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# Empirical cross-section dynamics in economic growth

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## 1. Introduction

Economic growth – across countries, regions, states – has recently been the subject of intensive empirical study; see among many others Barro (1991) and Barro and Sala-i-Martin (1991, 1992). Grossly over-simplifying, the aims of this work are two-fold: (a) to examine the determinants of long-run growth and (b) to check if, after conditioning on the hypothesized explanatory variables given under (a), per capita income converges towards a steady-state growth path, possibly differing across economies.[1] The standard approach in these studies is the following: (i) calculate the average growth rate over time for each economy; (ii) across economies regress that average growth rate computed in (i) on static conditioning variables – schooling, investment, government spending, political circumstances – and an initial income level.

The idea underlying this procedure is that the conditioning variables explain the permanent growth component or trend, while the initial condition controls for transitory dynamics. A very clear discussion of this appears in Barro and Sala-i-Martin (1992, pp. 224–227). The results from this analysis have been rich and provocative: ensuing insights range from verifying the neoclassical growth model's predictions to suggesting new directions for research in endogenous growth theory.

Implicit in this empirical work, however, is a view that every economy has a steady-state growth path, well-approximated by a time trend. Such a view is necessary for the time-averaged growth rate – the left-hand side in these cross-country regressions – to measure anything sensible, and thus for its covariation with proposed explanatory variables to indicate something stable.

The first part of this paper examines if the cross-country income data bear out such an implicit assumption: the answer turns out to be no. The second

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[1]Hereafter I use 'economy' to refer to either countries, regions, or states.

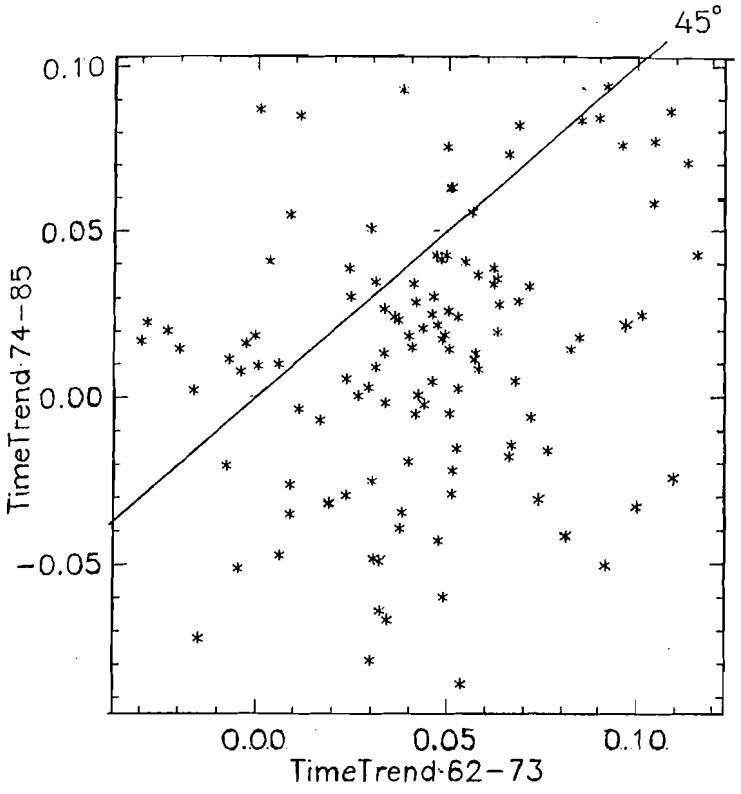


Fig. 1

part of the paper then turns to an alternative econometric strategy designed specifically to analyze the dynamics in the rich cross-section of country incomes. Conclusions and potential extensions are given in the last section.

