Development From the Viewpoint of Nonconvergence: History versus Expectations

1. Introduction

- So far we have considered *expectation-driven inertial self-reinforcement* based on the existence of complementarities and increasing returns.
  - Complementarities (or pecuniary externalities) and increasing returns may result in multiple equilibria and coordination failure.
    - The economy ends up in one equilibrium or the other depending on self-fulfilling expectations.
- An important problem with theories of multiple equilibrium is that they carry an unclear burden of *history*.
  - Suppose, for instance, that an economy has been in a low-level investment trap for decades.
  - Nothing in the multiple equilibrium theory prevents that very same economy from abruptly shooting into the high-level equilibrium today.

- *History versus Expectations*:
  - There seems to be a presumption that, somehow, *history* pins down the equilibrium, and
    - makes it difficult for firms, individuals or sectors to free themselves in a coordinated way from the low-level equilibrium trap.
  - At the same time it is also asserted that if somehow the *expectations* of the economic agents involved could be changed,
    - movement would occur from one equilibrium to the other.
- Recall that we started to study theories based on multiple equilibria and coordination failure to formalize the ideas of Rosenstein-Rodan (1943) and Hirschman (1958).
- But Rosenstein-Rodan (1943) and Hirschman (1958) were certainly concerned with the issue of “stickiness” of equilibria, that is, the issue of “inertia” associated with *inertial self-reinforcement.*

- This has to do with the fact that at any given moment of time, a *particular* equilibrium is in force, and
  - has possibly been in force in that society in the medium- or long-run past.
- What causes the past to stick?
- How is a particular equilibrium pinned down by the force of historical inertia?
- What will it take to unpin it?

- Unfortunately, the multiple equilibrium or coordination-game paradigm is not of much use in this regard beyond the demonstration that multiplicities may exist.
  - In some sense, it avoids altogether any answer to the question:
    - why is one society less developed than another, and what can be done about it?
  - For this would require a theory of
    - where the pessimistic beliefs originally came from, or
    - how they could be manipulated by policy interventions.

- The paradigm is also at a loss for *explaining* historical inertia:
  - Repeat a multiple equilibrium story and numerous dynamic equilibria emerge,
    - including those in which the society jumps between the bad and good equilibria in all sorts of deftly coordinated ways.

- We lack good economic theory that actually identifies the “stickiness” of equilibria that Rosenstein-Rodan (1943) and Hirschman (1958) were concerned with.
  - A small body of literature exists on this topic:
    - Krugman (1991),
    - Matsuyama (1991),
    - Adsera and Ray (1998),
    - Frankel and Pauzner (2000).

2. Krugman (1991)

- In models with external economies, when there are multiple equilibria, there is an obvious question: which equilibrium actually gets established?

- In the literature there is a broad division into two camps on this question:

1. On one side is the belief that the choice among multiple equilibria is essentially resolved by *history*:
   - past events set the preconditions that drive to one or another steady state.
2. On the other side, is the view that the key determinant of choice of equilibrium is *expectations*:
   - there is a decisive element of self-fulfilling prophecy.

- The contribution of Krugman (1991) is twofold:

(a) it points out the importance of the history versus expectations distinction;

(b) it shows how the parameters of the economy determine the relative importance of history and expectations in determining equilibrium.

2.1 A Simple Model with Multiple Equilibria

- A one-factor (labour, L) economy which is able to produce two goods:
  - C, a good produced with *constant returns*;
  - X, a good whose production is subject to an *externality*.
    - Assume that the larger the labor force engaged in X production (L_X), the higher is labor productivity in that sector:

$$\pi = \pi(L_X). \tag{1}$$

- Small-country assumption:

  This economy is able to sell both C and X at fixed prices on world markets.

- Normalization: By choosing units of goods and labor, we can normalize so that
  - one unit of labor produces one unit of C, and
  - the value of that unit is one.

$\Rightarrow$ Wage rate in the C sector is unity.

- In the X sector productivity depends on industry employment.
  - Since the economies of scale are external, each firm treats labor productivity as constant.

⇒ (Perceived) marginal product = average product.

⇒ Wage rate in the X sector is equal to the average product:

$$w = \pi\left(L_X\right). \tag{2}$$

  - Given the normalization, w is the wage rate in X relative to that in C.

- **Assumption:** $\pi\left(0\right) < 1$, and $\pi\left(\bar{L}\right) > 1$.
  - $\bar{L}$ is the economy's total labor supply.
  - Wage rate in the X sector would be
    - lower than that in the C sector if nobody were employed in X,
    - higher if everyone were employed in X.

