Econ 277B: Economic Development II
Semester II, 2020-21

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**Midterm Exam: Question 1 (13 June 2021)**

- Maximum marks: **50**
- Time allotted (including uploading on Moodle): **90 minutes**

- Consider a less developed country with $N$ sectors, each sector producing a distinct commodity. There is one *representative* price-taking consumer who supplies $L$ units of labour inelastically. The consumer's utility function is given by

$$u = x_1 \cdot x_2 \cdot \ ... \cdot x_N.$$

(a) [3 marks]

Show that if the aggregate income is $y$, then the demand function for any good $i$ is

$$x_i = \frac{y}{N \cdot p_i},$$

where $p_i$ is the price of good $i$.

- Let us treat the wage as numeraire. Now if the aggregate profit earned in the economy is given by $\overline{\pi}$, the aggregate income is

$$y = \overline{\pi} + L.$$

Note that $L$ is the total wage income in the economy because the wage is assumed to be equal to 1.

- The market structure within each sector is as follows. There exists a competitive fringe of firms that can convert 1 unit of labour into 1 unit of output. These are the "traditional" producers. In addition each sector has one (potential) "modern" firm that has to first incur a fixed cost of "burning up" $F$ units of labour and then can convert each unit of labour into $\alpha > 1$ units of output. Here "industrialization" of a sector is synonymous with the monopolist entering production in that sector.

- **Assumption:** The fixed cost, $F$, depends on how many sectors have already industrialized, $n$, that is, $F$ is a function of $n$, $F(n)$.
    - It means that the economy has *technological externalities* between its various sectors. The externality is *positive* if $F'(n) < 0$ and *negative* if $F'(n) > 0$.

- Assume that the competitive firms can enter and exit from the industry costlessly. Therefore, the competitive fringe of each sector has a perfectly elastic supply at price $= 1$ (recall that wage $= 1$, by definition).

- The monopolist in each sector decides whether to industrialize or to abstain from production. The monopolist maximizes profit taking the demand curve as given.

(b) [4 marks]

If a monopolist decides to industrialize, what is the optimal price it will charge? Give a clear explanation for your answer.

(c) [4 marks]

Suppose aggregate income is $y$, and the monopolist in some sector decides to industrialize. Derive, with a clear explanation, the expression for profit of the monopolist in that sector.

(d) [7 marks]

Let $y(n)$ be the aggregate income when $n$ sectors industrialize. Derive, with a clear explanation, that

$$y(n) = \left(\frac{N}{N - na}\right) \cdot (L - nF(n)),$$

where $a \equiv \dfrac{\alpha - 1}{\alpha}$. Provide a clear economic interpretation of the term $\left(\frac{N}{N-na}\right)$.

(e) [4 marks]

When $n$ sectors have industrialized, let $\pi(n)$ denote the monopolist's profit in each of these sectors. Derive, with a clear explanation, that

$$\pi(n) = \frac{aL - NF(n)}{N - na}.$$

(f) [6 marks]

Suppose that the technological externality is negative, that is, $F'(n) > 0$. Can there exist *multiple equilibria* with both *no-industrialization* and *full-industrialization* being equilibrium possibilities? Give a clear explanation for your answer.

(g) [22 marks: $10 + 12$]

Suppose that there exist *multiple equilibria* with both *no-industrialization* and *full-industrialization* being equilibrium possibilities. The ideas of the "big push" and the "critical minimum effort" were widely discussed in the literature on economic development in the fifties. For example, Leibenstein (1957) (*Economic Backwardness and Economic Growth*, New York: Wiley) had suggested that (a) there exists a certain minimum level of investment or effort such that a smaller investment is not sustainable and that (b) once this investment is undertaken, the economy will grow on its own.

(i) Explain clearly how the above model gives us insights into the idea of the "critical minimum effort".

(ii) Sketch a "balanced-budget tax and subsidy" policy of the government to implement the "critical minimum effort" that can eliminate the no-industrialization equilibrium.