## 2. Stable growth paths?

A defining feature of all empirical cross-country growth studies is the interest in how an inherently dynamic phenomena – long- or medium-run growth – varies across different economies in a cross-section. The standard approach, described above, collapses dynamic characteristics into a single summary statistic – an average or trend growth rate – and then asks how this statistic covaries with proposed explanatory factors in the cross-section. Such an approach is informative if permanent movements in income were well-described by smooth time trends, themselves largely unaffected by ongoing economic disturbances; or, if significant, large economic shocks occurred only at the beginning of the sample, and thus the smooth time-trends approximation is good, regardless of the true underlying structure.

Unfortunately, neither of these scenarios appears to well-describe the cross-country income data. Fig. 1 shows results from fitting linear time trends, country by country, to the log of per capita income for 118

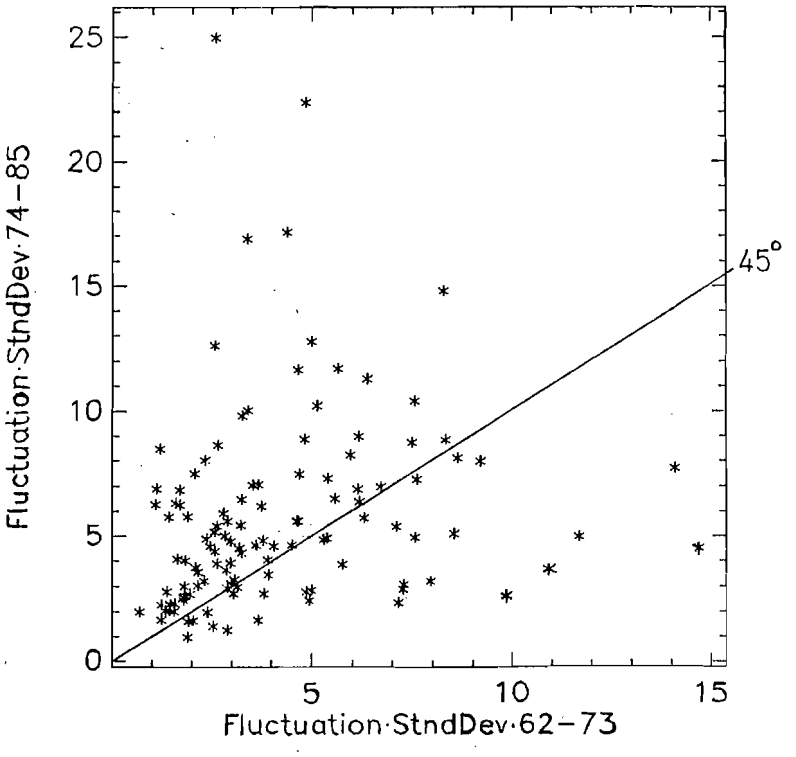


Fig. 2

countries.[2] It graphs the slope of each economy's time trend after 1973–1974 against that before. In practically all cases OLS and heteroskedasticity-robust standard errors show the change in trend to be significant. Of the 118 economies on this graph, 92 (or 78%) are below the 45° line – a considerable fraction of these, far below. Fig. 2 similarly graphs the standard deviation of (log) income fluctuations about the fitted (broken) trend line after 1973–1974 against the same measure before. Of the 118 economies, 85 have datapoints *above* the 45° line, indicating 72% of these economies experienced an increase – some substantial – in income variability. Imposing a smooth trend, i.e., not allowing a break, as done in traditional strategies only amplifies this increase.[3]

These graphs give a number of important messages. First, the data show instability in underlying long-run growth patterns: thus, assuming that each country has a stable growth path and then studying their cross-country

[2]The time sample is annual, 1962 through 1985; the regressions are estimated by OLS, with a change permitted in the coefficients on constant and time trend at 1973–1974. The data source and a list of the different economies studed here are given in a data appendix available from the author.