- **Multiple equilibrium:**

1. Nobody is employed in X ($\boldsymbol{L_X = 0}$):

   ◦ A worker considering producing X would find that she would receive a lower wage than she receives producing C.

   ⇒ There is an equilibrium in which the economy is specialized in the production of C.

2. Everyone is employed in the X sector ($\boldsymbol{L_X = \bar{L}}$):

   ◦ A worker considering producing C would find that this would involve a wage cut.

   ⇒ Specialization in X is also an equilibrium.

- Which equilibrium does the economy go to?
  - In expositions of this kind of model, one often appeals to a quasi-dynamic story of the kind illustrated in Figure I.
    - Assumption: Starting with some initial allocation of labor between the two sectors, labor moves toward the sector that offers the higher wage.
    - L_X^* denotes the employment in X when $w = 1$.
    - If the labor force in X is initially larger than L_X^*,

    $\Rightarrow$ the X sector will snowball until the economy is specialized in X.

    - If the labor force in X is initially smaller than L_X^*,

    $\Rightarrow$ the X sector will unravel, and the economy will end up specializing in C.
  - Thus *history*, which determines the initial conditions, determines the ultimate outcome.

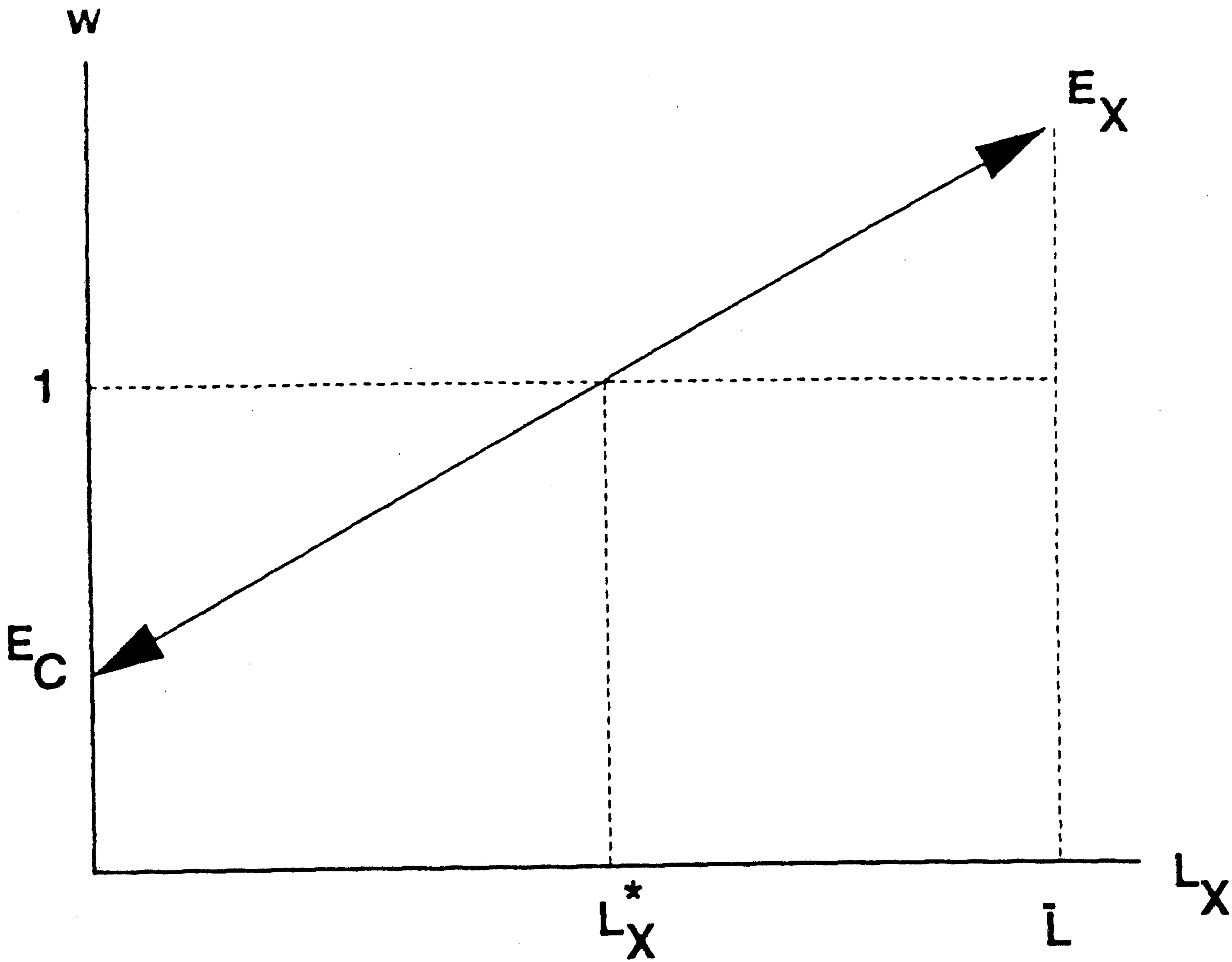

FIGURE I

- Problems with the quasi-dynamic story:

  Essentially the question is why labor should adjust slowly.

  - Suppose first that labor can move costlessly between the X and C sectors.
    - Then there is no reason why the initial distribution of labor should matter.
    - Whatever the initial position, all workers will move to the sector that they *expect* to yield the higher wage,
      - which is the sector that they *expect* all the other workers to move to.
    - Thus, in the absence of some cost of shifting labor, either equilibrium can be obtained as a *self-fulfilling prophecy*, whatever the initial position.
  - To make the initial position matter, then, it is necessary to introduce some cost of adjustment in shifting labor between sectors.

– As soon as we introduce this cost of adjustment,

  ◦ a worker's decision to shift between sectors becomes an *investment* decision,

    · which depends not only on the current wage differential but on *expected* future wage rates as well.

  ◦ But these future wage rates depend on the decisions of other workers;

    · if everyone expects many workers to move from C to X over time,

  $\rightarrow$ this will increase the attractiveness of moving from C to X even if there is no immediate effect on relative wage rates.

$\Rightarrow$ One cannot have dynamics without expectations.

  ◦ Once one has expectations playing a role, there is in this kind of model the possibility of *self-fulfilling prophecy*.

- Does this mean that the traditional view that history is crucial for determining equilibrium is completely wrong?
  - Is it always possible to reach either equilibrium if everyone expects it?
- The answer is no.
  - To see this, it is necessary to formulate the dynamics of the model explicitly.

2.2 Making the Model Dynamic

- The model is made explicitly dynamic by making the cost of shifting labor a function of the rate at which labor is moved between sectors:
  - Moving cost:

$$\frac{1}{2\gamma}\left(\dot{L}_X\right)^2,$$

    - γ is an inverse index of the cost of adjustment.

$\Rightarrow$ The national income of the economy at a given instant is

$$Y_t = \pi\left(L_X\right) \cdot L_X + \left(\bar{L} - L_X\right) - \frac{1}{2\gamma}\left(\dot{L}_X\right)^2. \tag{3}$$

- We suppose that individuals are able to borrow or lend freely on world markets at a given world interest rate r.

$\Rightarrow$ Their objective is to maximize the present value of output,

$$H = \int_0^\infty Y_t e^{-rt} dt. \tag{4}$$

- Note that (3) implies

$$\dot{L}_X = \sqrt{2\gamma \left[\pi\left(L_X\right) \cdot L_X + \left(\bar{L} - L_X\right) - Y_t\right]}. \tag{i}$$

- The dynamic optimization problem:

$$\underset{\{Y_t\}}{\text{Maximize}} \int_0^{\infty} Y_t e^{-rt} dt$$

subject to

$$\dot{L}_X = \sqrt{2\gamma \left[\pi\left(L_X\right) \cdot L_X + \left(\bar{L} - L_X\right) - Y_t\right]}.$$

  – Control variable: Y_t

  – State variable: L_X

- Current value Hamiltonian:

$$CVH = Y_t + q_t \cdot \sqrt{2\gamma \left[\pi\left(L_X\right) \cdot L_X + \left(\bar{L} - L_X\right) - Y_t\right]}$$

q_t: the shadow price placed on the "asset" of having a unit of labor in the X rather than the C sector.

- The first-order conditions are:

$$\frac{\partial CVH}{\partial Y_t} \;=\; 0, \tag{ii}$$

$$\frac{\partial CVH}{\partial L_X} \;=\; rq_t - \dot{q}, \tag{iii}$$

and the transversality condition:

$$\lim_{T\to\infty} \; q_T \cdot e^{-rT} \cdot L_X\left(T\right) = 0. \tag{iv}$$

- (ii) implies

$$1 - \frac{\gamma q_t}{\sqrt{2\gamma\left[\pi\left(L_X\right)\cdot L_X + \left(\bar{L} - L_X\right) - Y_t\right]}} = 0.$$

  – Using (i), this gives

$$\dot{L}_X = \gamma q_t. \tag{5}$$

– Labor moves at a rate determined by the equality of

∘ marginal moving costs ($\frac{\dot{L}_X}{\gamma}$), and

∘ a shadow price (q_t) that represents the difference in *private* value between having a unit of labor in the X sector and in the Y sector.

