Contractual Structure and Wealth Accumulation

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Can historical wealth distributions affect long-run output and inequality despite "rational" saving, convex technology and no externalities? We consider a model of equilibrium short-period financial contracts, where poor agents face credit constraints owing to moral hazard and limited liability. If agents have no bargaining power, poor agents have no incentive to save: poverty traps emerge and agents are polarized into two classes, with no interclass mobility. If instead agents have all the bargaining power, strong saving incentives are generated: the wealth of poor and rich agents alike drift upward indefinitely and "history" does not matter eventually. (D31, D91, I32, O17, Q15)

Whether historical levels of inequality and poverty constrain the current performance of economies is a matter of considerable interest in the economics of institutions, growth, and development [see, e.g., Robert J. Barro (1991), Alberto Alesina and Dani Rodrik (1994), Barro and Xavier Sala-i-Martin (1995), Daron Acemoglu et al. (1999), Stanley Engerman et al. (1999), and Michael R. Carter and Frederick J. Zimmerman (2000)]. Theoretical literature on this topic has recently focused on this question in the presence of capital-market imperfections [Glenn C. Loury (1981), Abhijit V. Banerjee and Andrew F. Newman (1993), Oded Galor and Joseph Zeira (1993), Lars Ljungqvist (1993), Ray and Peter A. Streufert (1993), and others]. Barring few exceptions, these theories provide explanations for the evolution, persistence, and macroeconomic consequences of wealth inequality based on technological indivisibilities in investment and the absence of strategic saving behavior.[1] In these models, the poor cannot escape poverty by investing in relevant assets, either via borrowing (owing to capital-market imperfections), gradual accumulation of capital stocks (owing to investment indivisibilities), or stepped-up savings (owing to habit persistence, myopia, limited rationality, or a warm-glow bequest motive in an overlapping-generations setting).

While it is generally agreed that the poor face significant borrowing constraints in most economies, the significance of investment indivisibilities or constraints on savings is open to debate. As Loury (1981) showed, historical levels of inequality cannot affect long-run output and inequality if the technology is convex and agents save strategically, if capital markets are entirely missing (for exogenous reasons). We reexamine this question in a context where capital-market imperfections are endogenously derived from underlying problems of moral hazard. We continue to maintain Loury's assumptions concerning convexity of the technology and strategic savings behavior.[2]

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[1] There is also a literature, exemplified by the work of Paul M. Romer (1986), Roland Bénabou (1996a, b) and Steven N. Durlauf (1996), which generates—via externalities—increasing returns (and therefore history dependence) at the societal level while retaining convexity at the individual level.

[2] A related paper of ours (Mookherjee and Ray, 2000) shows that nonconvexities at the individual level can arise from pecuniary externalities in human capital-accumulation decisions when capital markets are missing, agents save strategically and there are no indivisibilities.

In the presence of limited liability, moral hazard problems give rise to credit constraints for poor agents: they obtain positive but limited access to credit.[3] Credit access depends on the wealth of borrowers, which defines the amount of collateral they can post. Wealth accumulation would allow increases in future collateral, providing a motive for poor agents to save. The presence of uncertainty in returns from production potentially creates a supplementary precautionary motive for saving. On the other hand, the presence of moral hazard endogenously creates nonconvexities in the returns to asset accumulation, possibly restricting saving incentives.

In order to focus on the interplay between these various effects, we simplify the model in a variety of dimensions. We exogenously fix the set of agents (borrowers, workers, tenants, entrepreneurs) and principals (lenders, employers, landlords, financiers), and assume that (i) agents operate a convex production technology, (ii) agent-principal pairs are randomly matched in every period to negotiate over a short-term financial (credit, or interlinked credit-cum-tenancy) contract, and (iii) there are no externalities across agents. So as to focus cleanly on accumulation driven by contractual choice, we assume that agents discount future utility at a rate equal to some exogenous interest rate on savings. In the absence of any moral hazard, it will be optimal for every agent to maintain their wealth over time, so wealth distributions at any later date will exactly mirror the initial wealth distribution. It follows that wealth accumulation incentives in our model will be driven entirely by the nature of contractual distortions resulting from moral hazard.

We find that the pattern of wealth accumulation depends critically on the allocation of bargaining power. We consider two extreme cases: one in which principals, and the other in which agents, have all the bargaining power (i.e., one party makes a take-it-or-leave-it contract offer to the other). Poverty traps arise when principals have all the bargaining power.[4] Owing to their lack of collateral, poor agents are provided a "floor" contract awarding them rents (i.e., a payoff in excess of their outside opportunities), simply in order to provide adequate effort incentives: this "support system" is progressively withdrawn by principals as the agents become wealthier. This effectively creates a 100-percent marginal tax on limited degrees of wealth accumulation by the poor. At sufficiently high levels of wealth, however, the contractual rents disappear, as agents can post sufficient collateral and have high outside options (which rise with wealth): hence wealthy agents do recover some of the benefits of their saving. Consequently, a sharp nonconvexity in returns to savings emerges endogenously. For a range of suitable parameter values, this nonconvexity causes the poor to not save at all, precipitating a poverty trap. Wealthy agents, in contrast, have strong incentives to maintain wealth; consequently (conditional on not sliding into poverty) their wealth drifts upward. The wealth distribution is thus progressively polarized into two "classes" that grow further apart, with no interclass mobility. In this case long-run productivity and distribution depend strongly on wealth distributions in the distant past.

In the alternative setting where agents have all the bargaining power, strong incentives to save are generated *at all wealth levels.* All the benefits of incremental wealth, including those arising from the relaxation of credit constraints, accrue entirely to agents—poor and rich alike. This is supplemented by the precautionary savings motive. Irrespective of the potential nonconvexities that may arise from incentive constraints, no poverty trap can exist in such a

[3] Such models have been widely employed in different areas in recent literature. In the development economics literature, they have been employed to study effects of land reforms, contract regulations, subsidized credit, and public employment programs [Mukesh Eswaran and Ashok Kotwal (1986); Sudhir Shetty (1988); Bhaskar Dutta et al. (1989); Karla Hoff and Andrew B. Lyon (1995); Banerjee et al. (1997); and Mookherjee (1997a, b). In the macroeconomics and finance literature, they have been used to explain the presence of borrowing constraints, why external finance is more expensive than internal finance and why income distribution may have a role in explaining output fluctuations [Ben Bernanke and Mark Gertler (1990); Galor and Zeira (1993); Hoff (1994); Bengt Holmstrom and Jean Tirole (1997)]. Similar models have been used to explain why forms of industrial organization may depend on wealth inequality, and why worker cooperatives may occasionally perform superior to capitalist firms [Banerjee and Newman (1993); Samuel Bowles and Herbert Gintis (1994, 1995), Patrick Legros and Newman (1996); Banerjee et al. (2001)].

[4] This result can be viewed as an alternative formalization of the Amit Bhaduri (1973) hypothesis that rent extraction motives of landlords precipitate poverty traps for their tenants.

setting; agents accumulate wealth indefinitely, irrespective of initial conditions. In this case, therefore, wealth levels exhibit considerable upward mobility, and historical wealth distributions are irrelevant in the long run.

Section I introduces the model. Section II considers the case where principals have all the bargaining power; Section III considers the reverse situation. Sections IV and V respectively discuss possible extensions of the model, and related literature. Finally, Section VI concludes.

I. The Model

A. *Matching, Bargaining, and Power*

There is a large population of *principals* (e.g., landlords, lenders) and *agents* (e.g., tenants, borrowers) randomly matched in pairs at the beginning of any given period. A matched principal–agent pair selects a contract. At the end of the period, the match is dissolved. Then the next period arrives and the story repeats itself, ad infinitum. It is simplest to initially consider the setting where a matched pair do not have the option of seeking out alternative partners to contract with in the same period, should they happen to disagree on the contract. Hence the outside options of either partner are given by autarky in the current period, followed by the prospect of being matched with a fresh partner in future periods.

There are two key assumptions embodied here. One is that a matched principal–agent pair do not expect to be matched in future periods, so they enter into a short-term contract. One possible interpretation of the model is that these agents represent successive generations of an infinitely long-lived dynasty with altruistic preferences à la Barro (1974), in which case each period corresponds to a distinct generation. The implications of allowing long-term contracts will be discussed in Section IV.

The other important assumption is that in any given period a matched pair do not have the option of seeking out alternative partners within the same period. In Section I, subsection F we explain why this is inessential: the same results will continue to hold with strategic search for contracting partners.

Returning to the simple setting described above, the payoff of agents from a contract is analogous to the one-period return in an optimal growth exercise. Their wealth will represent a state variable, whose evolution will depend on outcomes of any given period. The agents will behave like Ramsey planners, maximizing a discounted sum of utilities and accumulating (or decumulating) wealth in the process. We assume there is no analogous state variable for the principals. Since they do not expect to contract with the same agent in future periods, they simply maximize net returns in the current period.

A matched principal–agent pair will have access to a "utility possibility frontier," representing efficient payoff combinations achievable to them from incentive-compatible contracts. Participation constraints correspond to the option of remaining in autarky in the current period, followed by the continuation value from next period onwards when the agent (or principal) is matched with a fresh partner. The precise description of the relevant constraints will be provided below.

We identify the *allocation of power* with the choice of a particular point on the (constrained) utility possibility frontier. Equivalently, one might identify the allocation of power with the implicit welfare weights on the utility of the two parties, which determine allocation of the surplus available after meeting the participation constraints. (To be sure, "power" also resides in the determination of these participation constraints or outside options. For instance, these would be influenced by the ability to strategically choose a contracting partner, a topic we postpone to Section IV, subsection B.)

The literature on noncooperative bargaining [summarized in Martin J. Osborne and Ariel Rubinstein (1990) or Abhinay Muthoo (1999)] provides a number of factors that affect the allocation of power (as defined above). These include factors that range from the individual (impatience, risk aversion) to the social and institutional (mirrored, for example, in the specification of the bargaining protocol). In addition, third parties may be persuaded to change their dealings with the other party in case of a bargaining breakdown (as in Kaushik Basu, 1986). To these factors one might add features that are less frequently modeled but just as pertinent, especially in developing countries: literacy, negotiation skills, access to legal resources, and contractual regulations. We treat all these as exogenous "institutional" factors, and study their impact on productive efficiency and the wealth accumulation process.

In particular, we will study two extreme allocations of bargaining power, corresponding to situations where one party receives a zero implicit welfare weight relative to the other. Effectively, thus, either the principal always makes a take-it-or-leave-it offer to the agent, or vice versa.

B. *Projects and Payoffs*

Turn now to a detailed description of the possible outcomes from any given matched pair of agent (A) and principal (P). A is a worker, entrepreneur, or tenant operating a project for which P leases out relevant assets and provides finance. The project can be operated at a scale α lying between 0 and 1. At scale α the project involves an upfront cost of αf, and yields a return αR (where $R > f$) with probability e. With probability $1 - e$, the project fails to generate a return. The probability e is affected by the (noncontractible) effort exerted by the agent at that date. Without loss of generality, we identify e with effort as well as the success probability. Notice there are constant returns to scale, subject to a constraint on the scale of the project, arising from limits on the time or attention of A. In particular, the production technology is convex.

A's current payoff is given by a von-Neumann-Morgenstern function $u(c) - D(e)$, where c denotes consumption and e effort. The utility function u is continuous, strictly increasing, and concave (note that linear utility *is* included as a special case). We impose limited liability: consumption must be nonnegative. Accordingly, $u(0)$ is the floor consumption utility and we normalize it to zero.

On the disutility of effort we impose the following restrictions: D is strictly increasing, strictly convex, satisfying $D(0) = D'(0) = 0$, $\lim_{e \to 1} D'(e) = \infty$, and $D'''(e) \geq 0$ for all $e \geq 0$.

The principal is risk-neutral, and his payoff is simply expected return net of consumption payments to the agent (a formal description follows shortly).

Finally, A discounts future utility using the factor δ. To abstract from time trends created purely from time preference, we impose the condition $\delta(1 + r) = 1$, where r is some (exogenously given) risk-free interest rate. P also discounts future utility, but with random matching of partners at every date this discount rate will not play any role in the model.

C. *Contracts*

Once matched, P and A negotiate a (short-term) contract. Without loss of generality we may suppose that P pays for the setup cost *ex ante* and receives outcome-contingent payments from A upon completion of the project.

A's *ex ante* wealth is assumed to be observable and collaterizable; the contract can be conditioned on this wealth.[5] In particular, A's *ex post* liability is limited to the sum of the project returns and collateral. Thus, given starting wealth, contractual payoffs can be identified with the *end-of-period* wealth of the agent, which we represent by x in the failure state and y in the success state.

Specifically, if p_s and p_f denote A's payments in success and failure states respectively, then $x = w - p_f$ and $y = \alpha R + w - p_s$, while limited liability requires $p_s \leq w + \alpha R$ and $p_f \leq w$. Then P's *net* returns are $p_f - \alpha f \equiv w - x - \alpha f$ and $p_s - \alpha f \equiv \alpha R + w - \alpha f - y$ under failure and success respectively, while the limited liability constraint reduces to:

$$(\text{LL}): (x, y) \geq 0.$$

To summarize, then, given some initial wealth w, a contract is identified by the collection (α, x, y), where α is the scale of the project, and x and y are end-of-period wealths satisfying (LL).

D. *Consumption-Saving Decisions*

Let w_t be starting wealth at some date t and let (α_t, x_t, y_t) be a contract. The agent's wealth at the end of the period is $z_t \in \{x_t, y_t\}$. A then

[5] The assumption that wealth can be observed is important for the results, or at least for the methodology we follow. If wealth is imperfectly observed (perhaps at some cost), or not observed at all, the analysis would be far more complicated. For instance, wealth revelation constraints would have to be additionally incorporated, and the principal could decide on a wealth investigation strategy. Agents' saving incentives would be affected since they may be able to "hide" future endowments.

