by her parents who also decide the extent of her education. She earns wage in the second period, consumes in that period and saves for her old age. She also decides to have children and educates them in the second period. Finally, she consumes the returns from her investment in the third period. An agent derives utility from her consumption in middle age and consumption in the old age. For simplicity we assume that the agents do not consume anything when young. The agent also derives utility from the number of her children as well as from the level of their human capital. For notation, we identify a generation by the period when it is in middle age. If an agent was born in period $t-1$ and is of middle age in period t, we call her a generation t agent.

Following the literature (see for example, Galor and Weil (2000), de la Croix and

Doepke (2003)), we assume that the utility of a generation t agent is given by

$$u(c_t^m) + \beta u(c_{t+1}^o) + v(n_t h_{t+1}),$$

where c_t^m and c_{t+1}^o are agent's consumption in the middle age and old age respectively. The agent discounts her utility from consumption in old age by β where $\beta \in (0,1)$. The utility from consumption is given by the function $u(.)$ which is assumed to be strictly increasing and concave, that is, $u'(.) > 0$ and $u''(.) < 0$. It also satisfies Inada conditions, that is, $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$. Since fertility is an important issue especially for the analysis of sustainability of PAYG type pensions, we model fertility as endogenous, along with the variable human capital that parents choose for their children. Thus our agents are assumed to be altruistic in nature. An agent's number of children is denoted by n_t and each child's human capital is denoted by h_{t+1}. The utility an agent derives from the number of her children and their human capital is denoted by $v(.)$. We assume that $v(.)$ also is strictly increasing, concave and satisfies Inada conditions.

An agent's human capital is a function of her level of education. An agent born in period t, for whom the parents have invested an amount e_t in education, has a human capital in period $t+1$ given by $h_{t+1} = h(e_t)$.[9] Human capital production is assumed to be a strictly increasing and concave function of agent's education level, that is, $h'(.) > 0$ and $h''(.) < 0$. We assume $h(0) > 0$, that is, the agent is endowed with some human capital even in absence of education. Further, we assume that the human capital production function satisfies Inada conditions. So the marginal effect of education on human capital is sufficiently high when education choice is close to zero, whereas the effect is close to zero at very high education level. Factor prices are assumed to be exogenously given: they are constant and fixed over time.[10]

The government is present in the economy. Initially, the only role the government plays is that of administering the PAYG pension system. An agent while working in her middle age pays a proportion of her income as tax to support pension for the old. In return, she receives pension support in her old age. In subsequent sections we suggest another role for the government to provide subsidy per child and education subsidy for correcting the pension externalities.

One key assumption in the paper is that the economy is dynamically efficient. A major implication of dynamic efficiency is the undesirability of the PAYG pension system. In a dynamically efficient economy, return on capital is higher than the product of growth rate of economy and growth rate of population. Therefore, capital markets give a higher

[9] With this notation, a generation t agent has $h_t = h(e_{t-1})$ amount of human capital.

[10] The importance of general equilibrium effects for sustaining intergenerational transfers is well documented in the literature (see, for example, Cooley and Soares (1999) and Boldrin and Rustichini (2000)). While focusing on the efficiency angle, we want to ensure that our results are not driven by the general equilibrium effects.

return than pension. Abel et al. (1989) show that the US and other OECD countries are dynamically efficient. Thus, in line with empirical evidence, we assume the economy to be dynamically efficient and this provides further motivation for phasing out a PAYG pension system.

3 Suboptimal Fertility in A PAYG Regime

We start with an economy where a pay-as-you-go pension scheme is in place with proportional pension tax at time t given by τ_t. An agent pays the pension tax in her middle age and receives pension support p_{t+1} in old age. We show that atomistic individual's fertility choice is suboptimal in this standard PAYG regime.

For a generation t agent with human capital h_t, her middle age budget constraint is given by

$$c_t^m + qe_t n_t + s_t = (1 - \tau_t - \chi n_t) w h_t.$$

The agent (with human capital h_t) can potentially supply one unit of labour inelastically and earn wh_t, where w is the exogenous wage rate per unit of human capital. However, raising children has an opportunity cost and each child takes away a fraction χ of labour which could otherwise be used to earn income in labour market. So an agent having n_t children bears an opportunity cost of $\chi n_t w h_t$. A proportion τ_t of the earning wh_t is paid to the government as tax for the pension benefits to the old generation. Per unit price of education is denoted by $q > 0$. Thus an agent with n_t children and educating each of them at the level e_t will incur an education cost of $qe_t n_t$. Agent's saving for old age is denoted by s_t.

In the old age the agent survives on her savings in the middle age that earns an exogenous gross interest rate $R > 1$, as well as pension support p_{t+1}. Thus, the agent's old age budget constraint is given by

$$c_{t+1}^o = s_t R + p_{t+1}.$$

The government balances its budget in every period. In period $t + 1$ it funds pension to the old, p_{t+1}, by taxing the earnings of n_t middle-aged of generation $t + 1$ at the rate τ_{t+1}. Thus the government's balanced budget constraint in period $t + 1$ is given by

$$p_{t+1} = \tau_{t+1} n_t w h_{t+1}.$$

3.1 Individualistic Choices in a PAYG Regime

In this standard PAYG regime an atomistic individual's fertility and education choices are given by the solution to the following problem of a generation t agent:

$$
\begin{aligned}
&\max_{\{s_t,e_t,n_t\}} u(c_t^m) + \beta u(c_{t+1}^o) + v(n_t h_{t+1}) \\
&\quad \text{subject to} \\
&\quad\quad c_t^m = (1 - \tau_t - \chi n_t) w h_t - q e_t n_t - s_t, \\
&\quad\quad c_{t+1}^o = s_t R + p_{t+1}, \\
&\quad\quad h_{t+1} = h(e_t).
\end{aligned}
\tag{A}
$$

Individuals act atomistically and do not take the government's pension budget constraint into account while solving their problem. The change in tax base to fund the pension due to one individual's fertility and education choice will be small and imperceptible to the individual. Therefore, parents take into account only the direct utility they enjoy from the quality and quantity of their children and ignore the pension benefits they receive as a result of investing in both the number and human capital of their children. For this reason they treat p_{t+1} as exogenously given while solving problem (A).

The solution to problem (A) is characterized by the middle age and old age budget constraints and the following first order conditions with respect to s_t, n_t and e_t respectively:

$$
\begin{aligned}
&u'(c_t^m) = \beta R u'(c_{t+1}^o), \\
&u'(c_t^m)(q e_t + \chi w h_t) = h(e_t) v'(n_t h(e_t)), \\
&u'(c_t^m) q n_t = n_t h'(e_t) v'(n_t h(e_t)).
\end{aligned}
\tag{1}
$$

The first condition is the standard Euler equation. An agent equates the marginal utility from consumption in middle age to that in old age. The second condition equates the marginal loss in consumption utility from having an extra child to the marginal gain in utility from having an extra child. The third condition equates the marginal loss in consumption utility from increasing one unit of children's education to the marginal gain in utility from the increase in children's human capital. Both the second and third conditions ignore the gain in utility due to increase in pension support working through the government's budget constraint.

Solution to the agent's problem exists where none of the consumption, fertility or education choices takes corner values. It follows from the first order conditions with respect to n_t and e_t that an agent's choice of her child's level of education is given implicitly by the following equation:

$$\frac{qe_t + \chi w h_t}{q} = \frac{h(e_t)}{h'(e_t)}. \tag{2}$$

In appendix A.1 we show the existence of a unique solution for the agent's education choice for her children and establish that child's education is increasing in parent's education.[11]

3.2 Optimal Choices in a PAYG Regime

An agent does not internalize her pension benefits and takes p_{t+1} as exogenous. This creates the well-documented free-riding problem (van Grozen et al. (2003)): having children and educating them have a positive externality of increasing the tax base for pension support which an individual agent does not take into account. The optimal fertility and education choices should internalize this positive externality. Alternatively, in the spirit of Boldrin and Jones (2002), the suboptimal individual choices can be interpreted as a competitive equilibrium where each agent takes as given the pension available to her and makes her education and fertility choices as the best response to maximize her utility. Then the optimal solution can be viewed as a cooperative equilibrium where the middle-aged agents jointly determine their old-age pension by choosing the quality and quantity of their children. So in the agent's optimal choice problem, pension in the old age budget constraint should show up as $\tau_{t+1} n_t w h_{t+1}$ instead of an exogenous p_{t+1}. An individual agent can control her pension in collaboration with other agents in the optimal solution. Thus, the optimal education and fertility choices are given by the solution to the following problem:

$$\begin{aligned}
&\max_{\{s_t, e_t, n_t\}} u(c_t^m) + \beta u(c_{t+1}^o) + v(n_t h_{t+1}) \\
&\quad \text{subject to} \\
&\qquad c_t^m = (1 - \tau_t - \chi n_t) w h_t - q e_t n_t - s_t, \\
&\qquad c_{t+1}^o = s_t R + \tau_{t+1} n_t w h_{t+1}, \\
&\qquad h_{t+1} = h(e_t).
\end{aligned} \tag{B}$$

