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Limited liability, contractual choice, and the tenancy ladder

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Abstract

We re-examine the connection between limited liability, contractual form and the agricultural tenancy ladder in a model with only labor moral hazard. We show that ex post limited liability does not explain sharecropping in this model, but that ex ante limited liability can do so. We characterize the nature of the tenancy ladder in either case. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In a well known contribution to the literature on agricultural contracts, Shetty (1988) has demonstrated the role of limited liability in explaining the observed positive relationship between tenants' wealth and their returns from tenancy contracts. His main contribution is the recognition of the ex post limited liability constraint—for sufficiently adverse realizations of output the rent cannot be paid completely, the liability of the tenant is limited by his amount of wealth—in

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influencing the terms of contracts and tenants' effort levels. However, Shetty does not fully characterize the complete structure of tenancy contracts in his model. We provide that analysis in this paper. In particular, we re-examine the implications of limited liability for contractual structure and the tenancy ladder.

We start with the case of an ex post limited liability constraint, as considered by Shetty (1988). Restricting ourselves to linear contracts—as is standard in this literature—we show that if the tenant's cropshare is unconstrained, wealthier tenants receive a fixed-rent contract, but less wealthy tenants receive a contract with cropshare greater than 1. Interestingly, the first-best outcome can be attained in this unconstrained cropshare case, even when the limited liability constraint binds. The reason is that if the cropshare can exceed 1, the problem of limited liability can be avoided by raising the fixed-rent and, at the same time, raising the cropshare above 1, without increasing the risk of default on this higher rental payment, and without reducing the tenant's incentive to apply the first-best effort.

However, as we discuss later in the paper, there are many reasons as to why in reality tenant's cropshare will be restricted to not greater than 1. Incorporating that restriction, we find that some potential but very poor tenants do not get any tenancy contract. All other tenants, who are wealthy enough to receive a tenancy contract, receive a fixed-rent contract. Thus, there is no room for sharecropping when there is a limited liability constraint in the ex post sense. We define a range of wealth such that, for tenants with wealth levels within that range, their effort levels increase with wealth, and the landlord prefers the wealthier tenants as his expected earning increases with the tenant's wealth. The wealthiest tenants, with wealth levels beyond this range, receive a fixed-rent contract, apply the efficient effort level, and the landlord earns the highest expected earning. Thus, a version of a tenancy ladder emerges, where the landlord prefers to do business with wealthier tenants.

Shetty (1988) establishes that for the tenants with wealth levels sufficient to guarantee landlords full payment for all output realizations, the optimal contract is a fixed-rental contract. But he does not derive the structure of contracts received by the less wealthy tenants.[1] Singh (1989) tries to address this question, but his argument that the contracts for the poorer tenants are share contracts, with the cropshare of the tenants between 0 and 1, has an error. Since then, there appears to be a possible misinterpretation in the literature:[2] limited liability a la Shetty gives

[1] He does seem to implicitly assume that they will be share contracts. For example, he states, "...wealthier tenants (up to A^0) are induced to put forth greater effort since they bear more of the output risk and therefore earn higher shares of their marginal product." (Shetty, 1988, p. 13). A similar statement is on p. 12 of his paper.

[2] Sengupta (1997) is an exception that does not suffer from this misinterpretation. See his footnote 18, p. 405, which also credits Debraj Ray.

rise to a separate explanation of the existence of sharecropping.[3] Thus, our paper clarifies this possible misinterpretation of Shetty (1988), and demonstrates that all contracts are rental contracts in his model.

The idea of the limited liability constraint affecting contract choice has remained quite influential in the literature. Basu (1992) uses the ex post limited liability constraint to explain the existence of share tenancy when there is moral hazard in the tenant's choice of technique. Sengupta (1997), using a slightly different model, clarifies Basu's point: moral hazard in the choice of technique is not in general sufficient; we also need moral hazard in the choice of effort to develop a consistent theory of sharecropping arising out of the limited liability constraint.

Laffont and Matoussi (1995) have taken an approach close to Shetty (1988), in the sense that they have considered a limited liability constraint and moral hazard in the choice of effort only. But they have interpreted the limited liability constraint in a very different way. They postulate an ex ante financial constraint— the fixed component of the rent has to be paid in advance. In Section 3 we illustrate how this ex ante limited liability constraint explains the emergence of *sharecropping* and the associated tenancy ladder through a simple example.

