Development From the Viewpoint of Nonconvergence: Expectations and Development

1. Introduction

- The first form of inertial self-reinforcement that we consider is driven by *expectations*.
- Expectation-driven inertial self-reinforcement is based on the existence of a particular type of *externality* known as *complementarity*:

  It is possible to order the actions spaces of all agents in a way so that a movement up the "action order" for some agents induce other agents to move up *their* action orders.
- Complementarities frequently manifest themselves in *coordination failure*:

  Situations in which the interactions across agents may lock them into inefficient action configurations, while at the same time there are other action configurations that are also self-justifying, but do better for all concerned.
  - That is, there are multiple equilibria driven by different degrees of expectations.

- Rosenstein-Rodan (1943) and Hirschman (1958) argued that economic development could be thought of as a massive coordination failure.
  - Several investments do not occur simply because other complementary investments are not made.
  - And these latter investments are not forthcoming simply because the former are missing.
- Thus, one might conceive of two equilibria *under the very same fundamental conditions*.
  - one in which active investment is taking place, with each industry's efforts motivated and justified by the expansion of other industries, and
  - another equilibrium involving persistent stagnation – the inactivity of one industry seeps into another.
- This serves as potential explanation of why similar economies may behave very differently,
  - depending on the nature of *expectations* held by agents in different sectors concerning the actions of each other.

- For this sort of situation to arise, there must be interactive effects or *externalities* across industries.
- These externalities can take two forms:

1. Two industries could be *linked*.
   - Demand Link: Expansion of one industry may provoke greater demand for the product of the other.
   - Supply Link: Expansion of one industry may facilitate the production of the second industry.
   - These links receive particular emphasis in the work of Hirschman (1958).
     - The old debate on “balanced” versus “unbalanced” growth, and related concept of leading sectors.
2. The externalities might take a more *indirect* form.
   - Industries generate income, and income generates demand for products of other industries.

- Since no individual firm internalizes these effects, a *coordination failure*, reinforced by pessimistic expectations, may generate a low level of economic activity.
  - On the other hand, an enhanced level of economic activity generates greater national income, and
    - the generation of national income may create additional demand to justify that activity.

- **Murphy, Shleifer and Vishny (1989)** presents a simple, coherent formalization of this second form of “indirect externalities” in a general equilibrium setting.
  - The authors go through a succession of models that attempt to capture ‘indirect externalities’ across firms and industries.

2. Murphy, Shleifer and Vishny (1989)

- This paper is noteworthy for its systematic theoretical exploration of the Rosenstein-Rodan hypothesis:

  Simultaneous industrialization of many sectors of the economy can be profitable for all of them even when no sector can break even industrializing alone.

- Analyze this idea in the context of an imperfectly competitive economy with aggregate demand spillovers.
  - Associate the "big push" with multiple equilibria and interpret big push into industrialization as a move from a bad to a good equilibrium,
    - from inefficient cottage production to Pareto superior factory production.

- Address the main question:

  What does it take for such multiple equilibria to exist?

2.1 A Simple Aggregate Demand Spillovers Model with a Unique Equilibrium

- Discuss a simple model in which profit spillovers across sectors are present,
  - but they are still not sufficient to generate the conditions for multiple equilibria.
- Start with this model to illustrate the fact that
  - the conditions for individually unprofitable investments to raise the profitability of investment in other sectors are more stringent than those loosely expressed in much of the big push literature of the 1940s and 1950s.

- A one-period economy with a representative consumer.
  - Utility function: Defined over a unit interval of goods indexed by q:

$$\int_0^1 \ln\ x(q)dq.$$

  - Preference structure implies all goods have the same expenditure shares.

$\Rightarrow$ When income is y, consumer can be thought of as spending y on every good $x(q)$.

  - The consumer is endowed with L units of labour, which he supplies inelastically.
    - He owns all the profits of this economy.
  - Income (with wage as numeraire):

$$y = \Pi + L, \tag{1}$$

Π is aggregate profit.

- Each good is produced in its own sector, and each sector consists of two types of firms:

1. Cottage Production:

   Each sector has a competitive fringe of firms that convert one unit of labour input into one unit of output with a constant returns to scale technology.

2. Mass Production:

   Each sector has a unique firm with access to an increasing returns technology.

   ◦ Fixed cost: Mass production requires a fixed cost of ‘burning up’ F units of labour.

   ◦ After incurring the fixed cost, the mass production technology converts each additional unit of labour to produce $\alpha > 1$ units of output.

– “Industrialization” of a sector is synonymous with the potential monopolist entering production in that sector.

– Firms in the competitive fringe incurs no fixed cost.

$\Rightarrow$ The competitive fringe of each sector has a perfectly elastic supply at price = 1 (since MC = wage = 1).

- The monopolist in each sector decides whether to industrialize or to abstain from production.
  - Monopolist maximizes profit taking the demand curve as given.
  - He industrializes only if he can earn a profit at the price he charges.
    - That price is equal to 1. (Why?)
  - Suppose aggregate income is y, and the monopolist industrializes.
    - Revenue: y
    - Cost: $\frac{y}{\alpha} + F$

  $\Rightarrow$ Profit:

$$\pi = \left(\frac{\alpha - 1}{\alpha}\right) y - F \equiv ay - F, \tag{2}$$

  - $\cdot$ $a \equiv 1 - \frac{1}{\alpha}$: difference between price and marginal cost, mark up.
  - When a fraction n of the sectors in the economy industrialize, aggregate profits are

$$\Pi(n) = n(ay - F). \tag{3}$$

– Substituting (3) into (1) yields aggregate income as a function of the fraction of sectors industrializing (n):

$$y(n) = \frac{L - nF}{1 - na}. \tag{4}$$

◦ $L - nF$: amount of labour used in the economy for actual production of output, after investment outlays.

◦ $\dfrac{1}{1 - na} > 1$ is the *multiplier* showing that an increase in effective labour raises income by more than one for one since expansion of low-cost sectors also raises profits.

· To see this more explicitly, note that

$$\frac{dy(n)}{dn} = \frac{\pi(n)}{1 - na}, \tag{5}$$

where $\pi(n)$ is the profit of the last firm to invest.

· When the last firm earns this profit, it distributes it to shareholders, who in turn spends it on all goods,

$\rightarrow$ raise profits in all industrial firms in the economy.

- · **Profit spillover:** The effect of the last firm's profit is therefore enhanced by the increases in profits of all industrial firms resulting from increased spending.
- ○ The multiplier is increasing in the number of firms that benefit from the spillover of the marginal firm.
  - · The more firms invest, the greater is the cumulative increase in profits and hence income resulting from a positive net present value investment by the last firm.

- Despite the fact that an individual monopolist ignores the profit spillover from its investment, there is a unique Nash equilibrium in which
  – either all sectors industrialize,
  – or no sector industrializes.
- Substituting (4) into (2) gives

$$\pi(n) = \frac{aL - F}{1 - na}.$$

- Since $1 - na > 0$, the sign of $\pi(n)$ is the same as the sign of $aL - F$.

$\Rightarrow$ The sign of $\pi(n)$ is independent of n.

- If $aL - F > 0$, then it is always worthwhile for a single monopolist to industrialize irrespective of the fraction of industrialized sector in the economy.

$\Rightarrow$ The only equilibrium is when all sectors industrialize.

- If $aL - F < 0$, then the only equilibrium is where no sector industrializes.
- In the no-industrialization equilibrium $\pi(n) < 0$, that is, a single firm loses money from industrializing.
  - Then there cannot be an equilibrium in which any firm invests.
    - Suppose that a single firm decides to invest.
    - Since it loses money, it only reduces aggregate income, making the profit from industrialization in any other sector even lower.
      - This makes the existence of the second equilibrium impossible.
    - As is clear from (5),
      - a firm's spillover is positive if and only if its own profits are positive.
      - The multiplier changes only the magnitude of the effect of a firm's investment on income, and *not* the sign.

2.2 A Model with a Factory Wage Premium

- When profits are the only channel of spillovers, the industrialized equilibrium cannot coexist with the unindustrialized one.
- In contrast, multiple equilibria arise naturally if an industrializing firm raises the size of other firms' markets even when it itself loses money.
  - This occurs when firms raise the profit of other industrial firms through channels other than their own profits.
- In this section we consider a model where industrialization raises the demand for manufactures because
  - workers are paid higher wages to entice them to work in industrial plants.
- Hence, even a firm losing money can benefit firms in other sectors because
  - it raises labour income and hence demand for their products.

- The model in this section comes close in its spirit to Rosenstein-Rodan (1943).
- To bring farm labourers to work in a factory, a firm has to pay a wage premium.
  - Unless the firm can generate enough sales to people other than its own workers, it will not be able to afford to pay higher wages.
- If this firm is the only one to start production,
  - its sales might be too low for it to break even.
- In contrast, if firms producing different products all invest and expand production together,
  - they can all sell their output to each other's workers, and so
  - can afford to pay a wage premium and still break even.
- The model in this section is constructed along these lines.

- Same set-up as in the last section; add the following assumption:
  - Work in a factory or in the industrialized sector is more arduous, and
  - higher wages – v units – are paid to compensate workers for disutility of such work.
- Competitive factory wage (w):

  Each monopolist must pay a wage that makes a worker indifferent between factory and cottage production employment:

$$w = 1 + v. \tag{6}$$

- Suppose aggregate income is y, and the monopolist industrializes.
  - Revenue: y
  - Cost: $\left(\frac{y}{\alpha} + F\right)(1 + v)$
  - Profit:

$$\pi = y\left(1 - \frac{1+v}{\alpha}\right) - F(1+v). \tag{7}$$

- As is clear from (7), for this model to be at all interesting, we must have

$$\alpha - 1 > v. \tag{8}$$

  - The productivity gain from using the IRS technology must exceed the compensating differential that must be paid to a worker.

- This model can have two equilibria:
  - a no-industrialization equilibrium, and
  - an industrialization equilibrium.

- **The no-industrialization equilibrium (cottage production):**
  - No firm incurs the fixed cost for fear of not being able to break even.
    - Population stays in cottage production.
  - Income = L, the wage bill of the cottage labour (no profits are earned).
  - For this to be an equilibrium, it must be the case that in no sector would a monopolist want to set up a factory if he has to pay the required factory wage.
    - That is, for no industrialization to be an equilibrium we must have

$$L\left(1-\frac{1+v}{\alpha}\right)-F\left(1+v\right)<0. \tag{9}$$

- **The industrialization equilibrium (factory production):**
  - All sectors industrialize.
  - By symmetry, the quantity of output produced in each sector is $\alpha(L-F)$.

  $\Rightarrow$ Revenue: $\alpha(L-F)$.

  ∘ Cost: $(L-F)(1+v)+F(1+v)=L(1+v)$.

  ∘ Profit: $\pi=\alpha(L-F)-L(1+v)$.
  - For this to be an equilibrium, profits must be positive:

$$\alpha(L-F)-L(1+v)>0. \tag{10}$$

  - When (10) holds, all firms *expect* a high level of income and sales resulting from simultaneous industrialization of many sectors,

  ∘ happy to incur the fixed cost, $F(1+v)$, to set up a factory.
  - This makes the expectation of industrialization *self-fulfilling*.

- **Multiple Equilibria and the "Big Push":**

  (9) and (10) show that there always exist some values of F for which *both* the equilibria exist:

$$\left(\frac{L}{1+v}\right)\left(1-\frac{1+v}{\alpha}\right) < F < L\left(1-\frac{1+v}{\alpha}\right). \qquad (*)$$

- Suppose the economy is characterized by a set of parameters such that (*) holds and the economy is currently in a situation where no industrialization has occurred.
  - This can be described as a 'poverty trap' or a 'low-level equilibrium trap'.
    - Thanks to (9), each sector finds it not worthwhile industrializing.
    - So no one does, and the state of poverty perpetuates.
  - But we know, thanks to (10), that if all sectors had industrialized, then the modern firm in each sector would be making profits.

- The reason for the multiplicity of equilibrium is:
  - The link between a firm's profit and its contribution to demand for products of other sectors is now broken.
    - Since a firm that sets up a factory pays a wage premium, it increases the size of the market for producers of other manufacturers
      - even if its investment loses money.
- The industrialized equilibrium is Pareto superior:
  - Since prices do not change, workers are equally well off as wage earners in the industrialized equilibrium,
    - but they also earn some profits.

$\Rightarrow$ They have higher income at the same prices and hence must be better off.

- The economy needs a “big push”, whereby it moves
  - from the unindustrialized equilibrium to the industrialized equilibrium
  - when all its sectors *coordinate* investments.
- Here is where the issue of *coordination* makes an appearance.
  - Each monopolist would invest if he were to *believe* that others would invest as well.
    - In the absence of such optimistic beliefs he would not invest.
- Whether or not such a coordinated equilibrium would arise depends on
  - the *expectations* that each entrepreneur holds about the others.
- To the extent that the formation of expectations is driven by past *history*, it may well be that
  - a region that is historically stagnant, continues to be so, whereas
  - another region that has been historically active, continues to flourish.
    - At the same time, there may be nothing that is intrinsically different between the two regions.

