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FINANCIAL MARKET GLOBALIZATION, SYMMETRY-BREAKING AND ENDOGENOUS INEQUALITY OF NATIONS

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This paper investigates the effects of financial market globalization on the inequality of nations. The world economy consists of inherently identical countries, which differ only in their levels of capital stock. Each country is represented by the standard overlapping generations model, modified only to incorporate credit market imperfection. An integration of financial markets affects the set of stable steady states, as it changes the balance between the equalizing force of the diminishing returns technology and the unequalizing force of the wealth-dependent borrowing constraint. The model is tractable enough to allow for a complete characterization of the stable steady states.

In the absence of the international financial market, the world economy has a unique steady state, which is symmetric and stable. In the presence of the international financial market, *symmetry-breaking* occurs under some conditions. That is, the symmetric steady state loses its stability and stable asymmetric steady states come to exist. In the stable asymmetric steady states, the world economy is endogenously divided into the rich and poor countries; the borrowing constraints are binding in the poor but not in the rich; the world output is smaller, the rich are richer and the poor are poorer in any of the stable asymmetric steady states than in the (unstable) symmetric steady state.

KEYWORDS: Broken symmetry, credit market imperfection, diminishing returns, structuralism, wealth-dependent borrowing-constraints.

1. INTRODUCTION

WHAT ARE THE EFFECTS of financial market globalization on the inequality of nations? The conventional wisdom suggests that an integration of national financial markets facilitates financial flows from rich countries to poor countries, thereby accelerating development in poor countries. According to this view, financial market globalization helps to reduce the inequality of nations. There is, however, the widely held belief that poor countries are unable to compete in integrated financial markets against rich countries, which can offer financial security to the lenders in an imperfect world. According to this view, whose intellectual origin can be traced back to structuralism of Nurkse (1953), Myrdal (1957), and Lewis (1977), financial market globalization magnifies inequality. The structualists often advocate that poor countries should impose capital controls to stem the outflows of domestic saving and that official aids from rich countries are needed for the development of poor countries. Some express an even more radical view that poor countries should jointly cut their links to rich countries and unite among themselves to escape poverty. It is difficult to evaluate the logical consistency of their argument, because there have

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been few attempts to formalize it. The lack of formality not only renders their argument subject to various interpretations, but also leads many mainstream economists to dismiss it as mere rhetoric or muddled thinking.[2] The structualists, on their part, dismiss standard economic theory, used by mainstream economists to illustrate conventional wisdom, as irrelevant, because they believe it fails to capture the complex reality of an imperfect world.[3] In short, the two camps seem unable to communicate with each other.

In the present paper, we take a small step toward reconciling these two conflicting views. To this end, we develop a framework within which to investigate the effect of financial market globalization on the inequality of nations in the presence of credit market imperfection. The world economy is made up of inherently identical countries that differ only in their initial levels of capital stock. Each country is represented by the Diamond overlapping generations model, modified to incorporate credit market imperfection. The model is set up in such a way that, in the absence of credit market imperfection, the only stable steady state is symmetric, both with and without integration.[4] The two key elements of this framework are the diminishing returns technology and endogenous borrowing constraints. The former makes the marginal productivity of investment higher in poor countries, which creates an equalizing force. The latter makes the domestic investment dependent upon the domestic wealth, which in turn depends on the domestic investment. This creates an unequalizing force. Financial market globalization affects the structure of stable steady states of the world economy, as it changes the balance between these two competing forces.

In the absence of the international financial market, the world economy has a unique steady state, which is symmetric and globally stable (in spite of credit market imperfection). This is because, with no international lending and borrowing, capital formation in each country is dictated entirely by domestic saving, and each country reaches the same steady state. The symmetric steady state is stable, because the domestic interest rate adjusts independently within each country to equate domestic saving and domestic investment, when different countries are hit by different shocks.

When the international financial market is introduced, *symmetry-breaking* occurs under some conditions.[5] That is to say, the symmetric steady state loses its stability and stable asymmetric steady states come to exist. The symmetric

[2]In so doing they seem to forget the fact that two of the structualists won Nobel Prizes in Economics.

[3]This view is aptly expressed in the title of Myrdal (1957).

[4]It is in part for this determinacy property that we chose the Diamond model as our basic setup. In the Cass infinitely-lived representative agent model, the steady state imposes no restriction on the distribution of wealth even when the credit market is perfect; see Becker (1980). This indeterminacy would make it inappropriate as a framework within which to evaluate the role of credit market imperfection.

