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# Economic underdevelopment

The case of a missing market for human capital

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This paper demonstrates that the coexistence of developed and underdeveloped countries can be a stationary equilibrium in a world economy with free trade in consumption goods and physical capital. An underdeveloped country is characterized by a high ratio of unskilled workers in the labor force, a small stock of physical capital, a low gross national product, a high rate of return on human capital and a corresponding large wage differential between skilled and unskilled workers. The critical assumptions are that future labor earnings cannot serve as collateral on a loan and indivisibilities in education.

#### 1. Introduction

Dualism is a term often used in the characterization of underdeveloped countries. It refers to asymmetries in these societies which cannot be found in the developed world. A commonly asserted dualism is the coexistence of a modern industrial sector and a backward agricultural sector. Another economic asymmetry has to do with earnings differentials between various types of labor. Such an example is provided by Psacharopoulos (1973), who, in the 1960s, compared the relative average earnings of individuals by educational level for a group of developed and less developed countries. In the latter countries he found that workers with a university education on the average earned 6.4 times as much as the typical worker who had completed primary school, while the corresponding figure for the developed countries was 2.4. A related empirical observation is the significantly higher private

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<sup>1</sup>United States, Canada, Great Britain, Netherlands, France and Norway constitute the group of developed countries, while the less developed countries include Malaysia, Philippines, Ghana, South Korea, Kenya, Uganda, Nigeria and India. See table 8.4 in Psacharopoulos (1973).

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and social rates of return on education in the less developed countries compared to the developed ones as documented in a comprehensive survey article by Psacharopoulos (1985). It seems therefore puzzling that substantially higher relative wage differences in the underdeveloped world do not trigger a reallocation of labor which would reduce those differentials. Or stated as a more general question, is the existence of a dual economy consistent with individual rationality?

In the classic Lewis (1954) model of a dual economy, the level of wages in the urban industrial sector is assumed to be constant and determined as a fixed premium over a constant subsistence level of wages in the traditional agricultural sector. The earnings differential is such that a rural worker is indifferent between keeping his current employment or starting to work in the industrial sector. The model's inadequacy to deal with 'excessive' relative wage differences is addressed by Harris and Todaro (1970), who assume a politically determined minimum urban wage at levels substantially higher than agricultural earnings. The resulting urban unemployment acts as an equilibrating force by reducing the expected earnings in the urban industry. Calvo (1978) shows how the urban wage can be determined endogenously after introducing a trade union into the Harris-Todaro model. However, it remains to be explored whether or not substantial relative wage differences can exist under pure competition. In doing so, we will combine Lewis' idea of economic dualism with Schultz's emphasis on the importance of human capital for understanding the situation in low-income countries. When these two economists shared the Nobel Prize in 1979, Schultz (1980) stressed also that poverty does not impair rationality and 'poor people are no less concerned about improving their lot and that of their children than rich people are'.

In the human capital literature, whose early contributors included Becker (1962) and Mincer (1958), educational decisions are based on maximizing behavior. A common assumption has also been that a perfect market for educational loans exists. This seems questionable, since the embodiment of human capital in people ought to affect its value as security for a loan. As Friedman (1962) pointed out, the productivity of human capital depends on the cooperativeness of the original borrower, and the prohibition of slavery makes it impossible to seize the capital from a borrower who does not honor his debt. It follows that credit constraints may be an important source of educational differences, which is also the conclusion in empirical work by Behrman et al. (1989) using U.S. data. Our paper adopts therefore the assumption of Loury (1981), which is that future labor earnings cannot serve as collateral on a loan.

The world economy in our model is assumed to be inhabited by agents who are identical with respect to preferences and innate abilities. An agent maximizes utility over an infinite horizon and can be thought of as

representing a dynastic family. All countries have access to the same technologies concerning the production of a single good and education. The good can be used for consumption or investment. It is produced with physical capital, skilled workers and unskilled workers as inputs in a constant-returns-to-scale production function. Any unskilled agent can become skilled when being educated by an already skilled worker. The international economy exhibits free trade in consumption goods and physical capital, while labor is assumed to be completely immobile between countries.

Within this framework we ask the question, can developed and underdeveloped countries coexist indefinitely in a competitive world equilibrium? That is, can we find an allocation and price system supporting such a dynamic general equilibrium? It should be noted that the model itself makes it feasible for a country to reach the development level of any other country. The countries are assumed to be identical with respect to technology and the innate abilities of their labor forces. Moreover, all countries have access to the international capital market to finance their stocks of physical capital or as an outlet for net national savings. It is therefore not a foregone conclusion how there can exist underdeveloped countries in a stationary world equilibrium. Finally, the focus on stationary outcomes is meant to further our understanding of economic development by first exploring under what circumstances economic underdevelopment can persist over time.

Given indivisibilities in education, this paper establishes a continuum of steady states depending on the distribution of human and nonhuman wealth. An underdeveloped country is characterized by a high ratio of unskilled workers in the labor force, a small stock of physical capital, a low gross national product, a high rate of return on human capital and a corresponding large wage differential between skilled and unskilled workers. The perpetuation of such a situation can be explained as follows. In a country with a high ratio of unskilled labor, the wage of an uneducated worker is very low relative to the wage of a skilled worker. This means that the cost of education is high in comparison to the labor earnings of an unskilled agent. It turns out that an unskilled worker with no or few assets chooses not to obtain an education. The loss of utility from foregone consumption while saving for educational expenditures outweighs the welfare from higher future earnings.<sup>2</sup>

The precise structure of the model is set out in the following section. Equilibrium prices and quantities in a country are characterized in section 3. Section 4 studies individuals' optimization behavior. Section 5 proves the

<sup>&</sup>lt;sup>2</sup>An economist from Ghana questioned the relevance of this model for his country where education is publicly subsidized. However, he later acknowledged that the low participation of the rural population in higher schooling was due to various costs related to education. These costs included tutoring necessary for passing entrance exams, lost labor income while studying, and higher living expenses in urban areas where institutes of higher learning are located.

existence of multiple steady states for a country, while a stationary world equilibrium is described in section 6. Section 7 discusses economic growth in the context of our model, and section 8 contains the conclusions.