3. Ciccone and Matsuyama (1996)

- The Murphy-Shleifer-Vishny formulation of multiple equilibria and low-level equilibrium trap is echoed in Ciccone and Matsuyama (1996):
  - Use a dynamic monopolistic competition model to show that
    - an economy that inherits a small range of specialized inputs can be trapped into a lower stage of development.
  - The limited availability of specialized inputs forces the final goods producers to use a labor intensive technology,
    - which in turn implies a small inducement to introduce new intermediate inputs.

- One critical aspect of economic development is that productivity growth is generally associated with
  - an ever greater indirectness in the production process, and
  - an ever increasing degree of specialization.
- In developed economies, consumer goods industries make superior use of
  - highly specialized capital goods, particularly in machinery, and
  - enjoy access to a wide variety of producer services, such as
    - equipment repair and maintenance,
    - transportation and communication services,
    - engineering and legal supports,
    - accounting, advertising, and financial services.
- Many underdeveloped economies, on the other hand, are characterized by
  - relatively simple production methods, and
  - a limited availability of specialized inputs.

- Attempts to transplant advanced technologies into the underdeveloped economies often meet disaster.
  - The vast network of auxiliary industries, taken for granted in industrialized economies, is not available.
- This paper emphasizes that there is a fundamental circularity between
  - the choice of technologies by consumer goods producers and
  - the variety of intermediate inputs available.
- With a wide range of specialized inputs and producer services,
  - firms in the consumer goods sector adopt more indirect and roundabout ways of production,
    - achieve high productivity.
  - The growing demand by the consumer goods industry in turn
    - creates a large market for intermediate goods, and
    - brings into being a host of specialized auxiliary industries to service its need.

- If the economy produces only a limited range of intermediate inputs and producer services,
  - the consumer goods industry is forced to use more primitive modes of production.

  $\Rightarrow$ A limited incentive to start up firms and introduce new goods in the intermediate inputs sector.
- This paper shows that this circularity is strong enough that
  - an economy that inherits a narrow range of intermediate inputs is trapped into a lower stage of economic development.

3.1 The Basic Model

- Time is continuous and extends from 0 to ∞.
- **Households:** Over an infinite horizon,
  - supply L units of labor inelastically;
  - consume the homogeneous final good (taken as the numeraire).
- At any moment t, households choose consumption (C_t) so as to

$$\text{Maximize } U_t \;=\; \int_t^\infty e^{-\rho(\tau-t)} \log\, C_\tau \, d\tau,$$

$$\text{s.t. } \int_t^\infty e^{-\int_t^\tau r_s\, ds}\, C_\tau \, d\tau \;\le\; L \int_t^\infty e^{-\int_t^\tau r_s\, ds}\, w_\tau \, d\tau + W_t.$$

  - $\rho > 0$: subjective discount rate,
  - r_t: rental rate,
  - w_t: the wage rate,
  - W_t: value of asset holding, consisting of ownership shares of profit making firms.

- The solution to this maximization problem is characterized by the Euler condition:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho, \text{ and} \tag{1}$$

  – the binding budget constraint:

$$\int_t^{\infty} e^{-\int_t^{\tau} r_s \, ds} \, (C_\tau - w_\tau L) \, d\tau = W_t. \tag{2}$$

- **Production:** Two sectors:
  – A final consumer good sector;
  – Intermediate goods sector.
- The final consumer good is produced by competitive firms.
  – Share the identical constant returns to scale production function

$$C_t = F(X_t, \ H_t).$$

  ◦ H_t: labour input;
  ◦ X_t: the composite of differentiated intermediate inputs or ‘producer services’.

- The composite of differentiated intermediate inputs has a form of symmetric CES:

$$X_t = \left[\int_0^{n_t} \left[x_t \left(i \right) \right]^{1-\frac{1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \ \sigma > 1. \tag{3}$$

  - $x_t \left(i \right)$: the amount of variety i used.
  - σ: elasticity of substitution between every pair of intermediate inputs.
  - At any moment only a subset of differentiated products, $[0, n_t]$, is available.
- This specification of product differentiation has one significant property:

  Increasing returns due to specialization in production (Ethier, 1982; Romer, 1987):
  - Total factor productivity increases with the range of differentiated inputs available.
    - Symmetry $\rightarrow$ it is efficient to use the same quantity of each variety, $x(i) = x$.

  $\Rightarrow$ If M is the total quantity of intermediate inputs used, then $M = nx$.
    - Then (3) implies $\dfrac{X}{M} = n^{\frac{1}{\sigma-1}}$.

    $\Rightarrow$ Productivity of intermediate goods increases with n since $\sigma > 1$.

- Each intermediate input is supplied by a single, atomistic firm.
  - Being the sole supplier, the firm has some monopoly power over its own product market.
    - It faces a downward-sloping demand for its product.
- Demand for each intermediate input i can be derived from the following **cost minimization exercise** of the producer of the final consumption good:

$$\left.\begin{array}{c} \underset{\{x_t(i)\},\ H_t}{\text{Minimize}} \displaystyle\int_0^{n_t} \left[p_t\left(i\right) x_t\left(i\right)\right] di + w_t H_t, \\ \\ \text{s.t. } C_t = F\left(X_t,\ H_t\right),\ \text{and } X_t = \left[\displaystyle\int_0^{n_t} \left[x_t\left(i\right)\right]^{1-\frac{1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}. \end{array}\right\} \tag{P}$$

- For Problem (P), the following two-stage minimization procedure is valid:
  - See Green (1964), pages 21-22, and Dixit and Stiglitz (1977), pages 298-299.

– **Stage I:** Define the price index of the intermediate goods composite as P_t, that is,

$$P_t X_t \equiv \int_0^{n_t} \left[p_t(i)\, x_t(i) \right] di, \tag{i}$$

and consider the following minimization problem:

$$\left. \begin{aligned} &\underset{X_t,\ H_t}{\text{Minimize}}\ \ P_t X_t + w_t H_t, \\ &\text{s.t. } C_t = F(X_t,\ H_t). \end{aligned} \right\} \tag{P.I}$$

– **Stage II:** Use the optimal value of X_t from stage I and solve the following minimization problem:

$$\left. \begin{aligned} &\underset{\{x_t(i)\}}{\text{Minimize}}\ \int_0^{n_t} \left[p_t(i)\, x_t(i) \right] di, \\ &\text{s.t. } X_t = \left[\int_0^{n_t} \left[x_t(i) \right]^{1-\frac{1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \right\} \tag{P.II}$$

- **Stage I:**
  - Define $\alpha_t \equiv$ factor share of intermediate inputs at t.
  - Solution to the first stage:

$$P_t X_t = \alpha_t C_t, \text{ and } w_t H_t = (1 - \alpha_t) C_t.$$

  - Perfectly competitive final goods sector $\Rightarrow \alpha_t = \dfrac{F_X(X_t,\ H_t) X_t}{F(X_t,\ H_t)}$.
  - Linear homogeneity of $F(X_t, H_t)$ implies that this expression solely depends on the relative factor price,

$$\frac{P_t}{w_t} = \frac{F_X(X_t,\ H_t)}{F_H(X_t,\ H_t)}.$$

  - Denote this relation by $\alpha_t = \alpha\left(\dfrac{P_t}{w_t}\right)$.

- **Stage II:**
  - The first-order condition for each input $x_t(i)$ is:

$$p_t(i) = \lambda \cdot [x_t(i)]^{-\frac{1}{\sigma}} X_t^{\frac{1}{\sigma}}. \tag{ii}$$

  - Solving (ii) for $x_t(i)$, we have

$$x_t(i) = \lambda^{\sigma} [p_t(i)]^{-\sigma} X_t. \tag{iii}$$

  - Substituting (iii) into X_t, we can solve for λ^{σ} as

$$\lambda^{\sigma} = \left[\int_0^{n_t} [p_t(i)]^{1-\sigma} di\right]^{\frac{\sigma}{1-\sigma}}. \tag{iv}$$

  - Using (iv), we get the demand for each intermediate input i:

$$x_t(i) = [p_t(i)]^{-\sigma} \left[\int_0^{n_t} [p_t(i)]^{1-\sigma} di\right]^{\frac{\sigma}{1-\sigma}} X_t. \tag{v}$$

– Recall that

$$P_t X_t \equiv \int_0^{n_t} \left[p_t\left(i\right) x_t\left(i\right)\right] di.$$

– Then (v) implies

$$p_t\left(i\right) x_t\left(i\right) = \left[p_t\left(i\right)\right]^{1-\sigma} \left[\int_0^{n_t} \left[p_t\left(i\right)\right]^{1-\sigma} di\right]^{\frac{\sigma}{1-\sigma}} X_t$$

$$\Rightarrow P_t X_t \equiv \int_0^{n_t} \left[p_t\left(i\right) x_t\left(i\right)\right] di = \left[\int_0^{n_t} \left[p_t\left(i\right)\right]^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} X_t$$

$$\Rightarrow P_t = \left[\int_0^{n_t} \left[p_t\left(i\right)\right]^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \tag{vi}$$

– Using (vi), the demand for each intermediate input i can be expressed as

$$x_t(i) = \left[\frac{p_t(i)}{P_t}\right]^{-\sigma} X_t. \tag{vii}$$

- $\dfrac{p_t(i)}{P_t}$ is the relative price of input i, relative to the price index of the intermediate goods composite, P_t.
- Clearly, due to this CES specification, demand for each input exhibits a constant price elasticity, σ.

- Producing a unit of each input requires a_x units of labor.

$\Rightarrow$ Marginal cost: $w_t a_x$.

– Normalization: $a_x \equiv 1 - \dfrac{1}{\sigma}$.

- Profit maximization implies each intermediate goods producer sets the price so as to equate marginal revenue with marginal cost, implying

$$p_t(i) = \frac{w_t a_x}{1 - \dfrac{1}{\sigma}} = w_t. \tag{4}$$

- Symmetry $\rightarrow$ all producers set the same price.
  - Using (4), the price index of the intermediate goods composite thus becomes

$$P_t = \left[\int_0^{n_t} \left[p_t\left(i\right) \right]^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = n_t^{\frac{1}{1-\sigma}} w_t. \tag{5}$$

  - Note that the effective relative factor price, $\dfrac{P}{w}$, decreases with n.
    - This is the mirror image of ‘increasing returns due to specialization’.
    - As a broader range of differentiated inputs are available, it becomes advantageous to use them more intensively as a group:
      - productivity of intermediate goods increases with n;
      - effective relative factor price, $\dfrac{P}{w}$, decreases with n.

- Using (5), the factor share of intermediate inputs, $\alpha_t = \alpha\left(\frac{P_t}{w_t}\right)$, can be expressed as a function of the product variety (n_t):

$$\alpha_t = \alpha\left(n_t^{\frac{1}{1-\sigma}}\right) \equiv A\left(n_t\right). \tag{6}$$

  – Note that

$$\alpha_t = \frac{F_X\left(X_t,\ H_t\right)X_t}{F\left(X_t,\ H_t\right)} = \frac{\left(\frac{X_t}{H_t}\right)\left(\frac{F_X(X_t,\,H_t)}{F_H(X_t,\,H_t)}\right)}{\left(\frac{X_t}{H_t}\right)\left(\frac{F_X(X_t,\,H_t)}{F_H(X_t,\,H_t)}\right)+1} = \frac{\left(\frac{X_t}{H_t}\right)\left(\frac{P_t}{w_t}\right)}{\left(\frac{X_t}{H_t}\right)\left(\frac{P_t}{w_t}\right)+1}$$

$\Rightarrow$ Elasticity of substitution between H and X greater (less) than one $\Rightarrow$ $A\left(n\right)$ is increasing (decreasing) in n.

  ○ Consider the case of an increasing $A(n)$ below, given the strong evidence that share of the producer services sector increases with the level of GNP, both in cross section and in time series.

- Since all intermediate inputs enter symmetrically in the final goods production,

$$p(i) = p(j)\,,\; x(i) = x(j)\,,\text{ and } \pi(i) = \pi(j)\,,\text{ for all } i, j,$$

  - π stands for operating profit.

- Then $n_t p_t x_t = \alpha_t C_t$, and thus

$$\pi_t = (p_t - w_t a_x)\, x_t = \frac{p_t x_t}{\sigma} = \frac{\alpha_t C_t}{\sigma n_t}.$$

- Then (6) implies

$$\pi_t = \frac{A(n_t)}{\sigma n_t} C_t. \tag{7}$$

  - An increase in the number of firms and available varieties has two effects on the profit of an incumbent firm working in opposite directions:

    1. For a given factor share of intermediate inputs in the final goods production, a larger set of competing varieties reduces the profit of each variety.
    2. The number of available varieties increases the factor share, and hence profit when the elasticity of substitution is greater than one.