[3]Plots like figs. 1 and 2 can be viewed as informal ways of examining non-stationarities like 'broken trends' and 'unit roots' in time series data.

variation produces results that are difficult to interpret. Second, the increasing fluctuation variability suggests that important disturbances – demand or productivity – are ongoing; a picture of the different economies as largely perturbed only by World War II, and thereafter adjusting towards steady-state equilibrium is flawed. Third, these graphs bear on the validity of conditional convergence, as studied in Barro and Sala-i-Martin (1991, 1992). Fitting separate time trends for each country, as I have done above, can be viewed as extreme successful conditioning in a cross-country growth regression, namely, one where the explanatory variables yields an $R^2$ of 1 in explaining the underlying growth trends. Yet, even then, each country's income shows rising, rather than falling, variability over time: shocks to each economy appear more important at the end of the sample than at the beginning.

## 3. Dynamically evolving distributions

In light of the previous section it is useful to turn to an alternative econometric strategy, one not tied to restrictive assumptions on the nature of long-run growth. Let $F_t$ denote the distribution of incomes across countries at time $t$; describe $\{F_t: \text{integer } t\}$'s evolution by the law of motion:

$$F_{t+1} = M \cdot F_t, \tag{1}$$

where $M$ maps one distribution into another, and tracks where in $F_{t+1}$ points in $F_t$ end up. Thus, $M$ encodes information on whether economies like Korea and the Philippines, say, which were close together in 1950 transit subsequently to widely different income levels. Notice that just carrying along aggregate statistics such as means or standard deviations of the sequence $\{F_t: \text{integer } t\}$ would not suffice, for that would hide intra-distribution movements.

Eq. (1) is like a standard first-order autoregression, except its values are distributions (rather than scalars or vectors of numbers), and it contains no explicit disturbance or innovation. By analogy with autoregression, there is no reason why the law of motion of $F_t$ need be first order, or why the relation need be time-invariant. Nevertheless, (1) is a useful first step for analyzing dynamics in $\{F_t\}$. Iteration yields (a predictor for) future cross-section distributions:

$$F_{t+s} = (M \cdot M \cdot \ldots M) * F_t = M^s \cdot F_t;$$

taking this to the limit as $s \to \infty$, one can characterize the likely long-run distribution of cross-country incomes. Convergence might manifest in $\{F_{t+s}\}$ tending towards a point mass; the world partitioning, in the long run, into

haves and have-nots might manifest in $\{F_{t+s}\}$ tending towards a two-point or bimodal distribution. The speed of convergence of the evolving distributions and their cross-sectional mobility properties can be studied from certain (spectral) characteristics of the operator $M$. In brief, variants of (1) allow answering a wealth of interesting questions about cross-sectional income dynamics.

To keep within its space allotment, this paper addresses only the simplest such question: what are the long-run tendencies of incomes across countries?

I took each country's per capita income relative to the world average as my basic data. Operator $M$ of (1) is approximated by discretizing the set of possible values of relative incomes into intervals at 1/4, 1/2, 1, and 2. (These choices – a priori reasonable to me – also turned out to divide the observed data into roughly equal-sized categories.) All relevant properties of $M$ are then described by a $5 \times 5$ Markov chain transition matrix whose $(j, k)$ entry is the probability that an economy in state $j$ transits to state $k$ – in the following I refer to this matrix and $M$ interchangeably. Low-numbered states correspond to low incomes: thus, for example, state 1 in table 1 comprises per capita incomes no greater than one-fourth the world average.

The table's first panel contains the one-step annual transition matrix, estimated by averaging the observed one-year transitions over every year from 1962–1963 through 1984–1985. The first column gives the total number of transitions with starting points in that income state. For example, the second row shows that over the entire sample – across 118 countries and 23 years – 643 observations fell in state 2, i.e., had incomes between one-fourth and one-half the world average. Of these, 92% remained in that same state in the following year.

Over this one-year horizon the predominant feature is – not surprisingly – high persistence: all diagonal entries exceed 90%; other entries are non-zero only for the first state off the main diagonal. From state 2, a representative economy is (marginally) more likely to fall behind than to go ahead; similarly for state 4 – this despite each higher cell's covering an ever-larger range.