In the optimal solution, an agent takes her pension support as endogenous. It is the product of the next period pension tax τ_{t+1}, each child's earning $w h_{t+1}$ and her number of children n_t. The solution to the problem (B) is given by the two budget constraints of middle age and old age and the following first order conditions with respect to s_t, n_t and

[11] It can easily be verified with a logarithmic utility function that the model can generate a negative income-fertility relationship once the economy crosses a certain income level, as is typically observed empirically. In our calibration exercise too we observe a negative income-fertility relationship.

e_t respectively:

$$\begin{aligned} &u'(c_t^m) = \beta R u'(c_{t+1}^o), \\ &u'(c_t^m)(qe_t + \chi w h_t) = \beta u'(c_{t+1}^o)\tau_{t+1} w h(e_t) + h(e_t) v'(n_t h(e_t)), \\ &u'(c_t^m) q n_t = \beta u'(c_{t+1}^o)\tau_{t+1} w n_t h'(e_t) + n_t h'(e_t) v'(n_t h(e_t)). \end{aligned} \tag{3}$$

The first condition is the standard Euler equation where agent smoothens her optimal consumption. Compared to the atomistic agent's solution, the second and third conditions include the marginal gain in utility through increase in pension. The second condition equates the marginal disutility from having an extra child to the marginal gain in utility from two channels. First, there is a direct gain in utility through an increase in number of children. Second, having more children increases the number of taxpayers in next generation who fund the pension support for current generation. This increases old-age consumption and utility. Similarly, the third condition equates the marginal disutility of providing one extra unit of child's education to the marginal gain in utility from this extra unit of education. This extra unit increases human capital of children which directly enhances parent's utility. It also increases the tax available to fund the pension support for generation t agents: as the children's generation has higher human capital, it earns more and pays more pension tax.

3.3 Suboptimality of Individualistic Fertility Choice

We denote the optimal fertility and education level of children which solve problem (B) by n_t^{opt} and e_t^{opt} respectively. Fertility and education level which solve an atomistic individual's problem (A) are denoted by n_t^{ind} and e_t^{ind} respectively with the subscript *ind* corresponding to individual's choices. In the following proposition we establish that when atomistic agents do not internalize the pension benefits, the fertility choice is suboptimal.

Proposition 1. *In a standard PAYG regime an agent's utility is lower under individualistic decision making than under optimal decision making. Although education choice remains the same, the suboptimality is generated by lower fertility choice under individualistic decision making, that is,* $n_t^{opt} > n_t^{ind}$ *and* $e_t^{opt} = e_t^{ind}$.[12]

Proof. See Appendix A.2.

[12] The fact that $e_t^{opt} = e_t^{ind}$ is due to the assumption that the weights attached to n and h are the same in the $v(.)$ function. If $v(.)$ takes the form $v(n_t^{\alpha} h_{t+1}^{\delta})$, then we have $e_t^{opt} < e_t^{ind}$ when $\alpha < \delta$ and $e_t^{opt} > e_t^{ind}$ when $\alpha > \delta$. For ease of exposition and following the literature (see for example, Galor and Weil (2000), de la Croix and Doepke (2003)), we continue our analysis with the assumption of equal weights. One important implication of having equal weights is that it can also represent the fact that altruistic agents derive utility from total potential income (wnh) of their children which is not possible under other specifications with unequal weights. We have verified that our results are robust to the specification with unequal weights.

4 Government Subsidies to Achieve the Optimal Choices

As proved in Proposition 1, agent's individualistic choices are suboptimal in fertility. To achieve the optimal fertility level the government must provide incentives to agent to have more children and these incentives may come in the form of a Pigouvian subsidy. Since children are normal goods in this model, lowering their effective cost will lead agents to have more children. But lowering only the cost of children will distort the choice of children's education as number of children and their human capital are substitutes. Therefore, the government must alter both the opportunity cost of having children as well as their education cost. We will show that the government can achieve the optimal choices by providing subsidies on child care and education which are financed by a lump sum tax.

Suppose in period t, the government provides an education subsidy of $\phi_{e,t}$ per unit of investment in education and a child care subsidy of $\phi_{c,t}$ per child. It finances these subsidies by a lump sum tax T_t. The government's budget constraint is given by

$$T_t = \phi_{e,t}e_t n_t + \phi_{c,t}n_t.$$

An agent's effective price of her child's education is now $(q-\phi_{e,t})$ and she also receives a per child subsidy $\phi_{c,t}$. However, she faces an extra tax burden T_t which she takes as exogenous. A generation t agent now faces the following problem:

$$\begin{aligned} &\max_{\{s_t,e_t,n_t\}} u(c_t^m) + \beta u(c_{t+1}^o) + v(n_t h_{t+1}) \\ &\quad \text{subject to} \\ &\qquad c_t^m = (1-\tau_t-\chi n_t)wh_t - T_t - (q-\phi_{e,t})e_t n_t + \phi_{c,t}n_t - s_t, \\ &\qquad c_{t+1}^o = s_t R + p_{t+1}, \\ &\qquad h_{t+1} = h(e_t). \end{aligned} \tag{C}$$

As before, the agent takes pension benefit p_{t+1} as exogenous and the government's balanced budget constraint requires

$$p_{t+1} = \tau_{t+1}n_t wh_{t+1}.$$

The subsidies on children and education reduce the marginal costs of education and child care, thus changing the first order conditions. The solution to this problem is given by the following first order conditions:

$$\begin{aligned} &u'(c_t^m) = \beta R u'(c_{t+1}^o), \\ &u'(c_t^m)((q-\phi_{e,t})e_t + (\chi wh_t - \phi_{c,t})) = h(e_t)v'(n_t h(e_t)), \\ &u'(c_t^m)(q-\phi_{e,t})n_t = n_t h'(e_t)v'(n_t h(e_t)). \end{aligned} \tag{4}$$

Comparing the first-order conditions (3) and (4) the following proposition defines the education and child care subsidies to match the optimal choices.

Proposition 2. *In the presence of PAYG pensions, the following results hold.*

(a) A child care subsidy alone or an education subsidy alone is not sufficient to achieve the optimal choices, that is, only one subsidy is not enough, both are needed to match the optimal choices.

(b) Subsidies on education ($\phi_{e,t}$) and child care ($\phi_{c,t}$) for which agent's individualistic fertility and education choices match the optimal choices are characterized by

$$\begin{aligned} R\phi_{e,t} &= \tau_{t+1} w h'(e_t^{opt}), \\ R(e_t^{opt}\phi_{e,t} + \phi_{c,t}) &= \tau_{t+1} w h(e_t^{opt}). \end{aligned} \tag{5}$$

Proof. See Appendix A.3.

The first equation under (5) implies that the optimal education subsidy is equal to the present value of agent's increase in pension from increasing one unit of child's education. The second equation says that the optimal child subsidy and education subsidy should be such that together their value should add up to the present value of the child's contribution towards the agent's pension. Hence, we establish that the government can take care of the pension externality of children by an appropriate education and child subsidy package. Another noteworthy point is that both education subsidy and child allowance are positive only if current generation's pension is positive, that is, $\tau_{t+1} > 0$. Since the only reason for childcare and education subsidies in this model is to correct for the positive externality generated by a child in a PAYG pension system, the subsidies are not required if the agent does not receive a pension support in old age.