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**Midterm Exam: Question 2 (13 June 2021)**

- Maximum marks: **10**
- Time allotted (including uploading on Moodle): **20 minutes**

- Take a very careful look at Table 2 and Table 3 (that are prepared from Table 1) along with Table 1. Does the evidence in Table 2 support the prediction of the convergence hypothesis? Give a clear explanation for your answer.

**TABLE 2-1 Real per Capita GDP for 56 Countries** TABLE 1
(in 1990 U.S. Dollars)

| | 1820 | 1870 | 1900 | 1913 | 1950 | 1973 | 1992 |
|---|---|---|---|---|---|---|---|
| **West European Countries** | | | | | | | |
| Austria | 1,295 | 1,875 | 2,901 | 3,488 | 3,731 | 11,308 | 17,160 |
| Belgium | 1,291 | 2,640 | 3,652 | 4,130 | 5,346 | 11,905 | 17,165 |
| Denmark | 1,225 | 1,927 | 2,902 | 3,764 | 6,683 | 13,416 | 18,293 |
| Finland | 759 | 1,107 | 1,620 | 2,050 | 4,131 | 10,768 | 14,646 |
| France | 1,218 | 1,858 | 2,849 | 3,452 | 5,221 | 12,940 | 17,959 |
| Germany | 1,112 | 1,913 | 3,134 | 3,833 | 4,281 | 13,152 | 19,351 |
| Italy | 1,092 | 1,467 | 1,746 | 2,507 | 3,425 | 10,409 | 16,229 |
| Netherlands | 1,561 | 2,640 | 3,533 | 3,950 | 5,850 | 12,763 | 16,898 |
| Norway | 1,004 | 1,303 | 1,762 | 2,275 | 4,969 | 10,229 | 17,543 |
| Sweden | 1,198 | 1,664 | 2,561 | 3,096 | 6,738 | 13,494 | 16,927 |
| Switzerland | na | 2,172 | 3,531 | 4,207 | 8,939 | 17,953 | 21,036 |
| United Kingdom | 1,756 | 3,263 | 4,593 | 5,032 | 6,847 | 11,992 | 15,738 |
| **Western Offshoots** | | | | | | | |
| Australia | 1,528 | 3,801 | 4,299 | 5,505 | 7,218 | 12,485 | 16,237 |
| Canada | 893 | 1,620 | 2,758 | 4,213 | 7,047 | 13,644 | 18,159 |
| New Zealand | na | 3,115 | 4,320 | 5,178 | 8,495 | 12,575 | 13,947 |
| United States | 1,287 | 2,457 | 4,096 | 5,307 | 9,573 | 16,607 | 21,558 |
| **South European Countries** | | | | | | | |
| Greece | na | na | na | 1,621 | 1,951 | 7,779 | 10,314 |
| Ireland | 954 | 1,773 | 2,495 | 2,733 | 3,518 | 7,023 | 11,711 |
| Portugal | na | 1,085 | 1,408 | 1,354 | 2,132 | 7,568 | 11,130 |
| Spain | 1,063 | 1,376 | 2,040 | 2,255 | 2,397 | 8,739 | 12,498 |
| Turkey | na | na | na | 979 | 1,299 | 2,739 | 4,422 |
| **East European Countries** | | | | | | | |
| Bulgaria | na | na | na | 1,498 | 1,651 | 5,284 | 4,054 |
| Czechoslovakia | 849 | 1,164 | 1,729 | 2,096 | 3,501 | 7,036 | 6,845 |
| Hungary | na | 1,269 | 1,682 | 2,098 | 2,480 | 5,596 | 5,638 |
| Poland | na | na | na | na | 2,447 | 5,334 | 4,726 |
| Romania | na | na | na | na | 1,182 | 3,477 | 2,565 |
| U.S.S.R. | 751 | 1,023 | 1,218 | 1,488 | 2,834 | 6,058 | 4,671 |
| Yugoslavia | na | na | na | 1,029 | 1,546 | 4,237 | 3,887 |
| **Latin American Countries** | | | | | | | |
| Argentina | na | 1,311 | 2,756 | 3,797 | 4,987 | 7,970 | 7,616 |
| Brazil | 670 | 740 | 704 | 839 | 1,673 | 3,913 | 4,637 |
| Chile | na | na | 1,949 | 2,653 | 3,827 | 5,028 | 7,238 |
| Colombia | na | na | 973 | 1,236 | 2,089 | 3,539 | 5,025 |
| Mexico | 760 | 710 | 1,157 | 1,467 | 2,085 | 4,189 | 5,002 |
| Peru | na | na | 817 | 1,037 | 2,263 | 3,953 | 2,854 |
| Venezuela | na | 783 | 1,311 | 1,733 | 3,478 | 5,017 | 5,949 |

