Econ 277C: Economic Development III — Tridip Ray

Semester II, 2022-23 — ISI, Delhi

Midterm Exam (5 March 2023)

- There are 2 questions; you have to answer *both* of them. You have 3 hours to write this exam.

1. [55 points]

- Consider an economy where, on the production side, there are two sectors: a final consumer good sector and an intermediate goods sector. The final consumer good, C, is produced by competitive firms who share the identical CES production technology:

$$C = F(X,\ H) = \left[X^{1-\frac{1}{\epsilon}} + H^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}},\ \epsilon > 1. \tag{1}$$

Here H is the labour input and X is the composite of differentiated intermediate inputs or 'producer services', and ϵ is the elasticity of substitution between X and H. The composite X takes a form of symmetric CES,

$$X = \left[\int_0^n [x(i)]^{1-\frac{1}{\sigma}}\, di\right]^{\frac{\sigma}{\sigma-1}},\ \sigma > 1, \tag{2}$$

where $x(i)$ is the amount of variety i used, σ is the elasticity of substitution between every pair of intermediate input varieties, and $[0, n]$ represents the range of intermediate inputs available in the marketplace.

- Each intermediate input is supplied by a single, atomistic firm. Due to the CES specification, the demand for each intermediate input i is $x(i) = \left[\dfrac{p(i)}{P}\right]^{-\sigma} X$, where P is the price index of the intermediate goods composite, that is, $P \equiv \left[\int_0^n [p(i)]^{1-\sigma}\, di\right]^{\frac{1}{1-\sigma}}$. Production of y units of each variety requires $(ay + F)$ units of labour; F is the fixed and a is the marginal labour requirement. Denoting the wage rate by w, the marginal cost is wa and fixed cost is wF. We use the normalization: $a \equiv 1 - \dfrac{1}{\sigma}$.

- Finally, the total labour supply in the economy is L.

(a) [6 points]

(i) Show that each intermediate input producer i sets the price $p(i) = w$.

(ii) Argue that $x(i) = x(j)$, for all i, j. Derive an expression of this common intermediate input (denote it by x) in terms of X and n.

Show that the *gross profit* of each intermediate input firm (that is, *gross* of the fixed cost), π, is proportional to output produced, that is, $\pi = \left(\frac{w}{\sigma}\right) x$.

(b) [8 points]

(i) Let $M = nx$ be the total intermediate inputs used. Show that the average productivity of intermediate inputs increases with n. Explain the economic implication of this result.

(ii) Derive the expression for effective relative factor price, $\frac{P}{w}$, and show that it decreases with n. Explain the economic implication of this result.

(iii) Derive the expression for the demand for X relative to H, $\frac{X}{H}$, and show that it increases with n. Does this make economic sense? Explain clearly.

(c) [9 points]

An equilibrium at the production side of the economy consists of (i) labour market clearing and (ii) free entry such that *net profit* (that is, after deducting the fixed cost) of each entering intermediate input firm is zero.

Using all the relevant information derived above, write down the labour market clearing condition and show that the *gross profit* of each intermediate input firm satisfies

$$\frac{\pi}{w} = \frac{L - nF}{(\sigma - 1)\, n + \sigma n^{\frac{\sigma - \epsilon}{\sigma - 1}}}.$$

(d) [32 points: $4 + 5 + 5 + 6 + 4 + 8$]

Define $\Pi(n) \equiv \dfrac{L - nF}{(\sigma - 1)\, n + \sigma n^{\frac{\sigma-\epsilon}{\sigma-1}}}$. When $\epsilon > \sigma$, it can be shown that, in the economically relevant range of n, the function $\Pi(n)$ has a single peak.

(i) Plot the $\Pi(n)$ function labeling all the relevant points clearly.

(ii) Use the figure to demonstrate clearly that there are *multiple equilibria*. Show all the equilibria in the figure clearly.

(iii) Explain clearly which of the equilibria are *stable* and which are *unstable*.

(iv) Show that there is a 'poverty trap' or a 'low-level equilibrium trap' in this economy. Explain clearly the economic mechanism leading to this poverty trap.

(v) Explain clearly how the mechanism gets reversed for the higher level equilibrium.

(vi) How will your answers to the earlier parts change when $\sigma > \epsilon$? Demonstrate clearly. Explain clearly the economic logic behind this difference in results?

2. [45 points]

Consider the following overlapping generations model with uncertainty.

- Time is discrete and runs to infinity. Each agent lives for two periods. There is a continuum of agents with mass 1 in each living generation, and agents of the same generation are all identical. The utility of an agent in generation t is given by

$$U(c_t, c_{t+1}) = \log\ c_t + \beta\ \log\ c_{t+1},$$

where $\beta \in (0,1)$ is the discount factor. Agents can only work in the first period of their lives, and they supply 1 unit of labour inelastically, earning the market determined wage rate w_t.

- The production side is characterized by a set of competitive firms, and is represented by the Cobb-Douglas technology. But the technology also includes a stochastic shock θ_t so that total output at time t is given by

$$Y_t = \theta_t K_t^{\alpha} L_t^{1-\alpha},\ 0 < \alpha < 1.$$

To simplify the analysis let us assume that capital depreciates fully after use. Also, the factor markets are competitive. Thus, the (gross) rate of return to saving, which equals the rental rate of capital, is given by

$$1 + r_t = R_t(\theta_t, K_t) = \alpha\theta_t K_t^{\alpha-1},$$

and the wage rate is

$$w_t(\theta_t, K_t) = (1-\alpha)\,\theta_t K_t^{\alpha}.$$

Note that factor prices depend only on the current values of θ and capital.