An important observation is worth mentioning here. The above result somewhat generalizes the "Siamese twins" results of van Grozen et al. (2003). In a model with fertility and pension, they prove the interdependence between childcare subsidy and pension in the presence of a market failure, precisely when externalities in public pension via fertility are ignored by the competitive equilibrium. The above results confirm that in a richer model where there is a trade off between the quality and quantity of children, a pension plan should be accompanied by both child care as well as education subsidies, choosing only one of them is not enough. From this we come to the conclusion that backward intergenerational goods like pension should be accompanied by all the forward intergenerational goods as these intergenerational transfers are like "Siamese twins". In fact a somewhat similar argument is also valid but gone unnoticed in Boldrin and Montes (2005) which shows that in the absence of a perfect credit market to borrow funds for education, an education-pension package can well replicate the complete market allocations, that is, a balance between the two oppositely directed intergenerational goods is a must as a solution. In Rangel (2003) too the interdependence between the forward and the backward intergenerational goods is very clear.

As Proposition 2 shows, in the presence of PAYG pensions, the government can always make an improvement by implementing a child subsidy and education subsidy scheme. This intergenerational arrangement will result in a rise in fertility to its optimal level. However, the problem is not over yet. From the discussion in the introduction it is clear that the PAYG pension system itself is under serious threat for both theoretical reasons as well as demographic, budgetary or political pressures. Thus, after achieving the optimal choices, an important and timely relevant issue is to show a gradual phase out of pensions in a Pareto way given the fiscal arrangements that we have in our setup. We devote the next section on this much discussed issue of phasing pensions out in a Pareto way.[13]

5 Phase Out of PAYG Pension

Interestingly, the externalities associated with PAYG pensions itself give us a way to address the phase out issue in a Pareto way. Since each generation can be made better off by the government subsidies scheme as discussed above, their pensions can be decreased in conjunction with correcting their fertility and education choices so that they get the same utility that they would have enjoyed under a PAYG regime with no government subsidies for child care or education. Therefore, the PAYG-only regime (that is, the PAYG regime without child care and education subsidies) provides the benchmark. The government can correct the pension externalities using the fiscal instruments mentioned above and take the agent's utility to a higher level. But the government is not necessarily obliged to adopt this policy, rather it can curtail pensions to some extent while ensuring that the agent gets the benchmark utility of PAYG-only regime. Once one generation pays a lower pension tax, we show that subsequent generations will have to pay lower pension taxes too and, eventually, pensions will be phased out in the process. Therefore, we use an inefficiency associated with the pension system itself (that arises through fertility) to phase out the pension system. For whatever reason PAYG pension system was introduced or justified, once pension has played its role, it can be phased out in finite time.[14]

[13]One not so interesting but technically plausible case is that government subsidies increase fertility to such an extent that the fertility rate n_t^{opt} (which is also the rate of growth of population) exceeds the rate of return on capital R. Then the economy becomes dynamically inefficient and the pension system remains desirable as its return exceeds the return from investment in capital market. In that case it would be better not to phase out PAYG pensions in the long run. However, we assume that such an implausible jump in fertility does not occur. Therefore, our goal is to phase out PAYG pensions precisely when the economy remains dynamically efficient throughout.

[14]Since this paper proposes a phasing out plan of pensions in a "Pareto way", we shed some light on the concepts of Pareto efficiency under endogenous fertility. In case of exogenous fertility, Pareto way is easy to understand as utility of the same agent is compared across two different scenarios. However, since fertility itself changes in our model, there are new agents in the phaseout scheme who were not born in the benchmark case where PAYG program continues forever. So the usual notion of Pareto efficiency cannot be used here as it cannot tell us whether the phaseout plan is better for the "extra" number of agents born. As mentioned in the

We now formally present our policy prescription. Consider a generation t agent whose indirect utility under the PAYG-only regime is denoted by V^*. This is the benchmark utility and the government ensures that each subsequent generation gets at least as much as V^* throughout the phaseout period. We denote the optimal and individualistic utilities for a generation t agent who faces a pension tax τ_t and whose children face a pension tax τ_{t+1} by $V^{opt}(\tau_t, \tau_{t+1})$ and $V^{ind}(\tau_t, \tau_{t+1})$ respectively. Our scheme works as follows.

Let us start from a steady state where a generation t agent pays a pension tax $\tau_t = \tau$ and whose children pay a pension tax $\tau_{t+1} = \tau$ so that $V^{ind}(\tau, \tau) = V^*$. First, a government subsidy scheme increases the utility of generation t agents by aligning their education and fertility choices with the optimal choices while maintaining the children's pension tax rate at τ. The government can always achieve this higher utility level by implementing the subsidy scheme discussed in Proposition 2. However, in order to phase the pensions out, the government must reduce the pension tax in period $t+1$. In part (a) of the following proposition we show that the government can do this while ensuring that the generation t agent is no worse off than it was under the PAYG-only regime by taking advantage of the increased utility in the subsidy regime.

Note that both the policy experiments – education and child subsidies as well as reduced rate of pension benefits – are introduced to the generation t agent. While the optimal subsidies increase the number of children, the reduced rate of pension benefits would result in a lower fertility as children are normal goods in this model. The question is the net impact on fertility. A related issue is the net effect on total pension benefits of the generation t agent, $p_{t+1} = \tau_{t+1} n_t w h_{t+1}$. In part (b) of the following proposition we argue that the net effects on fertility as well as total pension benefits are positive. The reason is that the government is affecting the fertility decision of the generation t agent without changing her pension tax rate τ_t. Since under individualistic choices the agent chooses her fertility, education and saving for a given pension tax and total pension benefits, any changes keeping total pension benefits constant or lower would decrease her utility. So total pension benefits must increase. As τ_{t+1} falls while education choice (and hence h_{t+1}) remains unaffected by the combined policy, it follows that the net effect on fertility is also positive.

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introduction, there are some recent studies dealing with the notion of Pareto efficiency under endogenous fertility. One notion is to compare utility of a representative agent across two scenarios, bypassing different population sizes across them. One such study is by Conde-Ruiz et. al. (2010) which ranks allocations exclusively on preferences of those agents who are actually born and calls it Milian efficiency. Michel and Wigniolle (2007) also ranks allocations using a similar notion and call it RC-efficiency where RC stands for representative consumer. Golosov et. al. (2007) captures this way of ranking through $\mathcal{A}$- efficiency which is concerned with agents common in both the economy. Further, they provide a notion of $\mathcal{P}$- efficiency which is concerned with the utility of all potential agents in the economy. Michel and Wigniolle (2007) proposes another criterion of CRC-efficiency where CRC stands for Children for Representative Consumers according to which one allocation dominates other if it RC-dominates other and it includes at least an equal number of children in each period. We demonstrate in Appendix A.6 that our phaseout plan ensures Pareto efficiency measured through these criteria.

Proposition 3. *Let $V^{ind}(\tau, \tau) = V^*$.*

(a) The government can reduce the pension tax of the generation $t+1$ agent to $\tau_{t+1} = \tau' < \tau$ while ensuring that the generation t agent is no worse off than what it was under the no-subsidy regime, that is, $V^{opt}(\tau, \tau') = V^*$.

(b) The combined policy of education and child subsidies and reduced rate of pension benefits results in a net increase in both fertility and total pension benefits of the generation t agent.

Proof. See Appendix A.4.

Proposition 3 sketches the impact of our scheme on the generation t agent. Next we demonstrate the effects on the subsequent generations. The generation $t+1$ agent faces a lower pension tax rate $\tau_{t+1} < \tau$. This lower tax rate, along with the internalization of pension externalities by education and child care subsidies, will make the agent better off as compared to the benchmark case of no subsidy. However, similar to the generation t agent, the government will decrease the rate of pension benefits of this generation (τ_{t+2}) further so that an agent of this generation also gets the same benchmark utility V^*. We continue in this way by reducing the rate of pension benefits of subsequent generations further and further while maintaining the benchmark utility V^* throughout the phaseout period. In the following proposition we first establish that the rate of fall in pensions in this way between two consecutive generations j and $j+1$ is given by $\frac{d\tau_{j+1}}{d\tau_j} = \frac{R}{n_j}$. Then we prove that fertility keeps falling throughout the phaseout period. Internalizing the pension externalities through subsidies leads to a rise in fertility. However, throughout the phaseout period, as the rate of pension benefits in the old age keep on falling, the size of externality keeps on decreasing and so do the subsidies. From equation (5) note that the subsidies $\phi_{e,t}$ and $\phi_{c,t}$ are increasing functions of the rate of pension benefit τ_{t+1}. Therefore, fertility rate keeps falling throughout the phaseout period as the subsidies keep falling. It follows that in a *dynamically efficient* economy with $n_t < R$, subsequent generations' fertility is also less than R. So the rate of fall in pensions becomes $\frac{d\tau_{j+1}}{d\tau_j} = \frac{R}{n_j} > 1$, that is, pensions keep falling at an ever-increasing rate. Hence in a *dynamically efficient* economy the government can phase out the pensions in a Pareto way in finite time. Finally, as both n_j and τ_{j+1} are falling, total pension benefits of the generation j agent, $p_{j+1} = \tau_{j+1} n_j w h_{j+1}$, keep falling throughout the phaseout period and becomes zero in finite time. We summarize these results in the following proposition.