[5]The notion of symmetry-breaking has found a wide range of applications in natural sciences. See Matsuyama (1995, 2002a) for its logic and its applications in economics.

steady state is unstable because, with the integration of financial markets, the interest rates in different countries must move together. Without offsetting changes in the domestic interest rates, the agents in the countries hit by relatively bad shocks are put at a disadvantage, and the domestic investment in these countries declines, creating a downward spiral of low-wealth/low-investment. The same force operates in the opposite direction within the countries hit by relatively good shocks, creating an upward spiral of high-wealth/high-investment. In the stable asymmetric steady states, the world economy is polarized into the rich and the poor and the borrowing constraint is binding in poor countries, but not in rich countries. Furthermore, the rich are richer and the poor are poorer and the world output is smaller than in the (unstable) symmetric steady state. Therefore, the symmetry-breaking case offers some support for the structualist view that globalization magnifies the inequality of nations, as well as for the popular belief that global capitalism is a mechanism through which some countries become rich at the expense of others. Contrary to the popular belief, however, the model suggests that poor countries cannot jointly escape from poverty by cutting their links to rich countries and that official aids from the rich would not eliminate the inequality. Just as in a game of musical chairs, some countries have to be excluded from being rich.

Demonstrating the possibility that globalization might cause symmetry-breaking is important, because it captures the structualist view and hence enables us to put their argument under logical scrutiny. What is equally important is that globalization does not always cause symmetry-breaking. The major advantage of the present framework is that it is simple and tractable enough to allow for a complete characterization of the stable steady states in the world economy, which enables us to express analytically both the sufficient and necessary condition for the symmetry-breaking case. (Roughly speaking, for a sufficiently large credit market imperfection, symmetry-breaking occurs when the productivity of the investment projects is neither too high nor too low.) The present model thus serves as an organizing framework for understanding and reconciling the two conflicting views of the world.

By offering a theory of endogenous inequality of nations, this paper examines how financial market globalization might change the *endogenous* components of heterogeneities across countries. Needless to say, there are exogenous sources of heterogeneities across countries, e.g., climate, natural endowments, location, etc. The logic of symmetry-breaking does not suggest that such exogenous heterogeneities are unimportant. On the contrary, symmetry-breaking is a magnification mechanism. It suggests that even small amounts of exogenous heterogeneities can be amplified to create large observed heterogeneities in a variety of endogenous variables.[6]

[6]See Matsuyama (1995) for more on this point.

As a theory of endogenous inequality of nations, the symmetry-breaking approach may be contrasted with an alternative, which may be called the "poverty trap" or "coordination failure" approach.[7] Consider any model of poverty traps that analyzes a country in isolation, either as a closed economy or as a small open economy, such as Murphy, Shleifer, and Vishny (1989), Azariadis and Drazen (1990), Matsuyama (1991), Ljungqvist (1993), Ciccone and Matsuyama (1996), and Rodríguez-Clare (1996). These studies show how some strategic complementarities create multiple equilibria (in static models) or multiple steady states (in dynamic models). It has been argued that such a model may explain diverse economic performance across inherently identical countries, simply because different equilibria (or steady states) may prevail in different countries. In other words, some countries suffer from coordination failures, locked into poverty traps, while others do not. Although the poverty trap approach suggests the possibility of co-existence of the rich and the poor, it does not suggest that such co-existence is the only stable pattern. Symmetric patterns are also stable. Without the broken symmetry, this approach does not capture the structualist view that the division of the world economy into the rich and the poor is an inevitable feature of the International Economic Order or of the Modern World System. Furthermore, it cannot yield any definite prediction regarding the effects of financial market globalization on the degree of inequality. Moreover, the two approaches have different policy implications. According to the poverty trap approach, the case of underdevelopment is an isolated problem, which can be treated independently for each country. According to the symmetry-breaking approach, it is a part of the interrelated whole, and needs to be dealt with at the global level, which is more in the spirit of structuralism.

The rest of the paper is organized as follows. Section 2 discusses more directly related work in the literature. Section 3 develops the building blocks of the model. Sections 4 and 5 provide the analysis for the autarky and small open economy cases, which serve as preliminary steps for the analysis of the world economy in Section 6. Section 7 discusses how robust the results are when different specifications are used. Section 8 concludes.