- **Entry and Start-up Operation:**

  The number of the specialist firms (and the range of producer services available) increases over time through the process of *entry*.

  - Initially, the economy inherits a given number of firms, n_0.
  - At any moment firms may enter freely into the intermediate goods sector, except that they need a *start-up operation*:
    - Requires the use of a_n units of labor per variety.
  - The entering firms finance start-up costs by issuing ownership shares.
  - Because of free entry, the value of an intermediate goods firm, v_t, never exceeds the start-up cost, $w_t a_n$, and
    - whenever some entry occurs, they are equalized.
  - Furthermore, the operating profit is always positive, so that no incumbent firm has an incentive to exit.
  - That is, in equilibrium, we have

$$w_t a_n \geq v_t, \;\; \dot{n}_t \geq 0, \;\; (w_t a_n - v_t)\, \dot{n}_t = 0. \tag{8}$$

- The market value of an intermediate goods producer is equal to the present discounted value of profits,

$$v_t \equiv \int_t^{\infty} e^{-\int_t^{\tau} r_s \, ds} \, \pi_\tau \, d\tau,$$

from which we obtain

$$\frac{\pi_t + \dot{v}_t}{v_t} = r_t. \tag{9}$$

  – The rate of return of holding ownership shares (dividend plus capital gain) is equal to the interest rate.

  – To derive (9), use the **Leibniz'a Rule** for differentiation of definite integrals:

    Suppose c is a parameter, and $F(c) = \int_{a(c)}^{b(c)} f(c, x) \, dx$. Then

$$\frac{dF(c)}{dc} = \int_{a(c)}^{b(c)} f_c(c, x) \, dx + f(c, b(c)) \cdot b'(c) - f(c, a(c)) \cdot a'(c).$$

    ◦ See Barro and Sala-i-Martin (2004), pages 624-625.

- **Labour market clearing** requires

$$L = a_n \dot{n}_t + H_t + n_t a_x x_t.$$

  – Since $n_t p_t x_t = n_t w_t x_t = \alpha_t C_t$, and $w_t H_t = (1-\alpha_t)\, C_t$, we get

$$\begin{aligned} L &= a_n \dot{n}_t + (1-\alpha_t)\left(\frac{C_t}{w_t}\right) + \left(1-\frac{1}{\sigma}\right)\alpha_t \left(\frac{C_t}{w_t}\right) \\ &= a_n \dot{n}_t + \left(1-\frac{A(n_t)}{\sigma}\right)\left(\frac{C_t}{w_t}\right). \end{aligned} \tag{10}$$

- **The national income account:**

  – Multiplying (10) by w_t we get $\quad w_t L + \dfrac{A(n_t)}{\sigma} C_t = C_t + w_t a_n \dot{n}_t.$

  – Using (7) and (8) we obtain the national income account:

$$w_t L + n_t \pi_t = C_t + v_t \dot{n}_t,$$

$\Rightarrow$ wage + profit = consumption + investment.

- **The transversality condition:**
  - The transversality condition requires:

$$\lim_{T\to\infty} n_T v_T e^{-\int_0^T r_s\,ds} = 0. \tag{12}$$

  - The national income account $\Rightarrow$

$$w_t L - C_t = v_t \dot{n}_t - n_t \pi_t = v_t \dot{n}_t + n_t \dot{v}_t - n_t v_t r_t = \frac{d}{dt}(n_t v_t) - n_t v_t r_t.$$

  - Integrating from time t to some time T this yields

$$\int_t^T \left[\frac{d}{dt}(n_t v_t) - n_t v_t r_t\right] dt = \int_t^T [w_t L - C_t]\, dt$$

$$\Rightarrow n_T v_T e^{-\int_0^T r_s\,ds} = n_t v_t + \int_t^\infty e^{-\int_t^\tau r_s\,ds} \left(w_\tau L - C_\tau\right)\, d\tau$$

$= 0$, using (2) and $n_t v_t = W_t$,

so that the transversality condition is satisfied.

3.2 The Market Equilibrium

- Describe the dynamic evolution of the economy in terms of two variables, n and $V = \dfrac{v}{C}$.
  - V represents the value of an intermediate inputs producing firm, measured in utility.
- Then

$$\begin{aligned}
\dot{V}_t &= \frac{\dot{v}_t}{C_t} - V_t \frac{\dot{C}_t}{C_t} \\
&= \frac{r_t v_t - \pi_t}{C_t} - V_t \left(r_t - \rho\right) \text{ [using (9) and (1)]} \\
&= \rho V_t - \frac{\pi_t}{C_t} \\
&= \rho V_t - \frac{A\left(n_t\right)}{\sigma n_t} \text{ [using (7)].}
\end{aligned}$$

- That is,

$$\dot{V}_t = \rho V_t - \frac{A\left(n_t\right)}{\sigma n_t}. \tag{13a}$$

- Next, (8) $\Rightarrow$ if $\dot{n}_t > 0$, then $w_t a_n = v_t$.
  - Then (10) $\Rightarrow$

$$\begin{aligned} \dot{n}_t &= \frac{L}{a_n} - \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\frac{C_t}{w_t a_n}\right) \\ &= \frac{L}{a_n} - \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\frac{1}{V_t}\right). \end{aligned}$$

  - Thus, the evolution of n_t is given by

$$\dot{n}_t = \max\left\{\frac{L}{a_n} - \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\frac{1}{V_t}\right),\ 0\right\}. \tag{13b}$$

- Finally, we have to consider the limiting behaviour of V_t and n_t as $t \to \infty$.
  - Recall the Euler condition: $\dfrac{\dot{C}_t}{C_t} = r_t - \rho$.
    - Integrating from time 0 to some time t we get

$$\log C_t - \log C_0 = \int_0^t r_s ds - \rho t, \text{ implying } C_t = C_0 e^{\int_0^t r_s ds - \rho t}.$$

– Then (12) implies

$$\lim_{t\to\infty} n_t v_t e^{-\int_0^t r_s\, ds} = 0$$

$$\Rightarrow \lim_{t\to\infty} n_t V_t C_t e^{-\int_0^t r_s\, ds} = 0$$

$$\Rightarrow \lim_{t\to\infty} n_t V_t C_0 e^{-\rho t} = 0,$$

that is,

$$\lim_{t\to\infty} n_t V_t e^{-\rho t} = 0. \tag{13c}$$

- **Market Equilibrium:**

  For any initial number of firms (n_0) the economy inherits, a market equilibrium of this economy is a path of $\{V_t,\ n_t\}$ that satisfies (13a), (13b) and (13c).

- The qualitative property of the equilibrium dynamics crucially depends on the shapes of the following two loci:
  - The VV locus (corresponds to $\dot{V}_t = 0$):

$$V_t = \frac{A(n_t)}{\rho \sigma n_t}. \tag{VV}$$

  - The NN locus (corresponds to $\dot{n}_t = 0$):

$$V_t = \frac{a_n}{L}\left(1 - \frac{A(n_t)}{\sigma}\right). \tag{NN}$$

- These two loci intersect at $n = n^*$ if and only if

$$\frac{A(n_t)}{\rho \sigma n_t} = \frac{a_n}{L}\left(1 - \frac{A(n_t)}{\sigma}\right),$$

that is, if and only if

$$\Phi(n^*) \equiv n^* \left(\frac{\sigma}{A(n^*)} - 1 \right) = \frac{L}{\rho a_n}, \tag{14}$$

– n^* is the steady-state varieties of differentiated intermediate inputs.

- Controlling for the factor share of intermediate inputs ($A(n)$),
  - the range of differentiated intermediate products increases with the size of the economy, measured by the total labor force.
  - This expresses the Smith-Young notion that *the division of labor depends on the extent of the market*.
- At the same time, increasing availability of specialized inputs may induce the final goods producers to use a more roundabout method of production.

$\Rightarrow$ Increase the size of the market for intermediate inputs.

  - That is, *the extent of the market also depends on the division of labor*.
- Because of this circularity, there may be multiple solutions to (14).

3.3 Market Equilibrium with Underdevelopment Traps

- Let $F(X,H)$ be a CES of the following form:

$$F(X,H)=\left[X^{1-\frac{1}{\epsilon}}+\beta^{\frac{1}{\epsilon}}H^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}},\ \epsilon>\sigma>1.$$

- To derive the VV and NN loci, we need to get the expression for $A\left(n_t\right)$ for this CES specification.

- Recall that $A\left(n_t\right)=\alpha_t=\dfrac{F_X\left(X_t,\ H_t\right)X_t}{F\left(X_t,\ H_t\right)}=\left[\dfrac{X_t}{H_t}\dfrac{P_t}{w_t}\right]\cdot\left[\dfrac{X_t}{H_t}\dfrac{P_t}{w_t}+1\right]^{-1}.$

- CES technology $\Rightarrow \dfrac{F_X\left(X_t,\ H_t\right)}{F_H\left(X_t,\ H_t\right)}=\dfrac{P_t}{w_t}$ implies $\dfrac{X_t^{-\frac{1}{\epsilon}}}{\beta^{\frac{1}{\epsilon}}H_t^{-\frac{1}{\epsilon}}}=\dfrac{P_t}{w_t}\Rightarrow\dfrac{X_t}{H_t}=\left[\beta\left(\dfrac{P_t}{w_t}\right)^{\epsilon}\right]^{-1}.$

$\Rightarrow$

$$\frac{1}{A\left(n_t\right)}=1+\frac{1}{\left(\frac{X_t}{H_t}\right)\left(\frac{P_t}{w_t}\right)}=1+\beta\left(\frac{P_t}{w_t}\right)^{\epsilon-1}=1+\beta n_t^{\frac{1-\epsilon}{\sigma-1}}.$$

- **The NN locus:** Defined by: $V_t = \frac{a_n}{L}\left(1 - \frac{A(n_t)}{\sigma}\right)$; given by

$$V_t = \frac{a_n}{L} - \frac{\frac{a_n}{L}}{\sigma\left(1 + \beta n_t^{\frac{1-\epsilon}{\sigma-1}}\right)}.$$

  – The NN locus is downward sloping.

- **The VV locus:** Defined by: $V_t = \frac{A(n_t)}{\rho\sigma n_t}$; given by

$$V_t = \frac{1}{\rho\sigma\left[n_t + \beta n_t^{\frac{\sigma-\epsilon}{\sigma-1}}\right]}.$$

  – $\left.\frac{dV_t}{dn_t}\right|_{VV} \gtreqqless 0$, according as $n_t \lesseqqgtr \check{n} \equiv \left[\frac{\beta(\epsilon-\sigma)}{\sigma-1}\right]^{\frac{\sigma-1}{\epsilon-1}}$.

  $\Rightarrow$ VV has a single peak at $\check{n}$.

- There are three generic cases to be distinguished, depending upon the effective labor supply, $\frac{L}{a_n}$.
- **Case 1: Sufficiently high start-up costs:**
  - Depicted in Figure 3, panel a.
  - Start-up costs are so high that NN lies above VV everywhere.
  - Any combination of n and V on the VV locus is a (trivial) steady state.
    - No entry takes place in this economy.
- **Case 2: Moderate start-up costs:**
  - Refer to Figure 3, panel b.
  - NN intersects VV twice, at S_L and S_H, both at the downward sloping part of VV.
  - The equilibrium path is unique for any initial condition.
  - Suppose the economy starts below S_L.
    - The narrow industrial base forces the final goods producers to use the labor intensive technology.