Proposition 4. *The following results hold during the pension phaseout period.*

(a) The rate of fall in pensions between two consecutive generations j and $j+1$ while maintaining the benchmark utility V^* is given by

$$\frac{d\tau_{j+1}}{d\tau_j} = \frac{R}{n_j}.$$

(b) $\left.\frac{dn_j}{d\tau_{j+1}}\right|_{V=V^*} > 0$, *that is, n_j falls with τ_{j+1} throughout the phaseout period. However, fertility never falls below the PAYG steady state level n^*.*

(c) *In a dynamically efficient economy with $n_t < R$, the rate of fall in pensions is given by*

$$\frac{d\tau_{j+1}}{d\tau_j} = \frac{R}{n_j} > 1.$$

That is, the pension tax keeps falling at an ever-increasing rate implying that pension tax reaches zero in finite time.

(d) *Similar to fertility, total pension benefits also keep falling throughout the phaseout period and become zero when pension tax reaches zero in finite time.*

Proof. See Appendix A.5.

6 Calibration

In this section we calibrate our model and present the paths of pension, fertility, income and consumption till the PAYG pensions are phased out. We assume that one period (or generation) has a length of 30 years. Utility of an agent is given by $\log(c_t^m) + \beta\log(c_{t+1}^o) + \gamma\log(n_t h_{t+1})$. This functional form of the utility and values of the parameters β and χ are taken from de la Croix and Doepke (2003). The quarterly discounting rate is assumed to be 0.99. Since one period in the model consists of 30 years, an agent discounts old age utility from consumption by 0.99^{120}. The opportunity cost of children, χ, is assumed to be 0.075, that is, each child takes around 7.5 percent of parent's time. Next we calibrate the parameter γ. OECD (2018a) estimates fertility in the United States to be 0.9. Thus the weight agents place on the quality and quantity of children, γ, is calibrated to 0.211 to ensure that fertility in the PAYG-only steady state, that is, in period 1, comes out to be 0.9. Human capital production function is assumed to take the functional form $h_{t+1} = (e_t + \theta)^\sigma$ and, following de la Croix and Doepke (2003), θ is taken to be 0.0119. Further we calibrate σ to 0.535. These values imply that elasticity of income with respect to education, that is, $\frac{\partial(wh(e_t))}{\partial e_t}\frac{e_t}{wh(e_t)} = \frac{\sigma e_t}{(e_t+\theta)}$, equals 0.49 which is comparable to the estimates in the literature.[15]

[15]For example, Angrist and Krueger (1991), using compulsory education as an instrumental variable, estimates the return to an additional year of schooling to be 7.5%. Krueger and Lindahl (2001) estimates the return to schooling in developed countries in the range of 8-10%. However, some studies using instrumental variables have reported higher marginal returns of schooling up to around 16% while some have reported lower returns around 5-6%. See Card (2001) for a comprehensive list. Elasticity of income with respect to education is the percentage increase in income divided by percentage increase in education. Assuming that return to additional year of schooling is 10% and an extra year of schooling raises education expenditure by 20%, elasticity comes out to be around 0.5. Since, both numerator and denominator vary in a range, de la Croix and Doepke (2003) pins down earnings elasticity of education to be between 0.4 and 0.8.

We assume that yearly interest rate is equal to 1.01 implying R to be equal to 1.01^{30}. Further we take wage rate w such that the total wage income per year, $wh_t(1-\chi n_t)/30$, is USD 45000 which is the median wage income of the US in the year. Our calibration ensures that the education share in the model in the initial steady state is equal to the estimate of education expenditure in the US which is 6.1 percent of GDP (see OECD (2018b)), that is, $\frac{qe_t n_t}{wh_t(1-\chi n_t)+\frac{s_{t-1}R}{n_{t-1}}} = 0.061$. For this to hold, we require per unit price of education $q = 0.195w$. The expression for education share is derived in the following way. Let us denote the number of middle-aged agents in period t by N_t. Then the total education expenditure in the economy is $N_t qe_t n_t$ and the total production in the economy, the sum of wage earnings by the middle-aged and returns on capital held by the old, is $N_t wh_t(1-\chi n_t) + N_{t-1}s_{t-1}R$. Dividing the education expenditure by the total production in the economy gives the share of education expenditure as a percent of GDP. To summarize, we calibrate model parameters σ, q and γ to ensure that fertility and education as a share of GDP match the US data, and we borrow other parameters from de la Croix and Doepke (2003).

We assume that before the intervention by the government to reduce the PAYG pension tax, the economy is at a steady state in period 1 with a PAYG pension tax of 15 percent of an agent's income. Throughout the phaseout exercise, the government ensures that an agent's utility in each period is equal to her utility under the steady state with PAYG pensions, that is, her utility in period 1. This steady state value is calculated by substituting the agent's optimal choices of consumption, children's education and fertility in the functional form of utility. Let this utility be V^* as denoted in section 5. The government introduces the education and child care subsidies in period 2 on the generation 2 agent, and, to ensure that her utility remains at V^*, reduces her rate of old-age pension benefits τ_3. Hence the pension tax in period 3 is less than the pension tax in period 2 and the phaseout scheme described in section 5 kicks in. Figure 1 and Figure 2 plot the paths of pension tax and fertility respectively. Figure 3 shows the path of total pension benefits $wh_{t+1}n_t\tau_{t+1}$ while Figure 4 depicts the paths of lifetime income and lifetime consumption discounted to middle age. Lifetime income $wh_t(1-\tau_t-\chi n_t)+\frac{wh_{t+1}\tau_{t+1}n_t}{R}$ includes after tax earnings in middle age and pension support in old age discounted by the gross interest rate R. Lifetime consumption is the sum of middle age consumption and old age consumption, also discounted by the interest rate R. As the pension externalities are internalized, both fertility and total pension benefits of the generation 2 agent increase, thus corroborating Proposition 3(b). After that both fertility and pension taxes decrease resulting in the fall in total pension benefits (Proposition 4).

From the path of pension tax τ_t, observe that the PAYG pension tax is 0 in the 9^{th} period. Thus the government needs 7 periods or 210 years (periods 2 to 8) to phase out