Section 4 provides a summary conclusion for our analysis. Throughout the paper, we follow the notation of Singh (1989).

2. Contracts under ex post limited liability

2.1. The set-up

Both the landlord and the tenant are risk-neutral. The production function is $Q = \theta Q(L)$, $Q' > 0$, $Q'' < 0$, where θ is a random variable with distribution function F, $\theta \in [\underline{\theta}, \overline{\theta}]$ and $E(\theta) = 1$.[4] The tenant has wealth W and receives an income $\alpha\theta Q(L) - C$ from the contract, where α is the tenant's cropshare[5] and C represents the fixed component of the contract, being the payment from the tenant to the landlord when $C > 0$. The liability of the tenant is limited, ex post, by the amount of his wealth, and we define θ_1 to be the value of θ such that the tenant

[3] For instance, Laffont and Matoussi (1995) comment, "In a contribution which is closest to our paper Shetty (1988) develops a model where sharecropping is explained by an ex post liability constraint... Even with risk-neutral tenants limited liability introduces non-concavities in the landlord's and tenant's payoff functions. Sharecropping mitigates within the relationship the associated insurance problem." (p. 382). See also Hayami and Otsuka (1993, p. 29), Sadoulet et al., (1994, p. 226), and Basu (1997, p. 260).

[4] This assumption simplifies the algebra, and is inessential.

[5] This seems to be a more common notation. Note that α is the landlord's share in Shetty's notation.

cannot make the agreed-on payment, $(1-\alpha)\theta Q(L)+C$, for any realization of θ less than or equal to θ_1. Formally, θ_1 is defined by:

$$\theta_1(\alpha,C,L;W):\begin{cases}\alpha\theta_1 Q(L)+W=C\\ \underline{\theta} \text{ if } \alpha\underline{\theta}Q(L)+W\geq C.\end{cases}$$

Note that θ_1 is decreasing in W, ceteris paribus.

In the presence of ex post limited liability, the expected earning of the tenant is $\int_{\theta_1}^{\bar{\theta}}[\alpha\theta Q(L)-C]\mathrm{d}F(\theta)+\int_{\underline{\theta}}^{\theta_1}(-W)\mathrm{d}F(\theta)-L$,[6] and that of the landlord is $\int_{\theta_1}^{\bar{\theta}}[(1-\alpha)\theta Q(L)+C]\mathrm{d}F(\theta)+\int_{\underline{\theta}}^{\theta_1}[\theta Q(L)+W]\mathrm{d}F(\theta)$.

In order to facilitate future reference, let us denote by L^{**} the efficient (first-best) level of labor input, that is, the amount of labor that maximizes the total expected earnings of the landlord and tenant, $Q(L)-L$. Thus, L^{**} is defined by $Q'(L^{**})=1$.

2.2. Contracts without the cropshare constraint

We first analyze the contracting problem without imposing any restriction on the cropshare, α. It will be interesting to see that the efficient effort choice can be achieved in this case even when the limited liability constraint is binding.

The contracting problem is:

$$\underset{\{\alpha,C,L\}}{\text{Maximize}} \quad \int_{\theta_1}^{\bar{\theta}}[(1-\alpha)\theta Q(L)+C]\mathrm{d}F(\theta)+\int_{\underline{\theta}}^{\theta_1}[\theta Q(L)+W]\mathrm{d}F(\theta)$$

$$\text{subject to:} \quad \text{(i)} \int_{\theta_1}^{\bar{\theta}}[\alpha\theta Q(L)-C]\mathrm{d}F(\theta)+\int_{\underline{\theta}}^{\theta_1}(-W)\mathrm{d}F(\theta)-L\geq K,$$

$$\text{(ii)} \int_{\theta_1}^{\bar{\theta}}[\alpha\theta Q'(L)]\mathrm{d}F(\theta)-1=0.$$

Constraint (i) is the participation constraint, and (ii) is the incentive constraint.

Before we start analyzing the contracting problem, we should clarify the following point. Throughout this paper we characterize the contracting problems by assuming that tenants compete for landlords, so tenants are driven down to their opportunity income, K. This is quite a standard in the principal-agent literature. Shetty (1988) assumes instead that landlords compete for tenants. This difference has no effect on the analysis of contract form—since solutions to both approaches characterize points on the same utility-possibility-frontier, the qualitative properties of the solutions are also the same.