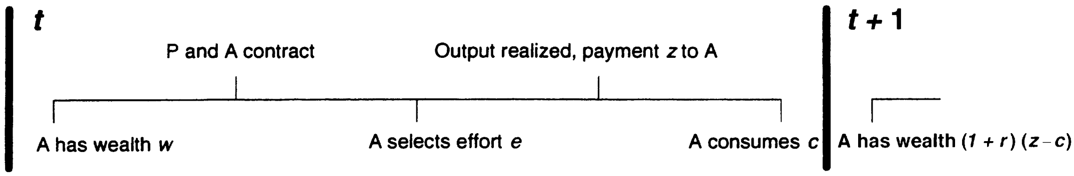

FIGURE 1. SEQUENCE OF EVENTS

chooses consumption $c_t \in [0, z_t]$. The resulting saving $z_t - c_t$ is invested at the risk-free rate $r > 0$, resulting in a level of wealth $w_{t+1} = (1 + r)(z_t - c_t)$ at the beginning of date $t + 1$.

Figure 1 depicts the sequence of events that we have described so far.

E. *Equilibrium*

We study Markov equilibrium (for any given allocation of bargaining power), in which contracts and agent effort are only conditioned on the *ex ante* (start-of-period) wealth of agents, and in which A's consumption strategy depends on *ex post* (end-of-period) wealth. Thus we employ the notation $(\alpha(w), x(w), y(w))$ for a "contract function," $e(w)$ for the effort function, and $c(z)$ for the consumption function. Refer to this collection as an *environment.*[6]

An environment induces a function $V(w)$, the present value utility for A at the *beginning* of any date when he has wealth w. We may think of this as A's *ex ante* value function. Then A's consumption strategy may be described as follows. Let w' denote his *ex ante* wealth for the next date. This corresponds to saving $\delta w'$, selected to solve the following problem:

$$(1)\quad B(z) \equiv \max_{0 \le \delta w' \le z} [u(z - \delta w') + \delta V(w')].$$

Here $B(z)$ denotes A's present value utility at the *end* of any date in which he attains end-of-period wealth z. (This is after project returns have been realized and contractual payments, if any, have been made.) Call this the *ex post* value function.

[6] The term intends to capture the fact that this describes the ongoing practice elsewhere in the economy when a particular principal–agent pair is matched.

Notice that *ex ante* and *ex post* value functions are related to each other as follows:

$$(2)\quad V(w) \equiv e(w)B(y(w)) + (1 - e(w))B(x(w)) - D(e(w)).$$

Observe that the specifications (1) and (2) automatically embody what one might call the *saving incentive constraint* (SIC):

(SIC) At every date, the agent solves (1).

But this is not the only restriction to be imposed. There are the familiar incentive and participation constraints, which we now describe. Say that the *effort incentive constraint* (EIC) is satisfied if specified effort e at every wealth level maximizes $eB(y) + (1 - e)B(x) - D(e)$, yielding

$$\text{(EIC)}\qquad D'(e) = \max\{B(y) - B(x), 0\}.$$

Then there are participation constraints. The *agent's participation constraint* (APC) is met at wealth w if

$$\text{(APC)}\quad eB(y) + (1 - e)B(x) - D(e) \ge B(w)$$

while the *principal's participation constraint* (PPC) is satisfied if

$$\text{(PPC)}\quad w - \alpha f + e(\alpha R - y) - (1 - e)x \ge 0.$$

If, under some environment, a contract-effort pair satisfies (EIC), (APC), and (PPC), we will say that it is *feasible* (relative to that environment).

We are now in a position to define an equilibrium. The concept varies depending on who has the power to make a take-it-or-leave-it offer.

When principals have all the bargaining power, say that an environment is a *P-equilib-*

rium if (a) (SIC) is satisfied, and (b) for every w, the stipulated contract-effort pair ($\alpha(w)$, $x(w)$, $y(w)$) and $e(w)$ maximizes P's payoff $w - \alpha f + e[\alpha R - y] - (1 - e)x$ over the set of contracts that are feasible relative to this environment.

When the allocation of bargaining power is reversed, the definition is modified as follows. An environment is an *A-equilibrium* if (a) (SIC) is satisfied, and (b) for every w, the stipulated contract-effort pair maximizes A's payoff $eB(y) + (1 - e)B(x) - D(e)$ over the set of contracts that are feasible relative to this environment.

Notice that both P- and A-equilibria select contractual sequences that are on the constrained Pareto frontier of equilibrium payoffs. In addition, observe that every feasible contract with $\alpha > 0$ but less than 1 can be weakly improved (for both P and A) by one with either $\alpha = 1$ or $\alpha = 0$.[7] Hence there is no loss of generality by restricting α to lie in $\{0, 1\}$. From now on we shall do so, recalling only that α was introduced in the first place to clarify that there are no technological indivisibilities that drive our analysis.

Notice that $\alpha = 0$ corresponds to no contract being offered at all, so such a case corresponds to $e = 0$ and $x = y = w$.

F. *Strategic Choice of Contracting Partner*

We shall argue now that the same characterization of equilibrium applies when matched partners have the option of searching for other partners to contract with in the same period, provided that there is no restriction on the number of agents that a principal can contract with (i.e., each principal has sufficient assets so that "capacity constraints" do not bind).

Assume that each agent can contract with at most one principal. Given the absence of capacity constraints for a principal, contracting with any agent does not crowd out the opportunity to contract with other agents. Hence there are no externalities across agents: the market for contracts with agents of a specific wealth level is independent of the market for contracts with agents of any different wealth level. Moreover, the utility attained by any agent will be a function only of her own wealth.[8]

In this setting, consider the following search scenario. Each principal "posts" a contract offer conditioned on the agent's wealth. Initially agents are randomly assigned to principals. An agent has the option of either accepting the contract posted by the principal she is assigned to, or searching for another principal, perhaps at some cost. If she decides to search, she visits another principal randomly selected from the pool of principals. Then the agent has the option of accepting the contract offer posted by the new principal, or continuing to search further, and so on.

It is easily checked that in this setting, *the A-equilibrium results if there is no search cost, and the P-equilibrium results whenever the search cost is positive.* In other words, in the absence of any search frictions the market for agents involves Bertrand competition among principals, resulting in the A-equilibrium; in the presence of search frictions the market effectively reduces to a set of segmented monopolies that result in the P-equilibrium. The reasoning is straightforward, so we only provide a brief outline.[9] If there are no search costs, then each A can costlessly search indefinitely and visit every single principal. Hence A can costlessly find the principal offering the contract that generates her the highest utility, and the "demand curve" for any principal is the same as in a Bertrand model. On the other hand, if search costs are positive, the model reduces to the celebrated search model of Peter A. Diamond (1971) which results in the monopoly outcome. The argument is as follows: (a) all principals will offer the same utility to agents of a given wealth level (otherwise the principal offering the contract with the highest utility can offer a

[7] If $eR - f > 0$, then putting $\alpha = 1$ and keeping x, y, e unchanged preserves feasibility and increases the lenders payoff, while leaving the borrower's payoff unchanged. On the other hand if $eR - f < 0$ then the same is true if α is set at 0 and x, y, e are left unchanged.

[8] This will no longer be true in the presence of capacity constraints: agent utility will depend on the distribution of wealth across the set of all agents in the market: this is discussed in Section IV, Subsection B.

[9] Note that since the contracts are short term, we can take as given the continuation payoffs that each party expects from the next period onwards as a function of current-period outcomes, and concentrate entirely on the current-period contract. Moreover, since there are no externalities across agents, we can treat the market for agents with a given level of starting wealth in isolation from all other agents.

more profitable contract with slightly lower utility without losing any clients), and (b) the common utility level offered must be the lowest consistent with participation constraints corresponding to autarky in the current period (otherwise each principal can lower the offered utility level slightly without losing any clients).

In the converse situation in which agents post offers (and principals search for contracting partners), it turns out that the A-equilibrium obtains *irrespective* of the search cost. This asymmetry results because each agent can contract with at most one principal, while each principal can contract with any number of agents. Since contracting with any agent does not crowd out the prospect of contracting with other agents, a principal treats offers from different agents in isolation from one another, and has an outside option equal to zero profit. So each agent will offer a contract which reduces the principal's payoff to zero, leading to the A-equilibrium.

Which of the two search models is more appropriate may well depend on the relative number of principals and agents. If there are relatively few principals and many agents, the model where principals post contract offers while agents search seems plausible. Conversely, if there are relatively few agents and many principals, the reverse situation where agents post offers and principals search would be natural. With positive search costs, the P-equilibrium will arise in the former situation, and the A-equilibrium in the latter setting.

G. *The Static Benchmark*

Our model has a particularly simple static counterpart, studied in more detail in Mookherjee (1997a, b).[10] In this situation, A simply consumes all end-of-period wealth, so that B (the *ex post* value function) is simply replaced by u (the one-period utility function) in all the incentive and participation constraints described above, and (SIC) is ignored.

It will be useful to consider the properties of the optimal static contract when P has all the bargaining power, and the agent has zero wealth. Then (APC) is implied by (LL) and (EIC), so it is optimal for P to set $x = 0$, and the problem reduces to selecting y and e to maximize $e[R - y]$, subject to the constraint that $D'(e) = u(y)$. Let (y^*, e^*) denote the solution to this problem.[11] Note that $e^* > 0$ (and consequently $y^* > 0$), since a small increase in y from 0 would increase P's profit. We make the following assumption:

ASSUMPTION [α]: $e^*[R - y^*] > f$.

Assumption [α] simply ensures that it is optimal to offer some non-null contract (in the static model) when agent wealth equals zero. The consequences of dropping this assumption are discussed in Section IV, subsection C.

Mookherjee (1997a, b) analyzes how the allocation of bargaining power and/or A's wealth affects the optimal contract and the induced effort in this static model. An important implication of the static model is that a P-equilibrium generates positive surplus to the agent when agent wealth equals zero (assuming that [α] is met). When agent wealth climbs, some—possibly all—of the resulting gain is expropriated by the principal, simply because the (APC) was not binding to begin with. In contrast, when A has all the bargaining power, all incremental gains in wealth—and possibly some additional gains resulting from better incentives—accrue to A.

These results suggest that in the dynamic model with wealth accumulation, savings will be affected in different ways under the two bargaining regimes. This is precisely the question investigated in this paper.

H. *The Dynamic First-Best Benchmark: Efficient Wealth Accumulation*

It is also useful to define a benchmark pattern of wealth accumulation that results in the absence of moral hazard, the fundamental market imperfection in the model. If agents' effort can be verified by contract enforcers, it can be written into the contract. Then since each principal is risk neutral, contracts will completely insure

[10] The same model is studied in Dutta et al. (1989), who also extend it to a dynamic context, but with no state variable of any kind.

[11] Under our assumptions, a maximum exists and is unique.

each agent against the uncertainty in the returns to their projects. In addition, the agent will be required to put in the "first-best" level of effort e^*, found by maximizing

$$eR - D(e)$$

and we can map out the entire first-best frontier by means of a fixed payment to the agent—call it i—satisfying the necessary conditions

$$D(e^*) \leq u(i) \qquad \text{and } i \geq 0.$$

The payment i will be determined by specific participation constraints and allocation of bargaining power.

The resulting Ramsey problem for an agent with starting wealth w is then:

$$\max \sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to the constraints that $w_0 = w$, and for each $t \geq 0$,

$$w_{t+1} = (1 + r)(w_t + i - c_t).$$

Because we have assumed that $\delta(1 + r) = 1$, the solution to this problem is to hold wealth constant. Hence any patterns of wealth accumulation or decumulation that may result in an equilibrium with moral hazard can be interpreted as resulting from the presence of moral hazard, in conjunction with the maintained assumptions about relative bargaining power.

II. Where P Has All the Bargaining Power

A. *Wealth Polarization*

The purpose of this section is to outline our main findings when the principal has all the bargaining power. It turns out to be extremely difficult to provide a fully general analysis. The main reason for this is that all incentive and participation constraints for the agent must now be defined—not by some exogenously given utility function as in the static model—but by a value function which is fully endogenous with respect to developments at some later date. This creates significant complications, which we tackle by means of two restrictions.

First, recall the static model and Assumption [α]. Under that assumption, the static P-equilibrium (with $w = 0$) yields a strictly positive payoff to the agent; denote this payoff by v^*. We impose

ASSUMPTION [β]:

$$\text{(3)} \quad \frac{1}{\delta}\left[(1 + \delta)u\left(\frac{R}{1 + \delta}\right) - u(R)\right] < v^*.$$

There are several ways to view this assumption. One might interpret it as stating that the success output R is not "too large," or that the static floor payoff v^* is not "too small," or even as a restriction on the discount factor. But [β] is perhaps best viewed as a restriction on the curvature of u (given all other parameters), stating that the agent does not have excessively strong preferences for consumption smoothing. For instance, [β] is always satisfied when u is linear.

It might be of interest to note that growth or technical change does not jeopardize this assumption, the reason being that R and v^* will typically move in step. For instance, if u exhibits constant relative risk aversion $\sigma \in (0, 1)$ and the effort disutility function is quadratic (say $D(e) = e^2/2$), [β] reduces to

$$\text{(4)} \quad \frac{\delta}{(1 + \delta)^\sigma - 1}\left[\frac{(1 + \sigma)^{1+\sigma}}{2 + \sigma}\right]^{1+\sigma} > 2$$

which depends only on A's discount factor and his degree of risk aversion.

Our second restriction is that we study P-equilibria with continuous *ex post* value functions. This class is not empty. Later, we will explicitly construct one such equilibrium.

PROPOSITION 1: *Assume* [α] *and* [β]. *In any continuous P-equilibrium,*

(*i*) *there exist wealth levels w below R where the agent earns rents: $V(w) > B(w)$; in particular this is true at zero wealth. For any $w > R$, the agents earns no rents: $V(w) = B(w)$.*
(*ii*) *If u is strictly concave, there exists a*

threshold $z^* > R$ *such that A maintains wealth* ($w(z) = z$) *if* $z > z^*$, *and decumulates it* ($w(z) < z$) *if* $R \leq z < z^*$. *If* $\lim_{c \to \infty}[u(c) - cu'(c)] > u(R)$ *then* $z^* < \infty$.