#### 2. The model

Consider a world economy inhabited by a continuum of infinitely lived agents. All agents are identical with respect to preferences and innate abilities. An agent's preferences at time  $t_0$  over consumption streams take the usual additively separable, discounted form,

$$\int_{t_0}^{\infty} e^{-\rho t} U(c_t) dt, \quad \rho > 0.$$
(1)

The instantaneous utility function U(c) is strictly increasing, concave and differentiable over the positive real numbers. The utility function is also assumed to satisfy the Inada condition

$$\lim_{c \to 0} U'(c) = \infty. \tag{2}$$

Each agent is endowed with a constant flow of time that can be devoted to work. As can be seen from the agent's preferences, there is no disutility associated with working.

The world economy produces a single good which can be used for either consumption or investment. A country's output at time t depends on its capital stock  $K_t$  and the employment levels in two different job categories. Let  $L_{\rm st}$  denote the employment level for the skill-intensive type of work, which can only be performed by skilled workers. On the other hand, the employment in the labor-intensive job category,  $L_{\rm ut}$ , can be made up of both unskilled and skilled workers. The good is then produced according to the aggregate production function

$$F(L_{st}, L_{ut}, K_t) = G(L_{st}, L_{ut})^{\alpha} K_t^{1-\alpha}, \quad 0 < \alpha < 1.$$
(3)

The production function exhibits constant returns to scale with positive marginal products, i.e., the function  $G(L_{\rm st},L_{\rm ut})$  is linearly homogeneous with positive first derivatives. It is also assumed that the marginal rate of substitution between different labor is diminishing everywhere:

$$\frac{F_1(L_s, L_u, K)}{F_2(L_s, L_u, K)} = \frac{G_1(L_s, L_u)}{G_2(L_s, L_u)} \quad \text{is strictly decreasing in} \quad \frac{L_s}{L_u}. \tag{4}$$

(An integer i as a subscript on a function denotes the partial derivative with respect to the ith argument.) Moreover the marginal product of unskilled labor approaches zero when the fraction of skilled labor to unskilled labor goes to zero, while the marginal product of skilled labor remains strictly positive in that limit (for any positive stock of physical capital). The corresponding restrictions on the function G are

$$\lim_{L_{\rm s}/L_{\rm u}\to 0} G_1(L_{\rm s}, L_{\rm u}) > 0, \tag{5a}$$

$$\lim_{L_{\rm s}/L_{\rm u}\to 0} G_2(L_{\rm s}, L_{\rm u}) = 0. \tag{5b}$$

For simplicity, physical capital is assumed to not depreciate over time unless it is being consumed. Concerning the skilled labor force, there is an education technology which is also common across countries. A number  $\gamma$  of skilled workers can instantaneously transform an unskilled worker into a skilled worker. This education allows the worker to remain skilled for  $\tau$  units of time.

The consumption good and physical capital are internationally traded without any transportation costs. However, labor is completely immobile between countries. Another critical assumption on feasible trades is that an agent's future labor earnings cannot serve as collateral on a loan. Physical capital can still be used as collateral security, but that is equivalent to selling and repurchasing capital in this world without uncertainty.<sup>3</sup>

We will focus on stationary equilibria for the described world economy. It turns out that a steady state, or, for that matter, a Pareto-optimal stationary allocation, will only exist if the education technology is sufficiently productive. In particular, the parameters must satisfy

$$1 - \rho \gamma > e^{-\rho \tau}, \tag{6}$$

which implies that  $\tau > \gamma$ .<sup>4</sup> If condition (6) is not satisfied, any constant stock of human capital would be unprofitable in an equilibrium. The rate of return

<sup>3</sup>Ljungqvist (1989) derives results similar to this paper in an overlapping generations framework. Following the approach of Becker (1974) and Barro (1974), the two-period lived agents in that model maximize an infinite-horizon objective function due to concern about their offspring. The critical assumption on feasible trades is that parents can only pass on nonnegative inheritance to their children.

<sup>4</sup>The parameters  $\rho$  and  $\gamma$  are both strictly positive, so assumption (6) implies that  $\rho\gamma \in (0, 1)$ . After taking the natural logarithm of (6) and calculating the Taylor series expansion of the left-hand side around  $\rho\gamma = 0$ , we arrive at

$$-\sum_{i=1}^{\infty} \frac{1}{i} (\rho \gamma)^{i} > -\rho \tau \Rightarrow \gamma + \sum_{i=2}^{\infty} \frac{1}{i} \rho^{i-1} \gamma^{i} < \tau \quad \Rightarrow \quad \gamma < \tau.$$

would fall short of the subjective discount factor  $\rho$  as will be shown in (17) below. Even feasibility is violated if  $\tau < \gamma$  as will be seen in (11) below. Such economies where human capital must necessarily vanish over time are left out from our analysis, i.e., condition (6) is assumed to hold throughout the paper.