Since the cropshare is unconstrained, it is possible for the landlord to push the tenant to his reservation income, K, so that the participation constraint holds with

[6] We assume that labor is measured in disutility units. This is without loss of generality.

equality.[7] As in Singh (1989), solving the constraints for $L(\alpha)$, $C(\alpha)$ and substituting in the objective function, the first-order condition for the maximization becomes:

$$\int_{\theta_1}^{\bar{\theta}}[-\theta Q(L)+\theta(1-\alpha)Q'(L)L_\alpha]\mathrm{d}F(\theta)+\int_{\theta_1}^{\bar{\theta}}C_\alpha \mathrm{d}F(\theta)$$
$$+\int_{\underline{\theta}}^{\theta_1}[\theta Q'(L)L_\alpha]\mathrm{d}F(\theta)=0. \tag{1}$$

Now using (ii) we can derive from (i):

$$\int_{\theta_1}^{\bar{\theta}}[\theta Q(L)-C_\alpha]\mathrm{d}F(\theta)=0. \tag{2}$$

Then from Eqs. (1) and (2), we get:

$$Q'(L)L_\alpha\left[1-\alpha\int_{\theta_1}^{\bar{\theta}}\theta \mathrm{d}F(\theta)\right]=0. \tag{3}$$

But $Q'(L)$ and L_α are non-zero. So finally, we get that the choice of α is given by:

$$\alpha=\frac{1}{\int_{\theta_1}^{\bar{\theta}}\theta \mathrm{d}F(\theta)}. \tag{4}$$

For the tenants with wealth levels high enough to guarantee full rental payments for all realizations of output, $\theta_1=\underline{\theta}$. Hence, for these wealthy tenants, $\alpha=1/\int_{\underline{\theta}}^{\bar{\theta}}\theta \mathrm{d}F(\theta)=1$, that is, they receive a fixed-rental contract. Shetty (1988) has noted this result, which is the standard one with a risk-neutral agent.

For the less wealthy tenants, however, $\theta_1>\underline{\theta}$. Since $\int_{\theta_1}^{\bar{\theta}}\theta \mathrm{d}F(\theta)<\int_{\underline{\theta}}^{\bar{\theta}}\theta \mathrm{d}F(\theta)=1$, for these tenants $\alpha>1$, that is, they receive a contract with cropshare greater than 1.[8]

Note that Eq. (4) implies $\int_{\theta_1}^{\bar{\theta}}\alpha\theta \mathrm{d}F(\theta)=1$, and substituting this into the incentive constraint (ii), we see that in response to such a contract the tenant's effort choice is such that $Q'(L)=1$, that is, efficiency in labor choice is achieved. Observe that the efficiency of labor choice occurs irrespective of whether the limited liability constraint binds or not.

We can determine the wealth level that distinguishes the 'wealthy' and 'less wealthy' tenants. Using the binding participation constraint (i), Eq. (4), and the fact that $\theta_1=\underline{\theta}$ for the wealthy tenants, we get that, for the wealthy tenants, $C=-K+Q(L^{**})-L^{**}$. Then, from the definition of θ_1, it follows that

[7] For more on this argument, see below.

[8] Singh (1989) fails to get this result because he errs in the calculation of conditional expectations.

$W \geq -\alpha\underline{\theta}Q(L^{**}) + C = Q(L^{**}) - L^{**} - K - \underline{\theta}Q(L^{**})$. We denote this distinguishing wealth level as $\overline{W} \equiv Q(L^{**}) - L^{**} - K - \underline{\theta}Q(L^{**})$.

Let us summarize the above findings with the proposition below.

Proposition 1. *In the presence of ex post limited liability, when the cropshare of the tenant is unconstrained, (a) tenants with wealth level greater than or equal to* $\overline{W}$ *receive a fixed-rental contract, (b) tenants with wealth level less than* $\overline{W}$ *receive a contract with cropshare greater than 1, and (c) the efficiency in labor choice is achieved irrespective of whether the limited liability constraint binds or not.*

For the wealthy tenants in this model, the limited liability constraint does not bind. So, in this model with risk-neutral landlords and tenants, it follows from the standard principal-agent analysis that for these tenants the cropshare will be 1, providing first-best incentives.