2. RELATED WORK IN THE LITERATURE

This paper focuses on credit market imperfection and the wealth-dependent borrowing constraint as the key mechanism behind symmetry-breaking. This is just one of many mechanisms through which structuralists believe that globalization magnifies the inequality of nations. Indeed, previous studies have focused on a different symmetry-breaking mechanism to capture the structuralist view. In Krugman (1981), Krugman and Venables (1995), and Matsuyama (1996), an integration of goods markets can lead to symmetry-breaking, dividing inherently identical countries into the rich and the poor.

[7] Matsuyama (2002a) discusses the differences between the two approaches in more detail.

The possibility that an integration of factor markets can lead to symmetry-breaking has also been extensively studied, although they are usually discussed in the context of regional integration within countries. The symmetry-breaking mechanism in all these studies is aggregate increasing returns, which create agglomeration economies. If this is the mechanism behind symmetry-breaking in the world economy, there are some efficiency gains from symmetry-breaking and the world as a whole may benefit from globalization and magnifying inequality. Even the countries that become poorer than others may gain from globalization. Furthermore, the effect would not depend on the form of globalization. Whether it takes place in financial markets, in factor markets, or in goods markets, globalization makes symmetry-breaking more likely in the presence of agglomeration economies. In the present paper, the technology satisfies diminishing returns at the aggregate level, so that symmetry-breaking generates efficiency losses. Thus, globalization makes some countries richer only at the expense of making the rest of the world poorer. Furthermore, the effect depends critically on the form of globalization. Financial market globalization (trade in financial assets) makes symmetry-breaking more likely, while factor market globalization (such as foreign direct investment and trade in physical capital, i.e., the capital good used in production) would make symmetry-breaking less likely.

Many recent studies have examined the role of the international financial market in the presence of credit market imperfection: see, for example, the work cited by Obstfeld (1998) and Tirole (2002a). They mostly focus on the issue of short-run volatility, motivated by recent economic crises in emerging markets. Only a few studies have addressed the effects of financial market globalization on the inequality of nations in the presence of credit market imperfection. In the static model of Gertler and Rogoff (1990), the country's wealth is given by an exogenous endowment. They examined how the distribution of the endowment across countries affects investment and financial capital flows, but, due to the static nature of the model, there is no feedback effect from the investment to the distribution. Boyd and Smith (1997) introduced such a feedback effect in an overlapping generations model of the world economy. Their model is so complicated that they had to assume that the borrowing constraint is always binding for all the countries, both in and out of the steady states, and even then, they had to rely on numerical simulation to prove the stability of asymmetric steady states. They also restricted their parameters in such a way that the symmetric steady state is always unstable. The model presented in this paper has the advantage of being tractable, which makes it possible to characterize all the stable steady states for the full set of parameter values, without making any auxiliary assumption.[8] In other words, the present model

[8]It turns out that one of the auxiliary assumptions that Boyd–Smith made would be untenable in the present model. The analysis shows that the borrowing constraint is not binding for the rich in all the stable asymmetric steady states, which necessarily exist when the symmetric steady state is unstable.

allows one to derive analytically the conditions for stability of the symmetric and asymmetric steady states and for the borrowing constraint to be binding in these steady states. This in turn makes it possible to examine the effects of changing the parameter values, making the model useful as an intuition-building device.[9]

Acemoglu and Zilibotti (1997, Section VI) and Martin and Rey (2001) demonstrated how incomplete markets (in the sense of Arrow–Debreu securities) could magnify the inequality of nations. The key mechanism in these models is that rich countries have better financial markets than poor countries, which provide more opportunities to diversify, and hence encourage more investment. In other words, the agents in poor countries do not enjoy equal access to the financial markets as those in rich countries. In the present paper, as well as in the models of Gertler–Rogoff–Boyd–Smith, it is assumed that countries do not differ in their degree of credit market imperfection. The key mechanism here is that globalization makes everyone have equal access to the financial markets, thereby forcing the agents in the poor countries, who have less wealth, to compete directly with those in rich countries for credit.