The table's second panel is like the first, but now it describes the one 23-year transition from 1962 through 1985; here, again, we see persistence, although less pronounced. For example, 7% of the economies originally at incomes between one-fourth and one-half of the world average (state 2) transited to incomes at world average or higher (states 4 and 5) over this longer horizon. This mobility is not all favorable though: of those same economies originally in state 2, over one-half *dropped* to even lower incomes. Looking down the neighborhood of the main diagonal suggests that at low incomes the greater tendency is to become even poorer, although some possibility for upward mobility always remains. At higher incomes, upward and downward mobilities just about balance; the highest income state

Table 1

Real GDP per capita (relative to world average).
First-order, time-stationary (1962 to 1984);
grid: (0, 1/4, 1/2, 1, 2, ∞); states: 5.

| | Upper endpoint | | | | |
|---|---|---|---|---|---|
| (Number) | 1/4 | 1/2 | 1 | 2 | ∞ |
| (456) | 0.97 | 0.03 | 0.00 | 0.00 | 0.00 |
| (643) | 0.05 | 0.92 | 0.04 | 0.00 | 0.00 |
| (639) | 0.00 | 0.04 | 0.92 | 0.04 | 0.00 |
| (468) | 0.00 | 0.00 | 0.04 | 0.94 | 0.02 |
| (508) | 0.00 | 0.00 | 0.00 | 0.01 | 0.99 |
| Ergodic | 0.24 | 0.18 | 0.16 | 0.16 | 0.27 |
| | From second-order specification | | | | |
| Ergodic | 0.28 | 0.15 | 0.12 | 0.15 | 0.30 |
| *23-year transition (1962–1985)* | | | | | |
| (17) | 0.76 | 0.12 | 0.12 | 0.00 | 0.00 |
| (29) | 0.52 | 0.31 | 0.10 | 0.07 | 0.00 |
| (35) | 0.09 | 0.20 | 0.46 | 0.26 | 0.00 |
| (17) | 0.00 | 0.00 | 0.24 | 0.53 | 0.24 |
| (20) | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |
| Ergodic | 0.16 | 0.05 | 0.10 | 0.12 | 0.57 |
| | From second-order specification (22-year transition) | | | | |
| Ergodic | 0.20 | 0.09 | 0.13 | 0.12 | 0.47 |
| *Stationary estimate, iterated 23 times* | | | | | |
| 1/4 | 0.61 | 0.27 | 0.09 | 0.03 | 0.00 |
| 1/2 | 0.37 | 0.32 | 0.20 | 0.09 | 0.02 |
| 1 | 0.14 | 0.23 | 0.31 | 0.25 | 0.07 |
| 2 | 0.04 | 0.11 | 0.25 | 0.39 | 0.22 |
| 5 | 0.00 | 0.01 | 0.04 | 0.12 | 0.82 |

however, appear persistent – the 95% probability of the richest remaining richest is by far the largest entry in this transition matrix.

This informal description suggests cross-country incomes tending towards extremes at both high and low endpoints. We can make this precise by examining the ergodic distributions implied by these transition functions.[4] Those distributions are given below the corresponding transition matrices. For both of the first two panels – $M$ estimated by averaging annual

[4]Nothing in the calculations enforces existence or uniqueness of an ergodic distribution: that precisely one such distribution was found is a consequence of the data. We should also emphasize that the steady-state distributions should *not* be read as forecasts of what will happen in the future – government policies might change; important, unforeseen events might occur. Rather, these distributions should be interpreted simply as characterizations of tendencies in the post-War history that actually realized.

transitions across time and over the long horizon – the implied ergodic distributions show first, a thinning in the middle, and second, an accumulation in *both* low and high tails. The two distributions show a higher probability at the upper tail — especially so for $M$ estimated over the long horizon.[5]

Recall that world per-capita income – the normalizing quantity – is a weighted average of each country's per capita income, assigning high weights to heavily-populated countries. Since incomes are non-negative, an accumulation at the high end of the distribution means that most of those countries getting richer have small populations. This behavior at the upper tail – convergence towards the richest countries, conditional on an economy being small and already relatively wealthy – should not distract from the distribution's behavior at the low end. Despite the cell here being the smallest it shows a similar accumulation in the ergodic distribution: countries caught in a low-income development trap remain there.