#### 3. Equilibrium prices and quantities in a country

Let  $w_{st}$  and  $w_{ut}$  denote the real wages at time t for the country's workers employed in the skill-intensive and labor-intensive jobs, respectively. In a competitive equilibrium, these two production factors are paid their marginal products:

$$w_{st} = F_1(L_{st}, L_{ut}, K_t) = \alpha G_1(L_{st}, L_{ut}) G(L_{st}, L_{ut})^{\alpha - 1} K_t^{1 - \alpha}, \tag{7a}$$

$$w_{\rm ut} = F_2(L_{\rm st}, L_{\rm ut}, K_t) = \alpha G_2(L_{\rm st}, L_{\rm ut}) G(L_{\rm st}, L_{\rm ut})^{\alpha - 1} K_t^{1 - \alpha}.$$
 (7b)

Since skilled workers can perform both types of work, it must also be true that  $w_{st}$  is at least as high as  $w_{ut}$ . A similar argument implies that the wage of educators is equal to  $w_{st}$ .

In a stationary world equilibrium, the internationally determined real interest rate will be equal to the rate of time preference  $\rho$ . Given this interest rate and the employment levels in the two job categories, we can find an expression for the equilibrium stock of capital,

$$\rho = F_3(L_{st}, L_{ut}, K_t),$$

i.e.,

$$K_{t} = [\rho^{-1}(1-\alpha)]^{1/\alpha}G(L_{st}, L_{ut}).$$
(8)

Since the production technologies for goods and education exhibit constant returns to scale, another implication of perfect competition is that 'pure' profits are zero and, therefore, the ownership of these industries is immaterial.

Normalize the country's total labor endowment to unity and let  $L_{\rm et}$  denote the number of skilled workers employed as educators. Since agents supply all labor inelastically, market clearing in the country's labor market is obtained when

$$L_{st} + L_{ut} + L_{et} = 1. (9)$$

If we let  $H_t$  denote the number of skilled workers at time t, feasibility requires also that

$$L_{st} + L_{et} \le H_t. \tag{10}$$

Due to international trade, there are no 'domestic' market clearing conditions for goods and physical capital. However, any flows of resources between countries must be consistent with individual agents satisfying their budget constraints as shown in section 4.

Since we will only analyze stationary equilibria, it is convenient to drop the time subscript from all variables. A country's steady state will then turn out to be fully characterized by its ratio of skilled workers H. This quantity is obviously related to the number of educators  $L_{\rm e}$ . After noting that rational agents will not incur the cost of education as long as their skills are intact, it follows that

$$H = \int_{0}^{\tau} \gamma^{-1} L_{e} dt = \frac{\tau L_{e}}{\gamma},$$

i.e.,

$$L_{\mathrm{e}}(H) = \frac{\gamma H}{\tau} \in (0, H). \tag{11}$$

 $L_{\rm e}(H)$  is strictly less than H by assumption (6), which ensures that any stationary ratio of skilled workers is feasible. Due to the resource cost associated with education, it must also be true in a steady state that no skilled workers are employed in the labor-intensive job category. After substituting (11) into (10) at equality, it can be seen that

$$L_{s}(H) = \frac{\tau - \gamma}{\tau} H \in (0, H), \tag{12a}$$

and by also using (9),

$$L_{\rm u}(H) = 1 - H.$$
 (12b)

It is then straightforward to express the capital stock in terms of H by substituting (12) into (8),

$$K(H) = \left[\rho^{-1}(1-\alpha)\right]^{1/\alpha} G\left(\frac{\tau-\gamma}{\tau}H, 1-H\right). \tag{13}$$

Similarly, the steady-state output level can be written as a function of H by substituting (12) and (13) into (3):

$$F(L_{\rm s}(H),L_{\rm u}(H),K(H)) = \left[\rho^{-1}(1-\alpha)\right]^{(1-\alpha)/\alpha}G\left(\frac{\tau-\gamma}{\tau}H,1-H\right). \tag{14}$$

The steady-state wages can also be expressed in terms of H by substituting (12) and (13) into (7):

$$w_{\rm s}(H) = \alpha [\rho^{-1}(1-\alpha)]^{(1-\alpha)/\alpha} G_1\left(\frac{\tau-\gamma}{\tau}H, 1-H\right),$$
 (15a)

$$w_{\rm u}(H) = \alpha [\rho^{-1}(1-\alpha)]^{(1-\alpha)/\alpha} G_2\left(\frac{\tau - \gamma}{\tau}H, 1 - H\right).$$
 (15b)

Finally, let r(H) denote the rate of return on human capital in a steady state, which can be computed from

$$\gamma w_{s}(H) = \int_{0}^{\tau} e^{-r(H)t} [w_{s}(H) - w_{u}(H)] dt.$$
 (16)

The left-hand side of this expression is the cost of education, while the right-hand side represents the present value of the increase in future labor income due to education.

## 4. Individuals' optimization behavior

This section examines what stationary prices are consistent with individuals' optimization behavior. In a steady state, it must be true that both skilled and unskilled workers *choose* to retain their respective educational status. We also know that an individual would prefer a constant consumption stream, since the rate of time preference is equal to the real interest rate.