(continued)

**TABLE 2-1 Real per Capita GDP for 56 Countries (continued)**
(in 1990 U.S. Dollars)

| | 1820 | 1870 | 1900 | 1913 | 1950 | 1973 | 1992 |
|---|---|---|---|---|---|---|---|
| **Asian Countries** | | | | | | | |
| Bangladesh | na | 531 | 581 | 617 | 551 | 478 | 720 |
| Burma | na | na | 647 | 635 | 393 | 589 | 748 |
| China | 523 | 523 | 652 | 688 | 614 | 1,186 | 3,096 |
| India | 531 | 558 | 625 | 663 | 597 | 853 | 1,348 |
| Indonesia | 614 | 657 | 745 | 917 | 874 | 1,538 | 2,749 |
| Japan | 704 | 741 | 1,138 | 1,334 | 1,873 | 11,017 | 19,425 |
| Pakistan | na | 531 | 687 | 729 | 650 | 981 | 1,642 |
| Philippines | na | na | 1,033 | 1,418 | 1,293 | 1,956 | 2,213 |
| South Korea | na | na | 850 | 948 | 876 | 2,840 | 10,010 |
| Taiwan | na | na | 759 | 794 | 922 | 3,669 | 11,590 |
| Thailand | na | 717 | 812 | 846 | 848 | 1,750 | 4,694 |
| **African Countries** | | | | | | | |
| Cote d'Ivoire | na | na | na | na | 859 | 1,727 | 1,134 |
| Egypt | na | na | 509 | 508 | 517 | 947 | 1,927 |
| Ethiopia | na | na | na | na | 277 | 412 | 300 |
| Ghana | na | na | 462 | 648 | 1,193 | 1,260 | 1,007 |
| Kenya | na | na | na | na | 609 | 947 | 1,055 |
| Morocco | na | na | na | na | 1,611 | 1,651 | 2,327 |
| Nigeria | na | na | na | na | 547 | 1,120 | 1,152 |
| South Africa | na | na | na | 1,451 | 2,251 | 3,844 | 3,451 |
| Tanzania | na | na | na | na | 427 | 655 | 601 |
| Zaire | na | na | na | na | 636 | 757 | 353 |
| World Average | 651 | 895 | 1,263 | 1,539 | 2,138 | 4,123 | 5,145 |

Source: Angus Maddison (1995), *Monitoring the World Economy 1820–1992*, Paris: OECD, Tables 1-3 and G-3, pp. 23–24, 228.

to understand how we arrived at our present economic conditions. The world averages at the bottom of the table were used in drawing part of Figure 2-1, which depicts the recent explosive improvement in living standards.

### 2.2.1 Recent Economic Growth

In order to better understand the changes in the levels of real per capita GDP since 1820, in Table 2-2 we present the average annual growth rates in real per capita GDP implied by Maddison's estimated levels of real per capita income from Table 2-1. Notice that most West European countries and the countries Maddison labels "Western Offshoots" enjoyed positive rates of growth throughout the 1820–1992 period. These economies have very high *levels* of real per capita output today because they have been growing relatively more rapidly than other countries for nearly 200 years. The growth rate of the U.S. economy has been quite consistent, ranging from 1.29% during the years 1820–1870 to 1.99% at the beginning of the twentieth century. A high standard of living is not built in a day!