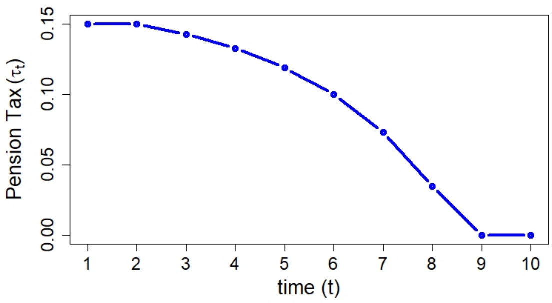

Figure 1. Pension tax

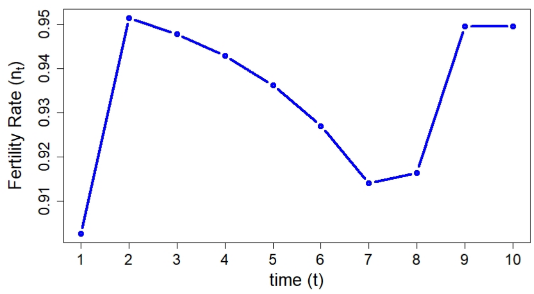

Figure 2. Fertility

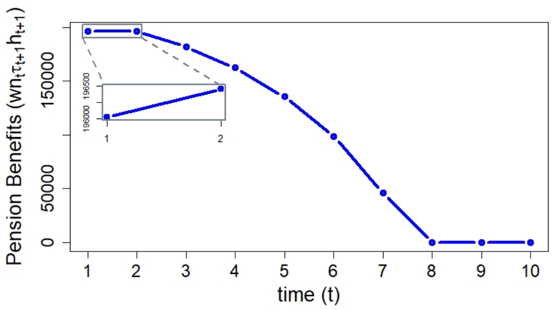

Figure 3. Pension Benefits

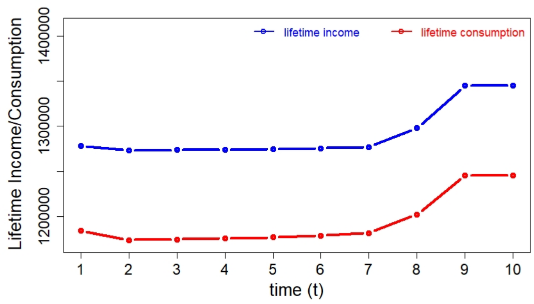

Figure 4. Lifetime Income and Consumption

pensions using the above strategies. Also note that τ_t is strictly decreasing and $\dfrac{d\tau_{t+1}}{d\tau_t} > 1$ during the phase out period (Proposition 4(c)). Therefore, the rate of fall in pension tax τ_t between two consecutive periods keeps on rising and the plot of pension tax over time keeps getting steeper. The fertility rate rises when the government implements education subsidy and child care subsidy (in period 2) because it decreases the relative cost of having children. However, the fertility rate is still less than R. So, even after providing incentives to agents to have more children, the economy remains dynamically efficient. After the phaseout of pensions, agents face no further pension tax. Therefore disposable income of agents increases, thus increasing the fertility rate as children are normal goods in this model. Hence, from period 9 onward, the fertility reaches a new steady state and takes a value higher than the steady state value of fertility under the PAYG regime.

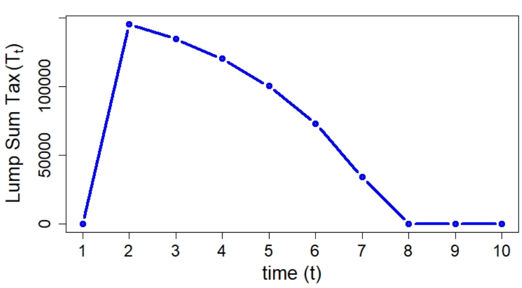

Figure 5. Lumpsum Tax

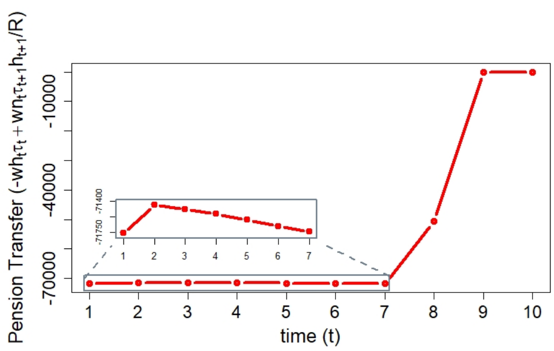

Figure 6. Net Pension Transfers

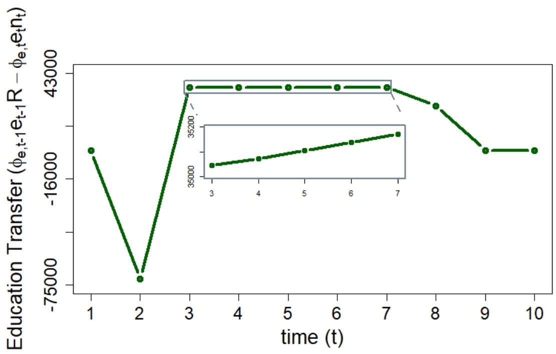

Figure 7. Net Education Transfers

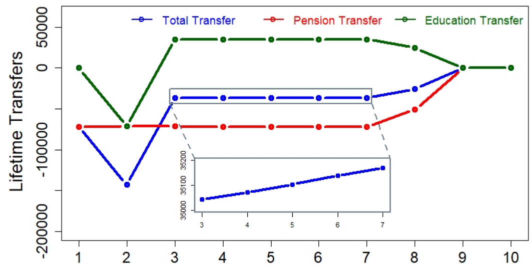

Figure 8. Net Transfers to Agent

To view the paths of net transfers to an agent, we plot the paths of net pension transfers, net education transfers and overall transfers. Figure 5 shows the path of lumpsum tax levied on the middle-aged which finances subsidies to the same generation to support the costs of raising and educating their children. It is zero in period 1 and takes a positive value as the tax-subsidy regime is introduced. Since this tax only funds the subsidies to take care of the externalities associated with children due to pension, it falls throughout the phaseout period as pension benefits fall. This corroborates an implication of equation (5) that the lumpsum tax a generation funds is proportional to the pension benefits it receives. For the path of net pension transfers $\left(-wh_t\tau_t + \dfrac{wh_{t+1}\tau_{t+1}n_t}{R}\right)$ depicted in Figure 6, observe that they are negative initially which is expected in a dynamically efficient economy. Negative net transfer on the account of social security cum medicare has been well observed in the data. Also for the initial year, our model guarantees around 5 percent negative net pension transfers as percentage of lifetime earnings which is reasonable (see Bommier et al. (2010)). After pension is phased out, net pension transfers go to zero. Let us now illustrate what happens in the phaseout period. In period 2, the middle-aged agent has higher pension benefits as the introduction of subsidies leads to higher fertility and the number of next generation taxpayers increases (Proposition 4(d)). From period 3 onwards, agent's pension tax falls. However, her pension benefits fall at an even faster rate as the next generation's pension tax falls more which is evident from the path of pension tax τ_t and from Proposition 4(c). Moreover, fertility is also falling throughout the phaseout period after a one time jump in period 2. Thus, net pension transfers decrease as pension benefits fall at a faster rate than the pension tax.

To analyze the path of net education transfers to an agent, we face a normative question of who benefits from an education subsidy. This is fundamentally an accounting issue. Since the parent is educating her children as she gets utility from her children's human capital, an education subsidy can be viewed as a transfer to the parent. On the other hand, a child can also be thought of as the direct beneficiary of an education subsidy. In the first case, net education transfer to an agent is zero as the parent gets an education subsidy which is funded by the lumpsum tax paid by her. We consider the second case where an education subsidy is a direct benefit to the child. In Figure 7, net education transfers to a generation t agent, $R\phi_{e,t-1}e_{t-1} - \phi_{e,t}e_t n_t$,[16] initially go from 0 for generation 1 agent (when there is no subsidy) to negative for generation 2 when subsidy is introduced. It is positive afterwards as each agent pays a part of education cost for n_t children which is less than the present value of benefit received by this agent as $R > n_t$ and ϕ_e is falling in the phaseout period. It keeps on increasing throughout the phaseout period. From equation (5) it follows that the education subsidy a generation t agent pays for is proportional to the pension benefits she will receive in her old age. Lagging by one period the equation also says that the education subsidy an agent receives is proportional to the tax she pays for funding older generation's pensions. Therefore, from period 3 onwards, an agent's net education transfers is proportional to the net pension payment she makes except that the sign is opposite. Therefore, net education transfers follow a path that is a mirror image of the path followed by net pension transfers. Finally, education transfers go from positive to zero when both education subsidy and pensions go to zero. Figure 8 plots net pension transfers, net education transfers, and total transfers on the account of education-pension which are just sum of net education transfers and net pension transfers.

7 Conclusion

For quite sometime the PAYG pension system is facing serious criticisms, mainly due to demographic shifts like a fall in fertility coinciding with a rise in longevity. Also, there is no efficiency rationale for pensions if the economy is dynamically efficient as all the economies are. But phasing pensions out without making any generation worse off has been a challenging task. This paper proposes a novel way out of the PAYG pension system in a Pareto way. An inefficiency within the pension system that is well acknowledged in the literature has been carefully used to phase it out without making any generation worse off. The paper combines three popular strands of literature related to PAYG pension. The first strand identifies the externalities associated with children and their human capital, and provides ways to deal with it. The second strand of literature tries to find a balance between differently directed intergenerational transfers, and it turns out that our policies have that balancing feature. The third strand of literature deals with phasing out of pension. To be precise, the paper recommends an education and childcare subsidy package to correct for the positive externality in a PAYG system and then takes advantage of this positive externality to propose a scheme that provides a way for subsequent generations' pension tax to be gradually reduced while ensuring that no generation is worse off than under the PAYG regime. In the process the PAYG pensions are completely phased out in finite time.