Let us first examine for what stationary wages a skilled worker is willing to continue to bear the cost of education. The answer is simply; whenever the rate of return on human capital is at least as high as the rate of return on the alternative of investing in physical capital, i.e.,  $r(H) \ge \rho$ . After using (16), this weak inequality can be rearranged to a restriction on the relative wage:

$$\frac{w_{s}}{w_{u}} \ge \frac{1 - e^{-\rho \tau}}{1 - e^{-\rho \tau} - \rho \gamma} \equiv \left(\frac{w_{s}}{w_{u}}\right)^{*} > 1.$$
 (17)

Condition (17) ensures that the wage differential between skilled and unskilled labor is high enough to compensate skilled workers for their investment in human capital. Given that (17) is satisfied, it is straightforward to derive the optimal consumption and savings of a skilled worker. The mentioned desire to smooth consumption over time leads the individual to choose a constant flow of savings q out of labor income, which together with compound interest is exactly sufficient to finance the educational expenditure  $\gamma w_s$  every  $\tau$  units of time, i.e.,

$$\int_{0}^{\tau} e^{\rho t} q dt = \gamma w_{s}.$$

The necessary flow of savings is therefore equal to

$$q = \frac{\rho \gamma w_{\rm s}}{e^{\rho \tau} - 1}.\tag{18}$$

In addition, let  $\bar{a}$  denote the individual's assets in excess of those used for financing education. It follows that the optimal consumption flow of such a skilled worker can be written as

$$c_{\rm s}(\bar{a}) = w_{\rm s} - q + \rho \bar{a}. \tag{19}$$

We now turn to the optimization behavior of an unskilled worker. Let  $a_0$  denote his assets. Given that the agent remains uneducated forever, the optimal consumption flow is

$$c_{\rm u}(a_0) = w_{\rm u} + \rho a_0$$
 (20)

with a corresponding life-time utility of

$$\int_{0}^{\infty} e^{-\rho t} U(c_{\mathbf{u}}(a_{0})) dt = \rho^{-1} U(c_{\mathbf{u}}(a_{0})).$$
(21)

Any unskilled worker is able to become skilled by acquiring education. However, if his assets are less than the educational expenses of  $\gamma w_s$ , the agent must first accumulate sufficient funds. During this accumulation phase it will

once again be optimal for the individual to choose a constant consumption stream, let say  $\hat{c} < w_u + \rho a_0$ . This consumption level will then determine the length of the accumulation period, denoted  $T(\hat{c})$ . The pair  $\hat{c}$  and  $T(\hat{c})$  represents a tradeoff between the reduction in consumption while assets are being accumulated and the rapidity at which the higher income of a skilled worker is realized. In particular, a shorter accumulation period enables an individual to acquire an education faster but reduces also his consumption level as an unskilled worker. The formal relationship is

$$\int_{0}^{T(\hat{c})} e^{\rho t} (w_{u} - \hat{c}) dt + e^{\rho T(\hat{c})} a_{0} = \gamma w_{s},$$

i.e.,

$$T(\hat{c}) = \rho^{-1} \ln \left( \frac{w_{\rm u} - \hat{c} + \rho \gamma w_{\rm s}}{w_{\rm u} - \hat{c} + \rho a_{\rm 0}} \right). \tag{22}$$

When the unskilled worker has obtained an education, the optimal consumption level is  $c_s(0)$  as defined in (19). The equality between the discount rate and the real interest rate makes it unattractive to acquire any assets in excess of those used for financing education. The optimal  $\hat{c}$  is therefore found by solving the following optimization problem:

$$\max_{\hat{c}} \int_{0}^{T(\hat{c})} e^{-\rho t} U(\hat{c}) dt + \int_{T(\hat{c})}^{\infty} e^{-\rho t} U(c_{s}(0)) dt,$$
(23)

subject to  $\hat{c} < w_u + \rho a_0$ ,

 $c_s(0)$  and  $T(\hat{c})$  as defined in (19) and (22),

given  $a_0$ .

The limiting value of the objective function when  $\hat{c}$  approaches  $w_{\rm u} + \rho a_0$  is clearly the life-time utility of an agent who remains uneducated forever, given by (21). If this latter utility level cannot be improved upon by obtaining a future education, an unskilled worker with insufficient assets,  $a_0 < \gamma w_{\rm s}$ , chooses to remain uneducated forever. After substituting (22) into (23), the condition for this can be seen to be

<sup>&</sup>lt;sup>5</sup>Any uneven consumption flow while saving for educational expenditures can be improved upon by cutting off the peaks and filling in the troughs at the interest rate  $\rho$ . It is clearly feasible for the individual to delay consumption by investing in physical capital. The ongoing accumulation of educational funds implies also that consumption smoothing is possible in the opposite direction through a reduction of early savings.

$$U(c_{\mathbf{u}}(a_0)) \ge \rho \frac{\gamma w_{\mathbf{s}} - a_0}{w_{\mathbf{u}} - \hat{c} + \rho \gamma w_{\mathbf{s}}} U(\hat{c}) + \frac{w_{\mathbf{u}} - \hat{c} + \rho a_0}{w_{\mathbf{u}} - \hat{c} + \rho \gamma w_{\mathbf{s}}} U(c_{\mathbf{s}}(0))$$
(24)

for all  $\hat{c} < w_u + \rho a_0$ .

## 5. Steady states and welfare in a country

As demonstrated in section 3, a country's steady state is fully characterized by its ratio of skilled workers H. We will now prove that there exist equilibrium wages as defined in (15), which are consistent with individuals' optimization behavior. Proposition 1 establishes what is the highest possible H in a steady state, while a continuum of steady states is proven to exist in Proposition 2. Proposition 3 compares prices and quantities across such stationary equilibria. Proposition 4 concludes that a Pareto-optimal stationary allocation is only obtained for the steady state with the highest ratio of skilled workers.