Japan provides an interesting contrast in that it began to grow only after 1870, when it abandoned its long isolation from the rest of the world. The data in Tables 2-1

**Table 2: Real Per Capita GDP (in 1990 US Dollars) and its Growth Rates for Selected Countries, 1870-1992**

| Countries | 1870 | 1950 | 1992 | Growth Rate (1870-1992) | Growth Rate (1950-1992) |
|---|---|---|---|---|---|
| | | | | | |
| South Korea | NA | 876 | 10,010 | | 5.800 |
| Taiwan | NA | 922 | 11,590 | | 6.027 |
| Japan | 741 | 1873 | 19425 | 2.677 | 5.569 |
| Italy | 1467 | 3425 | 16229 | 1.970 | 3.704 |
| Austria | 1875 | 3731 | 17160 | 1.815 | 3.633 |
| Finland | 1107 | 4131 | 14646 | 2.117 | 3.013 |
| Germany | 1913 | 4281 | 19351 | 1.897 | 3.592 |
| Norway | 1303 | 4969 | 17543 | 2.131 | 3.003 |
| France | 1858 | 5221 | 17959 | 1.860 | 2.941 |
| Belgium | 2640 | 5346 | 17165 | 1.535 | 2.777 |
| Netherlands | 2640 | 5850 | 16898 | 1.522 | 2.526 |
| Denmark | 1927 | 6683 | 18293 | 1.845 | 2.398 |
| Sweden | 1664 | 6783 | 16927 | 1.901 | 2.177 |
| UK | 3263 | 6847 | 15738 | 1.290 | 1.982 |
| Canada | 1620 | 7047 | 18159 | 1.981 | 2.254 |
| Australia | 3801 | 7218 | 16237 | 1.190 | 1.930 |
| Switzerland | 2172 | 8939 | 21036 | 1.861 | 2.038 |
| USA | 2457 | 9573 | 21558 | 1.780 | 1.933 |
| | | | | | |
| | | | | | |
| **For the 16 available countries, the correlation coefficient between growth rate of per capita GDP between 1870 and 1992 and the per capita GDP in 1870 is: -0.95.** | | | | | |
| | | | | | |
| | | | | | |
| **For these 18 countries, the correlation coefficient between growth rate of per capita GDP between 1950 and 1992 and the per capita GDP in 1950 is: -0.93.** | | | | | |
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**Table 3: Real Per Capita GDP (in 1990 US Dollars) and its Growth rates for 56 Countries, 1950-1992**

| Countries | 1950 | 1992 | Growth Rate (1950-1992) |
|---|---|---|---|
| | | | |
| Austria | 3731 | 17160 | 3.633 |
| Belgium | 5346 | 17165 | 2.777 |
| Denmark | 6683 | 18293 | 2.398 |
| Finland | 4131 | 14646 | 3.013 |
| France | 5221 | 17959 | 2.941 |
| Germany | 4281 | 19351 | 3.592 |
| Italy | 3425 | 16229 | 3.704 |
| Netherlands | 5850 | 16898 | 2.526 |
| Norway | 4969 | 17543 | 3.003 |
| Sweden | 6738 | 16927 | 2.193 |
| Switzerland | 8939 | 21036 | 2.038 |
| United Kingdom | 6847 | 15738 | 1.982 |
| Australia | 7218 | 16237 | 1.930 |
| Canada | 7047 | 18159 | 2.254 |
| New Zealand | 8495 | 13947 | 1.180 |
| United States | 9573 | 21558 | 1.933 |
| Greece | 1951 | 10314 | 3.965 |
| Ireland | 3518 | 11711 | 2.863 |
| Portugal | 2132 | 11130 | 3.935 |
| Spain | 2397 | 12498 | 3.932 |
| Turkey | 1299 | 4422 | 2.917 |
| Bulgeria | 1651 | 4054 | 2.139 |
| Czechoslovakia | 3501 | 6845 | 1.596 |
| Hungary | 2480 | 5638 | 1.955 |
| Poland | 2447 | 4726 | 1.567 |
| Romania | 1182 | 2565 | 1.845 |
| U.S.S.R. | 2834 | 4671 | 1.190 |
| Yugoslavia | 1546 | 3887 | 2.195 |
| Argentina | 4987 | 7616 | 1.008 |
| Brazil | 1673 | 4637 | 2.427 |
| Chile | 3827 | 7238 | 1.517 |
| Colombia | 2089 | 5025 | 2.090 |
| Mexico | 2085 | 5002 | 2.083 |
| Peru | 2263 | 2854 | 0.552 |
| Venezuela | 3478 | 5949 | 1.278 |
| Bangladesh | 551 | 720 | 0.637 |
| Burma | 393 | 748 | 1.532 |
| China | 614 | 3096 | 3.852 |
| India | 597 | 1348 | 1.939 |
| Indonesia | 874 | 2749 | 2.728 |
| Japan | 1873 | 19425 | 5.569 |
| Pakistan | 650 | 1642 | 2.206 |
| Philippines | 1293 | 2213 | 1.279 |
| South Korea | 876 | 10010 | 5.800 |