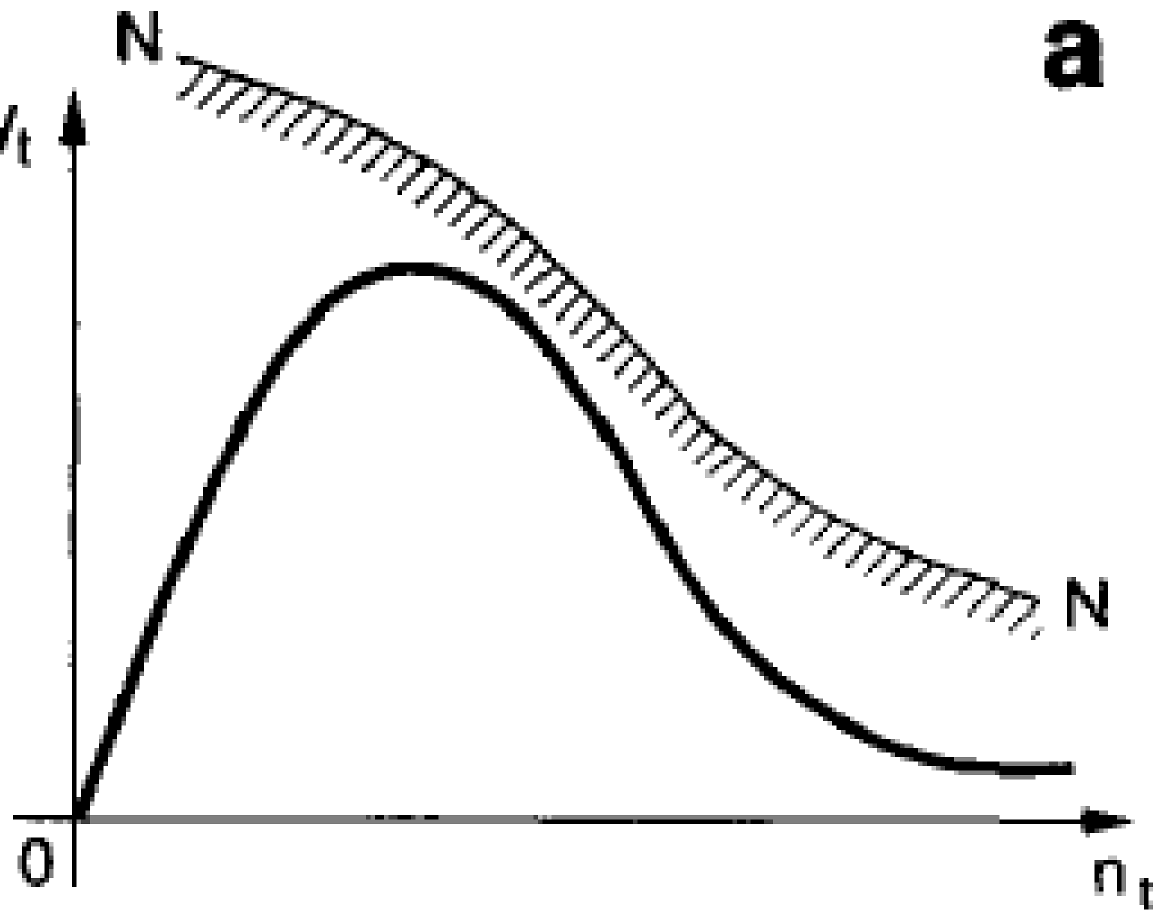

Figure 3: Panel a

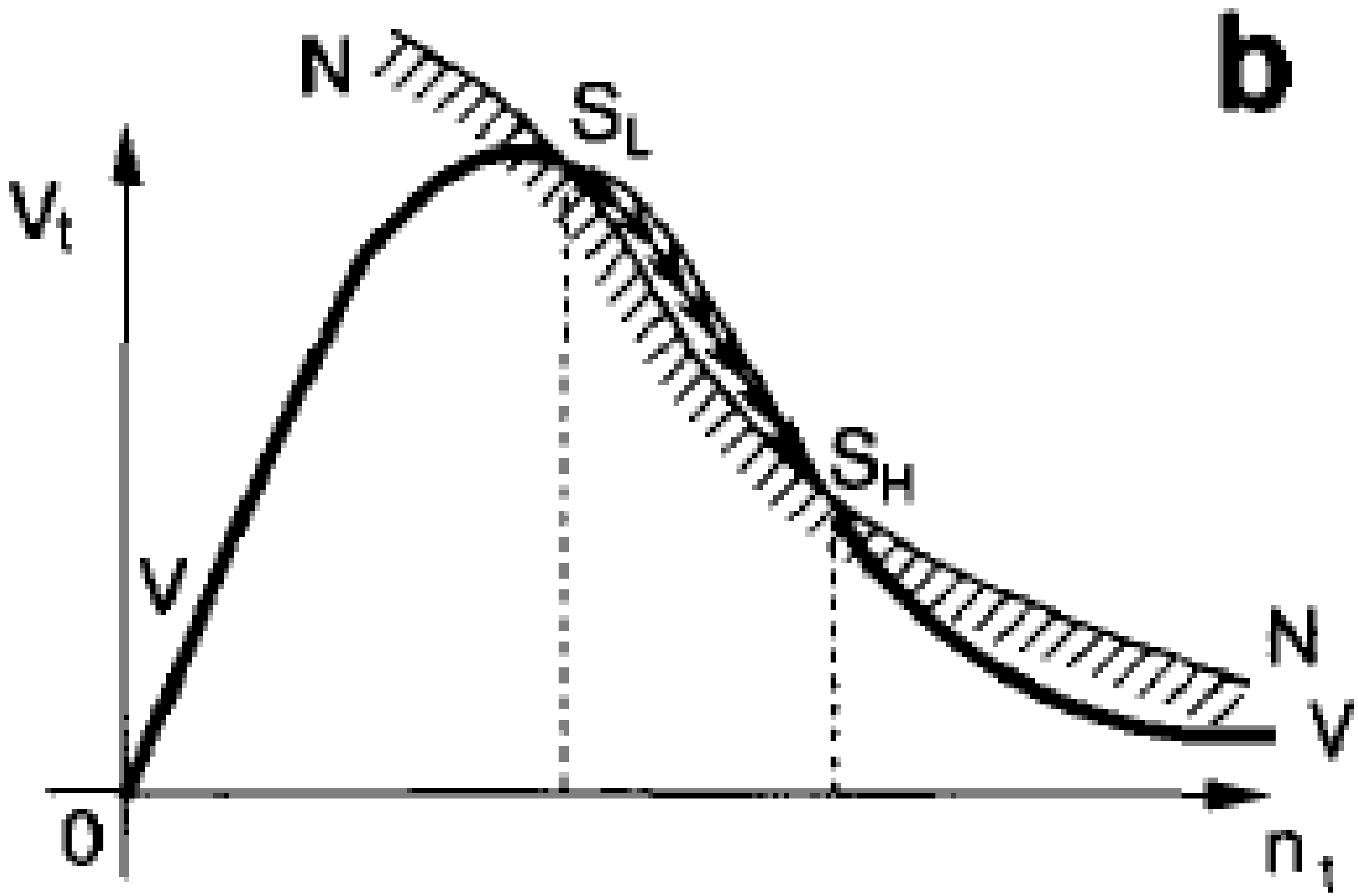

Figure 3: Panel b

$\Rightarrow$ Demand for intermediate inputs is too low to justify starting up new firms in the intermediate goods sector.

∘ The economy stays still on VV, and it is trapped in the lower stage of economic development.

– Suppose the economy starts slightly above S_L.

∘ The range of differentiated products available is sufficiently large.

∘ Induces final goods producers to make more intensive use of intermediate inputs.

∘ This generates a large market for intermediate products that lead new firms to enter.

$\Rightarrow$ The economy experiences

· an expanding variety of differentiated inputs,

· productivity growth, and

· a rising share of intermediate goods sector in employment.

∘ This cumulative process continues until the economy reaches the high level steady state, S_H.

- **Case 3: Small start-up costs:**
  - Refer to Figure 3, panel c.
  - NN intersects VV at its upward sloping part at S_L, the lower steady state.
  - This generates a possibility of *multiple equilibria*.
    - There exist two equilibria if the economy starts just below S_L.

1. The Pessimistic Equilibrium:
   - No entry is expected to occur, and the share of intermediate inputs is expected to remain small.
   - As a result, no entry takes place, and the economy stays still on VV.
2. The Optimistic Equilibrium:
   - *Optimistic expectations* that an increasing range of specialized intermediate products will lead to a rising share of the intermediate inputs market in the future induces new firms to enter.
   - Active entry *in fact* expands the range of intermediate goods.
   - The economy converges to the higher steady state, S_H.

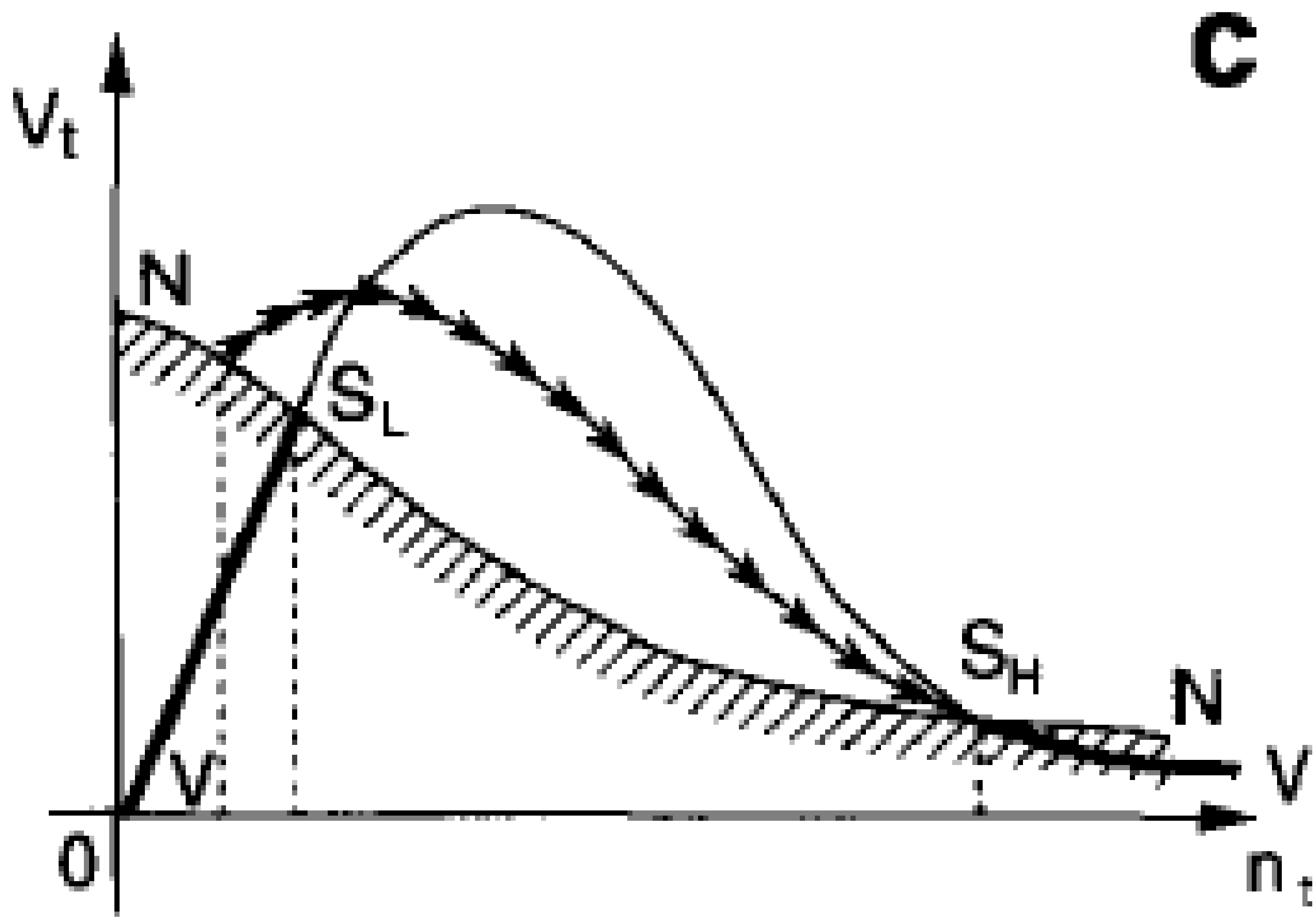

Figure 3: Panel c

- A take-off becomes possible as a result of the *self-fulfilling prophecy*.
  - The positive feedback between the entry and the rising share creates a virtuous circle along this equilibrium path.

- **The logic behind the existence of a development trap:**

1. Because of *start-up costs*, specialist firms that produce intermediate goods are subject to *dynamic increasing returns*.
   - The inducement to start up operations thus depends on the anticipated market size.
     - When high demand is expected, more firms enter, and
     - a wider range of specialized inputs will be available.
2. Starting up a new firm and introducing a new variety of intermediate inputs generate *positive externalities* that are not completely appropriated by individual firms.
   - The presence of such externalities leads to an insufficient inducement to start up firms and to introduce new products.

– These two factors, *start-up costs* and *positive externalities*, together imply the circularity between the degree of specialization and the market share of intermediate inputs, and
   - present possible barriers to economic development.

- **The circularity does not always imply a vicious circle of poverty.**
  - If the economy inherits a sufficiently broad range of specialized inputs and thus has more than the 'critical mass' of specialist firms,
    - the very fact that the relation is circular generates a *virtuous circle*.
  - Over time, the division of labor becomes far more elaborate, the production process more indirect,
    - involving an increasing degree of specialized inputs.
  - Through such a cumulative process, the economy experiences productivity growth and a rising standard of living.
  - The model thus suggests the existence of a *threshold* in economic development.
- When the economy starts *below the threshold* level,
  - a *coordinated entry* of specialist firms can push the economy above the threshold,
  - make it possible to break away from the development trap.
    - The economy escapes the trap due to a sort of *self-fulfilling prophecy*.