Proposition 1. The highest ratio of skilled workers consistent with a steady state in a country is  $H^*$  implied by

$$\frac{G_1((\tau - \gamma)/\tau^{-1} H^*, 1 - H^*)}{G_2((\tau - \gamma)/\tau^{-1} H^*, 1 - H^*)} = \left(\frac{w_s}{w_u}\right)^*,\tag{25}$$

where  $(w_s/w_u)^*$  is defined in (17). If (25) is not satisfied for any ratio of skilled workers in the unit interval,  $H^*$  is equal to one.

*Proof.* After substituting (15) into (17), the resulting expression (25) determines the highest H for which skilled workers would choose to remain educated. At this composition of the labor force, the equilibrium wages are such that the return on education net of 'depreciation' is equal to the real interest rate  $\rho$ . People are therefore indifferent between acquiring an education or investing in physical capital. This means also that unskilled workers have no incentive to change their educational status. An unskilled worker's life-time utility is even reduced if education would have to be preceded by a period of asset accumulation.

The existence of a unique  $H^* \in (0,1]$  is guaranteed by assumptions (4) and (5). The latter assumption implies that the limit of the left-hand side of (25) is infinity when H approaches zero, and the expression is decreasing in H by the former assumption. However, the model does not preclude the possibility that the rate of return on education does not fall below  $\rho$  even if all workers are educated.  $H^*$  would then be equal to one.

In addition to the highest possible steady state  $H^*$  in Proposition 1, the following proposition guarantees the existence of a continuum of steady states.

Proposition 2. There exists  $\overline{H} \in (0,1]$  such that any ratio of skilled workers  $H \in (0,\overline{H}]$  can be a steady state in a country whenever the unskilled workers have no assets.<sup>6</sup>

Proof. See appendix.

The continuum of steady states may seem surprising in light of assumption (4) that the relative wage of unskilled labor is lower the smaller H is. One might therefore think that unskilled workers would like to educate themselves and start earning the higher wage. But at very low ratios of skilled workers in the economy, the educational cost is also high compared to the labor income of an unskilled worker. It turns out that the loss of utility from foregone consumption while saving for educational expenditures outweighs the welfare gain from higher future earnings. Moreover, the proof of Proposition 2 is 'continuous' with respect to the unskilled workers' asset holdings. It can therefore be shown that a ratio of skilled workers strictly less than  $\overline{H}$  is consistent with a steady state as long as the unskilled workers have sufficiently few assets.

Proposition 3. The steady-state output level and stock of physical capital are increasing in the ratio of skilled workers in the labor force, while the wage of skilled labor in terms of unskilled labor and the rate of return on education are decreasing.

Proof. See appendix.

Proposition 4. Given lump-sum transfers being available, a Pareto-optimal stationary allocation is only obtained when the ratio of skilled workers is equal to  $H^*$  as defined in Proposition 1.

*Proof.* Since all agents have the same discount factor  $\rho$ , a Pareto-optimal stationary allocation must implement all investment opportunities with a rate of return greater than or equal to  $\rho$ . The international market for physical capital can be said to accomplish this objective for nonhuman capital. But according to the proofs of Propositions 1 and 3, there are investment opportunities in human capital earning a rate of return greater (less) than  $\rho$  whenever H is less (greater) than  $H^*$ . It is straightforward to construct a

<sup>6</sup>Please note that the highest possible steady state  $H^*$  in Proposition 1 may or may not be included in the set  $(0, \bar{H}]$  since  $H^* \ge \bar{H}$ .

Pareto-superior allocation for any stationary ratio of skilled workers other than  $H^*$  by gradually adjusting that ratio towards  $H^*$ .

## 6. World equilibrium

The existence proofs of a country's steady states were partial equilibrium arguments, since they only imposed market clearing in the domestic labor market. A world equilibrium requires also that the international good market and physical capital market clear. We will now examine under what circumstances, supply is equal to demand in the world capital market. It then follows that market clearing for international trade in goods is ensured by Walras' Law.

Consider any number of countries and suppose that each country has a stationary ratio of skilled workers consistent with a steady state as discussed in section 5. The steady-state capital stock in a country is then given by expression (8), which depends on the composition of the country's labor force. A condition for a stationary world equilibrium is that the implied world capital stock is willingly held by the agents. First of all, assets are demanded by skilled workers who are saving for future educational expenditures. These agents have been shown to optimally accumulate assets until it is time for them to obtain a new education. Despite this sawtooth time pattern of each skilled agent's asset holdings, it is straightforward to verify that a country's total assets used for financing education stay constant over time in a steady state. After summing up all such assets across countries, this demand for assets cannot be allowed to exceed the supply, i.e., the world capital stock. It follows that a stationary world equilibrium with interest rate  $\rho$  will only exist if the implied stock of physical capital is at least as large as agents' savings for future educational expenditures.

Any physical capital not used for financing education can be owned by anyone in the world, as long as the ownership is consistent with all agents choosing to retain their educational status. In particular, it has been shown that unskilled workers in an underdeveloped country must be relatively poor, since they would otherwise like to obtain an education. On the other hand, unskilled workers in a country with the highest possible ratio of skilled workers can own any amount of assets. At this composition of the labor force, the rates of return on human and nonhuman capital are equalized. As a result, agents are indifferent between obtaining an education or investing in physical capital.

In a steady state, countries' aggregate asset holdings and individuals' consumption levels stay constant over time. Any net flows of goods between countries will only arise from factor payments for foreign-owned physical capital. The international capital market equalizes the rates of return on

physical capital across countries. Agents are therefore indifferent to the location of their savings, and the allocation of the world's physical capital depends solely on countries' stocks of human capital. It follows that economic underdevelopment is caused by underinvestment in human capital. A reflection of this is the higher rates of return on education in less developed countries, which also correspond to larger relative wage differences between skilled and unskilled labor in these countries.