Each of the first two panels also reports a second ergodic distribution. To examine robustness of the tendency of incomes towards the two extremes, I experimented with Markov chain transition models having dynamics beyond the first order. Although details differ, the implied ergodic distributions from richer structures almost invariably carry the same message: thinning out of the middle-income economies in favor of the very rich and very poor. The second ergodic distribution given in these panels is generated from a second-order Markov chain (not reported here), estimated again by averaging across annual transitions and over the long horizon. The bimodal property of the ergodic distribution remains; the tendency for a poverty pile-up is now even more pronounced.

To check consistency of the short- and long-run first-order models, I iterated the one-period transition function of the short-run model to cover the same time span as that of the long-run one. The table's third panel reports this iterated one-step transition. Comparing diagonal entries in the second and third panels shows consistent under-prediction (except for state 2): the data display greater long-run persistence than predicted by the best-fitting first-order model. Also, every entry in the iterated transition function, except the two extreme endpoints, is strictly positive. By contrast, the long-run model shows no transitions between very low- and very high-income states – nine entries in the opposing off-diagonal corners are zero in the second panel (up to two decimal points). Again, this indicates higher

[5]One initially suspects this is due to (a) the way the cells are defined, the higher income cells covering a larger range, and (b) ongoing world-wide growth. Two points are worth noting: first, in the actual observed distributions – both in 1962 and over 1962–1984 – the modal points occur, not in the large-cover high-income states, but instead in the middle state (incomes between 50% and 100% of the world average). See the first column of each panel. Second, these distributions are for incomes relative to world average – world-wide growth is already absorbed in the normalization.

persistence – lower cross-sectional mobility – in the data over long horizons than predicted by a time-homogeneous first-order model.[6]

## 4. Conclusions and extensions

Numerous studies on the determinants of growth have treated the empirical evidence in a way that, I argue here, obscures interesting and important features. This paper provides an alternative framework for studying the dynamics of a rich panel of cross-economy incomes.

Despite its rough, first-pass nature, this study has produced interesting, robust characterizations of the tendency towards a two-camp world, divided between haves and have-nots, where escaping from the poverty trap is a low-probability proposition, either over short- or long-runs. There is greater persistence, at long horizons, than predicted by the best-fitting low-order time-homogeneous models. To refine these statements, to place more interpretation on the dynamics, to bring in conditioning information (explanatory variables), all appear to be useful and feasible research projects.[7] From a theoretical perspective, the empirical descriptions here suggest that economic growth might be insightfully studied using models of income distributions – where the distributions are across, rather than within, entire economies. From an econometric perspective, the more obvious statistical inadequacies here suggest fertile grounds for further research – broadly viewed, this work concerns data were time-series and cross-sectional dimensions have roughly the same order of magnitude. In probability theory, such data structures are known as *random fields*; econometric modelling of the dynamic- and cross-correlation properties of such structures is relatively unexplored. More narrowly, the transition functions above can be enriched by explicit time- and state-heterogeneous modelling; their mobility implications can also be studied using measures such as that in Geweke et al. (1986). The arbitrary discretizing grid used to construct the Markov chain transition matrix is simply a crude non-parametric estimator; sharper methods might yield correspondingly sharper insights. Also of interest is tracing out of the intermediate – rather than just the one-period or alternatively steady-state – dynamics of these evolving distributions.

[6]The diagonal under-prediction is familiar from Markov chain transition studies in sociology and occupational dynamics; see for instance Spilerman (1974) and Singer and Spilerman (1976).
[7]A start at some of these appears in Quah (1992).

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