4. Acemoglu and Zilibotti (1997)

- The positive externalities that we have come across so far are "pecuniary" rather than "technological".
  - Pecuniary externalities refer to the effects that are mediated by prices.
  - Example: Consider a fishery and a nearby oil refinery.
    - Technological externality: Fishery's productivity is affected by the emissions from the refinery.
    - Pecuniary externality: Fishery's profitability is affected by the price of oil.
- Pecuniary externalities are inconsistent with the traditional Arrow-Debreu paradigm of a complete set of perfectly competitive markets.
  - A full set of forward, contingent markets would enable these interdependencies to be mediated through the price mechanism.
    - That eliminates the possibility of multiple Pareto-ordered equilibria.

- This implies that these pecuniary externalities are particularly pervasive in early stages of development,
  - when well-developed financial markets are yet to emerge.
  - Acemoglu and Zilibotti (1997) formalizes this phenomenon.
- Acemoglu and Zilibotti (1997) offers a theory of development that links the degree of market incompleteness to capital accumulation and growth.
  - At early stages of development, the presence of indivisible projects limits the degree of risk spreading (diversification) that the economy can achieve.
    - The desire to avoid highly risky investments slows down capital accumulation.
    - The inability to diversify idiosyncratic risk introduces a large amount of uncertainty in the growth process.
- The typical development pattern will consist of
  - a lengthy period of “primitive accumulation” with highly variable output,
  - followed by takeoff and financial deepening, and
  - finally, steady growth.

- Although all agents are price takers and there are no technological spillovers,
  - the decentralized equilibrium is inefficient because
    - individuals do not take into account their impact on others' diversification opportunities.

4.1 Motivation

- *Slow* and *uncertain* progress at early stages of development.
  - "The advance occurred very slowly over a long period and was broken by sharp recessions. The right road was reached and thereafter never abandoned, only during the eighteenth century, and then only by a few privileged countries. Thus, before 1750 or even 1800 the march of progress could still be affected by unexpected events, even disasters." [Braudel, 1973]
  - Braudel (1982): Points out the presence of failed takeoffs:

    "... three occasions in the West when there was an expansion of banking and credit so abnormal as to be visible to the naked eye [Florence 1300s, Genoa late 1500s, and Amsterdam 1700s]. ... three substantial successes, which ended every time in failure or in some kind of withdrawal."

    - While the expansions were gradual, the collapses were abrupt, ignited by a few bankruptcies suggesting the presence of large undiversified risks.
  - North and Thomas (1973): Describes 14th and 15th centuries as times of "contractions, crisis and depression".

- *Slow* and *uncertain* progress even in today’s development experience.
  - Lucas (1988):

    Whereas “within the advanced countries, growth rates tend to be very stable over long periods of time,” for poorer countries, “there are many examples of sudden, large changes in growth rates, both up and down.”

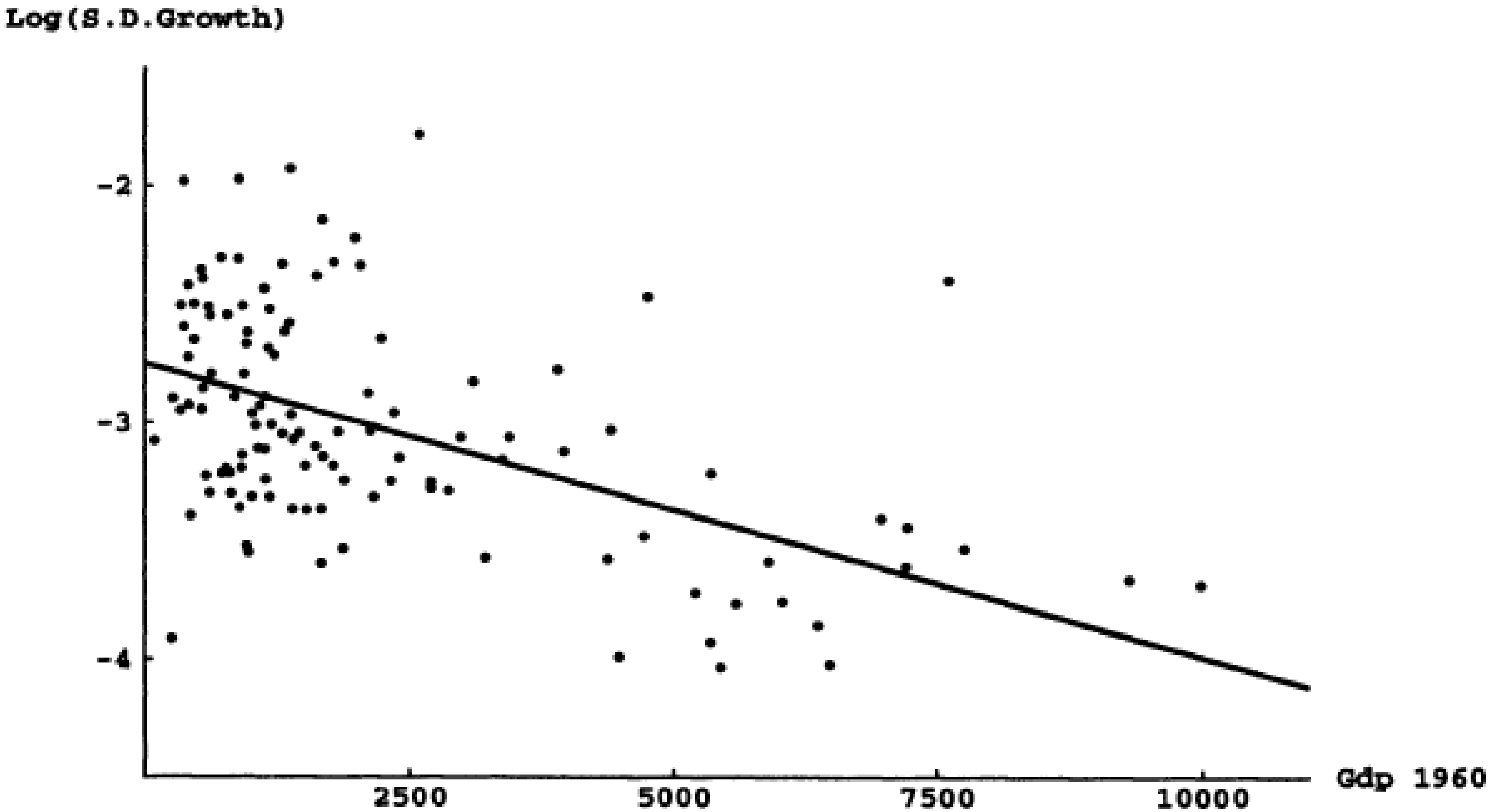

FIG. 1.—Variability of growth

TABLE 2

TRANSITION MATRIX (Quah 1993)

Relative GDP per Capita	Prob($\downarrow$)	Prob($\sim$)	Prob($\uparrow$)
$z < 1/4$	...	.76	.24
$1/4 < z < 1/2$	.52	.31	.17
$1/2 < z < 1$	.29	.46	.26
$1 < z < 2$	.24	.53	.24
$2 < z$	.05	.95	...

- **Why are early stages of development slow and face so much randomness?**
  - **Acemoglu and Zilibotti's (1997) argument**:

    These patterns can be predicted by the neoclassical growth model augmented with the natural assumptions of

    - micro-level indivisibilities, and
    - micro-level uncertainty.
  - **Observations:**
    1. Most economies have access to a large number of imperfectly correlated projects.

       $\Rightarrow$ A significant part of the risk can be diversified.
    2. A large proportion of these projects are subject to significant indivisibilities – minimum size requirements or start-up costs.
    3. Agents dislike risk.
    4. There exists less productive but relatively safe investment opportunities.
    5. Societies in the early stages of development have less capital to invest than developed countries.

– Implications:

1. At early stages of development, due to scarcity of capital, only a limited number of imperfectly correlated projects can be undertaken.
   - ○ Agents seek insurance by investing in safe but less productive assets.

   $\Rightarrow$ Poor countries will have lower productivity and slow development.

2. Diversification opportunities being limited, early stages of development will be highly random.
   - ○ Economic progress slow down further since many runs toward take-off will be stopped by crises.

3. Chance will play a very important role.
   - ○ “Lucky” economies receive good draws at early stages $\longrightarrow$ more capital $\longrightarrow$ achieve better risk diversification and higher productivity.

4.2 The Model and the Decentralized Equilibrium

- **The Environment:**
  - An overlapping generations model with competitive markets and nonaltruistic agents living for two periods.
    - A continuum of agents of mass $a > 1$ in each generation.
    - Agents of the same generation are all identical.
  - Production side of the economy consists of:
    - A single final-good sector:
      - Transforms capital and labour into final output.
    - A continuum of intermediate sectors (projects):
      - Transform savings of time t into capital to be used at $t+1$ without using labour.

- **Timing of Events:**
  - **Youth:**
    - Agents work in the final-sector firm and receive the wage rate.
    - Then they make their consumption, saving, and portfolio decisions.
      - Savings can be invested in risky securities or in a safe asset that has a nonstochastic gross rate of return, r.
    - Then the uncertainty unravels:
      - The security returns and the amount of capital brought forward to the next period are determined.
  - **Old Age:**
    - Capital that agents own in their retirement period is sold to final-sector firms (and fully depreciates after use).
    - Old agents consume this capital income.
  - Figure 2 summarizes this sequence of events.

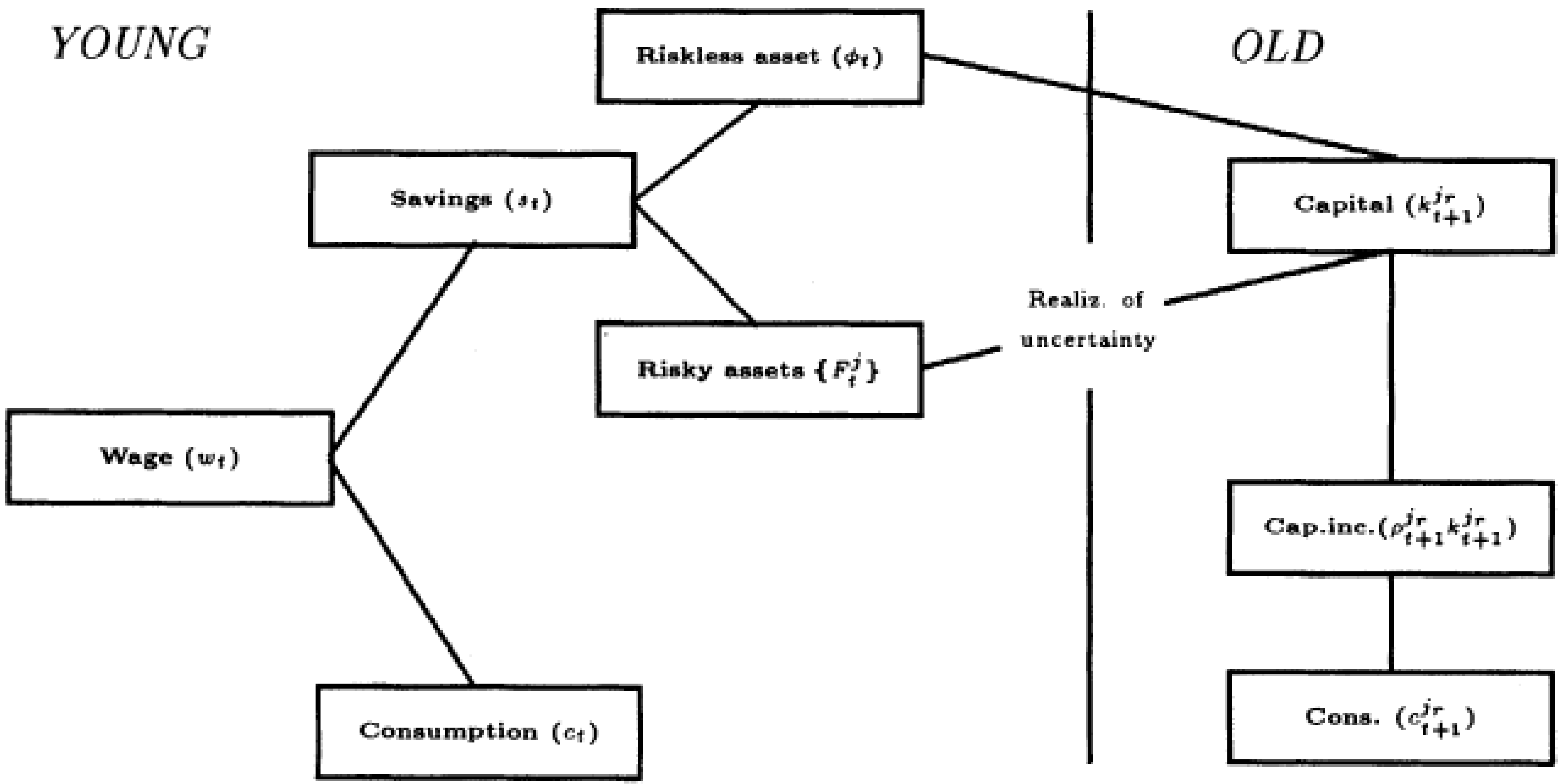

FIG. 2.—Timing of events (j_r stands for the realized state of nature)

- **Uncertainty:**
  - A continuum of equally likely states represented by $[0,1]$.
  - An investment of F^j in intermediate sector $j \in [0,1]$ pays
    - RF^j if state j occurs, *and* $F^j \geq M_j$;
    - *nothing* in any other state.
  - $R > r$: the risky projects are more productive than the safe investment.
  - The requirement $F^j \geq M_j$ implies some intermediate sectors require a certain minimum size, M_j, before being productive.
  - The distribution of minimum size requirement is given by:

$$M_j = \max \left\{0,\ \frac{D}{1-\gamma}(j-\gamma)\right\}.$$

    - Sectors $j \leq \gamma$ have no minimum size requirement.
    - For sectors $j \geq \gamma$, the minimum size requirement increases linearly.