#### 7. Economic growth

We have studied the implications of a missing credit market for human capital in a stationary environment. It is clearly desirable to extend the analysis to an economy exhibiting economic growth. The following proposition is suggestive with respect to the effects of neutral technological change.

Proposition 5. Suppose the production function in (3) is multiplied by a 'technology level' A, and assume that the marginal utility of consumption remains strictly positive in the limit when consumption approaches infinity. It is then possible to eliminate any steady state other than  $H^*$  by choosing A sufficiently large.

*Proof.* See appendix.

The proof uses the fact that neutral technological change raises the marginal products of all inputs by the same factor. This implies that the rate of return on education is unchanged for any given ratio of skilled workers in the labor force. However, the higher wage of unskilled workers makes economic underdevelopment less likely. At a sufficiently high income level, everyone would like to save for an education as long as the rate of return on human capital exceeds the interest rate in the market for physical capital.

Even though economic growth may eventually loosen the impact of a missing market for human capital, it leaves open the question how the process of growth itself is affected by such a market imperfection. The answer will depend on which 'growth mechanism' is chosen. The exogenous technological change in Solow's (1956) original neoclassical model has been superseded by the endogenous growth literature. Uzawa (1965) assumes that both intangible human capital and physical capital can be accumulated without limits making unbounded growth possible, while Arrow (1962) examines the effects of learning by doing. A more recent exploration of these concepts can be found in Romer (1986) and Lucas (1988). Another example of an endogenous growth model is the attempt by Becker and Barro (1988) to analyze fertility and capital accumulation decisions simultaneously within

a general equilibrium framework. In future work, we intend to reexamine the implications of these models when markets for human capital are incomplete.

#### 8. Conclusions

This paper has examined the effects of indivisibilities in education and a missing market for human capital, in a world economy with free trade in consumption goods and physical capital. Although technology and individuals' preferences are identical across countries, it is shown that both developed and underdeveloped countries can coexist in a stationary equilibrium. In fact, there is a continuum of steady states for the world economy corresponding to different distributions of human and nonhuman wealth. The perpetuation of economic underdevelopment is due to the inability to use future labor earnings as collateral on a loan and the nonconvexity in education. As a result, unskilled workers with little assets living in underdeveloped countries choose to remain uneducated despite the higher rates of return on education in these countries. The reason being that the loss of utility from foregone consumption while saving for educational expenditures, outweighs the welfare from higher future earnings.

Another model of an international economy with both skilled and unskilled labor is presented by Findlay and Kierzkowski (1983). Given a perfect student loan market, they show that skilled and unskilled workers attain the same utility level. This case corresponds to our unique Pareto-optimal production structure in Proposition 4, where an individual's welfare depends on the sum of his human and nonhuman assets but not on his educational status per se. The reason that the composition of the labor force can differ across countries in the model of Findlay and Kierzkowski, is the assumption of exogenously given levels of a specific educational input. Under our assumption that skilled labor is used to transform unskilled workers into skilled workers, it is shown with identical preferences that all countries are clones of each other. The market imperfection for human capital is therefore crucial for explaining economic underdevelopment in this framework when educational inputs are reproducible.

The constraint that future labor earnings cannot serve as collateral on a loan is also analyzed by Loury (1981), who writes down a model with human capital as the only intertemporal good. In the face of stochastic shocks to individuals' abilities, the economy is seen to converge to a unique income distribution. An important reason for multiple distributions being ruled out is the assumption that the recurrent education decision is a continuous choice variable. We have instead shown that indivisibilities in education can explain the persistence of economic underdevelopment, even when all agents can earn the market interest rate on any amount of savings chosen. We believe that the assumption of a lumpy education technology parallels more

closely to actual circumstances. The common practice, for whatever reason, is to provide education in 'packages' like high school and college degrees.

Our multiplicity of steady states resembles the idea of an underdevelopment trap by Azariadis and Drazen (1990). They assume that the technological rate at which individuals can accumulate human capital depends positively on the existing economy-wide stock of human resources. It is shown that a country can converge to a steady state with or without investments in human capital depending on whether or not the initial stock of human capital exceeds a critical threshold value. The positive externality of human capital, as in the earlier development paper by Lucas (1988), implies that rates of return on education are lower in less developed countries than in developed ones. Our model has the opposite implication, which is also supported by empirical work as mentioned in the introduction. To appreciate the differences in mechanism between the two types of models, we consider why an uneducated worker in an underdeveloped country would like to migrate to a developed country. In a model with technological externalities, the agent would enhance his own productivity by working in close proximity to highly educated individuals. In our model, his ability would not change, but he would earn a higher wage in the developed country because of the relative scarcity of unskilled labor compared to the stock of human and nonhuman capital.

Another implication of our analysis is a positive correlation between economic underdevelopment and income inequality within a country. The relationship is even exact when income derived from nonhuman assets is excluded. The income inequality can be said to reflect the severity of the credit constraint on human capital investments. This result is at variance with Kuznets' (1955) idea that inequality tends to increase in the early stages of economic development and to decrease in the later stages, the so-called 'Kuznets curve'. However, Fields and Jakubson (1990) argue that the existing empirical support of the Kuznets curve is entirely an artifact of the econometric method used. The inference is reversed as soon as country-specific effects are introduced in the estimation. Their conclusion that inequality tends to decrease with economic development is shown to be robust to alternative samples and functional form specifications.