[16] For a generation t agent, her net education benefits is the subsidy she gets $\phi_{1,t-1}e_{t-1}$ (which is multiplied by R to write it in terms of middle-age value) minus the education tax she pays in her middle-age, that is, $\phi_{e,t}e_t n_t$. Using equation (5) we can replace the subsidy values in terms of the next generation's pension tax to find a new expression for net education transfers: $e^{opt}wh'(e^{opt})(\tau_t - \frac{\tau_{t+1}n_t}{R})$. Note that this is proportional to the total pension transfers which is by design of optimal education subsidy in equation (5).

A Appendix

A.1 Existence and Uniqueness of Education Choice

Since both $u(.)$ and $v(.)$ satisfy Inada conditions, neither consumption nor fertility choice of an agent is zero. If either of these choices is zero, marginal utility from having one additional unit would be very high thus ensuring positive values. To check that an agent's choice of her children's education is non-zero, we proceed as follows.

Observe first that dividing the first order condition for n_t by the first order condition for e_t gives

$$\frac{qe_t + \chi w h_t}{q} = \frac{h(e_t)}{h'(e_t)}.$$

Rearranging terms the equation becomes

$$\frac{\chi w h_t}{q} = \frac{h(e_t)}{h'(e_t)} - e_t.$$

Let us denote the right-hand side as $g(e_t)$, that is, $g(e_t) \equiv \frac{h(e_t)}{h'(e_t)} - e_t$. Differentiating both sides with respect to e_t we get $g'(e_t) = \frac{-h(e_t)h''(e_t)}{(h'(e_t))^2} > 0$ as $h''(e_t) < 0$ by concavity of $h(e_t)$. Hence $g(e_t)$ is an increasing function of e_t and we prove next that it must be equal to the left-hand side for some positive and finite value of e_t.

For e_t close to 0, $h'(e_t)$ is sufficiently large by Inada conditions so that $g(e_t)$ is close to 0. By concavity of $h(e_t)$, $g(e_t) = \frac{h(e_t) - e_t h'(e_t)}{h'(e_t)} > \frac{h(0)}{h'(e_t)}$. For e_t large enough, again by Inada conditions, $h'(e_t)$ becomes small enough so that $g(e_t) > \frac{h(0)}{h'(e_t)} > \frac{\chi w h_t}{q}$. Hence the right-hand side, $g(e_t)$, is an increasing function of e_t and is less than the left-hand side for small e_t and exceeds the left-hand side for large e_t. It follows that it must be equal to the left-hand side for some positive and finite value of e_t which is the optimal education choice for children of an agent endowed with human capital h_t. As $g(e_t)$ is strictly increasing in e_t, the education level of children chosen by an agent is unique and increasing in parent's education e_{t-1}.[17]

A.2 Proof of Proposition 1

Proof. In the first order conditions (3) for the optimal choices, $u'(c_t^m)$ equals $\beta R u'(c_{t+1}^o)$ by the first condition. Substituting $u'(c_t^m)$ by $\beta R u'(c_{t+1}^o)$ in the second and third conditions

[17] Note that $h_t = h(e_{t-1})$ is strictly increasing in e_{t-1}.

we get the following two equations:

$$\beta u'(c_{t+1}^o)(R(qe_t + \chi wh_t) - \tau_{t+1}wh(e_t)) = h(e_t)v'(n_t h(e_t)),$$

$$\beta u'(c_{t+1}^o)(Rqn_t - \tau_{t+1}wn_t h'(e_t)) = n_t h'(e_t)v'(n_t h(e_t)).$$

Dividing the first equation by the second gives

$$\frac{R(qe_t + \chi wh_t) - \tau_{t+1}wh(e_t)}{Rqn_t - \tau_{t+1}wn_t h'(e_t)} = \frac{h(e_t)}{n_t h'(e_t)}.$$

From this, after some algebraic simplifications, we get

$$\frac{qe_t + \chi wh_t}{q} = \frac{h(e_t)}{h'(e_t)}.$$

This is the same equation that determines the child's education choice of an atomistic agent. Therefore the agent's individualistic choice of education for her children is no different from the optimal choice, that is, $e_t^{opt} = e_t^{ind}$.

Let us denote the utility of a generation t agent by $V_t(.)$, that is, $V_t(.) \equiv u(c_t^m) + \beta u(c_{t+1}^o) + v(n_t h_{t+1})$, where c_t^m, c_{t+1}^o and h_{t+1} are defined in the constraints of problem (B). Then

$$\frac{\delta V_t(.)}{\delta n_t} = -u'(c_t^m)(qe_t + \chi wh_t) + \beta u'(c_{t+1}^o)\tau_{t+1}wh(e_t) + h(e_t)v'(n_t h(e_t).$$

As an atomistic agent equates the first term on the right-hand side with the third term, the net marginal effect of fertility on utility, $\frac{\delta V_t(.)}{\delta n_t}$, equals $\beta u'(c_{t+1}^o)\tau_{t+1}wh(e_t) > 0$. Therefore the agent can increase her utility by modifying her fertility decision. Hence a better solution for the agent's problem exists in the neighborhood of her choices. This implies that $V_t^{opt} > V_t^{ind}$ and the agent's individualistic choices are not optimal.

Now we prove that $n_t^{opt} > n_t^{ind}$. Suppose, on the contrary, that $n_t^{opt} \leq n_t^{ind}$, that is, the optimal fertility choice n_t^{opt}, a solution to problem (B), is less than or equal to the agent's individualistic choice n_t^{ind}, a solution to problem (A). We show that this will lead to a contradiction. Consider first the first order condition of problem (A) with respect to n_t, that is, the second equation in (1):

$$u'(c_t^m)(qe_t + \chi wh_t) = h(e_t)v'(n_t h(e_t)). \tag{A1}$$

For problem (B), the first order condition with respect to s_t, the first equation in (3), implies $u'(c_{t+1}^o) = \frac{u'(c_t^m)}{\beta R}$. Substituting this value of $u'(c_{t+1}^o)$ in the second equation in

(3) gives

$$u'(c_t^m)\left(qe_t + \chi wh_t - \frac{\tau_{t+1}wh(e_t)}{R}\right) = h(e_t)v'(n_t h(e_t)). \tag{A2}$$

Individualistic and optimal choices of the agent must satisfy equations (A1) and (A2) respectively. We have proved that individualistic and optimal education choices of the agent are the same. Then, under the assumption that $n_t^{opt} \leq n_t^{ind}$, the right-hand side of equation (A2) must be greater than or equal to the right-hand side of equation (A1) by concavity of $v(.)$. It follows that

$$\begin{aligned}
& u'(c_t^{m,opt})\left(qe_t + \chi wh_t - \frac{\tau_{t+1}wh(e_t)}{R}\right) \geq u'(c_t^{m,ind})(qe_t + \chi wh_t) \\
\Rightarrow \quad & u'(c_t^{m,opt}) \geq u'(c_t^{m,ind}) \\
\Rightarrow \quad & c_t^{m,opt} \leq c_t^{m,ind} \qquad\qquad [\text{since } u(.) \text{ is concave}] \\
\Rightarrow \quad & c_{t+1}^{o,opt} \leq c_{t+1}^{o,ind}. \qquad\quad [\text{since } u'(c_t^m) = \beta R u'(c_{t+1}^o)]
\end{aligned}$$

Thus, under the assumption that $n_t^{opt} \leq n_t^{ind}$, optimal consumption decisions are less than the agent's individualistic consumption decisions. As the agent's optimal consumption and number of children are less in comparison with the atomistic choices, while child's education choice remains the same, we have $V_t^{opt} \leq V_t^{ind}$. This contradicts $V_t^{opt} > V_t^{ind}$. Hence, our assumption of $n_t^{opt} \leq n_t^{ind}$ is incorrect, implying that $n_t^{opt} > n_t^{ind}$. ■

A.3 Proof of Proposition 2

Proof. For the government subsidies to match the optimal choices, solutions to the optimal choices problem, problem (B), must be the same as the solutions to the problem with government subsidies, problem (C). As the solutions are characterized by first order conditions (3) and (4) respectively, we have to find $\phi_{e,t}$ and $\phi_{c,t}$ such that the first order conditions are the same. Equating the first order conditions with respect to e_t, the third equations in (3) and (4), gives $u'(c_t^m)\phi_{e,t}n_t = \beta u'(c_{t+1}^o)\tau_{t+1}wn_t h'(e_t)$. Using the Euler equation to substitute $u'(c_t^m)$ by $\beta R u'(c_{t+1}^o)$ we get $R\phi_{e,t} = \tau_{t+1}wh'(e_t^{opt})$, the first equation in (5). Similarly, equating the first order conditions with respect to n_t, the second equations in (3) and (4), and then using the Euler equation we get $R(e_t^{opt}\phi_{e,t} + \phi_{c,t}) = \tau_{t+1}wh(e_t^{opt})$, the second equation in (5).