• (iii) implies

$$rq_t - \dot{q} = \frac{q_t \gamma \left[\pi\left(L_X\right) - 1\right]}{\sqrt{2\gamma \left[\pi\left(L_X\right) \cdot L_X + \left(\bar{L} - L_X\right) - Y_t\right]}} = \pi\left(L_X\right) - 1 \quad \text{[using (i) and (5)].}$$

that is,

$$\dot{q} = rq - \pi\left(L_X\right) + 1. \tag{8}$$

– Since individuals do not internalize the increasing returns to scale present in X production, they take π as given.

∘ This is used in calculating $\frac{\partial CVH}{\partial L_X}$.

- Integrating forward we derive the shadow price from (8):

$$q_t = \int_t^{\infty} (\pi - 1)\, e^{-r(\tau - t)} d\tau. \tag{6}$$

- Rearranging (8) we get

$$r = \frac{(\pi - 1) + \dot{q}}{q}. \tag{7}$$

$\Rightarrow$ Interest rate must equal to the rate of return on the shadow asset consisting of

- ○ the difference in current earnings between labor in the X and C sectors ($\pi - 1$), and
- ○ the rate of capital gains on the shadow asset ($\dot{q}$).

- Equations (5) and (8) define a dynamic system in (L_X, q) space.
  - In Figures II and III the qualitative laws of motion of this system are shown by the small arrows.

- (5) $\Rightarrow \dot{L}_X = 0$ if $q = 0$;
  - whenever q is positive, L_X is rising,
  - whenever q is negative, L_X is falling.
- (8) $\Rightarrow \dot{q} = 0$ for the combinations of (L_X, q) such that

$$q = \frac{1}{r} \left[\pi \left(L_X \right) - 1 \right].$$

  - In Figures II and III these combinations are represented by the upward-sloping line marked $\dot{q} = 0$.
  - For these combinations of (L_X, q), q equals the capitalized value of a constant wage differential at the current rate.
  - A higher value of q can result only if q is expected to rise.
  - A lower value only if q is expected to fall.
- The two lines, $\dot{L}_X = 0$ and $\dot{q} = 0$, cross at $q = 0$, where $\pi \left(L_X \right) - 1 = 0$, that is, at $L_X = L_X^*$.