For the less wealthy tenants, however, the limited liability constraint binds. When there is a crop failure, the landlord appropriates from the tenant his wealth and the entire amount of crop produced (the landlord's share and also the tenant's share). Thus, since in bad states the tenant gets no marginal benefit, his incentives are reduced. A cropshare greater than 1 balances this out by paying more in good states.

Another way of explaining the intuition for the results is useful in understanding the role of limited liability.[9] Because the wealth of some tenants is limited, it may not be possible to use the fixed rent instrument alone in pushing such tenants to their reservation utility levels while attaining the first-best outcome. If α is unconstrained, and, in particular, can exceed 1 (the case under consideration in this subsection), this problem posed by the lack of wealth of some tenants can be circumvented. It is possible for the landlord to push these tenants to their reservation utility levels by raising the fixed rent, C, and, at the same time, raising α above 1. Since the tenant's cropshare can be raised above 1, increasing the fixed rent does *not* increase the risk of default on this rental payment or reduce the tenant's incentive to apply effort.

However, there are several compelling reasons as to why in reality the tenant's cropshare is restricted to be not greater than 1. Shetty suggests that the landlord promising a marginal share greater than 1 cannot be counted on to deliver. Certainly this is a possibility: keeping a share of output is different from actually receiving more than one has produced (at the margin). The latter perhaps relies more on the landlord's goodwill to deliver. The problem may be even more severe, however. In sharecropping (when α is between 0 and 1) both landlord and tenant benefit from more output, so neither has an incentive to destroy output. But if the tenant's cropshare is greater than 1, then the landlord is worse off from

[9] We are grateful to a referee for this suggestion.

having more output, ex post. So he has an incentive to destroy output. For example, he could send a gang of thugs at night to steal the crop when it is ripe enough to harvest. This might be another reason why contracts with a tenant's share greater than 1 are not observed.[10]

An alternative explanation for the restriction on the share is another kind of tenant moral hazard. A marginal share greater than 1 increases incentives for manipulating output. The sharecropper (with share less than 1) has an incentive to hide output, akin to the motives for tax evasion. The landlord will certainly try to detect this. The tenant with a pure rent contract has no such incentive, since he makes only a fixed payment. If α exceeds 1, however, the tenant has an incentive to overstate output in some cases. One can think of monitoring this as being more difficult (e.g. a neighbor's output is borrowed to stake a claim to more payment from the landlord, or last year's stocks are used, or chaff is included with the grain). These considerations might lead to the tenant's share being restricted to not exceed 1.

In Section 2.3, we characterize the contractual structure with this additional constraint, that tenant's cropshare is restricted to be weakly between 0 and 1.

2.3. Contracts with the cropshare constraint

Since it is easy to argue that $\alpha > 0$, we rewrite the contracting problem incorporating the cropshare constraint as follows:

$$\begin{aligned}
&\underset{\{\alpha, C, L\}}{\text{Maximize}} \quad \int_{\theta_1}^{\bar{\theta}} [(1-\alpha)\theta Q(L) + C] \mathrm{d}F(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L) + W] \mathrm{d}F(\theta) \\
&\text{subject to:} \quad \text{(i)} \int_{\theta_1}^{\bar{\theta}} [\alpha\theta Q(L) - C] \mathrm{d}F(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W) \mathrm{d}F(\theta) - L \geq K, \\
&\qquad \text{(ii)} \int_{\theta_1}^{\bar{\theta}} [\alpha\theta Q'(L)] \mathrm{d}F(\theta) - 1 = 0, \\
&\qquad \text{(iii)} \ \alpha \leq 1.
\end{aligned}$$

Here, (iii) is the cropshare constraint.