The income distribution matters also in the model of industrialization by Murphy et al. (1989). They interpret industrialization as the introduction of increasing returns technologies. The assumption that international trade is costly attaches then importance to the size and composition of domestic demand. Industrialization is seen to take place if incomes are distributed broadly enough to materialize as demand for mass-produced domestic manufactures. A higher concentration of incomes to the very rich is not conducive to industrialization, since it means a shift of aggregate demand away from high volumes to more variety of goods. As a consequence, fewer

industries may find it profitable to incur the fixed costs of introducing increasing returns technologies. These demand considerations are clearly absent in our model with unhampered international trade in goods, and the focus is instead on a relationship between the income distribution and the supply of human capital. At any rate, both models can be said to highlight economic interdependencies which are not present in a representative agent framework.

The fact that the steady state with the highest ratio of skilled workers is the only Pareto-optimal allocation implies that benevolent governments can do away with economic underdevelopment in our model. However, this may only be true if lump-sum transfers are available. Suppose, instead, that the economic reform must take the form of a student loan program. This would not only benefit the additional agents being educated but also the wage of workers remaining unskilled would increase due to the change in the labor force. On the other hand, the originally skilled workers would face a lower wage in terms of unskilled labor, and if there is an absolute decline in their income they would oppose the reform. Such an argument brings us back to the two economists referred to in the motivation of our paper. Both Lewis (1954, p. 409) and Schultz (1964, p. 196) spoke about underinvestment in human capital because of some agents' vested interests in maintaining the status quo. Romer (1990) resorts to similar reasoning when explaining import restrictions on producer durables, which slow down a country's economic growth but benefit domestic capital owners.

Lucas (1990) mentions that capital market imperfections can be a reason for countries remaining underdeveloped. Capital flows between countries are too small when there is no effective mechanism for enforcing international borrowing agreements. Our paper has shown that a similar constraint on households' financing of human capital can explain the same macroeconomic situation of underdevelopment. The analysis suggests that the observed migration pressure between countries due to restrictions on immigration has its counterpart in a lack of 'occupational migration' within underdeveloped economies because of imperfections in the process of human capital accumulation. The model is therefore consistent with Adelman's (1977) observation that newly industrialized countries, such as South Korea and Taiwan, implemented educational policies prior to their growth takeoffs in the early 1960s.

## **Appendix**

Proof of Proposition 2

As shown in the proof of Proposition 1, skilled workers choose to remain educated for any  $H \in (0, H^*]$ . We will now have to prove the existence of  $H^u$  such that an unskilled worker without assets would not like to acquire an

education for any  $H \in (0, H^{u}]$ . The proposition is then obviously true for  $\overline{H} = \min \{H^*, H^{u}\}$ .

According to (24), an unskilled worker without assets would not like to change his educational status if

$$U(c_{\mathbf{u}}(0)) \ge \frac{\rho \gamma w_{\mathbf{s}}}{w_{\mathbf{u}} - \hat{c} + \rho \gamma w_{\mathbf{s}}} U(\hat{c}) + \frac{w_{\mathbf{u}} - \hat{c}}{w_{\mathbf{u}} - \hat{c} + \rho \gamma w_{\mathbf{s}}} U(c_{\mathbf{s}}(0)) \quad \text{for all } \hat{c} < w_{\mathbf{u}}.$$

A sufficient condition is therefore that

$$U(c_{\mathrm{u}}(0)) \geqq U(\hat{c}) + \frac{w_{\mathrm{u}} - \hat{c}}{\rho \gamma w_{\mathrm{s}}} \, U(c_{\mathrm{s}}(0)) \quad \text{for all } \hat{c} < w_{\mathrm{u}}.$$

After imposing (18)–(20), this weak inequality can be written as

$$\frac{U(w_{\rm u}) - U(\hat{c})}{w_{\rm u} - \hat{c}} \ge \frac{U(\phi w_{\rm s})}{\rho \gamma w_{\rm s}} \quad \text{for all } \hat{c} < w_{\rm u}, \tag{A.1}$$

where

$$\phi = 1 - \frac{\rho \gamma}{e^{\rho \tau} - 1} > 0.$$

Assumption (6) ensures that the constant  $\phi$  is positive. By using (2), (5) and (15), the limits of the two sides can be found for H approaching zero:

$$\lim_{w_{\mathbf{u}} \to 0} \frac{U(w_{\mathbf{u}}) - U(\hat{c})}{w_{\mathbf{u}} - \hat{c}} \bigg|_{\hat{c} < w_{\mathbf{u}}} = \lim_{c \to 0} U'(c) = \infty; \tag{A.2a}$$

$$\lim_{w_s \to w_s} \frac{U(\phi w_s)}{\rho \gamma w_s},\tag{A.2b}$$

where

$$\underline{w}_{\mathrm{s}} = \alpha [\rho^{-1}(1-\alpha)]^{(1-\alpha)/\alpha} \lim_{L_{\mathrm{s}/L_{\mathrm{u}} \to 0}} G_1(L_{\mathrm{s}}, L_{\mathrm{u}}) > 0.$$

The limit in (A.2b) is always finite. This is obvious when  $\underline{w}_s$  is finite or the utility function is bounded from above. If  $\underline{w}_s$  is infinite and the utility function is unbounded, L'Hôpital's rule can be applied to obtain the limit  $(\rho \gamma)^{-1} \phi U'(\infty)$  which is still finite. We can then conclude that condition (A.1)

is satisfied for some interval  $(0, H^{u}]$ , i.e., an unskilled worker without assets chooses to remain uneducated if the ratio of skilled workers in the labor force is less than or equal to  $H^{u}$ .