Since $h'(.) > 0$, it follows from the first equation that $\phi_{e,t} > 0$. Substituting the value of $\phi_{e,t}$ from the first equation to the second equation we get

$$\phi_{c,t} = \frac{\tau_{t+1}w}{R}(h(e_t^{opt}) - e_t^{opt}h'(e_t^{opt})) > \frac{\tau_{t+1}wh(0)}{R} > 0$$

by the concavity of $h(.)$. Proposition 2(a) then follows from the fact that both $\phi_{e,t} > 0$ and $\phi_{c,t} > 0$. ■

A.4 Proof of Proposition 3

A.4.1 Proof of Proposition 3(a)

Proof. We start with $V^{ind}(\tau,\tau) = V^*$. In appendix A.2 we have shown that $V^{opt}(\tau,\tau) > V^*$. We prove this proposition by showing that $V^{opt}(\tau_t,\tau_{t+1})$ is strictly increasing in τ_{t+1}. Recall that $V^{opt}(\tau_t,\tau_{t+1})$ is the maximized value of the objective function in problem (B). It is clear from the agent's old age budget constraint in problem (B) that, other things remaining the same, an increase in τ_{t+1} expands the agent's budget set. Original optimal choice of $\{s_t, e_t, n_t\}$ being still available, an increase in τ_{t+1} certainly increases the agent's utility. Therefore, $V^{opt}(\tau_t,\tau_{t+1})$ is strictly increasing in τ_{t+1}. Thus, in order to guarantee $V^{opt}(\tau,\tau_{t+1}) = V^* < V^{opt}(\tau,\tau)$, the government should decrease τ_{t+1}, that is, $\tau_{t+1} < \tau$.
■

A.4.2 Proof of Proposition 3(b)

Proof. As a first step let us see that it would be enough if we can establish that the net effect of the combined policy of education and child subsidies and reduced rate of pension benefits on total pension benefits of the generation t agent, $p_{t+1} = \tau_{t+1}n_t w h_{t+1}$, is positive. We know that the subsidies do not affect the children's education choice which continues to be determined by equation (2). It is clear from equation (2) that τ_{t+1} does not have any effect on the generation t agent's education choice either. Thus wh_{t+1} remains unaffected by the combined policy. On the other hand τ_{t+1} falls. So if we can show that $p_{t+1} = \tau_{t+1}n_t w h_{t+1}$ increases, it would follow that n_t increases. In the next step we prove that p_{t+1} increases as a result of the combined policy.

Consider problem (A) where the generation t agent takes her pension benefit p_{t+1} as exogenous and makes her choices of fertility, savings and children's education. The choices are such that the government balances its pension budget constraint. Similarly consider problem (C) where the generation t agent takes as exogenous both her pension benefit p_{t+1} as well as her lump sum tax τ_t to finance the education and child care subsidies. Here the choices satisfy the government's subsidies-financing budget constraint and the pension budget constraint. Let us write the indirect utilities of the agent as a function of p_{t+1}, that is, $V^{ind}(p_{t+1})$ for problem (A) and $V^{opt}(p_{t+1})$ for problem (C). Before the subsidy scheme was introduced, the agent's indirect utility was at the benchmark level V^*, and let the resultant total pension benefit be denoted by p^*_{t+1}, so that $V^{ind}(p^*_{t+1}) = V^*$. Under the combined policy suppose that the resulting total pension benefit is p'_{t+1}. Note that choices under the combined policy is the solution to problem (C) where τ_{t+1} is reduced to ensure that $V^{opt}(p'_{t+1}) = V^*$. We prove that $p'_{t+1} > p^*_{t+1}$. Suppose, on the contrary, that $p'_{t+1} \leq p^*_{t+1}$. Then, from the budget sets for problems (A) and (C) it is clear that as long as the solution to problem (C) (with reduced τ_{t+1}) satisfies the government's subsidies-financing budget constraint and the pension budget constraint, the solution is also feasible under problem (A). It follows that the optimal choices under problem (A) must result in strictly higher utility than the choices under the combined policy, that is, $V^{ind}(p^*_{t+1}) \geq$

$V^{ind}(p'_{t+1}) > V^{opt}(p'_{t+1})$.[18] But this contradicts $V^{ind}(p^*_{t+1}) = V^* = V^{opt}(p'_{t+1})$. Hence, our supposition of $p'_{t+1} \leq p^*_{t+1}$ is incorrect, implying that $p'_{t+1} > p^*_{t+1}$. ∎

A.5 Proof of Proposition 4

It is enough to prove parts (a) and (b) of the proposition. The remaining parts follow easily from (a) and (b) and are already discussed in the text. We prove parts (a) and (b) in the following two steps.

Step 1: Proof of Proposition 4(a)

Proof. During the pension phaseout period the generation j agent faces pension tax τ_j and for generation $j+1$ agent it is τ_{j+1}. The government is always keeping an agent's utility at V^*, that is, $V^{opt}(\tau_j, \tau_{j+1}) = V^*$. Totally differentiating this equation we get

$$-u'(c^m_j)wh_j d\tau_j + \beta u'(c^o_{j+1})n_j wh(e_j)d\tau_{j+1} = 0.$$

Since optimal choices in n_j, s_j and s_j are made throughout the phaseout period, the indirect effects working through n_j, s_j and e_j are zero by the envelope theorem. Also, the optimal education and child care subsidies do not change the education choice of the agent implying that $h_j = h(e_j)$ throughout the phaseout period. Then, using the Euler equation $u'(c^m_j) = \beta R u'(c^o_{j+1})$, the above equation gives

$$\frac{d\tau_{j+1}}{d\tau_j} = \frac{R}{n_j}.$$

∎

Step 2: Proof of Proposition 4(b)

Proof. Now we show that fertility keeps falling throughout the pension phaseout period. During the phaseout period, the government sets pension tax rates such that

$$u(c^m_j) + \beta u(c^o_{j+1}) + v(n_j h(e_j)) = V^*. \tag{A3}$$

Also, the government subsidies ensure that any generation j agent's choices satisfy the following equations (which correspond to the first order conditions, equation (3), of the

[18] The first inequality follows from $p^*_{t+1} \geq p'_{t+1}$ and that indirect utility is increasing in pension benefits. The second (strict) inequality follows from the fact that the solutions to the optimization problems are unique as the objective function is strictly concave.

optimal education and fertility choices problem (B)):

$$u'(c_j^m) = \beta R u'(c_{j+1}^o), \tag{A4}$$

$$u'(c_j^m)(qe_j + \chi w h_j) = \beta u'(c_{j+1}^o)\tau_{j+1} w h(e_j) + h(e_j)v'(n_j h(e_j)), \tag{A5}$$

$$u'(c_j^m) q n_j = \beta u'(c_{j+1}^o)\tau_{j+1} w n_j h'(e_j) + n_j h'(e_j) v'(n_j h(e_j)). \tag{A6}$$

In appendix A.2 we have shown that equations (A5) and (A6) determine the optimal education choice given by equation (2). It is also clear from equation (2) that neither τ_j nor τ_{j+1} has any effect on the generation j agent's education choice either. Hence agent's education choice, e_j, remains unaffected throughout the pension phaseout period.

Next we argue, invoking the implicit function theorem, that equations (A4) and (A5) implicitly define n_j and s_j as functions of the parameters of the model. Rewriting equations (A4) and (A5) as $\mathcal{A}(s_j, n_j, e_j) = 0$ and $\mathcal{B}(s_j, n_j, e_j) = 0$ respectively, the sufficient condition requires that $\begin{vmatrix} \frac{\partial \mathcal{A}}{\partial n_j} & \frac{\partial \mathcal{A}}{\partial s_j} \\ \frac{\partial \mathcal{B}}{\partial n_j} & \frac{\partial \mathcal{B}}{\partial s_j} \end{vmatrix} \neq 0$. After some algebra we derive

$$\begin{vmatrix} \frac{\partial \mathcal{A}}{\partial n_j} & \frac{\partial \mathcal{A}}{\partial s_j} \\ \frac{\partial \mathcal{B}}{\partial n_j} & \frac{\partial \mathcal{B}}{\partial s_j} \end{vmatrix} = -(h(e_j))^2 v''(n_j h(e_j))(\beta R^2 u''(c_{j+1}^o) + u''(c_j^m))$$
$$- \beta u''(c_{j+1}^o) u''(c_j^m)(R(qe_j + \chi w h_j) - \tau_{j+1} w h(e_j))^2 < 0$$

by strict concavity of $u(.)$ and $v(.)$. Hence equations (A4) and (A5) define n_j and s_j as implicit functions of the parameters of the model, in particular, of the two parameters τ_j and τ_{j+1} that are changing during the pension phaseout period.