The incentive constraint can be solved for $L(\alpha)$ and substituted in the problem as earlier. But, since α cannot exceed 1, the participation constraint may not hold with equality. Let λ and μ be the Lagrange multipliers corresponding to the participation constraint (i) and cropshare constraint (iii), respectively. Then the

[10] This could well happen in places like Bihar, a state in India. We are grateful to Kaushik Basu for this idea.

first-order conditions (with the associated complementary slackness conditions) for the problem are given by:

$$\alpha: \quad \int_{\theta_1}^{\bar{\theta}} [-\theta Q(L) + (1-\alpha)\theta Q'(L) L_\alpha] \mathrm{d}F(\theta)$$
$$+ \int_{\underline{\theta}}^{\theta_1} [\theta Q'(L) L_\alpha] \mathrm{d}F(\theta)$$
$$+ \lambda \left[\int_{\theta_1}^{\bar{\theta}} [\theta Q(L) + \alpha\theta Q'(L) L_\alpha] \mathrm{d}F(\theta) - L_\alpha \right] - \mu = 0, \tag{5}$$

$$C: \quad \int_{\theta_1}^{\bar{\theta}} \mathrm{d}F(\theta) - \lambda \int_{\theta_1}^{\bar{\theta}} \mathrm{d}F(\theta) = 0. \tag{6}$$

Now, we consider three exhaustive and mutually exclusive cases depending on whether the two constraints bind or not.

Case I (Participation constraint does not bind). This case includes two sub-cases: (a) $\alpha < 1$, and (b) $\alpha = 1$.
Since the participation constraint does not bind, $\lambda = 0$. Hence, Eq. (6) implies $\int_{\theta_1}^{\bar{\theta}} \mathrm{d}F(\theta) = 0$, that is, $\theta_1 = \bar{\theta}$.
But if $\theta_1 = \bar{\theta}$, it follows from the expression of the tenant's expected earning that the tenant's labor choice, $L^* = 0$. Then, $Q(L^*) = 0$. It follows from the definition of θ_1 that $\alpha\theta_1 Q(L^*) + W = C$, implying that $C = W$, that is, the fixed component of the rent is equal to the tenant's wealth.
Since the participation constraint does not bind and $\theta_1 = \bar{\theta}$, we get from the participation constraint that $0 + \int_{\underline{\theta}}^{\bar{\theta}} (-W) \mathrm{d}F(\theta) - 0 > K$, that is, $W < -K$. So this case can arise when the tenant is so poor that his wealth level is less than $(-K)$, that is, he is indebted.
But since $\theta_1 = \bar{\theta}$ and $L^* = 0$, the landlord's expected earning in this case is W, which is less than $-K$. If, instead, the landlord does not enter into any contract with such a poor tenant (and presumably leaves the plot fallow), his expected earning is zero.
Conclusion: the landlord does not lease out his plot to tenants with wealth level less than $-K$, and hence Case I will not arise in a solution to the contracting problem.

Case II (Participation constraint binds, and $\alpha < 1$). Since $\alpha < 1$, it follows that $\mu = 0$. Also, Eq. (6) implies $\lambda = 1$.
Then it follows from Eq. (5), after some substitutions and rearrangement, that $Q'(L) L_\alpha \left[1 - \alpha \int_{\theta_1}^{\bar{\theta}} \theta \mathrm{d}F(\theta)\right] = 0$, implying that $\alpha = \dfrac{1}{\int_{\theta_1}^{\bar{\theta}} \theta \mathrm{d}F(\theta)} \geq 1$, a contradiction (since $\alpha < 1$ for the case under consideration).

Conclusion: Case II does not arise in a solution to the contracting problem. Also, note that Case I and Case II demonstrate decisively that a share contract does *not* arise when there is a limited liability constraint in the ex post sense.

Case III (Participation constraint binds, and $\alpha = 1$). Again, Eq. (6) implies that $\lambda = 1$. Then, after some substitutions and rearrangements, we get the following from Eq. (5): $Q'(L)L_{\alpha}\left[\int_{\underline{\theta}}^{\theta_1}\theta \mathrm{d}F(\theta)\right] = \mu$.
Since $\mu \geq 0$, there is no contradiction.
Conclusion: We get that this is the only case that solves the contracting problem. Thus, all the tenants receive a fixed-rent contract when there is ex post limited liability and tenant's cropshare is constrained not to exceed 1.

Next, we demonstrate how this fixed rent changes, and its associated effects on the tenant's labor choice and landlord's expected earning, by considering two subcases under Case III: (a) limited liability constraint (LLC) does not bind, and (b) limited liability constraint binds.