3. THE MODEL

The basic framework used is the Diamond overlapping generations model with two period lifetimes. A single final good is produced by two factors of production: labor, supplied by young agents, and physical capital, supplied by old agents. "Labor" should be interpreted broadly to include any endowment held by young agents, whose equilibrium value increases with the investment made by the older generation. "Physical capital" should be interpreted broadly to include human capital or any capital good used in production. The final good produced in period t may be consumed in period t or may be invested in the production of physical capital, which becomes available in period $t+1$. When physical capital is interpreted as human capital, this technology may be

[9]The present paper may remind some readers of the literature on wealth distribution across households; see Aghion and Bolton (1997), Banerjee and Newman (1993), Freeman (1996), and Matsuyama (2000a, 2000b). The last three studies in particular use the symmetry-breaking approach to explain endogenous inequality across households. Despite some resemblance, the present model differs fundamentally from these models. First, in all these models, the assumption that each household faces a nonconvex technology plays an essential role in generating the inequality among households. In the present model, the inequality among nations is generated despite the fact that each nation has a convex technology. Second, inequality is transmitted over time through bequest motives in these studies. Here, they are transmitted through nontraded factor markets that generate a home bias in the investment demand spillovers. These differences in the specifications lead to differences in the predictions, as well. For example, in the model of Matsuyama (2000a, 2000b), which uses the same specification of the credit market imperfection as the present model, endogenous inequality across households occurs when the productivity of the investment projects is sufficiently low. In the present model, endogenous inequality across nations occurs when the productivity of the investment projects is neither too low nor too high.

interpreted as education. Only the final good can be traded (intertemporally) between countries. Both factors of production are assumed nontradeable.

The technology of the final goods sector satisfies standard, neoclassical properties. It is given by a linear homogeneous production function, $Y_t = F(K_t, L_t)$, where K_t and L_t are aggregate domestic supplies of physical capital and labor in period t. Let $y_t \equiv Y_t/L_t = F(K_t/L_t, 1) \equiv f(k_t)$ where $k_t \equiv K_t/L_t$ and $f(k)$ is C^2 and satisfies $f'(k) > 0 > f''(k)$, $f(0) = 0$, and $f''(0) = \infty$. The factor markets are competitive, and the factor rewards for physical capital and for labor are equal to $\rho_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t) \equiv W(k_t)$, which are both paid in the final good. Note that $f''(k) < 0$ implies that a higher k_t increases w_t and reduces ρ_t. For simplicity, physical capital is assumed to depreciate fully in one period. This assumption is particularly reasonable when physical capital is interpreted as human capital.

Each generation consists of a continuum of homogenous agents with unit mass. (Sections 7.1 and 7.2 introduce heterogeneous agents.) Each agent is endowed with one unit of labor in the first period, which is supplied inelastically to the final goods sector, and consumes only in the second. Thus, $L_t = 1$, and the wage income, w_t, is also equal to the level of wealth held by the young agents at the end of period t. They allocate their wealth, w_t, in order to finance their consumption in period $t+1$. They have two options. First, they may lend it in the competitive credit market, which earns the gross return equal to r_{t+1} per unit. If they lend the entire wealth, their second-period consumption is equal to $r_{t+1}w_t$. Second, they may start an investment project. The project comes in discrete, nondivisible units, and each young agent can run only one project.[10] The project transforms one unit of the final good in period t into $R > 0$ units of capital in period $t + 1$. To avoid a taxonomical exposition, we focus on the case where

$$\text{(A1)} \qquad W(R) < 1.$$

As seen later, (A1) ensures that $w_t < 1$, so that the agent needs to borrow $1 - w_t > 0$ in the competitive credit market, in order to start the project. It is also assumed that the agent cannot start a project abroad (or it is prohibitively costly to do so). In other words, foreign direct investment is ruled out.[11]

The two assumptions, that factors are nontradeable and that agent cannot start a project abroad, are imposed to focus on the effects of financial market globalization, not those of factor market globalization. What is essential here

[10]Note that, even though each agent faces an indivisible investment technology, aggregate technology is convex, because there is a continuum of agents in each country that invest in the same indivisible project. The assumption that each agent can run at most one project is made for simplicity and can be dropped (see Section 7.2).

[11]This restriction is also reasonable if physical capital and the investment project are interpreted as human capital and education.

is that an imperfect integration of factor markets generates a home bias in the demand spillover effects of the domestic investment. A higher domestic investment increases the wealth of the domestic young agents more than the wealth of the foreign young agents.

We are now ready to look at the investment decision. Second period consumption, if the agent starts the project, is equal to $\rho_{t+1}R - r_{t+1}(1-w_t)$. This is greater than or equal to $r_{t+1}w_t$ (second period consumption if the agent lends the entire wage income) when the net present discounted value of the project, $\rho_{t+1}R/r_{t+1} - 1$, is nonnegative. This condition can be expressed as

$$(1) \qquad Rf'(k_{t+1}) \geq r_{t+1}.$$

Young agents are willing to borrow and to start the project when (1) holds. We shall call (1) *the profitability constraint*.