**Table 3: Real Per Capita GDP (in 1990 US Dollars) and its Growth rates for 56 Countries, 1950-1992**

| Countries | 1950 | 1992 | Growth Rate (1950-1992) |
|---|---|---|---|
| | | | |
| Taiwan | 922 | 11590 | 6.027 |
| Thailand | 848 | 4694 | 4.074 |
| Cote d'Ivorie | 859 | 1134 | 0.661 |
| Egypt | 517 | 1927 | 3.133 |
| Ethiopia | 277 | 300 | 0.190 |
| Ghana | 1193 | 1007 | -0.404 |
| Kenya | 609 | 1055 | 1.308 |
| Morocco | 1611 | 2327 | 0.876 |
| Nigeria | 547 | 1152 | 1.773 |
| South Africa | 2251 | 3451 | 1.017 |
| Tanzania | 427 | 601 | 0.814 |
| Zaire | 636 | 353 | -1.402 |
| | | | |
| | | | |
| **For all these 56 countries, the correlation coefficient between growth rate of per capita GDP between 1950 and 1992 and the per capita GDP in 1950 is: 0.01.** | | | |
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**Midterm Exam: Question 3 (13 June 2021)**

- Maximum marks: **40**
- Time allotted (including uploading on Moodle): **90 minutes**

- Consider the following model to explore the possibility of multiple equilibria and the role of history versus expectations in determining the equilibrium dynamics.
- Over an infinite horizon households supply $L$ units of labor inelastically and consume the homogeneous final good (taken as the numeraire). At any moment $t$, households choose consumption $(C_t)$ so as to maximize $U_t = \int_t^{\infty} e^{-\rho(\tau - t)} \log C_\tau \, d\tau$, subject to, $\int_t^{\infty} e^{-\int_t^{\tau} r_s \, ds} \, C_\tau \, d\tau \leq L \int_t^{\infty} e^{-\int_t^{\tau} r_s \, ds} \, w_\tau \, d\tau + W_t$, where $\rho > 0$ is the subjective discount rate, $r_t$ is the rental rate, $w_t$ is the wage rate, and $W_t$ is the value of asset holding consisting of ownership shares of profit making firms. The solution to this maximization problem is characterized by the Euler condition: $\frac{\dot{C}_t}{C_t} = r_t - \rho$, and the binding budget constraint: $\int_t^{\infty} e^{-\int_t^{\tau} r_s \, ds} \, (C_\tau - w_\tau L) \, d\tau = W_t$.
- On the production side there are two sectors: a final consumer good sector and an intermediate goods sector. The final consumer good is produced by competitive firms who share the identical constant returns to scale production function $C_t = F(X_t, \, H_t)$, where $H_t$ is the labour input and $X_t$ is the composite of differentiated intermediate inputs or 'producer services', which has a form of symmetric CES, $X_t = \left[ \int_0^{n_t} [x_t(i)]^{1 - \frac{1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma - 1}}$, $\sigma > 1$, where $x_t(i)$ is the amount of variety $i$ used, and $\sigma$ is the elasticity of substitution between every pair of intermediate inputs. At any moment only a subset of differentiated products, $[0, \, n_t]$, is available.
- Each intermediate input is supplied by a single, atomistic firm. Due to the CES specification, the demand for each intermediate input $i$ is $x_t(i) = \left[ \frac{p_t(i)}{P_t} \right]^{-\sigma} X_t$, where $P_t$ is the price index of the intermediate goods composite, that is, $P_t X_t \equiv \int_0^{n_t} [p_t(i) \, x_t(i)] \, di$.