Case III (a) (Participation constraint binds, $\alpha = 1$, and LLC does not bind). Since LLC does not bind, $\theta_1 = \underline{\theta}$. Then the incentive constraint implies $Q'(L) = 1$, that is $L = L^{**}$, where L^{**} denotes the efficient labor choice.
Since the participation constraint binds, we get $\int_{\underline{\theta}}^{\overline{\theta}}[\theta Q(L) - C]\mathrm{d}F(\theta) + 0 - L = K$, implying that $C = Q(L^{**}) - L^{**} - K$.
But, from the definition of θ_1, LLC does not bind means $\alpha\underline{\theta}Q(L) + W \geq C$, which implies that $W \geq -\underline{\theta}Q(L^{**}) + C = Q(L^{**}) - L^{**} - K - \underline{\theta}Q(L^{**}) \equiv \overline{W}$.
Conclusion: thus, we get that when the tenant is rich enough so that his wealth level is greater than or equal to $\overline{W}$, he receives a fixed-rent contract with the amount of fixed-rent (C) given as above, and puts the efficient effort.[11,12]

But what happens to the less wealthy tenants, that is, to the tenants with wealth level less than $\overline{W}$? Case III (b) addresses this question.

Case III (b) (Participation constraint binds, $\alpha = 1$, and LLC binds). Since LLC binds, from the definition of θ_1 it follows that:

$$\theta_1 Q(L) + W - C = 0. \tag{7}$$

[11] Note that this is the same wealth level $\overline{W}$ that distinguishes the wealthy and less wealthy tenants in Proposition 1.

[12] This is exactly Proposition 1 (p. 11) in Shetty (1988). The wealth level $\overline{W}$ is the counterpart of $A^0(\overline{P_L})$ and the fixed-rent $[Q(L^{**}) - L^{**} - K]$ is the counterpart of $\overline{R}(0, A^0; \overline{P_L})$ in that proposition. However, as noted earlier, Shetty does not characterize the equilibrium contract in the other cases we analyze.

The incentive constraint implies:

$$Q'(L)\int_{\theta_1}^{\bar{\theta}}\theta\,\mathrm{d}F(\theta)-1=0. \tag{8}$$

The participation constraint gives:

$$\int_{\theta_1}^{\bar{\theta}}[\theta Q(L)-C]\,\mathrm{d}F(\theta)+\int_{\underline{\theta}}^{\theta_1}(-W)\,\mathrm{d}F(\theta)-L-K=0. \tag{9}$$

Considering the wealth level, W, as a parameter, these three first-order conditions determine the fixed-rent (C), effort level (L) and cut-off level of uncertainty (θ_1)—all as functions of wealth level W. It is shown in the Appendix that $\frac{\partial\theta_1}{\partial W}<0$, $\frac{\partial L}{\partial W}>0$, and $\frac{\partial C}{\partial W}<0$. It is quite intuitive that, as the tenant's wealth level decreases, he becomes more prone to default, that is, the cut-off level of uncertainty (θ_1) increases. Since the tenant receives positive value to his marginal effort only when $\theta\in[\theta_1,\ \bar{\theta}]$, as θ_1 increases, his benefit from the marginal effort also decreases, and hence he puts less effort. Also, note that for these less wealthy tenants, the fixed rent (C) decreases as wealth increases. Since under adverse realizations of output the landlord appropriates the tenant's wealth, the wealthier the tenant, the more the landlord can afford to give him a greater concession in terms of a reduced fixed rent.

The above findings can be nicely tied up to one version of the tenancy ladder hypothesis. Since the participation constraint binds, substituting for the expression of C from Eq. (9) into the expression for landlord's expected return, we get that the landlord's expected return is $[Q(L(W))-L(W)-K]$. Hence, it follows from the above that as W decreases, L decreases, and the landlord's expected return decreases. It is in this sense that the landlord prefers to have wealthy tenants, suggesting a kind of tenancy ladder where wealthier tenants are preferred.

But how long will this continue, that is, what is the wealth level below which the landlord will stop leasing out his plot? (Recall from Case I that the landlord does not lease out his plot to a tenant with wealth level less than $-K$.) This wealth level is such that landlord's expected return is just zero: $Q(L(W))-L(W)-K=0$. Therefore, let $\underline{W}$ be the wealth level such that $Q(L(\underline{W}))-L(\underline{W})-K=0$.

We may therefore summarize the findings of this subsection with the proposition below.