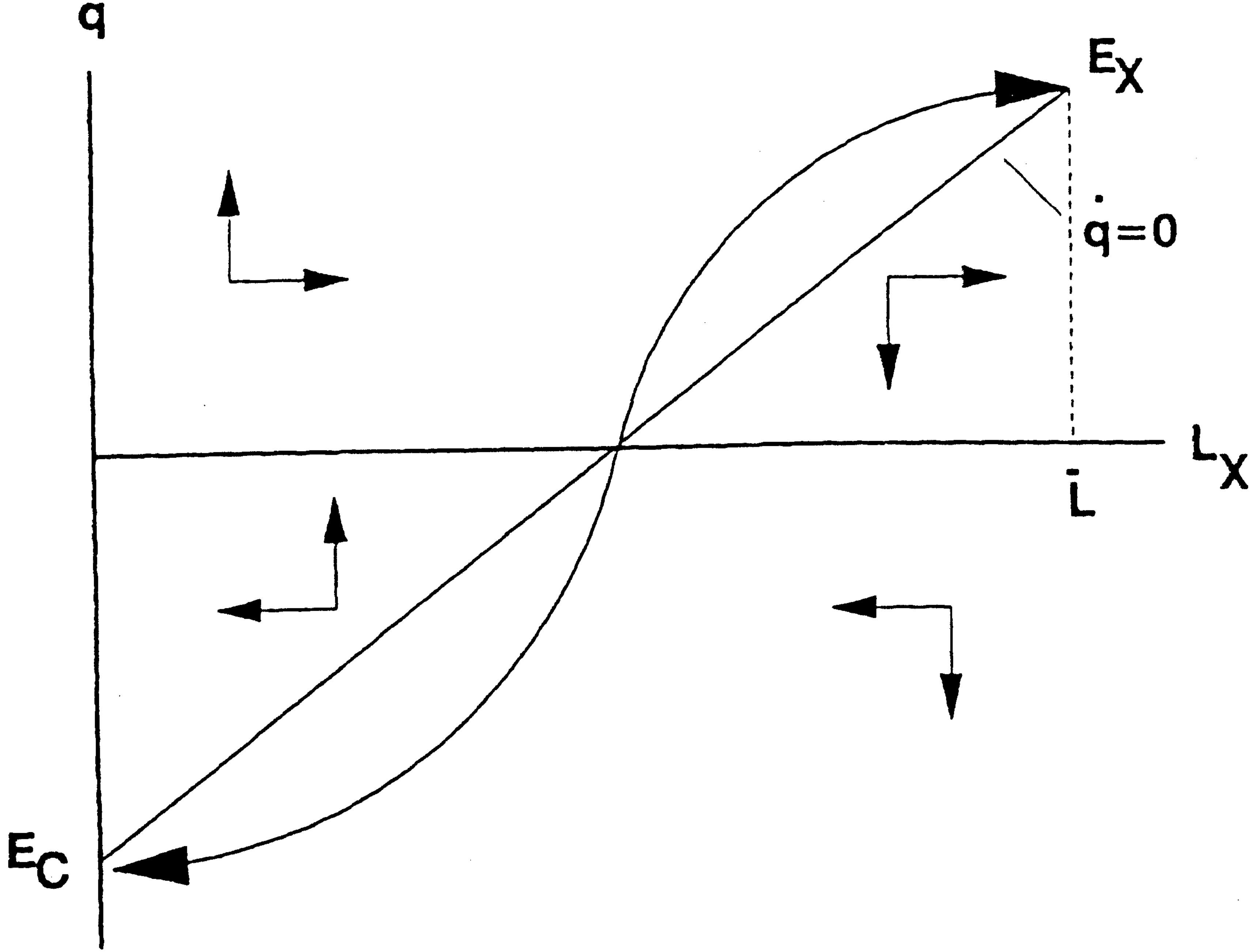

FIGURE II

- There are, two possible long-run equilibria of this model.
  - At one, illustrated by E_C, the economy specializes completely in production of C;
  - At the other, E_X, the economy specializes in X.
  - At each equilibrium q equals the present value of the difference between
    - what workers actually earn and
    - what an individual worker would earn if she decided to produce the other good indefinitely.
  - Note that ($q = 0$ and $L_X = L_X^*$) is an unstable steady state.
- We now ask what paths can lead to these equilibria, consistent with the laws of motion.
- Given the qualitative laws of motion shown in Figure II,
  - it is clearly possible to draw paths leading to the two equilibria that form the S-shaped locus shown in the figure.
    - The right half of the S represents a path that leads to E_X;
    - the left half a path that leads to E_C.

- Dynamic behaviour corresponding to the paths in Figure II:
  - Suppose that we are given an initial allocation of labor between the two sectors.
    - Then the initial value of q must be set at the unique value that puts the economy on the S-shaped curve.
    - From that point on, the economy would simply obey the dynamics,
      - converging to one or the other long-run equilibrium.
  - If $L_X > L_X^*$ initially, then the economy would gradually move to E_X;
  - If $L_X < L_X^*$ initially, then the economy would gradually converge to E_C.
  - Thus, the dynamics illustrated in Figure II confirm the quasi-dynamic story illustrated in Figure I.
    - Adding an explicit description of the decision to reallocate resources and of the implied role of expectations does not change much.

- The paths shown in Figure II are not, however, the only possible ones consistent with the qualitative laws of motion.

- Figure III illustrates that instead of a monotonic approach to each long-run equilibrium, the economy might follow equilibrium paths consisting of two interlocking spirals:
  - The spirals wind outward from the center of the figure and
    - eventually separate to head for the two long-run equilibria.
  - These paths do indeed obey the laws of motion indicated by the small arrows.
  - Also the two spirals never cross one another.
    - There is a unique path from any point.
    - Since the two paths end up in different places, they must not have any points in common.