(*iii*) *There is a poverty trap below R, i.e.,* $z_t \leq R$ *implies almost surely that* z_T *and* w_T *do not exceed R at any subsequent date T, and equal* 0 *infinitely often.*

Of particular interest in this proposition is the description of the poverty trap. Once *ex post* wealth falls below the value R, the proposition tells us that wealth can *never* escape this bound. In addition, agent wealth will visit zero infinitely often. The exact behavior of wealth is difficult to describe at this level of generality: for instance, it is quite possible that following certain values of $z < R$ the agent accumulates wealth and starts the following period with $w(z) > z$. What is claimed, however, is that such instances of wealth accumulation will be purely temporary.

In the *particular* P-equilibrium we later construct, the poverty trap does take the sharper form in which once $z \leq R$ at some date, *ex ante* wealth at *every* subsequent date equals zero.

Turn now to the induced dynamics of wealth in all regions, not just within the poverty trap. Notice that a P-equilibrium generates a first-order Markov process for wealth. Let

$$\mathcal{W} \equiv \{w \geq z^* | \text{ there is no feasible contract at } w\}$$

denote the set of wealth levels at which the agent maintains wealth, *and* no feasible contract exists. It should be clear that every wealth in $\mathcal{W}$ is absorbing: once the agent arrives at such a wealth, he is forced to "retire"; moreover, the agent optimally chooses to maintain this wealth forever thereafter. Note also that by Proposition 1, the event of achieving an *ex post* wealth below R is "absorbing," and $\mathcal{W} \cap [0, R]$ is empty. Thus (provided $\mathcal{W}$ is nonempty), these are two nonoverlapping (*ex post*) wealth classes with no mobility between them. We can go further to show that all wealths outside these two classes must be transient:

PROPOSITION 2: *Assume* [α], [β] *hold. Consider any continuous P-equilibrium, in which the wealth maintenance threshold* $z^* < \infty$. *Then from arbitrary initial wealth* w_0, *with probability 1* w_t *either enters a poverty trap or enters* $\mathcal{W}$, *or converges to* ∞. *The probability of transiting to the poverty trap is positive from any initial wealth less than* z^*. *In addition, if* $\mathcal{W}$ *is bounded, the probability that wealth goes to infinity from any* $w > \sup \mathcal{W}$ *is strictly positive.*

Hence the long-run wealth distribution must be concentrated entirely at most three disjoint noncommunicating wealth classes. If $\mathcal{W}$ is nonempty and bounded, then *each* of these classes will have positive mass as long as the initial distribution has support over the whole range of nonnegative wealths. On the other hand if $\mathcal{W}$ is empty, then wealth will either gravitate towards the poverty trap or grow arbitrarily large. In either case the limit wealth distribution will exhibit a high degree of polarization. Long-run wealth distributions will depend on historical distributions, and this dependence is particularly manifested in the possible persistence of poverty.

B. *Explanation and Discussion*

The fundamental observation that underlies many of the arguments is that poor agents will be offered a "floor" contract that awards them utility in excess of their outside option. This, in turn, follows from the limited liability constraint. When such agents acquire wealth, there will be a range over which the earlier contract continues to satisfy the agent's participation constraint, and thus remains feasible—implying that it is also the optimal contract from P's point of view. It follows that moderate increases in wealth are entirely expropriated by the principals. This imparts an (endogenous) nonconvexity to payoffs that hinders wealth accumulation.

To illustrate this as starkly as possible, we explicitly construct a P-equilibrium with a particularly simple structure. This equilibrium yields agent rents *only* over some interval of low wealth values, implying the following relation between the *ex ante* and *ex post* value functions:

$$V(w) = \max\{V(0), B(w)\}. \tag{5}$$

PROPOSITION 3: *Assume [α] and [β] hold. Then there exists a P-equilibrium with the following properties: there is $w^* > 0$ such that*

(*i*) *For all $w < w^*$, the static (P-) optimal contract-effort pair is offered: $e(w) = e^*$, $y(0) = y^*$, $x(0) = 0$ and the resulting floor (present value) utility is $V^* \equiv v^*/(1 - \delta)$. If $w \geq w^*$, then $V(w) = B(w)$.*

(*ii*) *If*

$$(6) \qquad \lim_{c\to\infty}[u(c) - cu'(c)] \geq v^*$$

there is an infinite increasing sequence of ex post wealth thresholds $z_1, z_2, \cdots \rightarrow z^* < \infty$ such that

$$B(z) = \begin{cases} \dfrac{1 - \delta^k}{1 - \delta}\, u\left(\dfrac{(1 - \delta)z}{1 - \delta^k}\right) + \delta^k V^* \\ \qquad\qquad \text{if } z \in [z_{k-1}, z_k] \\ \dfrac{1}{1 - \delta}\, u((1 - \delta)z) \\ \qquad\qquad \text{if } z > z^*. \end{cases}$$

If (6) *does not hold, then the agent fully consumes all ex post wealth and starts the next period with zero wealth.*

(*iii*) *For any $z < z^*$, the agent dissaves ($w(z) < z$), while for any $z > z^*$ the agents maintains wealth ($w(z) = z$). In particular, for any $z < z_1$, the agent saves nothing ($w(z) = 0$).*

(*iv*) *There is a strong poverty trap below w^*, i.e., $w_t < w^*$ implies $w_T = 0$ for all $T > t$ with probability* 1.

Apart from assuring existence, this particular P-equilibrium lends itself to easy interpretation. To begin with, note from part (i) that the value function satisfies the following version of (5): $V(w) = \max\{V^*, B(w)\}$. Thus, the *ex post* value function must satisfy a modified Bellman equation of the form:

$$(7) \quad B(z) = \max_{0 \leq \delta w \leq z} [u(z - \delta w) + \delta \max\{V^*, B(w)\}].$$

This allows us to interpret the *ex post* optimization problem as a Ramsey problem with an "exit option": at any date (starting from the next period), the agent can depart with some outside option V^*. (It is important to realize that this is only an interpretation that allows us to solve the mapping from *ex post* wealth to the following period's *ex ante* wealth, but says nothing about the actual evolution of wealth or contracts thereafter.)

Part (ii) of the proposition solves this modified Ramsey problem. As long as (6) is satisfied, the optimal saving strategy out of *ex post* wealth is characterized by an infinite sequence of thresholds $z_1, z_2, \cdots$ converging to some finite value z^*, with the property that for any wealth below z_1, A dissaves maximally. For intermediate wealth levels he plans to consume at a rate which would run down his wealth in a finite number of periods. For levels above z^* he fully maintains *ex post* wealth.

The resulting *ex post* value function B is continuous and strictly increasing. It is concave beyond z^* in the wealth maintenance region. Before this, however, every threshold constitutes a point where B is kinked and locally nonconvex. Crossing one of the thresholds causes the agent to slow down the rate at which the wealth is decumulated, by consuming less every period, which increases the marginal utility of the next increment in wealth. Figure 2 illustrates this function.

If, on the other hand, (6) is not satisfied, the situation is much easier to describe. Savings out of *ex post* wealth are always zero, so that $B(z)$ is simply $u(z) + \delta V^*$. Whether or not (6) is met depends on the parameters of the problem.[12]

In both cases, the *ex post* value function is defined entirely by the utility V^* corresponding to the floor contract. To complete the proof of equilibrium it suffices to check that the optimal contract designed by P for zero-wealth agents whose savings and effort incentives are defined by this value function, generates exactly the same floor contract. Assumption [β] ensures this is indeed the case.[13] We therefore have a bonafide P-equilibrium.

[12] A utility function of the form $u(c) = c^\gamma$, with $\gamma \in (0, 1)$, *always* satisfies (6). A linear utility function *never* does.

[13] Specifically, it ensures that R, the maximum return from the project, is less than the first threshold z_1 required to induce A to save anything. Since R exceeds the maximum payment that the principal might conceivably make to A (with zero initial wealth) in the event of success, this implies that such an agent will invariably consume his entire end-of-period wealth. This reduces the contracting problem to a static one, so P will indeed offer such agents the same contract as in a one-period setting. This, in turn, is the floor

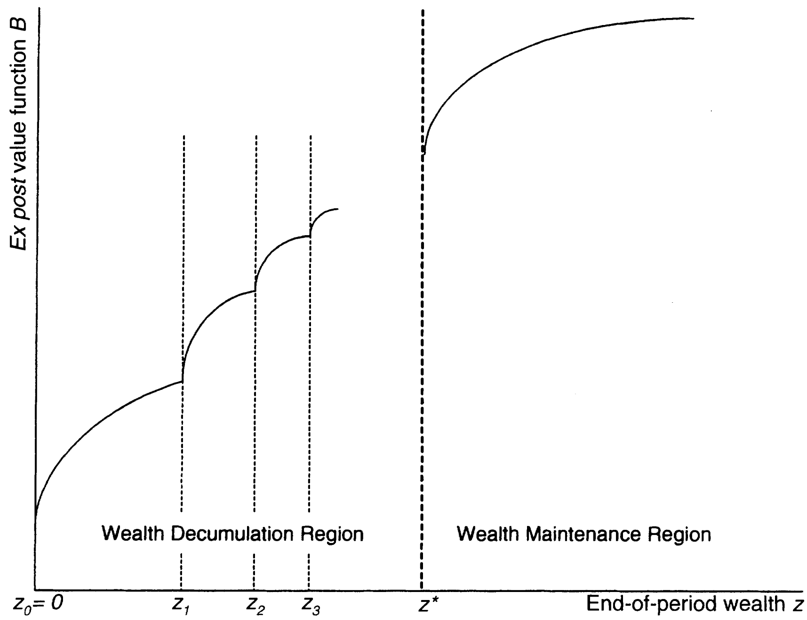

FIGURE 2. THE *EX POST* VALUE FUNCTION *B*

The wealth dynamics under this equilibrium take on a particularly simple form. When initial wealth falls below the threshold w^*, the agent receives the floor contract, in which *ex post* wealth will surely fall below R. Assumption [β] guarantees that R is less than the first "dissaving" threshold z_1. It follows that the agent will dissave maximally and start the next period with zero wealth, so that the same story is repeated ad infinitum thereafter.

What happens when the initial wealth exceeds w^*? When (6) fails the answer is obvious: the agent will invariably consume all current wealth, and enter the poverty trap the very next period. When instead there is enough preference for consumption smoothing so that (6) is satisfied, the agent will plan to maintain his wealth for sufficiently large realizations of *ex post* wealth. Nevertheless, over any bounded range of *ex ante* wealth levels for which a non-null feasible contract exists, the probability of a single failure is (uniformly) positive; hence, the probability of any given finite number of successive failures is also uniformly positive over the bounded region of starting wealths. Moreover, any such failure must result in a drop in tomorrow's *ex ante* wealth by some discrete amount. Hence from any initial wealth in this set, a finite string of successive failures will take the agent into the poverty trap, an event which therefore has uniformly positive probability over any compact range of wealths for which a feasible contract exists.

To be sure, if the agent happens to land in the interim at a maintenance wealth level at which no feasible contract exists, he stays there forever.

In the case where feasible contracts exist for an unbounded range of wealth levels, similar but more complicated arguments apply, and the Appendix may be consulted for the details.

Similar phenomena obtain in *any* P-equilibrium with continuous value functions. The reasoning there is more involved. The main difference in the nature of the equilibrium occurs at wealth levels below R, the gross return from the project in the successful state. In a general P-equilibrium, the agent could conceivably earn rents anywhere within this region, and in any pattern: in particular, no restriction can be derived on the slope of the value function V over this range. Hence no implication can be

contract used to construct the *ex post* value function, so these strategies do constitute a P-equilibrium.

derived regarding the nature of local saving incentives for poor agents, unlike the simple P-equilibrium. It is possible, for instance, that sufficiently poor agents save at a high rate in order to avail of rents accruing at higher wealth levels within this region.

What can be shown, however, is that rents *cannot* accrue to agents with wealth above R.[14] Thus the scope of any incentives to accumulate wealth among poor agents must be restricted to the region $[0, R]$. Moreover, Assumption $[\beta]$ implies that given any *ex post* wealth in this region, it is optimal for the agent to save or dissave in such a way as to arrive at a rent-generating contract within $[0, R]$ at some future date. The region $[0, R]$ collectively constitutes a generalized poverty trap—once the agent is in that region he can never escape it, and must arrive at 0 infinitely often thereafter. The rest of the wealth dynamics—for higher wealth levels outside this trap region—turns out to be qualitatively similar to the case of the simple P-equilibrium discussed above.

III. Where A Has All the Bargaining Power

In this section, we show that once the agent has all the bargaining power, no polarization of the wealth distribution is possible, in stark contrast to behavior exhibited under P-equilibrium.

We are able to establish these results quite generally. In particular—and unlike the analysis of P-equilibrium—we impose no parametric restrictions on the problem. For the most part, however, we do restrict attention to a subclass of equilibrium functions, for which the *ex post* value function $B(z)$ is right continuous; we refer to these below as *right-continuous A-equilibria.* Obviously, this class is broader than the class of all continuous equilibria.

We begin by showing that in any A-equilibrium, the agent is able to fully internalize all marginal gains to saving.