- Two important features of the formalization of uncertainty:

1. **Risk diversification:** Different projects are imperfectly correlated.

$\Rightarrow$ There is safety in variety.

– A convenient implication:

∘ If a portfolio consists of an equiproportional investment F in all projects $j \in \bar{J} \subseteq [0,1]$, and the measure of $\bar{J}$ is p, then the portfolio pays

· RF with probability p,

· *nothing* with probability $1-p$.

2. **Nonconvexity**: Captured by the minimum size requirement.

$\Rightarrow$ A trade-off between insurance and high productivity.

∘ If the production set were convex ($D = 0$),

· all agents would invest an equal amount in all intermediate goods sectors,

· diversify all the risks.

- **Preferences:**
  - Consumers' preferences over final goods:

$$E_t\, U\left(c_t, c_{t+1}\right) = \log\ c_t\ +\ \beta \int_0^1 \log\ \left(c_{t+1}^j\right)\ dj, \tag{1}$$

    j represents the states of nature.
  - Discount rate: β.
  - Rate of relative risk aversion = 1.
  - Realization of the state of nature affects consumption since it determines
    - how much capital each agent takes into the final-good production stage, and
    - the equilibrium price of capital.

- **Technology and Factor Prices:**
  - Output of the final-good sector:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}. \tag{2}$$

  - Normalize labor endowment at youth to $\frac{1}{a}$.

  $\Rightarrow L_t = 1,$ since the mass of agents is a and labour supply is inelastic.

  - Aggregate stock of capital depends on the realization of the state of nature:
    - If the state of nature is j, then

$$K_{t+1}^j = \int_{\Omega_t} \left(r\phi_{h,t} + RF_{h,t}^j \right) dh.$$

      - Ω_t: set of young agents at time t;
      - $F_{h,t}^j$: the amount of savings invested by agent $h \in \Omega_t$ in sector j;
      - $\phi_{h,t}$: the amount invested in the safe asset.

– Competitive factor markets $\Rightarrow$ equilibrium factor prices in state j are:

$$W_{t+1}^{j} = (1-\alpha)\,A\left(K_{t+1}^{j}\right)^{\alpha} = (1-\alpha)\,A\left[\int_{\Omega_t}\left(r\phi_{h,t}+RF_{h,t}^{j}\right)dh\right]^{\alpha}, \tag{3}$$

$$\rho_{t+1}^{j} = \alpha A\left(K_{t+1}^{j}\right)^{\alpha-1} = \alpha A\left[\int_{\Omega_t}\left(r\phi_{h,t}+RF_{h,t}^{j}\right)dh\right]^{\alpha-1}. \tag{4}$$

– Wage earning of an young agent (conditional on the realization of j):

$$w_t^j = \frac{W_t^j}{a}.$$

• **Intermediate Goods:**

– Intermediate sector firms are run by agents who compete to

∘ get funds by issuing *financial securities*, and

∘ sell them to other agents in the *stock market*.

– Each agent can run at most one project.

– More than one agent can compete to run the same project.

- **Portfolio Decisions: First Stage:**
  - Each agent $h \in \Omega_t$
    - takes the announcements of all other agents as given, and
    - announces his plan to
      - run at most one project in the intermediate sector, and
      - sell an unlimited quantity of the associated security.
  - 1 unit of security j entitles its holder to R units of $t+1$ capital in state of nature j.
  - $P_{j,h,t}$: unit price of security j (in terms of savings of t) issued by agent h.
  - Agent h is managing investments in project j on behalf of other agents:
    - for every unit of savings he collects from others,
      - invests $\dfrac{1}{P_{j,h,t}}$, and
      - keep the remaining $\dfrac{P_{j,h,t}-1}{P_{j,h,t}}$ as his commission.

– A first-stage strategy for an agent h at t is an announcement

$$Z_{h,t} = (j, P_{j,h,t}) \in [0,1] \times \mathbb{R}_+,$$

- ○ j is the project h intends to run,
- ○ $P_{j,h,t}$: the selling price of the corresponding security.

– $Z_{h',t} = \varnothing$ if h' decides to run no project.

– $Z_t : \Omega_t \to [0,1] \times \mathbb{R}_+$: announcements of all agents at t.

– The subset of all projects that at least one agent proposes to run at t:

$$J_t(Z_t) = \{j \in [0,1] : \exists\, h \text{ s.t. } Z_{h,t} = (j, P_{j,h,t})\}.$$

– Define the minimum price for each security j induced by the set of announcements Z_t by

$$P_t^j(Z_t) = \min_{\{h \text{ s.t. } Z_{h,t}=(j,P_{j,h,t})\}} (P_{j,h,t}) \,.$$

- **Portfolio Decisions: Second Stage:**
  - All agents behave competitively taking as given
    - the set of securities offered, and
    - the price of each security announced in the first stage.
  - Agents choose their savings s_t, demand for the safe asset ϕ_t, and demand for each security j, F_t^j, by solving the following optimization problem:

$$\max_{s_t,\, \phi_t,\, \left\{F_t^j\right\}_{0 \le j \le 1}} \log\ c_t \ +\ \beta \int_0^1 \log\ \left(c_{t+1}^j\right)\ dj \tag{5}$$

subject to

$$s_t \ =\ \phi_t + \int_0^1 \left(P_t^j(Z_t) \cdot F_t^j\right) dj, \tag{6}$$

$$c_{t+1}^j \ =\ \rho_{t+1}^j \cdot \left(r\phi_t + RF_t^j\right), \tag{7}$$

$$F_t^j \ =\ 0 \text{ for all } j \notin J_t(Z_t), \tag{8}$$

$$c_t + s_t \le w_t + v_t. \tag{9}$$

∘ v_t is the commission the agent gets for running a project.

· For all $h \in \Omega_t$ such that $Z_{h,t} = \varnothing$, we have $v_{h,t} = 0$.

· For an agent $h \in \Omega_t$ who runs project j,

$$v_{h,t} = \left(\frac{P_{j,h,t} - 1}{P_{j,h,t}} \right) \hat{F}^{j,h,t},$$

$\hat{F}^{j,h,t}$: total amount of fund agent h raises.

– In this stage, each agent takes w_t, P_t^j, ρ_{t+1}^j, and the set of risky assets $J_t(Z_t)$ as given.

- **Static Equilibrium: Definition**
  - Given K_t, an equilibrium at time t is a set of
    - first-stage announcements Z_t^*,
    - second-stage savings and portfolio decisions s_t^*, ϕ_t^*, and $\left\{F_t^{j*}\right\}_{0 \leq j \leq 1}$, and
    - factor returns $\left\{W_{t+1}^j\right\}_{0 \leq j \leq 1}$ and $\left\{\rho_{t+1}^j\right\}_{0 \leq j \leq 1}$

    such that

(a) given any Z_t, w_t, and $\left\{\rho_{t+1}^j\right\}$, each agent h chooses s_h^*, ϕ_h^*, and $\left\{F_h^{j*}\right\}$ in the *second stage* by solving (5) subject to (6) – (9);

(b) in the *first stage*, given the set of first-stage announcements and the decision rules s^*, ϕ^*, and $\left\{F^{j*}\right\}$ of all other agents in the second stage, every agent h makes the optimal announcement $Z_{h,t}^*$; and

(c) $\left\{W_{t+1}^j\right\}$ and $\left\{\rho_{t+1}^j\right\}$ are given by (3) and (4).

- **Static Equilibrium: Characterization**

  Two useful observations:

1. Preferences being logarithmic, saving rule is (irrespective of the risk-return trade-off):

$$s_t^* \equiv s^* (w_t) = \frac{\beta}{1+\beta} w_t. \tag{10}$$

   – Given this, an agent's optimization problem can be broken into two steps:

   ◦ Step 1: amount of savings is determined;

   ◦ Step 2: an optimal portfolio is chosen.

2. Free entry into the intermediate goods sector $\Rightarrow v_{h,t} = 0$ for all t, h.

   ◦ Reason: There are more agents than projects ($a > 1$).

   ◦ $v_{h,t} = \left(\frac{P_{j,h,t} - 1}{P_{j,h,t}} \right) \hat{F}^{j,h,t} = 0 \Rightarrow P_{j,h,t} = 1$ for all $j \in J_t$.

• **Lemma 1.** *Let Z_t^* be the set of equilibrium announcements at time t. Then*

(i) $F_t^{j*} = F_t^{j'*}$ *for all* j, $j' \in J_t(Z_t^*)$, *and*

(ii) $J_t(Z_t^*) = [0,\ n_t(Z_t^*)]$ *for some* $n_t(Z_t^*) \in [0, 1]$.

(i): Balanced Portfolio: Since an agent is facing the same price for all the traded *symmetric* securities, he purchases an equal amount of each.

(ii): When only a subset of projects can be opened in equilibrium, “small projects” are opened before “large projects”.

$\Rightarrow$ If sector j^* is open, all sectors $j \leq j^*$ must also be open.

∘ Intuition:

· All *feasible* portfolios have the same return, but the variability decreases with the number of open projects.

$\rightarrow$ *Risk-averse* agents want to have the maximum number of projects open subject to feasibility.

· Minimum size requirements (M_j) $\Rightarrow$ “small projects” are chosen first.

- **Optimal Portfolio Decision:**
  - Given the amount of savings s_t^*, the optimal portfolio choice problem is:

$$\max_{\phi_t,\, \left\{F_t^j\right\}_{0\leq j\leq 1}} \int_0^1 \log\ \left[\rho_{t+1}^j \cdot \left(r\phi_t + RF_t^j\right)\right]\ dj$$

  subject to

$$\phi_t + \int_0^1 \left(P_t^j(Z_t) \cdot F_t^j\right) dj = s_t^*.$$

  - Lemma 1 implies that the return profile of an agent is:

$$\begin{cases} r\phi_t & \text{with probability } 1 - n_t \\ r\phi_t + RF_t & \text{with probability } n_t. \end{cases}$$

$\Rightarrow$ Expected period $t+1$ utility is: $n_t \log\left[\rho_{t+1}^{q_G}\left(r\phi_t + RF_t\right)\right] + \left(1 - n_t\right)\log\left[\rho_{t+1}^{q_B}\left(r\phi_t\right)\right].$

  - ○ $\rho_{t+1}^{q_B}$: MP of capital in “bad” state when the realized state is $j > n_t$, and no risky investment pays off;
  - ○ $\rho_{t+1}^{q_G}$: MP of capital in “good” state when the realized state is $j \leq n_t$.

$\Rightarrow$ Given Lemma 1 and $P_t^j(Z_t) = 1$, the portfolio choice problem can be written as:

$$\max_{\phi_t,\, F_t} \; n_t \log \; \left[\rho_{t+1}^{q_G} \left(r\phi_t + RF_t\right)\right] + (1 - n_t) \log \; \left[\rho_{t+1}^{q_B} \left(r\phi_t\right)\right] \tag{11}$$

$$\text{subject to,} \qquad \phi_t + n_t F_t = s_t^*. \tag{12}$$

– In solving programme (11) - (12),

  ○ n_t and ρ_{t+1}^j are taken as parametric by the agent, and

  ○ s_t^* is given by (10).

– Solution: The optimal portfolio choice is:

$$\phi_t^* = \left[\frac{(1 - n_t)\, R}{R - rn_t}\right] s_t^*, \tag{13}$$

$$F_t^{j*} = \begin{cases} F_t^* \equiv \left[\dfrac{R - r}{R - rn_t}\right] s_t^* & \text{for all } j \leq n_t \\ \\ 0 & \text{for all } j > n_t. \end{cases} \tag{14}$$

– Figure 3:

Expresses the aggregate demand for each risky asset, $aF^*(n_t)$, as a function of the proportion of securities that are offered, n_t.

  ◦ Obtained by aggregating (14) over all agents.