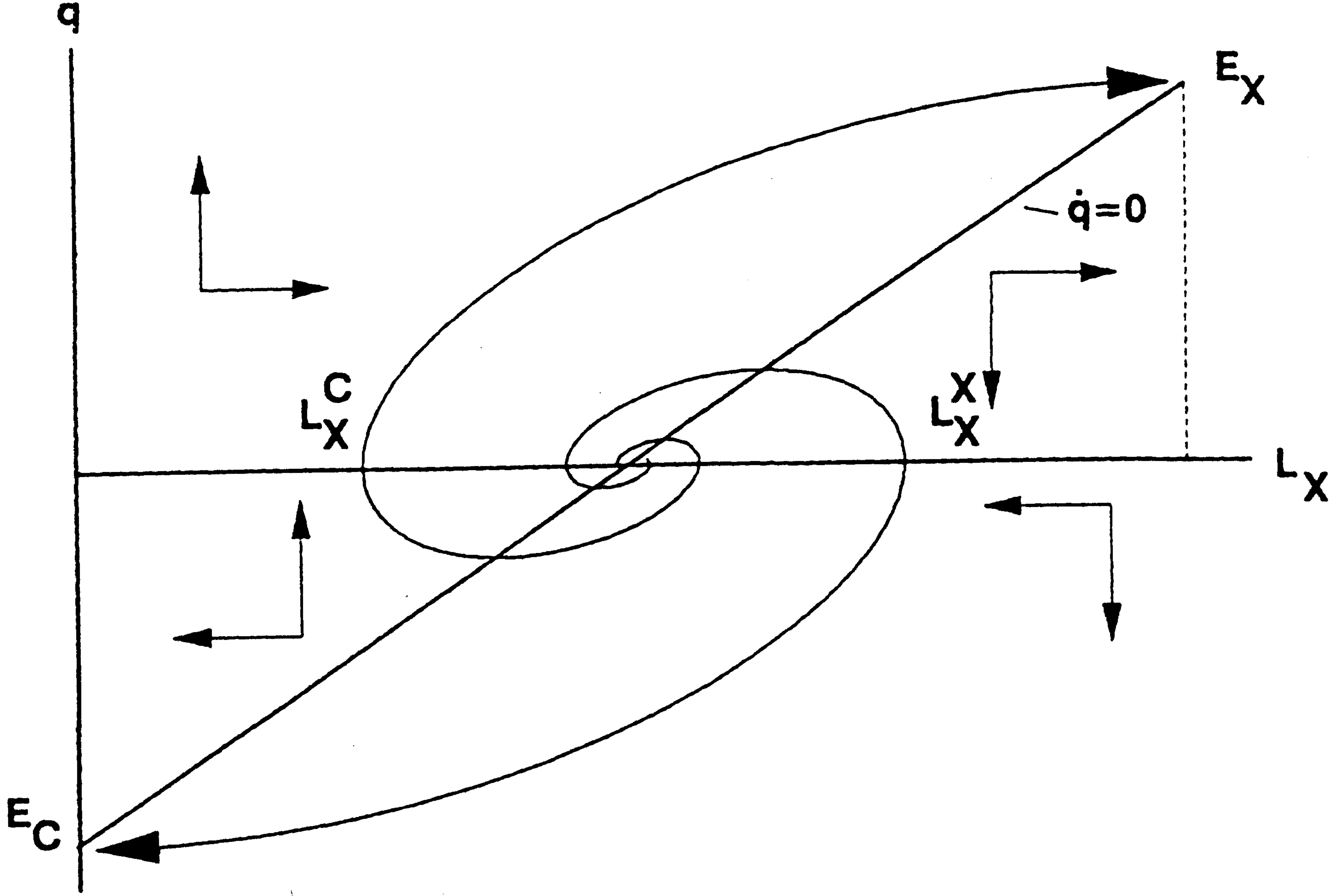

FIGURE III

2.3 S-Curve versus Spirals

- Let us first confirm that both Figure II and Figure III are possible descriptions of equilibrium paths, and find out under what circumstances each description prevails.
- For that purpose it is necessary to place some more structure on the model.
  - The simplest structure is a linear one:
    - we suppose that the function $\pi(L_X)$ takes the particular form,

$$\pi(L_X) = 1 + \beta(L_X - L_X^*). \tag{9}$$

- With this structure, the dynamic system defined by (5) and (8) constitutes a pair of linear differential equations:

$$\begin{pmatrix} \dot{L}_X \\ \dot{q} \end{pmatrix} = \begin{bmatrix} 0 & \gamma \\ -\beta & r \end{bmatrix} \begin{pmatrix} L_X \\ q \end{pmatrix} + \begin{pmatrix} 0 \\ \beta L_X^* \end{pmatrix}.$$

  - The behaviour of the system is determined by the characteristic roots of the matrix

$$\begin{bmatrix} 0 & \gamma \\ -\beta & r \end{bmatrix}.$$

– The characteristic roots are:

$$\rho = \frac{1}{2}\left[r \pm \sqrt{r^2 - 4\beta\gamma}\right]. \tag{10}$$

- Two qualitative cases:

1. $\boldsymbol{r^2 > 4\beta\gamma}$:

⇒ There are two real positive roots.