PROPOSITION 4: (*i*) *Consider any A-equilibrium. Let* $c(z) \equiv z - \delta w(z)$ *denote the consumption strategy of the agent. Then at any wealth* w:

$$(8)\quad \lim_{\varepsilon \to 0+} \frac{V(w+\varepsilon) - V(w)}{\varepsilon} \geq u'(\max\{c(y(w)), c(x(w))\}).$$

(*ii*) *If the equilibrium is right continuous, then at any* w:

$$(9)\quad \lim_{\varepsilon \to 0+} \frac{V(w+\varepsilon) - V(w)}{\varepsilon} \geq \left[e(w) \frac{1}{u'(c(y(w)))} + (1 - e(w)) \frac{1}{u'(c(x(w)))} \right]^{-1}$$

and the consumption strategy has the property that at any z:

$$(10)\quad \frac{1}{u'(c(z))} \leq e(w(z)) \frac{1}{u'(c(y(w(z))))} + (1 - e(w(z))) \frac{1}{u'(c(x(w(z))))}.$$

The reasoning underlying these results is quite simple: any increment in *ex ante* wealth of the agent can be rebated to him *ex post* without upsetting the incentive constraints in a way that jeopardizes the profitability of the principal. The value of increased *ex post* wealth is, in turn, bounded below by the marginal utility of consumption, as the agent always has the option of immediately consuming the incremental wealth. Hence the rate of increase of V is bounded below by the corresponding marginal utility of (the higher of the two possible values of) consumption at the end of the period, as expressed by (8). Right continuity of the value function B ensures that the increased *ex ante* wealth can be rebated to the agent to result in an uniform increase in *ex post* value in both states, success and failure, which leaves effort incentives entirely unchanged. Hence the *expected* marginal utility of *ex post* consumption forms a lower bound to the return to incremental *ex ante* wealth. Since the optimal consumption-saving

[14] This is where the continuity of the value function B plays a key role.

strategy of the agent trades off current consumption against starting the next date with a higher wealth, the "Euler equation" (10) follows. It incorporates the precautionary motive for saving that arises from the uncertainty in project outcome at future dates: roughly speaking, the marginal utility of current consumption is related to the expected marginal utility of consumption at the following date. Here it takes the form of an inequality because of the possibility of a corner solution (where the agent consumes all current wealth), and because we only have a lower bound on the rate of return to saving.

Notice that effects of augmenting future wealth equal the rate of return on savings, *plus* the surplus-enhancing benefits of relaxing credit constraints, which are especially sharp for poor agents. One would therefore expect poor agents to save aggressively, rendering poverty traps less likely. In particular, if u is linear and the agent has no preference for consumption smoothing, then the rate of return on saving given by (8) is everywhere at least as large as the marginal utility of current consumption. In such a context it is always a best response for the agent to save *all* current wealth, at least up to the wealth level where the contract itself attains full efficiency. The contrast with savings policies in the simple P-equilibrium is especially striking: there agents with linear utility functions never save anything, ensuring the perpetuation of poverty.

We now use Proposition 4 to establish more general behavior for the class of all concave functions, not just the linear ones. Begin with the idea of a "strong" poverty trap, the existence of which was established for the simple P-equilibrium. Recall that this is a situation in which zero *ex ante* wealth constitutes an absorbing state.

PROPOSITION 5: *If u is strictly concave, there cannot be any A-equilibrium with a strong poverty trap.*

(10) can be used (in the case where the equilibrium is right continuous) to gain some intuition for this result. If there is a strong poverty trap, the agent must consume his entire *ex post* wealth in both success and failure states. Moreover, his consumption thereafter must follow a stationary distribution, exactly equal to the *ex post* wealth distribution that results from the contract offered to a zero-wealth agent. If u is strictly concave, condition (10) must be violated in the successful state (the marginal utility of current consumption is too low relative to the expected marginal utility of consumption at the succeeding date, i.e., the agent is saving too little).

More generally, (10) implies that consumption cannot converge to an invariant distribution with a bounded support. This also suggests the impossibility of a more general poverty trap, as described in Proposition 1, in which zero wealth is not an absorbing state, but a recurrent state instead. The reasoning outlined in the previous paragraph suggests that from any initial wealth level, the stream of possible future consumption levels (hence also wealth) will be unbounded above. Hence, irrespective of initial conditions, all agents can possibly become arbitrarily wealthy.

This suggests something more than just the absence of a poverty trap (weak or strong), and indeed an "upward mobility" result can be established for any right-continuous equilibrium. Consider first the case where a feasible contract exists at all wealth levels, and marginal utility is bounded away from zero. Then *with probability 1* every agent—rich or poor—must eventually become arbitrarily wealthy:

PROPOSITION 6: *Assume that u is strictly concave, and that $u'(\infty) > 0$. Consider any right-continuous A-equilibrium in which a feasible non-null contract exists at all wealth levels. Then wealth almost surely converges to ∞ from any initial level.*

The outline of the argument is as follows. Under the stated assumptions, (10) reduces to the statement that $1/u'(c_t)$ forms a submartingale; hence c_t converges almost surely. In the event that it converges to something finite, agents cannot be motivated to supply effort by the prospect of higher levels of consumption, but instead by the prospect of higher levels of future wealth. This must eventually result in overaccumulation of wealth, i.e., the agent would not be pursuing an optimal consumption-saving strategy. Hence consumption must converge to infinity almost surely, implying the same must be true of wealth.

What can we say about the case in which a feasible contract does not exist at all wealth levels? It is natural to expect that if sufficiently high levels of consumption reduce marginal utility of further consumption to negligible levels, agents will "retire" upon achieving certain thresholds of affluence.

PROPOSITION 7: *Consider any right-continuous A-equilibrium, in which a feasible non-null contract exists if and only if wealth lies below some threshold W^*. Then from any initial wealth $w \in [0, W^*)$, the wealth at some future date will exceed W^* with positive probability.*

Hence irrespective of initial conditions every agent must "retire" with positive probability at some future date. However, it is difficult to ensure that retirement is an absorbing state, so one must entertain the possibility that agents retire for a finite number of periods and then return to productive activity. In this case "affluent retirement" is a recurrent event for all agents, irrespective of their initial wealth. The net result is similar to the previous case: the economy exhibits a high degree of upward mobility.

IV. Extensions

In this section we discuss possible extensions of our model, and their possible implications for the robustness of our results.

A. *Long-Term Relationships*

The random-matching assumption enabled us to ignore long-term contracts. Depending on the context, this may or may not be good modeling strategy. If different periods correspond to different generations of agents, this is a reasonable approach: one cannot expect parents to enter into contracts that bind their children. If instead they correspond to different dates in the life of the same agent, the assumption may be restrictive. Moreover, even if only short-term contracts are allowed (say, owing to the inability of parties to commit to long-term contracts), additional dynamic complications from the principal's side may appear if a principal expects to contract repeatedly with the same agent in future periods. This is particularly pertinent to the P-equilibrium, since one context where principals may naturally be thought of as having disproportionate bargaining power is when the market is monopolistic.

Suppose, then, that a *single* P possesses a monopoly over contracting rights with a set of agents. Suppose, moreover, that this monopolist owns enough productive assets and wealth to be able to transact simultaneously with all the agents; hence there is no need for agents to compete with one another for the right to obtain a contract. Then the notion of P-equilibrium is appropriate only if P is completely myopic. *Otherwise each agent's wealth will also constitute a state variable for P.* He will offer contracts with the possible manipulation of the agent's future wealth in mind. Consequently, the appropriate notion of equilibrium must incorporate the effect of the current contract on future profits of P owing to dependence of those profits on the agent's future wealth.

In order to analyze the resulting complications, we need to evaluate the effect of agent's wealth on P's profit. There are two contrasting effects. On the one hand, with greater wealth, the limited liability constraint is attenuated, permitting high-powered incentives and therefore reducing P's exposure in the event of a project failure. On the other hand, a wealthier agent's willingness to exert effort decreases owing to wealth effects in the demand for leisure. The trade-off between these two effects is difficult to resolve in general. If the second effect dominates, then P benefits from contracting with poorer agents, and the tendency toward a poverty trap is further accentuated by dynamic considerations on P's side. Conversely, if the first effect dominates, a poverty trap would be less likely.

Suppose we were to abstract from the second effect by assuming that the agent's utility over consumption is linear. Then wealth has no effect on the demand for leisure. The preceding argument suggests that in this case a poverty trap is least likely. *It turns out, however, that our results on P-equilibria extend to this case:* with a single monopolist principal, indefinite repetition of the static optimal contract and zero saving by A (at *all* wealth levels) is a perfect equilibrium (when P makes take-it-or-leave-it offers). Indeed, if the game has a finite termination date, this is also the *unique* subgame-perfect equilibrium outcome.[15] The contracts offered by P award rents to A in the form of the

[15] When the relationship is infinitely repeated, other equilibria may conceivably appear. We conjecture that our analysis would continue to apply if we selected the

floor contract below the threshold w^*, and no rents for wealthier agents. The agent's *ex ante* value function then reduces to $V(w) = \max\{V(0), w\}$, inducing A to entirely consume any wealth at the end of any period, in order to avail of the floor $V(0)$ at the next period. A's incentives in the dynamic setting are then exactly the same as in a static setting; anticipating this, it is optimal for P to offer the static optimal contract at all dates, as the current contract has no effect at all on A's future wealth.

The argument of the two preceding paragraphs suggest that reintroducing concavity into the agent's (consumption) utility function will only strengthen the existence of poverty traps. Unfortunately this is not something that we have formally been able to establish.[16]

In the reverse situation, there is a single monopolist agent dealing repeatedly with a large number of principals. Then our analysis of the A-equilibrium applies *exactly*. For in such a situation, A will make a take-it-or-leave-it offer which will push each P down to their outside option, where they earn zero profits. Dynamic considerations will arise only on A's side, and contracts will be chosen to maximize A's present value utility, subject to breakeven constraints for the principals, as represented in the definition of A-equilibrium.

In summary, our results extend to contexts where long-term contracts are allowed, in the case where agents have no preference for consumption smoothing. And we conjecture that they are reinforced if agents do have preferences for smoothing, though this remains to be formally verified.

B. *Alternative Models of the Market for Contracts*

Our model of search for contracting partners was based on (i) an asymmetry between the two sides of the market: one side posted offers while the other side searched for partners; and (ii) the absence of capacity constraints on the side of the principals. We now discuss the implications of alternative formulations on both of these dimensions.

Begin by dropping (i), while retaining (ii). Consider instead a pairwise matching approach (as in Ariel Rubinstein and Asher Wolinsky, 1985). Within any given period, there are many successive rounds; if a pair matched in any given round fail to agree on a contract, they remain in the pool of market participants that are randomly rematched in the next round. Given pairwise matching, if there are a different number of principals and agents, then some of those on the "long side of the market" will not be matched in the current round and wait to be matched in future rounds. Delays in matching may result in a small cost to either party, or a shrinkage of the available surplus. This model permits the exploration of an alternative source of bargaining power, based on outside options rather than implicit welfare weights. The outside options will depend on the imbalance between the number of agents and principals, since this affects the waiting times for each type of agent, in the event of disagreement with the current partner.

To abstract from the source of bargaining power studied so far, suppose that the symmetric Nash bargaining solution applies to select a contract for any matched pair, with disagreement payoffs given by their outside options from returning to the pool of unmatched participants at the next round. Suppose, moreover, that there are relatively few principals and many agents, so the latter are on the long side of the market. Then the disagreement payoff of an agent decreases with increased imbalance between the number of principals and agents, since this increases the time an agent must wait until the next match (in the event of a current disagreement). Indeed, if the imbalance is sufficiently great, the disagreement payoff converges to the autarkic utility level. On the other hand, the disagreement payoff for a principal is always effectively zero, owing to the absence of capacity constraints. It follows that the greater the imbalance between the two sides, the more profitable will be the selected contract for the principal.

In fact, if the agent's wealth is sufficiently low, the limited liability constraint for the agent

equilibrium that maximizes the principal's payoff from the class of all subgame-perfect P-equilibria.

[16] The difficulties stem from discontinuous upward jumps in the agent equilibrium value functions. Such jumps correspond to (local) zones of high-powered incentives that the principal may want to offer to the agent. But such zones can only exist if there are *further* jumps in the value function at even higher levels of wealth. This sort of bootstrapping can possibly be ruled out, but this is beyond the scope of the present paper (and—presently—its authors).

tends to bind, and the contract selected by the Nash bargaining solution will be the profit-maximizing contract.[17] But then, just as in this paper, the effects of small (agent) wealth increments will accrue entirely to the principal, cutting down the incentive to save. To be sure, at higher wealth levels the selected contract will diverge from the profit-maximizing contract, owing to the fact that the latter awards the agent no utility gain relative to her disagreement payoff (as the participation constraint defined by autarky binds, instead of the limited liability constraint). Now the agent will participate in the marginal gains resulting from any wealth increment. A nonconvexity similar to that in the P-equilibrium in the agent's value function and saving incentives reappears in this setting, even though the exact equilibrium will diverge from the profit-maximizing contract at high wealth levels.

In contrast, if the principals are on the long side of the market, then as the cost of delay across successive rounds becomes small the equilibrium will converge to the A-equilibrium. Since each agent expects to be rematched in every successive round, the delay costs they encounter are small. With strategies conditioned only on the current number of unmatched participants on either side (a standard assumption made in the Rubinstein-Wolinsky model), it follows that agents will obtain almost all the surplus in such a case, in both absolute and marginal terms.

Next, we discuss the implications of dropping assumption (ii) concerning lack of capacity constraints. Each capacity-constrained principal will try to seek out agents with "profitable characteristics." In particular, the options available to any agent will depend not only on her own wealth, but also those of other agents. This complicates the model considerably, since the appropriate state variable involves the entire wealth distribution across agents.

However, one observation is easy to make: the A-equilibrium must arise whenever each principal contracts with one agent, there are more principals than agents, and the principals engage in Bertrand competition with one another (as in the search model in Section I, Subsection F with zero search costs). The reason is that an equilibrium must involve every principal earning zero profits (since at least one principal, P_1 say, must end up not contracting with any agent, and thus earn zero profit; any other principal earning positive profit could be undercut by P_1).

The real complications arise in the case with fewer principals than agents: then some agents will have to stay in autarky, while others obtain a contract. The selection of agents that end up with a contract will depend on their relative wealth levels: their value functions will be interdependent in a complex manner. In particular, savings incentives can now appear for low-wealth agents, since an increase in their future wealth will increase their chances of obtaining a contract in future periods which awards them rents owing to limited liability. So our results concerning the P-equilibrium are unlikely to obtain in this setting.

C. *Market Exclusion*

Assumption [α] ensured that agents with zero wealth would not be excluded from the market. In its absence, there will exist an interval of wealth levels such that only the null contract (with $\alpha = 0$) is feasible.[18] Such agents will therefore be excluded from the market. If the "entry wealth threshold" $\bar{w}$ is not too large,[19] then at $\bar{w}$ the P-optimal contract will award a strictly positive surplus to both parties: this corresponds to a phenomenon analogous to a *tenancy ladder* (Shetty, 1988).