### Proof of Proposition 3

Given the equilibrium expressions for the allocation of labor in (12) and the wages in (15), assumption (4) implies that the wage of skilled labor in terms of unskilled labor is decreasing in the ratio of skilled workers in the labor force. After dividing both sides of (16) by  $w_s(H)$ , it then also follows that the rate of return on education is decreasing in H. To establish that the steady-state output level and stock of physical capital are increasing in H, it must be shown according to (13) and (14) that the function G is increasing in H for the relevant domain, i.e., for all values of H which can constitute steady states. When totally differentiating G with respect to H, it can be seen that G is an increasing function as long as

$$G_1\left(\frac{\tau-\gamma}{\tau}H, 1-H\right) / G_2\left(\frac{\tau-\gamma}{\tau}H, 1-H\right) > \frac{\tau}{\tau-\gamma}.$$
 (A.3)

In a steady state, the left-hand side is equal to relative wage of skilled labor in terms of unskilled labor. The proof can therefore be completed by demonstrating that the lower bound on the steady-state wage in (17) is greater than the right-hand side of (A.3), i.e.,

$$\left(\frac{w_{\rm s}}{w_{\rm u}}\right)^* \equiv \frac{1 - {\rm e}^{-\rho\tau}}{1 - {\rm e}^{-\rho\tau} - \rho\gamma} > \frac{\tau}{\tau - \gamma} \Leftrightarrow {\rm e}^{-\rho\tau} > 1 - \rho\tau.$$

The inequality holds trivially for  $\rho\tau \ge 1$ . If  $\rho\tau \in (0,1)$ , take the natural logarithm of both sides and calculate the Taylor series expansion of the right-hand side around  $\rho\tau = 0$ . The result is the obviously true statement that

$$0 > -\sum_{i=2}^{\infty} i^{-1} (\rho \tau)^{i}$$
.

# Proof of Proposition 5

Let H be a steady state for some given technology level A, i.e. conditions (17) and (24) are satisfied for the corresponding equilibrium wages  $w_{\rm s}$  and  $w_{\rm u}$ . It can then be shown that condition (24) will eventually be violated when A

goes to infinity unless H is equal to  $H^*$ . In particular, we will show that even unskilled workers without assets would like to start saving for an education at a sufficiently high A. First, choose an arbitrary savings plan implied by some consumption level  $\hat{c} < w_u$ . Second, multiply the resulting consumption allocation by the same factor as the contemplated increase in A, let say  $\lambda$ . This is clearly feasible since all wages are raised by  $\lambda$  in the case of neutral technological change. Our new version of (24) becomes

$$U(\lambda c_{\mathbf{u}}(0)) \ge \frac{\rho \gamma \lambda w_{\mathbf{s}}}{\lambda w_{\mathbf{u}} - \lambda \hat{c} + \rho \gamma \lambda w_{\mathbf{s}}} U(\lambda \hat{c}) + \frac{\lambda w_{\mathbf{u}} - \lambda \hat{c}}{\lambda w_{\mathbf{u}} - \lambda \hat{c} + \rho \gamma \lambda w_{\mathbf{s}}} U(\lambda c_{\mathbf{s}}(0)).$$

After imposing (18)–(20), this can be rearranged to read

$$\frac{U(\lambda w_{\bar{\mathbf{u}}}) - U(\lambda \hat{c})}{U(\lambda \phi w_{\bar{\mathbf{s}}}) - U(\lambda w_{\bar{\mathbf{u}}})} \ge \frac{w_{\bar{\mathbf{u}}} - \hat{c}}{\rho \gamma w_{\bar{\mathbf{s}}}},\tag{A.4}$$

where  $\phi$  is defined in (A.1). The limit of the left-hand side when A, and therefore  $\lambda$ , goes to infinity is

$$\lim_{\lambda \to \infty} \frac{U(\lambda w_{\rm u}) - U(\lambda \hat{c})}{U(\lambda \phi w_{\rm s}) - U(\lambda w_{\rm u})} = \lim_{\lambda \to \infty} \frac{w_{\rm u} U'(\lambda w_{\rm u}) - \hat{c} U'(\lambda \hat{c})}{\phi w_{\rm s} U'(\lambda \phi w_{\rm s}) - w_{\rm u} U'(\lambda w_{\rm u})} = \frac{w_{\rm u} - \hat{c}}{\phi w_{\rm s} - w_{\rm u}}. \quad (A.5)$$

Besides applying L'Hôpital's rule, we have used the assumption in Proposition 5 that the marginal utility of consumption remains positive when consumption approaches infinity. The marginal utilities in the numerator and denominator of (A.5) must therefore converge to the same number and cancel out. Finally, substitute this limit back into (A.4),

$$\frac{w_{\mathrm{u}} - \hat{c}}{\phi w_{\mathrm{s}} - w_{\mathrm{u}}} \ge \frac{w_{\mathrm{u}} - \hat{c}}{\rho \gamma w_{\mathrm{s}}} \Leftrightarrow \frac{w_{\mathrm{s}}}{w_{\mathrm{u}}} \le \frac{1 - \mathrm{e}^{-\rho \tau}}{1 - \mathrm{e}^{-\rho \tau} - \rho \gamma} \equiv \left(\frac{w_{\mathrm{s}}}{w_{\mathrm{u}}}\right)^{*}. \tag{A.6}$$

A limiting stationary equilibria with skilled and unskilled labor must satisfy both conditions (17) and (A.6), i.e. the only permissible relative wage is  $(w_s/w_u)^*$  and  $H^*$  is the unique steady state according to Proposition 1. [If  $H^*$  is equal to one, there are no unskilled workers and condition (A.6) becomes irrelevant.]

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