– Demand for each security grows as the measure of open sectors increases.

  ◦ When more securities are available, the risk diversification opportunities improve and consumers become willing to

    · reduce their investments in the safe asset, and

    · increase their investments in the risky projects.

– Equations (10), (13), and (14) completely characterize the *second-stage* decision rules of savers.

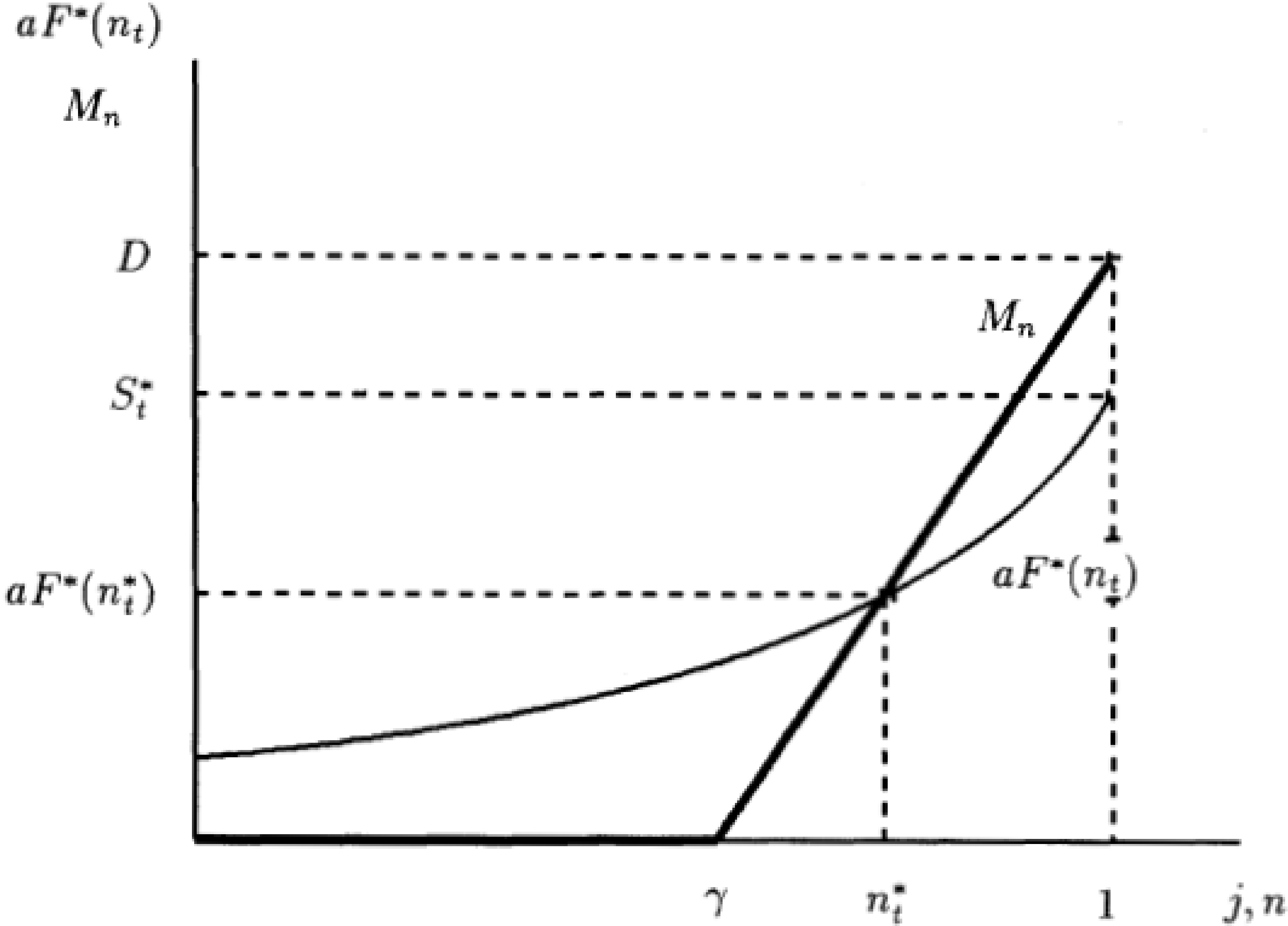

FIG. 3.—Static equilibrium

- **Static Equilibrium:**

  Recall that the static equilibrium is defined conditional on K_t.

  – Given K_t, (10) and (3) imply

  $$s_t^* = \frac{\beta}{1+\beta} w_t = \frac{\beta}{1+\beta}\left(\frac{W_t}{a}\right) = \frac{\beta}{1+\beta}\left[\frac{(1-\alpha)\,AK_t^{\alpha}}{a}\right].$$

  – Define $\Gamma \equiv \left(\frac{\beta}{1+\beta}\right) A\,(1-\alpha)\,.$

  – Then aggregate savings, $as_t^* = \Gamma K_t^{\alpha}$.

  – When $K_t > (D/\Gamma)^{1/\alpha}$, aggregate savings $> D$,

  ∘ there are enough funds to open all the projects, $\Rightarrow n_t^*\,(K_t) = 1$.

  – When $K_t \leq (D/\Gamma)^{1/\alpha}$, aggregate savings $\leq D$,

  ∘ $n_t^*\,(K_t) < 1$; only projects in $[0, n_t^*\,(K_t)]$ are open.

- Figure 3: $n_t^* (K_t) < 1$ is shown as the intersection of
  - the aggregate demand curve for each risky asset, $aF^* (n_t)$, and
  - the curve tracing the minimum size requirements, M_n.
- When $n > n_t^*$, if one proposes to open one more sector,
  - each agent invests more in risky projects,
  - but not sufficient to cover the minimum size requirement of the proposed sector.
- When $n < n_t^*$, if one proposes to open one more sector,
  - can raise enough funds, and
  - make some positive profit v.
- Thus, the equilibrium must be at n_t^*.
- Proposition 1 below characterizes the static equilibrium conditional on K_t.
- **Assumption 1.** $R \geq (2 - \gamma) r$.
  - Assumption 1 is important in ensuring uniqueness of the static equilibrium.

• **Proposition 1.** *Suppose that Assumption 1 holds and let*

$$n_t^*\left(K_t\right)=\begin{cases}\dfrac{\left(R+r\gamma\right)-\left\{\left(R+r\gamma\right)^2-4r\left[\left(R-\gamma\right)\left(1-\gamma\right)\frac{\Gamma}{D}K_t^{\alpha}+\gamma R\right]\right\}^{\frac{1}{2}}}{2r} & \text{if } K_t\leq\left(\dfrac{D}{\Gamma}\right)^{\frac{1}{\alpha}} \\ 1 & \text{if } K_t>\left(\dfrac{D}{\Gamma}\right)^{\frac{1}{\alpha}}.\end{cases} \tag{15}$$

Then there exists a unique equilibrium such that,

– *in the first stage*,

  ○ *for all* $h\in\Omega_t$, *either* $Z_{h,t}^*=\varnothing$ *or* $Z_{h,t}^*=\left(j,1\right)$, *where* $j\in\left[0,n_t^*\right]$; *and*

  ○ *for all* $j\in\left[0,n_t^*\right]$, *there exists* $h\in\Omega_t$ *such that* $Z_{h,t}^*=\left(j,1\right)$.

– *In the second stage*, $s_t^*=\dfrac{\beta}{1+\beta}\left[\dfrac{\left(1-\alpha\right)AK_t^{\alpha}}{a}\right]$,

and ϕ_t^*, F_t^{j*} *are given by* (13) *and* (14).

– *Factor returns are given by* (3) *and* (4).

- **Dynamic Equilibrium:**
  - Proposition 1 characterizes the (static) equilibrium allocation & prices for given K_t.
  - A dynamic equilibrium is a sequence of static equilibria linked to each other through the law of motion of K_t.
  - By Proposition 1, each agent's portfolio consists of
    - an equiproportionate investment F_t^* in all projects $j \in J_t(Z_t^*) = [0, n_t^*]$ (measure of $J_t(Z_t^*) = n_t^*$), and
    - an amount ϕ_t^* in the riskless asset.

$\Rightarrow$ The return profile of an agent is:

$$\begin{cases} r\phi_t^* & \text{with probability } 1 - n_t^* \\ r\phi_t^* + RF_t^* & \text{with probability } n_t^*. \end{cases}$$

$\Rightarrow$ The aggregate return profile is:

$$\begin{cases} ar\phi_t^* & \text{with probability } 1 - n_t^* \\ a\left(r\phi_t^* + RF_t^*\right) & \text{with probability } n_t^*. \end{cases}$$

- This becomes the profile of next period's capital, K_{t+1}.

- **Equilibrium Law of Motion:**
  - The equilibrium law of motion of K_t is:

$$K_{t+1} = \begin{cases} \dfrac{r\left(1-n_t^*\right)}{R-rn_t^*} R\Gamma K_t^\alpha & \text{with probability } 1-n_t^* \\ \\ R\Gamma K_t^\alpha & \text{with probability } n_t^*, \end{cases} \tag{16}$$

    where $n_t^* = n_t^*\left(K_t\right)$ is given by (15).

  - K_t follows a Markov process in which K_{t+1} depends on whether the economy is lucky in period t,
    - which happens when the risky investments pay off, with probability n_t^*.
  - The probability of being lucky, $n_t^*\left(K_t\right)$, changes over time.
    - As the economy develops,
      - it can afford to open more sectors,
      - the probability of transferring a large capital stock to the next period increases.

- **Dynamics of Development:**

  Define two reference steady states:

  – **QSSB:** The "quasi steady state" of an economy that *always has unlucky draws*.

  ○ An economy would converge to this quasi steady state if it follows the optimal investments characterized above,

  · but the sectors invested never pay off because of bad luck.

  – **QSSG:** The "quasi steady state" of an economy that *always receives good news*.

  ○ The sectors invested always pay off.

  – The capital stocks of these two quasi steady states are:

$$
\begin{aligned}
K^{\mathsf{QSSB}} &= \left\{ \frac{r\left[1 - n^*\left(K^{\mathsf{QSSB}}\right)\right]}{R - rn^*\left(K^{\mathsf{QSSB}}\right)} R\Gamma \right\}^{\frac{1}{1-\alpha}}, \\
K^{\mathsf{QSSG}} &= (R\Gamma)^{\frac{1}{1-\alpha}}.
\end{aligned}
\tag{18}
$$

– If $n^*\left(K^{\text{QSSG}}\right) = 1$, that is, uncertainty is completely removed,

$\Rightarrow$ there would never be bad news upon reaching K^{QSSG},

$\Rightarrow$ the good quasi steady state would be a *real* steady state, denoted by K^{SS}:

· a point, if reached, from which the economy would never depart.

$\Rightarrow$ K^{SS} exists if savings corresponding to K^{QSSG} is sufficient to ensure a balanced portfolio of investments of at least D in all intermediate sectors:

$$K^{\text{QSSG}} > \left(\frac{D}{\Gamma}\right)^{\frac{1}{\alpha}},$$

that is,

$$D < \Gamma^{\frac{1}{(1-\alpha)}} R^{\frac{\alpha}{(1-\alpha)}}. \tag{19}$$

- Figure 4 describes the dynamics of development.
  - Very low levels of capital (**Region I**):
    - ○ Positive growth even conditional on bad news.
      - · Both 'bad draws' and 'good draws' curves lie above the 45-degree line.
  - **Region II:**
    - ○ Positive growth conditional only on good draws.
      - · The 'bad draws' curve is below the 45-degree line.
  - Regions I and II are separated by K^{QSSB}.
    - ○ Not a steady state, but the economy will spend some time around it.
      - · When below, an economy grows toward it.
      - · When above, output $\downarrow$ upon receiving bad shocks.
      - · Probability of bad news very high when just above.
      - · As good news is received, $K \uparrow$, and probability of a further lucky draw $\uparrow$.
      - · Even when it grows, the economy is still exposed to large undiversified risks and experiences some setbacks.