In this case we conjecture that the poverty trap would take the form of wealth level $\bar{w}$ constituting a recurrent state. Agents with wealth below $\bar{w}$ would seek to accumulate assets up to $\bar{w}$ via saving in order to be eligible to enter the market; conversely agents with wealth above $\bar{w}$ would take care to avoid falling below $\bar{w}$ to avoid exclusion. Agents with wealth $\bar{w}$ would have no (local) incentives to increase

[17] This occurs if the agent's payoff at the profit-maximizing contract is sufficiently large relative to the agent's autarkic utility (owing to limited liability). Then the utility gain for the agent at the profit-maximizing contract relative to the disagreement payoff is large relative to the payoff gain for the principal, so the profit-maximizing contract will be the Nash bargaining solution.

[18] Specifically, all agents with wealth less than $\bar{w}$ will be excluded, where $\bar{w} \equiv f - e^*[R - u^{-1}(D'(e^*))]$.

[19] The specific condition is $e^*D'(e^*) - D(e^*) \geq u(\bar{w})$.

their wealth above $\bar{w}$. In the successful state they would save just enough to ensure that they start the next period with exactly $\bar{w}$. A failure would, however, cause their wealth to fall to zero, whence they would tend to save their way back to $\bar{w}$ in due course, and the same cycle will be repeated thereafter.

D. *Endogenous Entry and Exit*

A more ambitious extension of the model would endogenously determine entry into the ranks of principals and agents. For instance, wealthy agents could become principals and conversely poor principals could become agents, and the allocation of bargaining power at any date could depend on the relative numbers of principals and agents. If we combine this approach with that described in this paper, both wealth polarization and sustained accumulation might constitute steady states of such an extended model. Specifically, poverty traps and wealth polarization are consistent with the presence of a few wealthy agents (who become principals) and a large number of poor agents, which restricts entry into the ranks of the principals, reinforcing the disproportionate bargaining power of the latter. Conversely, sustained accumulation is consistent with an ever-present and large pool of principals (owing to the significant upward wealth mobility in the equilibrium) that compete vigorously with one another, causing agents to have most of the bargaining power.

V. Related Literature

The relation to the imperfect capital-market models of Loury (1981), Banerjee and Newman (1993), and Galor and Zeira (1993) have already been discussed above. Tomas Piketty (1997) considers a model with diminishing returns to (divisible) capital, but assumes there are two levels of effort, and that savings rates are constant and exogenous. His model has no poverty traps, but owing to pecuniary externalities has multiple steady states, whereby long-run output can depend on initial inequality. Philippe Aghion and Patrick Bolton (1997) use a nonconvex technology, and an exogenous savings rate: the latter is assumed high enough to ensure that wealth is ergodic; they focus on inequality dynamics generated by pecuniary externalities across agents in the intermediate term. Maitreesh Ghatak et al. (1997) analyze a model with an exogenous investment threshold and endogenous savings in a two-period setting. They focus on the effects of credit-market imperfections on the effort and saving incentives of young agents in a competitive setting where agents have all the bargaining power.

In contrast to these papers, our model is characterized by a convex technology, endogenous savings decisions in an infinite-horizon framework, absence of pecuniary externalities across agents, and varying allocations of contractual bargaining power. Whether or not poverty traps arise depends on the endogenous emergence of investment thresholds, which are determined exclusively by institutional characteristics of the economy.

R. Glenn Hubbard et al. (1994, 1995) show that social insurance programs with means testing discourage saving by poor households, owing to the phase-out of safety nets as they accumulate assets. Moreover, they show that such a model can replicate empirical patterns concerning savings rates across households of different wealth levels. This mirrors the effect of the "floor" contract available to poor agents in the P-equilibrium in our model. The main difference is that our model concerns a laissez-faire market economy rather than the effect of an exogenous government welfare program. In particular it is in the private interest of P's with a lot of bargaining power to offer contracts similar to a means-tested social insurance program which is phased out as agents accumulate assets. This is consistent with the description of "patron-client" relationships characterizing unequal hierarchical societies, in which the role of quasi-monopolistic landlords and lenders with their clients simultaneously incorporates patterns of "patronage and exploitation" frequently emphasized by sociologists (e.g., Jan Breman, 1974).

Other literature on the dynamics of inequality in a asymmetric information contracting framework includes Edward J. Green (1987), Jonathan Thomas and Tim Worrall (1990), Andrew Atkeson and Robert E. Lucas, Jr. (1992), Cheng Wang (1995), and Christopher Phelan (1998). These papers study efficient insurance where agent endowments are private information and follow an independently and identically distributed (i.i.d.) process. In this literature, wealth constraints do not play any role and agents cannot save by assumption. Moreover the incentive problem arises from private information (unobservable endowments) rather than moral hazard (unobservable effort); and their focus is on a social planning problem of designing an efficient long-term mechanism. In contrast, we study a sequence of equilibrium short-period contracts in a decentralized setting. The results also differ markedly: the insurance model tends to generate (almost) all agents drifting down into poverty. In the corresponding "competitive" case, our model produces a diametrically opposite conclusion: agents' wealth drift upwards indefinitely, irrespective of initial conditions.[20]

VI. Concluding Comments

Our model suggests that poverty traps can arise even when agents are farsighted and save strategically, and there are no technological nonconvexities or externalities. The extent of wealth mobility then depends in a fundamental way on the institutional characteristics of the economy. In this paper, we have concentrated on one of those characteristics: the unequal allocation of bargaining power. Our principal result is that incentives for poor agents to accumulate wealth are less than in a first-best setting (without moral hazard or uncertainty) if they have too little bargaining power, and greater than first-best if they have all the bargaining power. The basic reason is that in the former case the principal appropriates most of the gain in surplus when agent wealth rises, as the limited liability constraint forces the principals to provide incentive rents to poor agents. For exactly the mirror-image set of reasons, an A-equilibrium generates overaccumulation: an increase in wealth generates an increase in efficiency in the contract (once again owing to the limited liability constraint), but this time the agent pockets it all. This "marginal efficiency" improvement may be thought of as an additional rate of return on wealth accumulation. Added to r (the rate of interest), the equilibrium generates an "effective" rate of return that makes for unbounded wealth accumulation under the A-equilibrium.

In future research it would be worthwhile to explore more fully some of the extensions described in Section IV, e.g., existence of long-term relationships, capacity constraints on the principals' side, and endogenous entry into the ranks of the principals and agents. We also abstracted from the possibility of asset sales that might permit an agent to purchase the right to become a principal. For instance, in an agricultural setting a tenant may be able to purchase land from a landlord for purposes either of self-cultivation or becoming a landlord. Land markets are known, however, to be thin in most developing countries, a phenomenon which needs to be theoretically explained. Mookherjee (1997a) provides one explanation using a static version of the model in this paper, based on credit constraints faced by poor tenants. In a dynamic setting, however, such credit constraints can conceivably be overcome via saving, as stressed by Carter and Zimmerman (2000) in the context of a numerical model calibrated to Nicaraguan data. The implications of potential land-market transactions in a dynamic setting deserves to be analyzed in future research.

Finally, it would be worthwhile to explore some empirical and policy implications of our analysis. Is there any evidence suggesting the role of institutional factors affecting relative bargaining power on savings and upward mobility? If so, what policy measures might affect these institutional variables? Social insurance programs where benefits are phased out as agents become wealthier may encourage the formation of poverty traps. Poverty-reduction strategies should perhaps devote greater attention to institutional reforms such as asset redistributions, legal and literacy reforms that distribute contractual bargaining power more equitably. Detailed policy implications must ultimately be based on careful empirical analysis of the role of these factors in investment incentives.

[20] This owes partly to the operation of precautionary motives for "private" saving in our model. A key role is also played by the limited liability constraint in our model, which ensures that the incentive problem remains nonnegligible for poor agents. Despite these differences, it is interesting to note that both models are characterized by a similar "Euler" equation governing the intertemporal distribution of consumption. The dynamic properties of the two models thus appear to be similar in many respects. We thank Ethan Ligon for this observation.

APPENDIX

For the sake of brevity, we omit details at several points. A complete Appendix is available at ⟨http://econ.bu.edu/dilipm/wkpap.htm/cswaapp.pdf⟩

In what follows, Assumptions [α] and [β] are maintained throughout, though they do not apply to all the lemmas or propositions. The normalization $u(0) = 0$ is also employed without comment. Finally, recall that v^* is the payoff to the agent under the one-shot P-optimal contract, and $V^* \equiv (1 - \delta)^{-1}v^*$.

LEMMA 1: *In any P-equilibrium, if* $y \leq y'$, *then* $B(y') - B(y) \geq u(y') - u(y)$.

PROOF:

Letting $c(z)$ denote A's optimal consumption at wealth z, we have $B(y) = u(c(y)) + \delta V(w(y))$, while $B(y') \geq u(c(y) + y' - y) + \delta V(w(y))$, implying $B(y') - B(y) \geq u(c(y) + y' - y) - u(c(y)) \geq u(y') - u(y)$, the last inequality resulting from the concavity of u and $c(y) \leq y$.

LEMMA 2: *In any P-equilibrium,* $V(0) > B(0) > 0$.

PROOF:

Suppose that P selects $x = 0$ and $y = y^*(>0)$ from the static optimal contract. In response, A will choose $e \geq e^*$, since $B(y) - B(0) \geq u(y) - u(0)$ for all y (by Lemma 1). This will generate a profit for P no smaller than that of the static optimal contract, so Assumption [α] implies that there does exist a feasible and profitable contract at $w = 0$. But any such contract *must* yield the agent strictly positive return (because either x or y or both must be strictly positive). So $V(0) > 0$.

But $B(0) = u(0) + \delta V(0) = \delta V(0)$. Because $V(0) > 0$, we conclude that $V(0) > B(0)$.

LEMMA 3: *Consider the Ramsey problem with exit option* V^* *depicted in* (7).

(*i*) *If*

$$\text{(A1)} \quad \lim_{c\to\infty}[u(c) - cu'(c)] > (1 - \delta)V^*$$

is violated, then the solution is (for all z*):*

$$\text{(A2)} \quad c(z) = z, \quad B^*(z) = u(z) + \delta V^*.$$

(*ii*) *If, on the other hand,* (A1) *is satisfied, the solution is described as follows. Let* $T(z)$ *equal the optimal date of "exit" for an agent with initial wealth* z, *with* $T(z) = \infty$ *if the agent never exits. There exists an infinite sequence* $\{z_k\}_{k=0}^{\infty}$, *with* $0 = z_0 < z_1 < z_2 < \cdots$, *and* $z^* \equiv \lim_{k\to\infty} z_k < \infty$, *such that* $T(z) = k$ *for* $z \in [z_{k-1}, z_k]$ *(with indifference holding for adjacent values of T at the endpoints), and* $T(z) = \infty$ *for all* $z \geq z^*$. *Moreover, under* [β], *we have* $R < z_1$.

The associated value function in case (*ii*) *is*

(A3)

$$B^*(z) = \frac{1 - \delta^k}{1 - \delta}\, u\left(\frac{(1 - \delta)z}{1 - \delta^k}\right) + \delta^k V^* \quad \text{for all } z \in [z_{k-1}, z_k]$$

$$= \frac{1}{1 - \delta}\, u((1 - \delta)z) \quad \text{for all } z \geq z^*.$$

PROOF:

Consider the related problem of selecting a real number $x \in [1, 1/(1 - \delta)]$ to maximize $\psi(x; z) \equiv xu(z/x) + [1 - (1 - \delta)x]V^*$. Here x corresponds to $(1 - \delta^k)/(1 - \delta)$, where the exit date k is treated as a continuous variable in $[1, \infty)$. Since it is optimal to smooth consumption perfectly until the exit date, A will consume until the exit date at the steady level of $c(z) = z(1 - \delta)/(1 - \delta^k)$, thereby running down wealth to 0 at k, and exiting with V^*. This generates the value function

$$B(z) = \frac{1 - \delta^k}{1 - \delta}\, u\left(\frac{z(1 - \delta)}{1 - \delta^k}\right) + \delta^k V^*.$$

Notice that

$$\psi_x = u\left(\frac{z}{x}\right) - xu'\left(\frac{z}{x}\right)\frac{z}{x^2} - (1 - \delta)V^*$$

$$= u\left(\frac{z}{x}\right) - \frac{z}{x}u'\left(\frac{z}{x}\right) - (1 - \delta)V^*$$

so the concavity of u implies that ψ is concave in x, for any z.

Note, moreover, that if (A1) fails with a strict inequality, then $\psi_x(x; z) < 0$ for all x, so then the optimal value of $x = 1$, i.e., $k = 1$. Then A consumes all current wealth and exits at the next date, implying (i). [It is easily shown that the same is the case when (A1) holds as an equality.]

If (A1) is satisfied, there exists m such that $u(m) - mu'(m) = (1 - \delta)V^*$. Define $\bar{z} = m/(1 - \delta)$. Then $z \geq \bar{z}$ implies $z/x \geq \bar{z}(1 - \delta) = m$ for all $x \in [1, 1/(1 - \delta)]$. Hence $\psi_x(x; z) \geq 0$ for all x, so optimal $x = 1/(1 - \delta)$, or $k = \infty$. Conversely, $z < \bar{z}$ implies that $\psi_x(1/(1 - \delta); z) < 0$, so it is optimal for the agent to exit at some finite date. In particular, $z \leq m$ implies that optimal $x = 1$ (so that $k = 1$). And $z \in (m, m/(1 - \delta))$ implies that the agent must exit at some date $k > 1$.

To calculate the exact exit date, the concavity of ψ implies that it suffices to look at the two integer solutions for k generating values of z/x closest to m. Every finite k will therefore be an optimal exit date for some z, and the exact switch points can be calculated by the condition of indifference between adjacent exit dates.