Producing a unit of each intermediate input requires $a_x$ units of labor, so that the marginal cost is constant and equal to $w_t a_x$. We use the normalization: $a_x \equiv 1 - \frac{1}{\sigma}$.

- Profit maximization implies each intermediate goods producer sets the price so as to equate marginal revenue with marginal cost, implying

$$p_t(i) = \frac{w_t a_x}{1 - \frac{1}{\sigma}} = w_t.$$

Using this, the price index of the intermediate goods composite becomes

$$P_t = \left[ \int_0^{n_t} [p_t(i)]^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = n_t^{\frac{1}{1-\sigma}} w_t,$$

implying that the effective relative factor price, $\frac{P}{w}$, decreases with $n$. It is easy to see that the factor share of intermediate inputs, $\alpha_t \equiv \frac{P_t X_t}{C_t}$, is a function of $\frac{P_t}{w_t}$. Since $\frac{P_t}{w_t}$ is a function of the product variety ($n_t$), $\alpha_t$ is also a function of $n_t$; define it by $\alpha_t \equiv A(n_t)$. It follows that $A(n_t)$ is increasing (decreasing) in $n_t$ whenever the elasticity of substitution between $H_t$ and $X_t$ is greater (less) than one.

  - Since $p_t(i) = w_t$, for all $i$, it follows that $p_t(i) = p_t(j)$, $x_t(i) = x_t(j)$, and $\pi_t(i) = \pi_t(j)$, for all $i, j$, where $\pi_t$ stands for operating profit, $\pi_t = (p_t - w_t a_x) x_t$. It can be shown (you don't need to show this) that

$$\pi_t = \frac{A(n_t)}{\sigma n_t} C_t. \tag{1}$$

- The number of the specialist firms (and the range of producer services available) increases over time through the process of *entry*. Initially, the economy inherits a given number of firms, $n_0$. At any moment firms may enter freely into the intermediate goods sector, except that they need a *start-up operation* which requires the use of $a_n$ units of labor per variety. The entering firms finance start-up costs by issuing ownership shares. Because of free entry, the value of an intermediate goods firm, $v_t$, never exceeds the start-up cost, $w_t a_n$, and whenever some entry occurs, they are equalized. Furthermore, the operating profit is always positive, so that no incumbent firm has an incentive to exit. That is, in equilibrium, we have

$$w_t a_n \geq v_t, \ \dot{n}_t \geq 0, \ \ (w_t a_n - v_t) \dot{n}_t = 0. \tag{2}$$

The market value of an intermediate goods producer is equal to the present discounted value of profits, $v_t \equiv \int_t^\infty e^{-\int_t^\tau r_s \, ds} \, \pi_\tau \, d\tau$, from which we obtain

$$\frac{\pi_t + \dot{v}_t}{v_t} = r_t. \tag{3}$$

- It can be shown (you don't need to show it) that the labour market clearing condition implies that

$$L = a_n \dot{n}_t + H_t + n_t a_x x_t = a_n \dot{n}_t + \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\frac{C_t}{w_t}\right). \tag{4}$$

- Finally, it can be checked (you don't need to check it) that the transversality condition of the households' maximization problem also holds:

$$\lim_{T \to \infty} n_T v_T e^{-\int_0^T r_s \, ds} = 0. \tag{5}$$

- We describe the dynamic evolution of the economy in terms of two variables, $n$ and $V = \frac{v}{C}$, where $V$ represents the value of an intermediate inputs producing firm, measured in utility.