From the above discussion it follows that we can track the changes in the endogenous variables c_j^m, c_{j+1}^o and n_j due to adjustments in the parameters τ_j and τ_{j+1} during the phaseout period through the system of equations (A3), (A4) and (A5), and the fact that e_j remains unchanged. In what follows we use these three equations and $de_j = 0$ to establish that fertility keeps falling throughout the phaseout period.

Substituting the value of $u'(c_{j+1}^o)$ from (A4) into (A5), we get

$$u'(c_j^m)(R(qe_j + \chi w h_j) - \tau_{j+1} w h(e_j)) = R h(e_j) v'(n_j h(e_j)). \tag{A7}$$

Totally differentiating (A3), (A4) and (A7) respectively, and using $de_j = 0$, we get

$$u'(c_j^m) dc_j^m + \beta u'(c_{j+1}^o) dc_{j+1}^o + v'(n_j h(e_j)) h(e_j) dn_j = 0, \tag{A8}$$

$$u''(c_j^m)dc_j^m = \beta R u''(c_{j+1}^o)dc_{j+1}^o, \tag{A9}$$

$$u''(c_j^m)(R(qe_j+\chi wh_j)-\tau_{j+1}wh(e_j))dc_j^m - wh(e_j)u'(c_j^m)d\tau_{j+1} = R(h(e_j))^2v''(n_jh(e_j))dn_j. \tag{A10}$$

Substituting the value of dc_{j+1}^o from (A9) into (A8) we get

$$u'(c_j^m)dc_j^m + u'(c_{j+1}^o)\frac{u''(c_j^m)}{Ru''(c_{j+1}^o)}dc_j^m + v'(n_jh(e_j))h(e_j)dn_j = 0$$

$$\Rightarrow \quad dc_j^m = \frac{-v'(n_jh(e_j))h(e_j)}{u'(c_j^m) + \frac{u'(c_{j+1}^o)u''(c_j^m)}{Ru''(c_{j+1}^o)}}dn_j.$$

We substitute this expression of dc_j^m in terms of dn_j into equation (A10) to derive the expression for changes in n_j in response to the adjustments in τ_j and τ_{j+1} while maintaining an agent's utility at V^* during the phaseout period:

$$\left.\frac{dn_j}{d\tau_{j+1}}\right|_{V=V^*} = \frac{-wh(e_j)u'(c_j^m)}{\dfrac{(R(qe_j+\chi wh_j) - wh(e_j)\tau_{j+1})u''(c_j^m)v'(n_jh(e_j))h(e_j)}{u'(c_j^m) + \dfrac{u''(c_j^m)u'(c_{j+1}^o)}{Ru''(c_{j+1}^o)}} + R(h(e_j))^2v''(n_jh(e_j))} > 0.$$

The numerator is negative as it contains a negative sign and other terms including $u'(.)$ are positive. The denominator has two terms. The second term has $v''(.) < 0$. Hence the second term is negative. The first term has $u''(.) < 0$, $v'(.) > 0$ and another term $(qe_j + \chi wh_j)R - wh(e_j)\tau_{j+1}$. This term is the total cost of a child minus total monetary returns from the child. From equation (A7) it follows that this term is positive. This makes the first term in the denominator negative too. Each of the two terms being negative makes the denominator negative. Both the numerator and denominator of $\left.\frac{dn_j}{d\tau_{j+1}}\right|_{V=V^*}$ being negative, n_j falls with τ_{j+1} throughout the phaseout period.

Finally we prove that fertility never falls below the PAYG steady state level. To prove this claim, we focus on two time periods - the phaseout period and the post-phaseout period (the new steady state with no pension). To show that fertility in a no PAYG steady-state regime is strictly greater than fertility in a PAYG steady state regime with pension $\tau_t = \tau$ for all t, we use the first order conditions of agent ((1)) to prove that $\frac{dn}{d\tau} < 0$. Totally differentiating the first two equations in ((1)) and using the fact that education choice is unchanged between the PAYG steady state and the post-phaseout regime we get

$$u''(c_t^m)dc_t^m = \beta R u''(c_{t+1}^o)dc_{t+1}^o,$$
$$u''(c_t^m)(qe_t + \chi w h_t)dc_t^m = (h(e_t))^2 v''(n_t h(e_t))dn_t.$$

These two equations ensure that c_t^m, c_{t+1}^o and n_t either increase or decrease together in response to a change in τ. Moreover, for a pension tax τ, combining the agent's middle-age and old-age budget constraints as a lifetime budget constraint we get

$$c_t^m + \frac{c_{t+1}^o}{R} + (qe_t + \chi w h_t)n_t = w h_t(1-\tau) + \frac{p_{t+1}}{R}. \tag{A11}$$

For any $\tau > 0$, net income through PAYG pension is negative in a dynamically efficient economy. So the post-phaseout period has a higher income than the PAYG pension period and both consumption and fertility are higher in the post-phaseout period as they either increase or decrease together.

To prove that fertility during the phaseout period is not lower than fertility in the PAYG period, it suffices to show that fertility in the last period of the phaseout at least equals fertility in the PAYG period as we have already established that fertility falls throughout the phaseout period. In the last period of phaseout k, $\tau_{k+1} = 0$. Hence it follows from (5) that there is no government subsidy in period k. Subsidies being zero, it follows from comparing the first order conditions that there is no difference between optimal and individualistic choices of agents in the margin. The only way differences can occur is through income effect as the agents pays and receives the same pension tax and benefit τ in the PAYG period whereas she pays some pension tax in period k but does not receive any pension benefit. But this income effect is also neutralized by forcing that the agent gets the utility V^*, the benchmark utility in PAYG regime. The reason is the following. For agents making individualistic choices given a lifetime income, the indirect utility is increasing in lifetime income. Since both the PAYG generation agent and generation k agent make individualistic choices and gets indirect utility V^*, their lifetime incomes must be the same. For the same lifetime income, the agent in last period of phaseout makes the same choices as an agent in PAYG regime. So, in the last period of phaseout, fertility equals fertility in a PAYG regime. This completes our argument that fertility in the phaseout period never falls below the PAYG fertility level. ■

A.6 Efficiency under Endogenous Fertility

Our phaseout plan guarantees that each agent born enjoys at least V^* utility which is also the benchmark utility enjoyed under the PAYG scheme. After the phaseout, since income of a representative agent is higher, her utility is also higher than V^*. On the other hand, in the absence of pension, any agent in any generation gets V^*. Clearly, any agent who is born in the phaseout period has at least as much utility as an agent born in the PAYG economy, and some agents have higher utility. If we apply the notion of $\mathcal{A}$-efficiency, then the phaseout economy $\mathcal{A}$- dominates the PAYG economy. Moreover, as representative agent is at least as good as PAYG forever utility level under phaseout plan and some generations are better off, phaseout scheme dominates PAYG forever according to any of the representative agent efficiency criterion. On the other hand, the notion of $\mathcal{P}$-efficiency is concerned with the utility of all potential agents in the economy. Therefore, if phaseout plan has to $\mathcal{P}$- dominate PAYG forever economy, every potential agent has to be better off in our plan. Under the assumption that an unborn agent prefers to be born and get V^* instead of remaining unborn, it is easy to see that this also holds in our setup. First note that once the phaseout scheme kicks in, more agents are born under this scheme in comparison to an economy with PAYG forever, for all generations (see proposition 4(b)). These extra agents who are born are no worse off in our scheme while those unborn in both cases are indifferent between both the schemes. On the other hand, all those who are born under both the schemes are at least as good in phaseout in comparison to PAYG forever while some are better off as argued for $\mathcal{A}$- domination. Hence, we have shown that no potential agent in phaseout is worse off and some are clearly better off. Hence, the phaseout economy $\mathcal{P}$- dominates an economy with PAYG forever. Further, it also CRC-dominates an economy with PAYG forever.

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