Finally, note from Assumption [β] that for all $z \leq R$,

$$\frac{1}{\delta}\left[(1 + \delta)u\left(\frac{z}{1 + \delta}\right) - u(z)\right] < v^*$$

because $(1 + \delta)u(c/(1 + \delta)) - u(c)$ is nondecreasing in c. Rearranging this inequality and recalling that $V^* = (1 - \delta)^{-1}v^*$, we see that

$$u(z) + \delta V^* > (1 + \delta)u\left(\frac{z}{1 + \delta}\right) + \delta^2 V^*$$

which means that exit at date 1 is strictly preferred to exit at date 2. By the concavity of ψ, it follows that exit at date 1 is uniquely optimal, proving that $R < z_1$.

The reader is reminded that in what follows, a *continuous* P-equilibrium refers to a P-equilibrium with continuous *ex post* value function B.

LEMMA 4: *For any continuous P-equilibrium, let w^n be a sequence of wealth levels converging to $\hat{w}$, with corresponding contracts (x^n, y^n, e^n) converging to $(\hat{x}, \hat{y}, \hat{e})$. Then $(\hat{x}, \hat{y}, \hat{e})$ is an optimal contract for P at $\hat{w}$.*

PROOF:

Recalling that B and D are continuous, this follows from a simple application of the maximum theorem to the principal's (constrained) optimization problem.

LEMMA 5: *For a continuous P-equilibrium and wealth w, suppose that a non-null contract is offered and $V(w) = B(w)$. Then $x(w) < w < y(w)$. Moreover, for any compact interval $[0, \bar{w}]$, there exists $\eta > 0$ such that (i) $x(w) < w - \eta$ and (ii) $y(w) > w + \eta$, whenever $w \in [0, \bar{w}]$ and a non-null contract is offered with $V(w) = B(w)$.*

PROOF:

The first part follows from the need to maintain (APC) and (PPC). The uniform version follows from Lemma 4.

LEMMA 6: *For any continuous P-equilibrium and any compact interval $[0, \bar{w}]$, there is $\varepsilon > 0$ such that whenever $w \in [0, \bar{w}]$ and a non-null contract is offered, we have $e(w) \in (\varepsilon, 1 - \varepsilon)$.*

PROOF:

That $e(w) \in (0, 1)$ follows from the need to maintain (APC) and (PPC). As in Lemma 5, the uniform version is a consequence of Lemma 4.

LEMMA 7: *Consider a P-optimal contract for some continuous B. If (APC) is not binding at some w, so that $V(w) > B(w)$, then the P-optimal contract has $x(w) = 0$ and $y(w) < R$.*

PROOF:

Suppose that $V(w) > B(w)$. It can be verified that the principal's return conditional on success must be higher than his return conditional on failure; that is,

$$\text{(A4)} \qquad w + R - y(w) > w - x(w).$$

With (A4) in hand, and (APC) not binding, it is easy to show that $x(w) = 0$ (we omit the details). Now use this fact along with (A4) to conclude that $y(w) < R$.

LEMMA 8: *If $w \geq R$, then $V(w) = B(w)$.*

PROOF:

Suppose not; then $V(w) > B(w)$. By Lemma 7, $x(w) = 0$ and $y(w) < R \leq w$. But then

$$V(w) = e(w)B(y(w)) + [1 - e(w)]B(x(w)) - D(e) < B(y(w)) < B(R) \leq B(w)$$

which is a contradiction to (APC).

LEMMA 9: *If $V(w) > B(w)$, then $V(w) \geq V^*$.*

PROOF:

By Lemma 7, if $V(w) > B(w)$, then $x(w) = 0$. So $\{e(w), y(w)\} = (e, y)$ solves

$$\max_e e[R - h(e)] \tag{A5}$$

subject to $eD'(e) - D(e) \geq B(w)$ [which simply combines (APC) and (EIC)].[21] Recall that the one-shot P-optimal contract can be identified with the unconstrained solution e^* to

$$\max_e e[R - h^*(e)]. \tag{A6}$$

Let $e(w)$ be a solution to (A5). Using Lemma 1, it can be verified that $e(w) \geq e^*$. Now observe that

$$V(w) = \delta V(0) + [e(w)D'(e(w)) - D(e(w))]. \tag{A7}$$

Setting $w = 0$ in (A7), using the fact that $V(0) > B(0)$ (Lemma 2) so that $e(0) \geq e^*$ (as asserted above), and noting that $eD'(e) - D(e)$ is increasing in e, we conclude that

$$V(0) = (1 - \delta)^{-1}[e(0)D'(e(0)) - D(e(0))] \geq (1 - \delta)^{-1}[e^*D'(e^*) - D(e^*)] = V^*. \tag{A8}$$

Now consider any w such that $V(w) > B(w)$. With (A8) in mind and again using $e(w) \geq e^*$, we may conclude that

$$V(w) = \delta V(0) + [e(w)D'(e(w)) - D(e(w))] \geq \delta V^* + [e^*D'(e^*) - D(e^*)] = V^*.$$

Therefore $V(w) \geq V^*$ for all w with $V(w) > B(w)$, as claimed.

In what follows, we consider another Ramsey problem with exit, this one more general than the one described in Lemma 3. Recall that in any equilibrium, we have

$$B(z) \equiv \max_{0 \leq \delta w \leq z} [u(z - \delta w) + \delta \max\{V(w), B(w)\}]. \tag{A9}$$

This induces the following exit problem: for each initial wealth level z, choose an *exit date* $T(z) \geq 1$ and an *exit wealth* $\iota(z)$ such that after $T(z)$ periods of consumption and saving, the agent simply takes the outside option $V(\iota(z))$. To be sure, $V(\iota(z)) > B(\iota(z))$, otherwise the exit interpretation is meaningless. (Notice that the *existence* of such a wealth level is guaranteed by Lemma 2.) If no wealth w with $V(w) > B(w)$ is ever reached, set $T(z)$ equal to infinity.

Because $\delta = 1/(1 + r)$ and u is concave, it is optimal to hold consumption constant until the exit date. This means that if $T \leq \infty$ denotes the exit date, the sequence of wealths until exit at wealth w is given by the difference equation

$$w_{t+1} = (1/\delta)\left[w_t - \frac{(z - \delta^T w)(1 - \delta)}{1 - \delta^T}\right] \tag{A10}$$

where w_0 is just z. Indeed, this is the unique policy if u is strictly concave.

LEMMA 10: *There exists a finite integer M such that for every $z \in [0, R]$, exit occurs in the generalized Ramsey problem at some date $T(z) \leq M$.*

PROOF:

Consider the special Ramsey problem as described in Lemma 3, with the exit option set

[21] It is understood that the corresponding value of y will be read off from $D'(e) = B(y) - B(0)$. Here $h(e) \equiv g^{-1}(D'(e))$ and $h^*(e) \equiv g^{x-1}(D'(e))$, where $g(y) \equiv B(y) - B(0)$ and $g^*(y) \equiv u(y) - u(0)$.

equal to V^*. The last statement in part (ii) of that lemma assures us that under Assumption $[\beta]$, it is optimal to exit at date 1, provided $z \in [0, R]$. In particular, date 1 exit is preferred to indefinite maintenance.[22] Applying Lemma 9, this immediately shows that $T(z) < \infty$. The uniform bound over a compact set of wealths involves minor technicalities that are omitted.

LEMMA 11: *If exit from $z \in [0, R]$ occurs in the generalized Ramsey problem at some wealth $\iota(z) \le z$, then exit must occur at date* 1.

PROOF:

Let exit occur at some wealth $z' = \iota(z)$, where $z' \le z$. It will suffice to prove that for all $T \ge 2$,

$$\text{(A11)} \quad \delta(1-\delta^{T-1})V(z') > \frac{1-\delta^T}{1-\delta} u\left(\frac{(z-\delta^T z')(1-\delta)}{1-\delta^T}\right) - u(z-\delta z').$$

[The reason is that it is optimal to hold consumption steady until exit, so that the condition (A11) above is just a sufficient condition for exit at $T = 1$.]

Suppose, on the contrary, that (A11) is false for some $T \ge 2$. Because that the agent *could* have spent these T periods running his wealth down to zero and taking $V(0)$ instead, it must be the case that

$$\text{(A12)} \quad \frac{1-\delta^T}{1-\delta} u\left(\frac{(z-\delta^T z')(1-\delta)}{1-\delta^T}\right) + \delta^T V(z') \ge \frac{1-\delta^T}{1-\delta} u\left(\frac{z(1-\delta)}{1-\delta^T}\right) + \delta^T V^*,$$

where we use Lemma 9 to replace $V(0)$ by V^*.

Combining the negation of (A11) with (A12) and eliminating $V(z')$, we see that

$$\text{(A13)} \quad \frac{1-\delta^T}{1-\delta} u\left(\frac{(z-\delta^T z')(1-\delta)}{1-\delta^T}\right) - u(z-\delta z') - \frac{(1-\delta^T)(1-\delta^{T-1})}{(1-\delta)\delta^{T-1}} \times \left[u\left(\frac{z(1-\delta)}{1-\delta^T}\right) - u\left(\frac{(z-\delta^T z')(1-\delta)}{1-\delta^T}\right)\right] \ge \delta(1-\delta^{T-1})V^*.$$

It is possible to check that the left-hand side of (A13) is nonincreasing in z'. Thus setting $z' = 0$ in (A13), we have

$$\frac{1-\delta^T}{1-\delta} u\left(\frac{z(1-\delta)}{1-\delta^T}\right) - u(z) \ge \delta(1-\delta^{T-1})V^*.$$

But this contradicts Assumption $[\beta]$.

LEMMA 12: *If exit from z in the generalized Ramsey problem occurs at some wealth level $\iota(z) > z$, then for all intervening wealth levels w until exit, both $x(w)$ and $y(w)$ lie in $[0, R]$ (assuming a non-null contract is offered at those wealths).*

PROOF:

If u is concave, then

$$\text{(A14)} \quad \text{If } z < z' \equiv \iota(z), \text{ then } w_t < z' \quad \text{for all } 0 < t < T.$$

[This is easy to check from (A10) when u is strictly concave. When u is (weakly) concave the argument is a bit more complicated, and is omitted.]

It follows that if some intermediate wealth level w is attained, then $w < z' = \iota(z)$ and $V(w) = B(w)$. Now $V(z') > B(z')$, so by the monotonicity of B,

$$\text{(A15)} \quad V(z') > B(z') \ge B(w) = V(w).$$

On the other hand, we know that $x(z') = 0$, that $y(z') \le R$, and that $x(w) \ge 0$ (assuming that a contract is offered at w). So the only way

[22] This is true because indefinite wealth maintenance must be an optimal strategy in the generalized Ramsey problem without exit, *provided* the exit option is not taken.

in which (A15) can be satisfied is by having both $x(w)$ and $y(w)$ lie in $[0, R]$.

In what follows, we move away from the generalized Ramsey problem, which is artificial insofar as it pertains to planned wealth levels in the future in the absence of any uncertainty. It is useful in defining optimal saving decisions but not the actual evolution of future wealths. Every reference to a process of wealths is now (unless otherwise stated) to the *actual* wealth evolution of the agent in the P-equilibrium. Notice, however, that for every end-of-period wealth z, tomorrow's beginning wealth is given exactly by some policy function induced by the Ramsey problem.

LEMMA 13: *If end-of-period wealth $z \in [0, R]$, then the stochastic process of all subsequent wealths in any continuous P-equilibrium must lie in $[0, R]$ almost surely.*

PROOF:

Consider any $z \in [0, R]$. Let $\hat{z}$ denote the random variable that describes end-of-period wealth next period, conditional on z today. Recall that $\hat{z}$ is determined by the conjunction of two processes. First, a choice is made for next period's starting wealth; call it w_1. This is done by following *exactly* some first-period solution to the artificial Ramsey problem. Next, a contract may be offered at w_1, leading to the random variable $\hat{z}$ that describes next period's end-of-period wealth. If a contract is not offered, then $\hat{z}$ is simply equal to w_1.

Suppose, first, that exit takes place at date 1. Then $V(w_1) > B(w_1)$. By Lemma 8, it must be that $w_1 < R$. Moreover, by Lemma 7, we have $x(w_1) = 0$ and $y(w_1) < R$, so that $\hat{z} \in [0, R]$ almost surely in this case.

Suppose, next, that exit takes place at some finite time greater than 1 (these are the only two possibilities: by Lemma 10, finite exit *must* occur). Then, by Lemma 11, it must be the case that $w_1 < \iota(z)$, so that by Lemma 12, both $x(w_1)$ and $y(w_1)$ lie in $[0, R]$, if a non-null contract is offered. So $\hat{z} \in [0, R]$ once again. Moreover, since $w(z) < R$ (see Lemma 8), $w_1 \in [0, R]$ as well.

Finally, if no contract is offered at w_1, then $\hat{z} = w_1$ and by the same argument as in the previous paragraph, $\hat{z} \in [0, R]$.

LEMMA 14: *From any end-of-period wealth $z \in [0, R]$, agent wealth must visit 0 infinitely often (with probability one) in any continuous P-equilibrium.*

PROOF:

For any initial end-of-period wealth z, recall that the policy function (or any selection from the policy correspondence) of the artificial Ramsey problem gives us next period's beginning wealth—say w_1—after which end-of-period wealth $\hat{z}$ is either w_1 (if no contract is offered), or (if a contract is offered) the random variable which takes values $y(w)$ with probability $e(w)$ and $x(w)$ with probability $1 - e(w)$.

Define S to be the smallest integer greater than $M \times R/\eta$, where M is given by Lemma 10 and η is given by Lemma 5. Pick some agent with initial wealth z. Consider a sample path in which—over the next S periods—there are successes for this agent whenever contracts are offered. We claim that the agent's wealth must visit some value w for which $V(w) > B(w)$ within these S periods.

Suppose that the claim is false. Then it must be that at any end-of-period wealth z_t along this sample path ($0 \leq t < S$), beginning wealth next period must satisfy $w_{t+1} \geq z_t$. [This follows from Lemma 11. If $w_{t+1} < z_t$, then $V(w_{t+1}) > B(w_{t+1})$.] It follows that S cannot contain a subset of more than R/η periods for which a contract is offered. (If it did, then, by Lemma 5 and the assumption that only successes occur, wealth would wander beyond $[0, R]$, a contradiction to Lemma 13.) But then, by the definition of S, it must contain more than a set of M *consecutive* periods for which the null contract is optimal.

