Econ 277B: Economic Development II — Tridip Ray
Semester II, 2020-21 — ISI, Delhi

Final Exam: Question 1 (12 August 2021)

- Maximum marks: **35**
- Time allotted (including uploading on Moodle): **75 minutes**

- Consider a small open economy in a one-good world. The good can be used for either consumption or investment. The good can be produced by two technologies, one which uses skilled labour and capital, and the other using unskilled labour only. Production in the *skilled* labour sector is described by

$$Y_t^s = F\left(K_t, L_t^s\right),$$

where Y_t^s is output in skilled labour sector at time t, K_t is the amount of capital at time t and L_t^s is labour input in skilled labour sector at time t. F is a concave production function with constant returns to scale. Production in the *unskilled* labour sector is described by

$$Y_t^n = w_n \cdot L_t^n,$$

where Y_t^n and L_t^n are output and unskilled labour input respectively, and $w_n > 0$ is the marginal product of unskilled labour.

- Individuals live for two periods each in overlapping generations. They can either work as unskilled in both periods of life or invest in human capital when young and be skilled workers in the second period of life. An individual supplies one unit of labour in each of the working periods.

- Acquiring skill requires a fixed time investment of one period and a fixed resource investment of $h > 0$ units. However, skill formation is an uncertain process in the sense that even after spending the time and resources a person may *not* be successful in acquiring skill. In particular, a person who makes the required time and resource investments will be a skilled worker in the next period with probability $0 < p < 1$, and will remain unskilled with probability $(1 - p)$.

- Each individual has one parent and one child. In each generation there is a continuum of individuals of size normalized to 1. The population at time t is described by a (probability) distribution function $G_t(x)$, which gives the measure of the population with wealth less than x. People care about their children and leave them bequests. It is also assumed that people consume in the second period of life only. The individual utility function is

$$u = c^{\alpha} b^{1-\alpha}, \quad 0 < \alpha < 1,$$

  where c is consumption in second period and b is bequest. Denoting the second-period income realization (sum of wage and interest incomes, if there is any) by y, indirect utility then takes the form δy, where $\delta \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

- Capital is assumed to be perfectly mobile so that both firms and individuals have perfect access to international capital markets. The world rate of interest, r, is constant over time. Individuals can lend any amount at this rate. *Credit market imperfection* shows up while borrowing. A borrowing individual can evade debt repayments. Lenders can avoid such defaults by keeping track of borrowers, but this is a costly affair. These costs create a difference between the borrowing and lending interest rates: the borrowing interest rate for individuals, i, is strictly higher than the lending rate r. Unlike individuals, firms are unable to evade debt repayment, due to reasons such as immobility, reputation, etc. Hence, firms can borrow at the lenders' interest rate r.

1. [3 marks]

   - Argue that $\frac{K_t}{L_t^s}$ is constant over time.
   - Argue that wage of skilled labour, w_s, is also constant over time.

- Next we describe individual optimal decisions and the resulting wealth dynamics.

2. [32 marks: $5+3+2+5+9+8$]

   (a) Derive the expression for *expected* indirect utility of an individual who inherits a wealth x_t from her parent and

   - decides to work as unskilled and not invest in human capital,
   - invests in human capital and is a lender ($x_t \geq h$),
   - invests in human capital and is a borrower ($x_t < h$).

(b) Define $\overline{w} \equiv pw_s + (1-p)\,w_n$. Argue that if $\overline{w} - h\,(1+r) < w_n\,(2+r)\,,$ then all individuals prefer to work as unskilled.

- Since this is a case with limited interest, we assume that

$$\overline{w} - h\,(1+r) \geq w_n\,(2+r)\,. \qquad \text{(Assumption 1)}$$

(c) Argue that borrowers (individuals with $x_t < h$) invest in human capital as long as

$$x \geq \frac{w_n\,(2+r) + h\,(1+i) - \overline{w}}{i-r} \equiv f.$$

- We make the following assumption to ensure that $f > 0$:

$$w_n\,(2+r) > \overline{w} - h\,(1+i)\,. \qquad \text{(Assumption 2)}$$

(d) In view of your answers to parts (a) to (c) above, describe the corresponding optimal choice of occupation for the different ranges of wealth levels: $x_t < f$; $f \leq x_t < h$; $x_t \geq h$.