Proposition 2 ((Tenancy Ladder)). *In the presence of ex post limited liability, when the cropshare of the tenant is constrained to be weakly between 0 and 1, we have the results given below.*

(a) Tenants with wealth level below $\underline{W}$ are excluded from receiving any tenancy contract.
(b) All the tenants with wealth level above $\underline{W}$ receive a fixed-rental contract.
(c) For the tenants with wealth levels greater than $\underline{W}$ but less than $\underline{W}$, the effort levels increase with wealth, and the landlord prefers the wealthier tenants as his expected earning increases with the tenant's wealth level.
(d) Tenants with wealth level greater than or equal to also receive a fixed-rent contract with the amount of fixed-rent given by: $Q(L^{**}) - L^{**} - K$. These tenants provide the efficient effort level, and the landlord's expected earning is the highest from entering into tenancy contracts with these rich tenants.

The intuition for the fixed-rent contract for less wealthy tenants (tenants with wealth levels greater that $\underline{W}$ but less than $\overline{W}$) can be understood as follows. Recall from our discussion of Proposition 1 that because the wealth of these tenants is limited, the first-best outcome is attained only by raising their cropshares above 1. Now that α is constrained not to exceed 1, it is clear that the first-best outcome cannot be achieved, that is, these tenants put too little effort when there is a cropshare constraint and the limited liability constraint binds. Since a fixed-rent contract induces the highest effort level among all possible tenurial contracts, the landlord offers the fixed-rent contracts to these less wealthy tenants in order to get as close as possible to the first-best.

So far, we have seen that there is no room for share contracts when there is limited liability in the ex post sense. In Section 3, we show, through an example, how sharecropping can arise when there is ex ante limited liability.

3. Ex ante limited liability and sharecropping: an example

Laffont and Matoussi (1995) have posed the limited liability constraint in a very different way. They postulate an ex ante financial constraint that says that the tenant must pay the fixed component of the rent. The logic is that the landlord wants to obtain as much of a cash advance from the tenant as possible in the form of the fixed rent. Thus, this amount of fixed rent is bounded above by the tenant's wealth.[13] That is, in our notation, $C \leq W$. To illustrate how this ex ante limited liability constraint explains the emergence of sharecropping and the associated tenancy ladder, we consider the following example.

[13] Laffont and Matoussi (1995) consider the upper bound to be the tenant's working capital. We equate this working capital to the tenant's wealth in order to provide a clean exposition of the tenancy ladder.

Both the landlord and the tenant are risk-neutral. The production function is $Q = \theta L$, with $E(\theta) = 1$. The tenant's disutility from work is $1/2bL^2$, so his expected utility is $\alpha L - C - 1/2bL^2$, while that of the landlord is $(1-\alpha)L + C$. So the contracting problem is:

$$\begin{aligned}
&\underset{\{\alpha,C,L\}}{\text{Maximize}} && (1-\alpha)L + C \\
&\text{subject to,} && \text{(i) } L = \underset{\{L'\}}{\text{Max}} \quad \alpha L' - C - 1/2bL'^2 : \text{Incentive constraint,} \\
& && \text{(ii) } \alpha L - C - 1/2bL^2 \geq K : \text{Participation constraint,} \\
& && \text{(iii) } C \leq W : \text{Limited liability constraint.}
\end{aligned}$$

The incentive constraint gives the solution: $L = \alpha/b$. Substituting this into the objective function and the participation constraint, the contracting problem becomes:

$$\begin{aligned}
&\underset{\{\alpha,C\}}{\text{Maximize}} && \frac{\alpha(1-\alpha)}{b} + C \\
&\text{subject to,} && \text{(ii) } \frac{\alpha^2}{2b} - C \geq K : \text{Participation constraint,} \\
& && \text{(iii) } C \leq W : \text{Limited liability constraint.}
\end{aligned}$$

This problem can be analyzed straightforwardly (details are omitted, but available from the authors). First, we can show that at least one of the constraints must bind. Next, if only participation constraint binds, it must be the case that $W > C$, that is, $W > 1/2\text{b} - K$. If only the limited liability constraint binds, then $W < 1/8b - K$. For intermediate values of wealth, both constraints bind. We summarize the full results in the following table:

Tenant's wealth (W)	Tenant's cropshare (α)	Fixed component of rent (C)	Landlord's expected utility (U_L)
$W < \frac{1}{8b} - K$	$\frac{1}{2}$	W	$U_L < \frac{3}{8b} - K$
$W = \frac{1}{8b} - K$	$\frac{1}{2}$	W	$\frac{3}{8b} - K$
$\frac{1}{8b} - K < W < \frac{1}{2b} - K$	$\frac{1}{2} < \sqrt{2b(K+W)} < 1$	W	$\frac{3}{8b} - K < U_L < \frac{1}{2b} - K$
$W \geq \frac{1}{2b} - K$	1	$\frac{1}{2b} - K$	$\frac{1}{2b} - K$

The above example can be generalized to the following proposition, which presents a variant of Proposition 1 of Laffont and Matoussi, but with a novel interpretation of possible equilibria in terms of a tenancy ladder:

Proposition 3. *When there is an ex ante limited liability constraint, (a) wealthier tenants (for whom the limited liability constraint does not bind) receive a fixed-rent contract and put forth the efficient effort level, (b) less wealthy tenants (for whom the limited liability constraint binds) receive a share contract, and make an inefficient effort choice, and (c) as the tenant's wealth increases, his cropshare (and effort level) increases and the landlord's expected utility also increases.*

Thus, sharecropping arises in the presence of an ex ante limited liability constraint. Also, note that the landlords prefer to have wealthy tenants since their expected utility increases with the tenant's wealth. Again this might be interpreted as a tenancy ladder. The intuition for this outcome, and its difference from the ex post constraint case, is as follows.[14] In the ex ante constraint case, landlords have less scope to extract surplus from tenants by using the fixed rent component of the contract. This pushes them to use share contracts, where otherwise they would set the tenant's share at 1 (or higher if that is possible) and use the fixed component for collecting income.

4. Conclusion

In this paper, we have provided a complete characterization of contracts in Shetty's model of a tenancy ladder with ex post limited liability. We have shown that sharecropping can never arise in that model. However, as we demonstrated in Section 3, sharecropping does come about with ex ante limited liability. In either case, a version of a tenancy ladder can be characterized, as we have shown above.

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[14] We are indebted to one of the referees for this intuition.

Appendix A. Comparative static results under Case III (b)

We assume that the sufficient condition for maximization is satisfied, so that the Jacobian determinant of the first-order conditions (7), (8) and (9), denoted by Δ, has a negative sign, that is, $\Delta < 0$. Differentiating the first-order conditions totally, we get:

$$\begin{bmatrix} \theta_1 Q'(L) & -1 & Q(L) \\ Q''(L)\int_{\theta_1}^{\bar{\theta}} \theta \mathrm{d}F(\theta) & 0 & -Q'(L)\theta_1 f(\theta_1) \\ 0 & -\int_{\theta_1}^{\bar{\theta}} \mathrm{d}F(\theta) & 0 \end{bmatrix} \begin{bmatrix} \mathrm{d}L \\ \mathrm{d}C \\ \mathrm{d}\theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} -\mathrm{d}W \\ 0 \\ \left[\int_{\underline{\theta}}^{\theta_1} \mathrm{d}F(\theta)\right] \mathrm{d}W \end{bmatrix}.$$

Note that the partial derivative of Eq. (9) with respect to L is $Q'(L) \int_{\theta_1}^{\bar{\theta}} \theta \mathrm{d}F(\theta) - 1$, which is equal to zero by Eq. (8), and the partial derivative of Eq. (9) with respect to θ_1 is $-f(\theta_1)[\theta_1 Q(L) - C + W]$, which is again equal to zero by Eq. (7).

Now we can derive the following comparative static results:

$$\frac{\partial \theta_1}{\partial W} = \frac{-Q''(L)\int_{\theta_1}^{\bar{\theta}} \theta \mathrm{d}F(\theta)}{\Delta} < 0, \text{since } Q''(L) < 0, \text{and } \Delta < 0.$$

$$\frac{\partial L}{\partial W} = \frac{-Q'(L)\theta_1 f(\theta_1)}{\Delta} > 0, \text{since } Q'(L) > 0, \text{and } \Delta < 0.$$

$$\frac{\partial C}{\partial W} = \frac{-F(\theta_1)}{1 - F(\theta_1)} < 0.$$

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