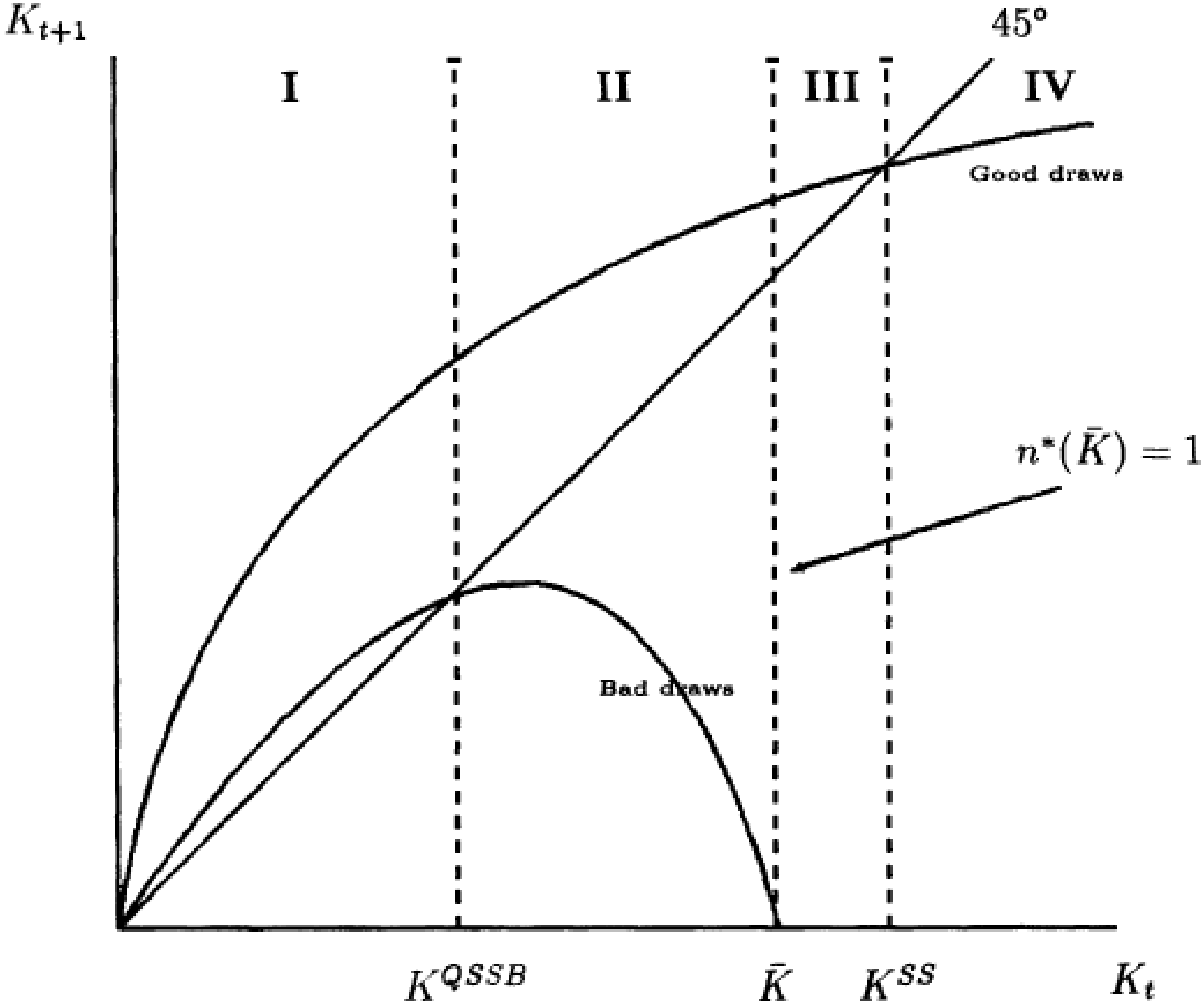

FIG. 4.—Capital accumulation

– The economy eventually enters **Region III** (provided (19) holds):

  ◦ All idiosyncratic risks are removed;

    · all sectors are open;

    · equal amount is invested in all sectors.

  ◦ There is deterministic convergence to K^{SS}.

• We have the following proposition summarizing the dynamics of development:

Proposition 2.

Suppose that (19) *is satisfied. Then* $\operatorname*{plim}_{t\to\infty} K_t = K^{\mathsf{SS}}$.

– When (19) is satisfied, the equilibrium stochastic process has a unique ergodic set, which is just a point, K^{SS}.

– Take-off will occur almost surely, though it will take longer and may be painfully slow for unfortunate countries.

- **Variability of Growth Rates:**
  - Recall the equilibrium law of motion of K_t:

$$K_{t+1} = \begin{cases} \dfrac{r\left(1-n_t^*\right)}{R-rn_t^*} R\Gamma K_t^{\alpha} & \text{with probability } 1-n_t^* \\ \\ R\Gamma K_t^{\alpha} & \text{with probability } n_t^*. \end{cases}$$

  - Taking logs, rewrite the law of motion of K_t as

$$\Delta \log \left(K_{t+1}\right) = \begin{cases} \log\Gamma + (\alpha-1)\log\left(K_t\right) + \log\left[\dfrac{r\left(1-n_t^*\right)}{R-rn_t^*} R\right] & \text{with probability } 1-n_t^* \\ \\ \log\Gamma + (\alpha-1)\log\left(K_t\right) + \log R & \text{with probability } n_t^*. \end{cases}$$

  - Define the random variable:

$$\sigma\left(n^*\left(K_t\right)\right) = \begin{cases} \dfrac{r\left(1-n_t^*\right)}{R-rn_t^*} R & \text{with probability } 1-n_t^* \\ \\ R & \text{with probability } n_t^*. \end{cases}$$

⇒ The law of motion of K_t becomes

$$\Delta \log \ (K_{t+1}) = \log \Gamma + (\alpha - 1) \log (K_t) + \log [\sigma (n^* (K_t))] \,. \tag{20}$$

⇒ Capital (and output) growth volatility is entirely determined by the stochastic component σ

- ○ after removal of the deterministic "convergence effects" induced by the neoclassical technology.

– For growth variability, a natural candidate is the variance of σ conditional on the proportion of sectors open.

– The random variable $\sigma\left(n^*\left(K_t\right)\right)$ defines the "total factor productivity" (conditional on the proportion of sector open).

– The expected "total factor productivity" is:

$$\sigma^e\left(n^*\left(K_t\right)\right)=\left(1-n^*\right)\frac{r\left(1-n^*\right)}{R-rn^*}R+n^*R. \tag{17}$$

∘ The expected productivity of an economy depends on its level of development and diversification.

∘ As $n^*\uparrow$, $\sigma^e\left(n^*\left(K_t\right)\right)\uparrow$.

– Define the variance of σ given K_t as V_n.

∘ Want to determine how this volatility measure evolves as a function of n^* (and K).

∘ Two forces to consider:

(i) As economy develops, more savings are invested in risky assets;

(ii) As more sectors open, idiosyncratic risks are better diversified.

– We have $V_n \equiv var\left[\sigma\left(n^*, \cdot\right) \mid n^*\right] = n^*\left(1 - n^*\right)\left[\frac{R\left(R - r\right)}{R - rn^*}\right]^2$.

∘ $\frac{\partial V_n}{\partial n^*} = \frac{\left[R\left(R - r\right)\right]^2}{\left(R - n^* r\right)^3}\left(R - 2Rn^* + n^* r\right)$.

⇒ sign $\left(\frac{\partial V_n}{\partial n^*}\right)$ = sign $\left(R - 2Rn^* + n^* r\right)$.

– If $n^* > \frac{R}{2R - r}$, then $\frac{\partial V_n}{\partial n^*} < 0$.

∘ We know from (15) that $n^* > \gamma$.

⇒ If $\gamma > \frac{R}{2R - r}$, then V_n is decreasing in n^* everywhere.

– Otherwise, it will be nonmonotonic: inverse U-shaped with $n^* = \frac{R}{2R - r}$ maximizing V_n.

– Since (15) implies $\frac{\partial n^*}{\partial K} \geq 0$ for all K, the following proposition follows.

• **Proposition 3.**

(a) If $\gamma \geq \dfrac{R}{2R-r}$, then $\dfrac{\partial V_n}{\partial K_t} \leq 0$, for all K_t.

(b) If $\gamma < \dfrac{R}{2R-r}$, then there exists K' such that $n^*(K') = \dfrac{R}{2R-r} < 1$, and

$$\frac{\partial V_n}{\partial K_t} = \begin{cases} \leq 0 & \text{for all } K_t \geq K' \\ > 0 & \text{for all } K_t < K'. \end{cases}$$

(a): Growth variance uniformly decreases with capital accumulation if

- ○ either γ is large enough,
- ○ or productivity of risky projects is sufficiently higher than the safe asset.

(b): Variability exhibits an inverse U-shaped relation with respect to the capital stock, and is decreasing for K_t large enough.

– Growth variability is decreasing in income at the later stages of development.

- **Quantitative Significance:**
  - Theory: Interaction between micro indivisibilities and risk aversion leads to a slow and random path of development.
    - Economy fluctuates in a state of low productivity before achieving full diversification and higher productivity.
  - Simulation Exercise: How important and long-lasting are these effects?
    - How many periods it takes for a set of simulated economies to start from K^{QSSB} and reach full diversification.
  - Parameter specifications:
    - 3 cases: $\alpha = 0.35, 0.5, 0.65.$
    - $R = 2$, $\gamma = 0.25$, $\Gamma = 2$.
    - r chosen so that $Y^{SS}/L = 15 \cdot \left(Y^{QSSB}/L\right)$ (15-fold difference between US and Senegal's per-capita income in 1985).
    - D adjusted to ensure $n\left(K^{\text{SS}}\right) = 1.$

- Runs 100 simulations in each case and calculates a number of statistics on the speed of convergence to full diversification.
- The **simulation results** are reported in Table 3.
  - Effects of indivisibilities are long-lasting; less so with strong diminishing returns.
  - Compare the convergence speed of deterministic neoclassical model with the average of this model:
    - $\alpha = 0.35$: speed decreases by a factor of 3;
    - $\alpha = 0.5$: speed decreases by a factor of 5;
    - $\alpha = 0.65$: speed decreases by a factor of 10.
  - The differences between the transition length of *lucky* versus *unlucky* countries are very large.
    - The tenth-most unlucky country would take more than 3 times as long to "industrialize" as the tenth-luckiest economy.
  - Overall, the effects described by this model appear very persistent and quantitatively significant.

TABLE 3

SIMULATIONS: SPEED OF CONVERGENCE

Case	Mean (T) (1)	Standard Deviation (T) (2)	$[Q_T(10\%),$ $Q_T(90\%)]$ (3)	$\min[T]$ (4)
$\alpha = .35$	19.47	11.56	[7, 30]	6
$\alpha = .50$	44.91	23.67	[21, 72]	9
$\alpha = .65$	116.24	63.04	[65, 195]	11

- **Inefficiency of the Decentralized Equilibrium:**
  - The decentralized equilibrium characterized above is not Pareto optimal.
  - The reason for the inefficiency is the presence of *pecuniary externality* due to missing markets.
    - ○ As an additional sector opens, all existing projects become more attractive relative to the safe asset because the amount of undiversified risks they carry is reduced.
  - $\Rightarrow$ Risk-averse agents are more willing to buy the existing securities.
  - Since each agent ignores his impact on others' diversification opportunities, the externality is not internalized.
  - It is important to reiterate at this point that in this model markets are not assumed to be missing;
    - ○ instead, the range of open markets is endogenously determined in equilibrium.

4.3 Concluding Remarks on Acemoglu and Zilibotti (1997)

- [These remarks are based on Mookherjee and Ray (2000), section 2.4.]
- There is a view on convergence that has not received as much attention in the literature as it deserves.
  - Consider the stochastic version of the neoclassical growth model.
    - It predicts convergence, to be sure, but how "soon" is the long-run?
    - Why do ergodic distributions receive so much attention, if they do not matter to the relevant future of current generations?
- Acemoglu and Zilibotti (1997) do not display this preoccupied focus on the steady state or on eventual convergence to it.
  - The appropriate stochastic process governing economic evolution *is* ergodic in their model.
  - They describe, instead, the arduous and difficult period that an economy can go through in the process of transition to this ergodic distribution.

- An added attraction of their model is the endogenous explanation for incompleteness of the market structure of an economy, and of how this evolves in the process of development.
  - Formalize the ideas of Scitovsky (1954) concerning the role of pecuniary externalities in the development process.

- What Acemoglu and Zilibotti (1997) bring out with particular power is the following observation:
  - Poor societies may languish in their state of poverty for an inordinately long period of time before finally receiving a series of lucky draws that pulls them into the limiting distribution.
    - This is because poor societies generate low levels of savings, an low levels of savings make for limited diversification.
    - It is therefore possible – perhaps even likely – that poor societies will be often faced with calamitous outcomes in which
      - very low incomes are generated, while these outcomes reinforce, in turn, the likelihood of a similar calamity being repeated in the next period.

- To be sure, sooner or later, there will be a string of lucky successes, which will create high incomes.
  - The resulting high savings will then create a self-reinforcing move towards greater diversification,
    - insulating the society from low income shocks in the future.

- Notice that *ultimately*, all societies converge.
  - But that convergence may be a long time coming,
    - and is not half as interesting as the lingering, self-reinforcing phase that precedes the diversification-based jump to maturity.

- This is why Acemoglu and Zilibotti (1997) is an excellent example of "self-reinforcement as slow convergence".

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6. Acemoglu, Daron and Fabrizio Zilibotti (1997), "Was Prometheus Unbound by Chance? Risk, Diversification and Growth", *Journal of Political Economy*, 105(4), 709-751.