  It can be shown (you don't need to show it) that, for any initial number of firms ($n_0$) the economy inherits, a market equilibrium of this economy is a path of $\{V_t, \ n_t\}$ that satisfies

$$\dot{V}_t = \rho V_t - \frac{A(n_t)}{\sigma n_t}, \tag{6a}$$

$$\dot{n}_t = \max\left\{\frac{L}{a_n} - \left(1 - \frac{A(n_t)}{\sigma}\right)\left(\frac{1}{V_t}\right), \ 0\right\}, \tag{6b}$$

  and

$$\lim_{t \to \infty} n_t V_t e^{-\rho t} = 0. \tag{6c}$$

- The qualitative property of the equilibrium dynamics crucially depends on the shapes of the following two loci:
  - the VV locus (corresponds to $\dot{V}_t = 0$): $V_t = \frac{A(n_t)}{\rho \sigma n_t}$, and
  - the NN locus (corresponds to $\dot{n}_t = 0$): $V_t = \frac{a_n}{L}\left(1 - \frac{A(n_t)}{\sigma}\right)$.

- To explore the possibility of multiple equilibria and the role of history versus expectations in determining the equilibria let $F(X, H)$ be a CES of the following form:

$$F(X_t, H_t) = \left[X_t^{1-\frac{1}{\epsilon}} + \beta^{\frac{1}{\epsilon}} H_t^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \ \epsilon > 1. \tag{7}$$

- It can be shown (you don't need to show it) that with this CES specification the expression for $A(n_t)$ is given by

$$\frac{1}{A(n_t)} = 1 + \beta n_t^{\frac{1-\epsilon}{\sigma-1}}.$$

  - Then the NN locus is given by

$$V_t = \frac{a_n}{L} - \frac{\frac{a_n}{L}}{\sigma\left(1 + \beta n_t^{\frac{1-\epsilon}{\sigma-1}}\right)},$$

    and the VV locus is given by

$$V_t = \frac{1}{\rho\sigma\left[n_t + \beta n_t^{\frac{\sigma-\epsilon}{\sigma-1}}\right]}.$$

(a) [12 points] Consider the case when $\sigma > \epsilon > 1$.

Draw the phase diagram showing the NN and VV loci clearly plotting $n_t$ on the $x$-axis and $V_t$ on the $y$-axis. You must show the horizontal and vertical arrows demonstrating the movements of $n_t$ and $V_t$ in all the different regions generated by the NN and VV loci.

Describe the dynamic evolution of the economy for any initial number of firms ($n_0$) that the economy inherits.

Is there any possibility of multiple equilibria? Explain clearly. [Recall that for any initial number of firms ($n_0$) the economy inherits, a market equilibrium of this economy is a path of $\{V_t,\ n_t\}$ that satisfies (6a), (6b) and (6c).]

(b) [15 points] Consider the case when $\epsilon > \sigma > 1$.

(i) Consider first the subcase where $a_n$ and $\rho$ are so high that the NN locus lies above the VV locus everywhere.

Draw the phase diagram with the horizontal and vertical arrows in all the different regions generated by the NN and VV loci.

Describe the dynamic evolution of the economy for any initial number of firms ($n_0$) that the economy inherits.

Is there any possibility of multiple equilibria? Explain clearly.

(ii) Consider next the subcase where $a_n$ and $\rho$ are moderate so that the NN locus intersects the VV locus twice, both at the downward sloping part of VV.

Draw the phase diagram with the horizontal and vertical arrows in all the different regions generated by the NN and VV loci.

Describe the dynamic evolution of the economy for any initial number of firms ($n_0$) that the economy inherits.

Is there any possibility of multiple equilibria? Explain clearly.

(iii) Consider finally the subcase where $a_n$ and $\rho$ are small enough so that the NN locus intersects the VV locus once at the upward sloping part of VV and then again at the downward sloping part of VV.

Draw the phase diagram with the horizontal and vertical arrows in all the different regions generated by the NN and VV loci.

Describe the dynamic evolution of the economy for any initial number of firms ($n_0$) that the economy inherits.

Is there any possibility of multiple equilibria? Explain clearly.

(c) [13 points]

Recall that Krugman (1991) ("History versus Expectations", *Quarterly Journal of Economics*, 106, 651-667) has identified three factors deciding the roles of history versus expectations in determining the nature of equilibrium dynamics: rate at which agents discount the future, strength of the external economy and speed of adjustment.

Identify the counterparts of the above three factors in terms of the parameters of the model described above.

Discuss clearly how these three factors explain the roles of history vis à vis expectations in determining the nature of equilibrium dynamics for the model described above.