⇒ The system is unstable and must steadily diverge from ($q = 0$ and $L_X = L_X^*$).

∘ The possible paths to the two equilibria, ($q = -\dfrac{\beta L_X^*}{r}$ and $L_X = 0$) and ($q = \dfrac{\beta\left(\bar{L} - L_X^*\right)}{r}$ and $L_X = \bar{L}$), form the S-curve in Figure II.

2. $\boldsymbol{r^2 < 4\beta\gamma}$:

⇒ There are two complex roots with positive real parts.

⇒ The system is unstable, but diverges from ($q = 0$ and $L_X = L_X^*$) in expanding oscillations.

∘ The possible paths form the interlocking spirals of Figure III.

- What is the economic meaning of the case illustrated in Figure III?
  - Note that the spirals define a range of values of L_X from L_X^C to L_X^X, from which either long-run equilibrium can be reached.
  - Which one is reached depends on expectations.
    - ◦ For any initial position in this range, there exists at least one set of *self-fulfilling expectations* leading to either long-run outcome.
      - · In particular, there are the simple paths defined by the outer arms of the two spirals that lead most rapidly to either long-run position.

⇒ The case of complex roots, which corresponds to Figure III, is also the case in which over some range *expectations rather than history are decisive*.

- We refer to this range of L_X from which either equilibrium can be reached, $\left[L_X^C, L_X^X\right]$, as the *overlap*.
- Outside the overlap, *history is decisive*:
  - ◦ For $L_X < L_X^C$, there is a unique path leading to E_C;
  - ◦ For $L_X > L_X^X$, there is a unique path leading to E_X.

- Inside the overlap there may be more than one set of expectations that leads to each equilibrium.
  - If people expect a direct path to E_X, that will happen.
    - But, for some values of L_X, there are also self-fulfilling cyclical paths.
    - Indeed, as L_X gets close to L_X^*, there is an infinite number of possible paths in each direction.
  - Thus, the possible dynamics are surprisingly complex.
- In general, many things can happen if there is an overlap and the initial position of the economy is inside it.
  - All that we can usefully say is that when there is an overlap,
    - the economy must eventually go to one equilibrium or the other;
    - self-fulfilling expectations can lead it in either direction.
- It is clear from the above analysis that the basic question of the respective roles of history and expectations resolves itself in this model into the question of the overlap:
  - Does an overlap exist, and how wide is it?

2.4 Existence and Size of the Overlap

- If there is no overlap, then history is always decisive in this model.
- If there is an overlap, then
  - history determines the outcomes if L_X lies outside the overlap, but
  - expectations decide the outcome if L_X lies inside.
- So we must be interested in the factors determining the existence and width of the overlap.
- Recall that an overlap exists if and only if $r^2 < 4\beta\gamma$.
- The existence of an overlap depends on three parameters:
  - r: the interest rate,
  - β: represents the strength of the external economies,
  - γ: measures the speed of adjustment.

1. If r is sufficiently large, then there will be no overlap, and history will dominate expectations.
   – r is sufficiently large $\Rightarrow$ the future is heavily discounted,

   $\Rightarrow$ individuals will not care much about the future actions of other individuals, and

   ∘ this will eliminate the possibility of self-fulfilling prophecies.

2. A small β eliminates the possibility of self-fulfilling expectations.
   – β small $\Rightarrow$ external economies are small.

   $\Rightarrow$ there will not be enough interdependence among decisions.

3. If γ is small, so that the economy adjusts slowly, then history is always decisive.
   – If adjustment is slow, factor rewards will be near current levels for a long time whatever the expectations,

   $\Rightarrow$ factor reallocation always follows current returns.

- We expect that the same factors – r, β and γ – will also determine the width of the overlap.
  - Determining the width of the overlap explicitly, even in the linear case, is an algebraic nightmare.

- The effect of γ on the width of the overlap may be demonstrated using a simple geometric argument.
  - Figure IV shows the outermost part of a spiral converging to E_X.
  - Point A on this spiral where it crosses $q = 0$ determines the lower boundary of the overlap.
  - Suppose $\gamma \uparrow$.
    - ○ (8) $\Rightarrow$ $\dot{q}$ remains unaffected.
    - ○ But (5) $\Rightarrow$ at any positive q, the rate at which L_X rises would be increased.

  $\Rightarrow$ A path starting at point A would start to diverge to the right of the original path leading to E_X, and

  · would do so increasingly over time.