- Consumption and Savings Decisions:

Savings by an individual of generation t, s_t, are determined as a solution to the following maximization problem:

$$\underset{\{c_t, c_{t+1}, s_t\}}{\text{Maximize}}\ \log\ c_t + \beta\ \log\ c_{t+1},$$

subject to

$$c_t + s_t \leq w_t,$$

and

$$c_{t+1} \leq (1 + r_t)\, s_t = R_{t+1} s_t.$$

(a) [5 points] Show that the law of motion of capital stock is given by

$$K_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)}\theta_t K_t^{\alpha}.$$

- The stochastic shock θ_t can take two values, $0 < \theta_L < \theta_H = 1$. Therefore, in every period there is a positive probability that the economy loses capital and drops down the income ranking. We assume the existence of an economy-wide capital accumulation externality, such that as the economy reaches more advanced stages of development, proxied by its stock of accumulated capital, the likelihood of an economic setback decreases. That is, we postulate that $p_t \equiv Prob\,(\theta_t = \theta_H)$ is given by

  $$p_t = p(K_t), \text{ where } p(0) > 0, \text{ and } p'(K_t) > 0.$$

  It is also assumed that there exists $\overline{K} < \infty$, such that $p(K_t) = 1$, for all $K_t \geq \overline{K}$.

- With the above formulation of the stochastic shock θ_t the law of motion of K_t becomes

  $$K_{t+1} = \begin{cases} \dfrac{\beta(1-\alpha)}{(1+\beta)}K_t^{\alpha} & \text{with probability } p(K_t), \\ \dfrac{\beta(1-\alpha)}{(1+\beta)}\theta_L K_t^{\alpha} & \text{with probability } 1 - p(K_t). \end{cases}$$

  Define the map $g(K) \equiv \dfrac{\beta(1-\alpha)}{(1+\beta)}K^{\alpha}$, and let K_a denote the fixed point of the bad-outcome map $\theta_L g(K)$ and K_b denote the fixed point of the good-outcome map $g(K)$.

(b) [5 points] Illustrate the law of motion of K_t by plotting the bad-outcome and the good-outcome maps in the K_t-K_{t+1} plane showing K_a and K_b in the figure clearly.

- The dynamic evolution of the economy could be represented as in the above figure where $[K_a, K_b]$ is the *stable set* for the capital stock, that is, as $t \to \infty$, $Prob\,\{K \in [K_a, K_b]\} = 1$. In what follows we focus on the case where there is a unique stationary distribution μ^* measuring the probability that $K = \tilde{K}$, for all $\tilde{K} \in [K_a, K_b]$.

(c) [11 points] Consider an economy where $K_a < \overline{K} < K_b$.

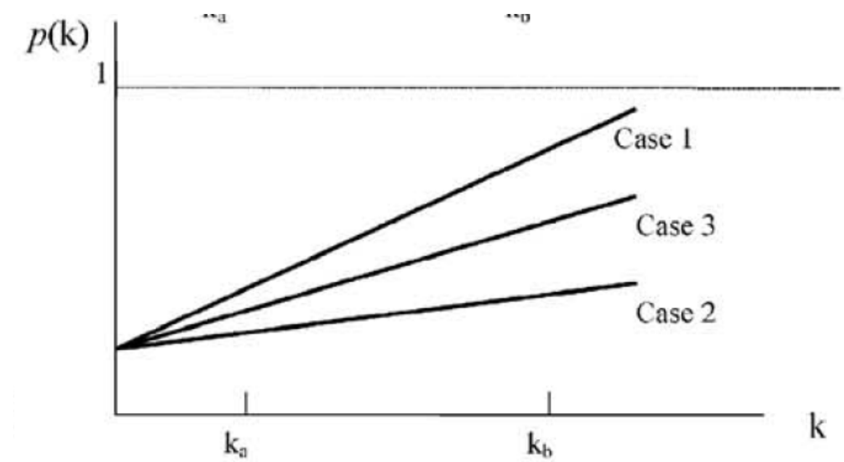
  (i) Illustrate the law of motion of K_t by plotting the bad-outcome map, the good-outcome map and the three reference points $\overline{K}$, K_a and K_b in the K_t-K_{t+1} plane.

  (ii) Analyze the dynamics of development of this economy describing its behaviour in transition as well as the *shape* of the resulting stationary distribution μ^*.

  (iii) Does history matter? Does luck matter? Explain clearly.

(d) [24 points]

Consider another economy where $K_a < K_b < \overline{K}$. Consider the behaviour of $p(K_t)$ distinguished by the three cases depicted in the following figure.



- Case 1: $p(K_a)$ is already substantially high and then it grows higher as K goes to K_b.
- Case 2: $p(K_a)$ is low and it is still quite low at $p(K_b)$.
- Case 3: $p(K_a)$ is low, but $p(K_b)$ is quite high.

- **Question:** For each of the three cases above, (i) analyze the dynamics of development of the economy describing its behaviour in transition as well as the *shape* of the resulting stationary distribution μ^*; (ii) draw the stationary distribution μ^*; (iii) relate the dynamics of development to empirically observed development experiences of different countries.