(e) Derive the dynamic equation showing the corresponding wealth (bequest) dynamics and draw the phase diagram (plot x_t on x-axis and x_{t+1} on y-axis). [You must depict the *actual* bequest lines (with 'success' and 'failure'). Show the *expected* bequest line also for reference.]

- To draw the bequest lines, use the following two assumptions:

$$(1-\alpha)\,(1+r) < 1, \qquad \text{(Assumption 3)}$$

$$(1-\alpha)\,(1+i) > 1. \qquad \text{(Assumption 4)}$$

(f) Comment on the nature of long-run wealth dynamics. In particular, is it possible to have a threshold level of initial wealth such that dynasties below this threshold go to a different steady-state compared to the dynasties above this threshold? Explain your answer very carefully.

[**Hint:** Pay attention to the amount of bequest under no investment, $x = 0$, and that under investment but with failure at $x = h$.]

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Final Exam: Question 2 (12 August 2021)

- Maximum marks: **25**
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- Consider a small open economy in a one-good world. The good can be used for either consumption or investment. The good can be produced by two technologies, one which uses skilled labour and capital, and the other using unskilled labour only. Production in the *skilled* labour sector is described by

$$Y_t^s = F\left(K_t, L_t^s\right),$$

where Y_t^s is output in skilled labour sector at time t, K_t is the amount of capital at time t and L_t^s is labour input in skilled labour sector at time t. F is a concave production function with constant returns to scale. Production in the *unskilled* labour sector is described by

$$Y_t^n = w_n \cdot L_t^n,$$

where Y_t^n and L_t^n are output and unskilled labour input respectively, and $w_n > 0$ is the marginal product of unskilled labour.

- Individuals live for two periods each in overlapping generations. They can either work as unskilled in both periods of life or invest in human capital when young and be skilled workers in the second period of life. An individual supplies one unit of labour in each of the working periods.

- Acquiring skill requires a fixed time investment of one period and a fixed resource investment of $h > 0$ units. However, skill formation is an uncertain process in the sense that even after spending the time and resources a person may *not* be successful in acquiring skill. In particular, a person who makes the required time and resource investments will be a skilled worker in the next period with probability $0 < p < 1$, and will remain unskilled with probability $(1 - p)$.

- Each individual has one parent and one child. In each generation there is a continuum of individuals of size normalized to 1. The population at time t is described by a (probability) distribution function $G_t(x)$, which gives the measure of the population with wealth less than x. People care about their children and leave them bequests. It is also assumed that people consume in the second period of life only. The individual utility function is

$$u = c^{\alpha} b^{1-\alpha}, \quad 0 < \alpha < 1,$$

where c is consumption in second period and b is bequest. Denoting the second-period income realization (sum of wage and interest incomes, if there is any) by y, indirect utility then takes the form δy, where $\delta \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

- Capital is assumed to be perfectly mobile so that both firms and individuals have perfect access to international capital markets. The world rate of interest, r, is constant over time. Individuals can lend any amount at this rate. *Credit market imperfection* shows up while borrowing. A borrowing individual can evade debt repayments. Lenders can avoid such defaults by keeping track of borrowers, but this is a costly affair. These costs create a difference between the borrowing and lending interest rates: the borrowing interest rate for individuals, i, is strictly higher than the lending rate r. Unlike individuals, firms are unable to evade debt repayment, due to reasons such as immobility, reputation, etc. Hence, firms can borrow at the lenders' interest rate r. It is easy to argue that wage of skilled labour, w_s, is also constant over time.

- Define $\overline{w} \equiv pw_s + (1-p)w_n$. We make the following assumptions.

$$\overline{w} - h(1+r) \geq w_n(2+r). \qquad \text{(Assumption 1)}$$

$$w_n(2+r) > \overline{w} - h(1+i). \qquad \text{(Assumption 2)}$$

$$(1-\alpha)(1+r) < 1, \qquad \text{(Assumption 3)}$$

$$(1-\alpha)(1+i) > 1. \qquad \text{(Assumption 4)}$$

- In our analysis so far (in Question 1) we have assumed that each parent finances her child's education *privately* (with the help of the imperfect credit market, of course). Consider now the following intergenerational *tax-and-transfer policy* that works very similar to a *public* education system. In each period t, the government taxes the second-period income (sum of wage and interest incomes, if there is any) of the old agents at the rate τ_t and transfers the tax revenue to the young agents in equal amounts. If we use R_t to denote the equal amount of transfer received by each young agent in period t, then

$$R_t = \tau_t \cdot \left[\int y_t dF_t(y_t)\right],$$

where $F_t(y)$ is the distribution function of the second-period income (sum of wage and interest incomes, if there is any). The tax rate τ_t is endogenously determined in each period through majority voting. The young agent treats this transfer R_t in exactly the same way as she treats her bequest in the private education regime.