In this case, wealth simply follows the sequence in the artificial Ramsey problem. But Lemma 10 assures us that within M periods, a wealth level w *will* be reached for which $V(w) > B(w)$. This proves the claim.

Let $\iota(z)$ denote the first wealth level for which—following the path described above for S periods—$V(\iota(z)) > B(\iota(z))$. Let $C(z)$ denote the number of times a contract is offered until the wealth level $\iota(z)$ is reached. Consider the event $E(z)$ in which successes are obtained each time, and the first failure occurs at $\iota(z)$. Define $q(z) \equiv \varepsilon^{C(z)+1}$, where ε is given by Lemma 6. Then the probability of the event $E(z)$ is bounded below by $q(z)$. But $E(z)$ is clearly contained in the event that wealth hits zero starting from z. We have therefore shown that the probability of this latter event is bounded away from zero in z [because $q(z) \geq \varepsilon^S$].

The lemma follows from this last observation.

LEMMA 15: *In any continuous P-equilibrium with strictly concave u, $w(z_1) = z_1$ for some $z_1 > R$ implies $w(z) = z$ for all $z > z_1$.*

PROOF:

Otherwise there exists $z_2 > z_1$ such that exiting at some future date is optimal at z_2, while wealth maintenance is optimal at z_1. Hence there exists integer T and $z' \leq R$ such that

$$\text{(A16)}\quad B(z_2) = \frac{1-\delta^T}{1-\delta} u\left(\frac{(z_2 - \delta^T z')(1-\delta)}{1-\delta^T}\right) + \delta^T V(z') \geq \frac{u(z_2(1-\delta))}{1-\delta}$$

while $z' < R < z_1$ implies

$$\text{(A17)}\quad \frac{1-\delta^T}{1-\delta} u\left(\frac{(z_1 - \delta^T z')(1-\delta)}{1-\delta^T}\right) + \delta^T V(z') \leq \frac{u(z_1(1-\delta))}{1-\delta}.$$

This contradicts the fact that

$$\frac{1-\delta^T}{1-\delta} u\left(\frac{(z - \delta^T z')(1-\delta)}{1-\delta^T}\right) - \frac{u(z(1-\delta))}{1-\delta}$$

is strictly decreasing in z, by the strict concavity of u.

LEMMA 16: *In any continuous P-equilibrium with strictly concave u, $w(z) \leq z$ for all $z > R$.*

PROOF:

We know that either an agent will smooth wealth indefinitely or (by virtue of Lemma 8) will plan to exit at some wealth not exceeding R. Moreover, by strict concavity, (A10) holds, so that wealth approaches the exit wealth monotonically over time. In particular, $w(z) < z$ in this case.

LEMMA 17: *Assume that u is strictly concave and*

$$\text{(A18)}\quad \lim_{c\to\infty}[u(c) - cu'(c)] > u(R).$$

Then, in any continuous P-equilibrium, there exists $z^* \in (R, \infty)$ such that $z > z^*$ implies $w(z) = z$ and $z \in [R, z^*)$ implies $w(z) < z$.

PROOF:

The optimality of nonmaintenance of wealth at $z > R$ implies the existence of T and exit wealth $z' \leq R$ such that

$$\text{(A19)}\quad \frac{1-\delta^T}{1-\delta} u\left(\frac{(z - \delta^T z')(1-\delta)}{1-\delta^T}\right) - \frac{u(z(1-\delta))}{1-\delta} + \delta^T V(z') \geq 0.$$

Since $z' \geq 0$, this requires

$$\text{(A20)}\quad \frac{1}{1-\delta}\left[(1-\delta^T) u\left(\frac{z(1-\delta)}{1-\delta^T}\right) - u(z(1-\delta))\right] + \delta^T V(z') \geq 0.$$

But the concavity of u implies that

$$\text{(A21)}\quad (1-\delta^T) u\left(\frac{z(1-\delta)}{1-\delta^T}\right) - u(z(1-\delta)) \leq \delta^T[z(1-\delta)u'(z(1-\delta)) - u(z(1-\delta))].$$

Combining (A20) and (A21), we conclude that

$$\text{(A22)}\quad (1-\delta)V(z') \geq u(z(1-\delta)) - z(1-\delta)u'(z(1-\delta)).$$

Now we know that $V(z') > B(z')$ (by the definition of an exit wealth), and for such wealths w we know that $x(w) = 0$ and $y(w) \leq R$ (Lemma 7). Consequently, $V(z')$ cannot exceed $(1-\delta)^{-1}u(R)$. Using this information in (A22), it follows that a *necessary* condition for nonmaintenance is

$$u(R) \geq u(z(1-\delta)) - z(1-\delta)u'(z(1-\delta)).$$

Using (A18) and the fact that $u(c) - cu'(c)$ is increasing in c, define z^{**} such that equality holds in the expression above. Then $w(z) = z$ for all $z > z^{**}$. On the other hand, $w(z) < z$ for z in a (right) neighborhood of R, since Assumption $[\beta]$ ensures that exit is optimal in the Ramsey problem for such a neighborhood of R. Hence there exists $z^* \in (R, z^{**}]$ defined by $\inf\{z|w(z) = z\}$ such that it is optimal for the agent to maintain wealth above z^* and decumulate below, this very last observation being a consequence of Lemma 15.

PROOF OF PROPOSITION 1:

Part (i) follows from Lemmas 3 and 8. Part (ii) follows from Lemmas 15, 16, and 17. Part (iii) follows from Lemmas 13 and 14.

LEMMA 18: *Consider any continuous P-equilibrium and any* $\bar{w} < \infty$. *Then there exists* $\kappa > 0$ *such that for any initial wealth* $w_0 \in [R, \bar{w})$ *not in* $\mathcal{W}$, w_t *enters* $[0, R]$ *or the set* $\mathcal{W}$ *at some date with probability at least* κ. *In particular, if* $w_0 < z^*$ *then,* w_t *enters* $[0, R]$ *at some date t with probability at least* κ.

PROOF:

Since w_0 is not in $\mathcal{W}$, either a contract is offered at w_0, or $w_0 < z^*$ and a contract is not offered at w_0. Consider the event that a failure results whenever a contract is offered and the agent's wealth exceeds R. Note that in such an event the agent's wealth falls monotonically (conditional on wealth exceeding R and not entering $\mathcal{W}$), by virtue of the description of the agent's saving behavior (if a contract is not offered then wealth must lie below z^*, in which case the agent's starting wealth in the next period is smaller) and given Lemmas 5 and 8 (in case a contract is offered failure implies the agent's wealth drops from the beginning to end of the period). Moreover, in this event wealth falls by at least η every time a contract is offered. If the number of times a contract is offered exceeds

$$K \equiv \frac{\bar{w} - R}{\eta} + 1,$$

the agent's wealth must—at some time—fall below R. Note also that if a contract is not offered at some intervening wealth less than z^*, his wealth falls deterministically until such time that it either drops below R or arrives at a level where a contract is offered. Hence with probability at least $\kappa \equiv \varepsilon^K$, where ε is the uniform lower bound on the probability of failure given by Lemma 6, the agent's wealth must eventually either fall below R or enter $\mathcal{W}$.

Finally, note that if $w_0 < z^*$, notice that exactly the same event can be constructed to yield the second part of the lemma.

LEMMA 19: *Consider any continuous P-equilibrium and an unbounded sequence of wealth levels* $w_n \to \infty$ *for each of which a feasible contract exists. Suppose also that u is strictly concave and* $z^* < \infty$. *Then there exists* $\bar{W} > z^*$ *and a number* $\tau > 0$ *such that for any* $w > \bar{W}$:

(*i*) $x(w) > z^*$
(*ii*) $e(w) \geq \underline{e} \equiv R/f$
(*iii*) $e(w)y(w) + [1 - e(w)]x(w) \geq w + \tau$.

PROOF:

Omitted.

PROOF OF PROPOSITION 2:

By the first part of Lemma 18, the probability of entering either $\mathcal{W}$ or the poverty trap is bounded away from zero over any compact interval of initial wealths of the form $[0, \bar{w}]$. By the second part of that lemma, the probability of entering the poverty trap is bounded away from zero if initial wealth is less than z^*.

To complete the proof of the proposition, it suffices to prove that the only remaining limit event is one in which wealth goes to infinity, and that this has positive probability whenever initial wealth w exceeds sup $\mathcal{W}$.

To this end, we make the following claim: there exists $w^* > 0$ and $\theta > 0$ such that if $w \geq w^*$, then $\text{Prob}(w_t \to \infty) \geq \theta$.

To prove this claim, pick any $w \geq \bar{w}$, where $\bar{w}$ is given by Lemma 19, and consider the events

$$\mathcal{L}(w) \equiv \{w_0 = w;\ w_t \geq \bar{w} \quad \text{for all } t\}$$

and

$$\mathcal{G}(w) \equiv \{w_0 = w;\ w_t \to \infty\}.$$

Using Lemma 19, it is easy to verify that

(A23)

$$\text{Prob}[\mathcal{G}(w)|\mathcal{L}(w)] = 1 \text{ for all } w > \bar{w}.$$

The reason for this is that to the right of $\bar{w}$, the process is akin to a Markov process in which, for every w, there are two possible continuation values for end-of-period wealth. The expected value exceeds w by some amount bounded away from zero [see Lemma 19, part (iii)]. Moreover, the probability of the success wealth is also bounded away from zero [see part (ii) of that lemma]. Finally, since all wealths lie above z^* [part (i) of Lemma 19], next period's starting wealth equals this period's ending wealth. Standard arguments then show that conditional on staying above $\bar{w}$, the process must converge to infinity almost surely, which is exactly (A23).

The same argument actually reveals something stronger: that if w is large enough (and sufficiently larger than $\bar{w}$), the process will stay above $\bar{w}$ forever with probability that is bounded away from zero.[23] In other words, there exists $w^* > 0$ and $\varepsilon > 0$ such that for all $w \geq w^*$,

$$\text{(A24)} \qquad \text{Prob}[\mathcal{L}(w)] \geq \varepsilon.$$

Combining (A23) and (A24), we may conclude that for all $w \geq w^*$,

$$\text{(A25)} \qquad \text{Prob}[\mathcal{G}(w)] \geq \varepsilon.$$

Next, observe that starting from any wealth $w > \sup \mathcal{W}$, it is possible to hit a starting wealth that exceeds w^* with probability bounded away from zero (take η as given in Lemma 5, and simply look at the event in which K successes occur, where K is the smallest integer exceeding $[w^* - \sup \mathcal{W}]/\eta$). It follows [applying (A25)] that there exists $\varepsilon' > 0$ such that for all $w > \sup \mathcal{W}$,

$$\text{Prob}[w_0 = w;\ w_t \rightarrow \infty] \geq \varepsilon'.$$

Combining this observation with the fact that over any compact interval, the probability of entering $\mathcal{W}$ or the poverty trap is bounded away from zero, the proof of the proposition is complete.

LEMMA 20: *If Assumption [β] holds, then $e(0) = e^*$ and $y(0) = y^*$ maximizes $e[R - y]$ subject to $D'(e) = \max\{B^*(y) - B^*(0), 0\}$ and $y \geq 0$, where B^* denotes the ex post value function in the Ramsey problem with exit option V^*.*

PROOF:

Note that we can restrict the range of feasible values of y to $[0, R]$, since any $y > R$ is strictly dominated by $y = R$. By Lemma 3, part (ii), it follows that $y < z_1$. That is, $B^*(y) = u(y) + \delta V(0) = u(y) + B^*(0)$. So $B^*(y) - B^*(0) = u(y)$, and the problem reduces to the static optimal contracting problem.

PROOF OF PROPOSITION 3:

Lemma 3 describes the solution B^* to the Ramsey problem with exit option V^*. Assuming for the moment that B^* will be the *ex post* value function under the constructed P-equilibrium, it should be clear—from Lemma 5 itself—that (ii) and (iii) of the proposition are immediately satisfied.

By definition, B^* satisfies the functional equation

$$B^*(z) = \max_{0 \leq \delta w \leq z} [u(z - \delta w) + \delta \max\{V^*, B^*(w)\}].$$

Thus—if $B^*(z)$ is to be an equilibrium—it must be that the *ex ante* value function satisfies

$$\text{(A26)} \qquad V(w) = \max\{V^*, B^*(w)\}.$$

Part (i) of the proposition would follow right away from (A26), and part (iv) would only require the additional assistance of Lemma 7.

So all that remains to be proved is that if the principal does take the value function B^* as given to solve his constrained optimization problem, then indeed, the resulting *ex ante* value function satisfies (A26). The easy verification of this claim is omitted.

LEMMA 21: *In any A-equilibrium, (PPC) binds at every wealth level w.*

[23] The formal proof of this result requires a simple coupling argument which we omit.

PROOF:
Omitted.

LEMMA 22: *In any A-equilibrium V is strictly increasing.*

PROOF:
Obviously V is nondecreasing. Suppose there exist w_1, w_2 with $w_2 > w_1$ such that $V(w_2) = V(w_1)$. Then $\{x(w_1), y(w_1), e(w_1)\}$ is feasible at w_2, and hence must also be optimal at w_2. But here (PPC) does not bind, which contradicts Lemma 21.