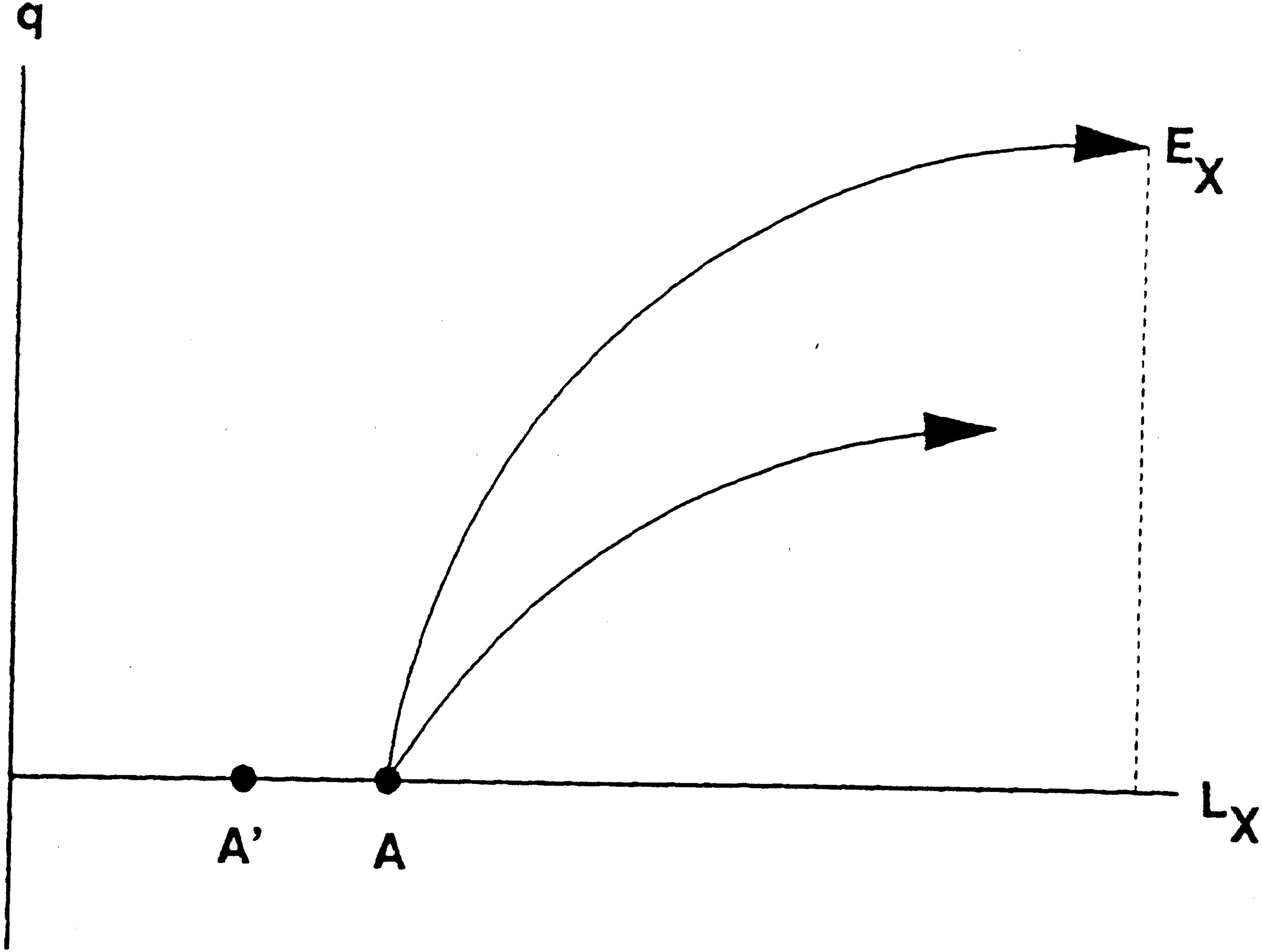

FIGURE IV

$\Rightarrow$ In order to reach E_X with a higher γ we would have to start somewhere farther to the left of A, say at A'.

$\Rightarrow$ Width of the overlap increases.

- The result that as $\gamma \uparrow$, width of the overlap increases, should not be surprising.
  - We noted at the beginning that in the absence of adjustment costs history is irrelevant:
    - any equilibrium can be reached through convergent expectations.
  - We now see that the slower the rate at which the economy adjusts, the more likely it is that history matters;
    - if adjustment is slow enough, history is always decisive.