In this case an individual's decision-making involves a two-step procedure. She first chooses the level of consumption that maximizes her utility for a given post tax income, treating the transfer received by her child as given. Since marginal utility from consumption is positive, this would imply that she consumes her entire post tax income, that is, $c_t = (1 - \tau_t) y_t$. Thus, the utility function of a period-t old agent is a function of the tax rate, τ_t. In the next step, she chooses the tax rate, τ_t, so as to maximize this utility function.

(a) [5 marks] Derive the utility function of a period-t old agent as a function τ_t and determine her preferred tax rate.

(b) [10 marks] Point out the crucial difference in the transition dynamics under this intergenerational tax-and-transfer regime as compared to the private education regime.

(c) [10 marks] Comment on the long-run (steady-state) behaviour of the economy under this intergenerational tax-and-transfer regime and compare it with that of the private education regime.

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Final Exam: Question 3 (12 August 2021)

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- Define $\overline{w} \equiv p w_s + (1-p) w_n$. We make the following assumptions.

$$\overline{w} - h(1+r) \geq w_n (2+r). \qquad \text{(Assumption 1)}$$

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$$(1-\alpha)(1+r) < 1, \qquad \text{(Assumption 3)}$$

$$(1-\alpha)(1+i) > 1. \qquad \text{(Assumption 4)}$$

- We consider an education financing policy known as the *income contingent loans* (ICL). Income contingent loan scheme is available to anyone who wishes to borrow and invest in human capital. This scheme is different from loans available in the capital markets in three respects. (a) It is purely an education loan, which means that the loan amount is not in the hands of the agents. The loan giving agency (which is the government in our model) transfers the loan amount (h per agent) directly to the educational institutions.

(b) The loan repayment is in the form of additional taxes. (c) Repayment takes place *only if* the agent is successful and works as a skilled labour. In case of a failure an individual is exempted from repayment.

The government, a credible agency, can borrow from the international capital market at the lenders' interest rate r. It then passes the education investment amount, h, to the individual at the same repayment interest rate r. Repayment is in the form of taxation such that the government balances its budget. Suppose L agents take the education loan so that the government's repayment cost is $L \cdot h(1+r)$. To recover this cost the government imposes a proportional tax, τ, on skilled wage income w_s. But this tax has to be paid only when the educational investment is successful (which occurs with probability p). In the even of a failure, no repayment is asked for. Hence the tax revenue collection from these L agents is $p \cdot L \cdot \tau w_s$. Balanced budget requires $p \cdot L \cdot \tau w_s = L \cdot h(1+r)$, implying

$$\tau w_s = \frac{h(1+r)}{p}.$$

(a) [8 marks] Proceeding as in Question 1, part 2, describe an agent's optimal choice of occupation for different ranges of wealth levels.

(b) [8 marks] Derive the dynamic equation showing the corresponding wealth (bequest) dynamics and draw the phase diagram (plot x_t on x-axis and x_{t+1} on y-axis). [You must depict the *actual* bequest lines (with 'success' and 'failure'). Show the *expected* bequest line also for reference.]

(c) [8 marks] Comment on the nature of long-run wealth dynamics under the ICL scheme.

(d) [8 marks] Compare the long-run behaviour of the economy under the ICL scheme with the private education regime and with the intergenerational tax-and-transfer regime.

(e) [8 marks] Suppose the government would like to reduce the variance of the long-run wealth distribution under the ICL scheme. Suggest some modifications in the ICL scheme to achieve that in a budget-balanced way (as in the original ICL scheme). Could the variance be completely eliminated? Compare the long-run behaviour of the economy under this modified ICL scheme with the private education regime and with the intergenerational tax-and-transfer regime.