PROOF OF PROPOSITION 4:
Consider part (i). Divide the proof into three cases: (a) $R > y - x$; (b) $R < y - x$; (c) $R = y - x$. We illustrate (a), the arguments in the remaining cases being very similar:

Case (a): Take any positive $\varepsilon < e[R - (y - x)]$, where e denotes the effort assigned at w. Construct a contract ($\tilde{y} = y + \varepsilon/e$, $\tilde{x} = x$), and let $\tilde{e}$ denote the associated effort response. Then by construction (PPC) is satisfied at wealth $w + \varepsilon$ by the new contract ($\tilde{y}$, $\tilde{x}$), if the agent were to continue to select effort e. Since B is strictly increasing, the agent's optimal effort response $\tilde{e} \geq e$. Given $R > \tilde{y} - \tilde{x}$, (PPC) must continue to be satisfied at $\tilde{e}$. Hence the new contract is feasible at wealth $w + \varepsilon$. Since the effort e is still available to the agent,

(A27)
$$V(w + \varepsilon) \geq eB(\tilde{y}) + (1 - e)B(\tilde{x}) - D(e).$$

Note that $B(\tilde{y}) - B(y) \geq u(c_s + \varepsilon/e) - u(c_s)$ if $c_s \equiv c(y)$ since it is always feasible for the agent to entirely consume any increment in end-of-period wealth. Hence

(A28)
$$V(w + \varepsilon) - V(w) \geq e\left[u\left(c_s + \frac{\varepsilon}{e}\right) - u(c_s)\right]$$
$$\geq \varepsilon u'\left(c_s + \frac{\varepsilon}{e}\right)$$
$$\geq \varepsilon u'\left(\max\{c_s, c_f\} + \frac{\varepsilon}{e}\right)$$

where (c_s, c_f) denotes $(c(y), c(x))$. The result then follows upon dividing through and taking limits with respect to ε.

Now turn to part (ii). By assumption,

$$V(w) = eB(y) + (1 - e)B(x) - D(e) \tag{A29}$$

where B is right continuous at $y = y(w)$ and $x = x(w)$, with e denoting $e(w)$. Then for small enough $\varepsilon > 0$, there exist positive incremental payments $\Delta_y(\varepsilon)$, $\Delta_x(\varepsilon)$ that solve the following two equations:

$$e\Delta_y + (1 - e)\Delta_x = \varepsilon.$$

$$B(y + \Delta_y) - B(y) = B(x + \Delta_x) - B(x).$$

Moreover, $\Delta_y(\varepsilon)$ and $\Delta_x(\varepsilon)$ both tend to 0 as $\varepsilon \to 0+$, and so does $\psi(\varepsilon) \equiv B(y + \Delta_y(\varepsilon)) - B(y)$. By construction, the contract $y + \Delta_y(\varepsilon)$, $x + \Delta_x(\varepsilon)$ elicits the same effort response e as the previous contract; hence it is feasible at wealth $w + \varepsilon$. Therefore:

$$V(w + \varepsilon) - V(w) \geq \psi(\varepsilon). \tag{A30}$$

Since $B(z + \Delta) - B(z) \geq u(c(z) + \Delta) - u(c(z))$, it follows that for $z = y, x$:

(A31)
$$\lim_{\varepsilon \to 0+} [\psi(\varepsilon) - \Delta_z(\varepsilon)u'(c(z))] \geq 0.$$

Weighting inequality (A31) by the probability of the corresponding outcomes and adding across the two states, it follows that

$$\lim_{\varepsilon \to 0+} [\psi(\varepsilon) - \theta^{-1}\varepsilon] \geq 0 \tag{A32}$$

where θ denotes

$$\left[e\,\frac{1}{u'(c(y(w(z))))} + (1 - e)\,\frac{1}{u'(c(x(w(z))))}\right].$$

The first inequality in part (ii) of the proposition then follows from combining (A30) and (A32).

Finally, to establish the second inequality in (ii), note that if this result is false at some z, $c(z)$ must be positive, so it is feasible for the agent to consume a little bit ($\varepsilon > 0$) less. This would cause wealth at the following date to be

$w(z) + \varepsilon/\delta$ instead of $w(z)$. It must therefore be the case that for every small $\varepsilon > 0$:

$$\text{(A33)} \quad u(c(z)) - u(c(z) - \varepsilon) \geq \delta\left[V\left(w(z) + \frac{\varepsilon}{\delta}\right) - V(w(z))\right].$$

Taking limits with respect to ε, and using the first inequality in (ii) (which we have already established), we obtain a contradiction.

PROOF OF PROPOSITION 5:

Suppose there is an equilibrium with a strict poverty trap, which requires that $w(y(0)) = 0 = w(x(0))$. If u is strictly concave, a standard revealed-preference argument implies that $w(\cdot)$ must be nondecreasing. We also know that $y^* \equiv y(0) > x^* \equiv x(0)$ is necessary for the agent to exert effort and thus satisfy (PPC). Hence $w(z) = 0$ and $B(z) = u(z) + \delta V(0)$ for all $z \in [0, y^*]$. So B is differentiable at x^*.

If B is right differentiable at y^*, then using $u'(y^*)$ to denote the right derivative at y^*, (10) of Proposition 4 implies that

$$\frac{1}{u'(y^*)} \leq \left[e(0)\,\frac{1}{u'(y^*)} + (1 - e(0))\,\frac{1}{u'(x^*)}\right]$$

upon using the hypothesis that $c(y^*) - y^* = c(x^*) - x^* = 0$. Since $1/u'$ is strictly increasing, and e less than 1 [otherwise (PPC) will be violated], this inequality contradicts $y^* > x^*$.

If B is not right differentiable at y^*, then note that the rate of increase of B at y^* is bounded below by $u'(y^*)$, since it is feasible for the agent to entirely consume all incremental wealth. Now modify the proof of Proposition 4 to infer that the inequality in (10) must be strict at y^*, which will again generate a contradiction.

LEMMA 23: *In any A-equilibrium, let $\mathcal{M}$ denote the set of wealth levels for which a non-null contract is offered. Then*

(*i*) *if either $\mathcal{M}$ is bounded or $u'(\infty) > 0$,* $\inf_{w \in \mathcal{M}} e(w) > 0$.
(*ii*) *if $\mathcal{M}$ is bounded,* $\sup_{w \in \mathcal{M}} e(w) < 1$.

PROOF:

If (i) is false, there is a sequence w_n and corresponding non-null contracts (x_n, y_n, e_n) with $e_n \to 0$. This implies $y_n - x_n \to 0$. (PPC) then implies that y_n (and hence also x_n) must be less than w_n, but this contradicts (APC). If (ii) is false we can find a sequence with $w_n \to w$ and $e_n \to 1$, so $\lim_n y_n = \infty$. Since (PPC) binds for each n, $\lim_n w_n = \infty$, contradicting the boundedness of $\mathcal{M}$.

PROOF OF PROPOSITION 6:

Since $u'(\infty) > 0$, Proposition 4 implies that $1/(u'(c_t))$ forms a submartingale in any right-continuous A-equilibrium, where c_t denotes the consumption of the agent at date t. Hence $1/(u'(c_t))$ converges almost surely. Since u is strictly concave, this implies that c_t converges almost surely to a (possibly infinite valued) random variable $\tilde{c}$.

We claim that almost surely $\tilde{c} = \infty$, implying that $z_t \to \infty$, and hence that $w_t \to \infty$.

Because $u'(\infty) > 0$ by assumption, part (i) of Lemma 23 applies. Define $m \equiv D'(\inf_{w \in \mathcal{M}} e(w))$. Then $m > 0$. Pick any $Z < \infty$ and integer $T > 1/m[B(Z) - B(0)] - 1$. Also select any nonnegative integer q. Define the event

$$\mathcal{B}_q(Z) \equiv \{\tilde{c} \in [q, q + 1) \text{ and } z_t < Z \text{ for all } t\}.$$

Note that $z_{t-1} < Z$ implies that $w_t < W \equiv Z/\delta$. Applying both parts of Lemma 23, there are bounds $\underline{e}$, $\bar{e} \in (0, 1)$ for effort levels arising in any contract corresponding to wealth in $[0, W]$.

Next, select $\eta \in (0, (1 - \delta)m)$. Note that since $u(0) = 0$, continuity and concavity of u imply that u is uniformly continuous. Hence we can find $\varepsilon > 0$ such that $|u(c) - u(c')| < \eta$ whenever $|c - c'| < \varepsilon$.

Conditional on wealth w at the beginning of date 0, define for any positive integer T the T-step-ahead possible realizations of z, c, and w under the given A-equilibrium in the following manner. Let $n_t \in \{s, f\}$ denote the outcome of the project t dates ahead, and let n^t denote the history of project outcomes $(n_t, n_{t-1}, \dots, n_0)$ between dates 0 and t. Given the equilibrium we can find the T-step-ahead realizations as functions of the history of the outcomes of the project between 0 and T: $z^T(n^T, w)$, $c^T(n^T, w)$, $w^T(n^{T-1}, w)$. Define $C(w, T) = \{c | c = c^t(n^t,$

$w), t \leq T\}$, the set of possible realizations of consumptions T-steps ahead.

Then define the event

$$\mathcal{A}(Z) \equiv \{e_t \in (\underline{e}, \bar{e}) \text{ for all } t \text{ and } \lim_{t\to\infty} \text{diam } C(w_t, T) = 0\}.$$

LEMMA 24: *For any Z and any integer q*:

(A34) $$\text{Prob}[\mathcal{A}(Z) | \mathcal{B}_q(Z)] = 1.$$

We omit the proof since it involves some technical details. The basic idea underlying it is quite simple: if the diameter of $C(w_t, T)$ does not converge to 0, this means that the variance of consumption T-steps ahead does not vanish, so consumption cannot converge to a finite limit.

LEMMA 25: *For any Z and any q*:

(A35) $$\text{Prob}[\mathcal{A}(Z) \cap \mathcal{B}_q(Z)] = 0.$$

PROOF:

The proof rests on the following claim.

Claim: Let s^t (respectively, f^t) denote t-step-ahead histories in which the project results in a success (failure) in every period. Then for any Z and any q:

$$\text{Prob}[B(z^T(s^T, w_t)) - B(z^T(f^T, w_t)) > m(T+1) \text{ for all } t > t^* \text{ for some } t^* | \mathcal{A}(Z) \cap \mathcal{B}_q(Z)] = 1.$$

To prove this claim, consider any path in $\mathcal{A}(Z) \cap \mathcal{B}_q(Z)$, and select t^* such that diam $C(w_t, T) < \varepsilon/2$ for all $t > t^*$. Note that along any such path, $e_t > \underline{e}$ at all t; hence the effort incentive constraint implies $B(z^0(s^0, w)) - B(z^0(f^0, w)) > m$ for all w. So the inequality

(A36)

$$B(z^k(s^k, w_t)) - B(z^k(f^k, w_t)) > m(k+1)$$

holds for $k = 0$ for all t. We shall show that if it holds for $k - 1$ it holds for k as well.

Use z^S and z^{SS} to denote $z^{k-1}(s^{k-1}, w_t)$, $z^k(s^k, w_t)$, respectively. Similarly use z^F and z^{FF} to denote $z^{k-1}(f^{k-1}, w_t)$, $z^k(f^k, w_t)$, respectively. And use z^{FS}, z^{SF} to denote $z^k(f, s^{k-1}, w_t)$ and $z^k(s, f^{k-1}, w_t)$, respectively.

Next note that

$$V(w(z^S)) - V(w(z^F)) = \frac{1}{\delta}[B(z^S) - B(z^F)] - \frac{1}{\delta}[u(c(z^S)) - u(c(z^F))].$$

By the induction hypothesis $B(z^S) - B(z^F) > km$. Moreover, since $t > t^*$, we have ensured by construction that

$$\frac{1}{\delta}[u(c(z^S)) - u(c(z^F))] < \frac{1}{\delta}\eta.$$

Since $\eta < (1 - \delta)m$:

$$V(w(z^S)) - V(w(z^F)) \geq \frac{1}{\delta}[km - \eta] > km.$$

Since $V(w(z^S)) = \max_e[eB(z^{SS}) + (1 - e) \times B(z^{FS}) - D(e)]$ and $V(w(z^F)) = \max_e[eB(z^{SF}) + (1 - e)B(z^{FF}) - D(e)]$, it is evident that $V(w(z^S)) - V(w(z^F)) > km$ implies that $\max\{B(z^{SS}) - B(z^{SF}), B(z^{FS}) - B(z^{FF})\} > km$. Since $B(z^{SS}) - B(z^{FS}) \geq m$ and $B(z^{SF}) - B(z^{FF}) \geq m$, it then follows that $B(z^{SS}) - B(z^{FF}) > (k+1)m$, establishing the claim.

We are now in a position to prove Lemma 25. In the event $\mathcal{B}_q(Z)$, $B(z_t)$ is bounded above by $B(Z)$. Since we selected $T > (1/m)[B(Z) - B(0)] - 1$, the claim above implies that $z^T(s^T, w_t)$ exceeds the upper bound Z for all $t > t^*$. Since given event $\mathcal{A}(Z)$, a string of T successive successes will almost surely occur infinitely often, it follows that the event $\mathcal{A}(Z) \cap \mathcal{B}_q(Z)$ has zero probability.

Combining the results of Lemmas 24 and 25 it follows that $\text{Prob}[\mathcal{B}_q(Z)] = 0$ for any Z and q. If we define the event $\mathcal{B}_q \equiv \cup_{k=1}^{\infty} \mathcal{B}_q(k) = \lim_k \mathcal{B}_q(k)$ that z_t is bounded while $\tilde{c}$ lies in $[q, q+1)$, this implies that $\text{Prob}[\mathcal{B}_q] = 0$. Hence if consumption converges to a limit in $[q, q + 1)$, almost surely $z_t \to \infty$; i.e., $w_t \to \infty$. But in this event Proposition 4 implies that the asymptotic rate of increase of V will be bounded below by $u'(q + 1)$. On the other hand V is bounded above by the value function $\tilde{V}$ corresponding to the case where effort disutility function is identically zero, whose asymptotic rate of increase equals $u'(\infty) < u'(q + 1)$, and we obtain a contradiction. Hence consumption converges to a limit between q and $q + 1$ with zero probability. Since this is true for all integers q, it follows that almost surely consumption will converge to ∞.

PROOF OF PROPOSITION 7:

Proceed in a manner analogous to that in the proof of the previous proposition. If the result is false, then wealth and consumption are bounded with probability 1, which ensures that $1/(u'(c_t))$ again forms a submartingale, so c_t converges almost surely. Lemma 23 ensures that effort is bounded away from zero and one, so all finite step histories will occur infinitely often with probability 1. If the agent receives a contract at all dates, then the agent's wealth must be unbounded with probability 1 in order to provide necessary effort incentives at all dates, and we obtain a contradiction.

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