The credit market is competitive in the sense that both lenders and borrowers take the equilibrium rate, r_{t+1}, as given. It is not competitive, however, in the sense that one cannot borrow any amount at the equilibrium rate. The borrowing limit exists because the borrowers can pledge only up to a fraction of the project revenue for the repayment. More specifically, the borrower would not be able to credibly commit to repay more than $\lambda\rho_{t+1}R$, where $0 < \lambda < 1$. Knowing this, the lender would lend only up to $\lambda\rho_{t+1}R/r_{t+1}$. Thus, the agent can start the project only if $1 - w_t \leq \lambda\rho_{t+1}R/r_{t+1}$, or

$$(2) \qquad \lambda Rf'(k_{t+1}) \geq r_{t+1}(1 - W(k_t)).$$

We shall call (2) *the borrowing constraint*.[12] It is also assumed that the same commitment problem rules out the possibility that different agents may pool their wealth to overcome the borrowing constraint. Young agents in period t start the project only when both (1) and (2) are satisfied. In other words, they must be both willing and able to borrow. The parameter, λ, captures the credit market friction in a parsimonious way. If it were zero, agents would never be able to borrow and hence must self-finance their projects entirely. If it were equal to one, the borrowing constraint would never be binding whenever the agents want to borrow. By setting it between zero and one, this specification allows us to examine the whole range of intermediate cases between the two extremes. The reader may thus want to interpret this formulation simply as a black box, a convenient way of introducing the credit market imperfection in a dynamic macroeconomic model, without worrying about the underlying causes

[12]One may also call (2) *the self-financing or collateral constraint*, because it can be rewritten as $w_t \geq C_{t+1} \equiv 1 - \lambda\rho_{t+1}R/r_{t+1}$, where C_{t+1} may be interpreted as the downpayment or collateral requirement.

of imperfections.[13]

The two constraints, (1) and (2), can be summarized as

$$(3) \qquad R \geq R_t \equiv \begin{cases} (r_{t+1}/f'(k_{t+1}))(1 - W(k_t))/\lambda & \text{if } k_t < K(\lambda), \\ r_{t+1}/f'(k_{t+1}) & \text{if } k_t \geq K(\lambda), \end{cases}$$

where R_t may be interpreted as the project productivity *required* in order for the project to be undertaken in period t, and $K(\lambda)$ is defined implicitly by $W(K(\lambda)) = 1 - \lambda$. Note that which of the two constraints is binding depends entirely on k_t. The borrowing constraint (2) is binding if $k_t < K(\lambda)$; the profitability constraint (1) is binding if $k_t > K(\lambda)$. Thus, the investment is borrowing constrained only at the lower level of domestic wealth. The critical value of k, $K(\lambda)$, is decreasing in λ, with $K(1) = 0$ and $K(+0) = R^+$, where R^+ is given by $W(R^+) = 1$. Thus, the less imperfect the credit market, the less important the borrowing constraint becomes, and if the credit market is perfect ($\lambda = 1$), the borrowing constraint is never binding.

4. THE AUTARKY CASE

Let us first consider the case of autarky. Without international lending and borrowing, domestic investment (by the young) must be equal to domestic saving (by the young) in equilibrium.[14] From (3), domestic investment is equal to zero if $R_t > R$, and to one, if $R_t < R$, and may take any value between zero and one if $R_t = R$. Domestic saving is equal to $W(k_t)$, which is less than one, if $k_t < R$, from (A1). Thus, in equilibrium, $R_t = R$ and the aggregate investment is made equal to $W(k_t)$. Thus, the fraction of young agents who become borrowers and start the project is equal to $W(k_t)$, while the rest, $1 - W(k_t)$, become lenders. If $k_t \geq K(\lambda)$, young agents are indifferent between borrowing and lending. When $k_t < K(\lambda)$, on the other hand, they strictly prefer borrowing

[13] Nevertheless, it is possible to give any number of moral hazard stories to justify the assumption that borrowers can pledge only up to a fraction of project revenue. The simplest story would be that they strategically default, whenever the repayment obligation exceeds the default cost, which is proportional to the project revenue. Alternatively, each project is specific to the borrower, and requires his services to produce R units of physical capital. Without his services, it produces only λR units. Then, the borrower, by threatening to withdraw his services, can renegotiate the repayment obligation down to $\lambda \rho_{t+1} R$. See Kiyotaki and Moore (1997). It is also possible to use the costly-state-verification approach used by Bernanke and Gertler (1989) and Boyd and Smith (1997), or the ex-ante moral hazard approach used by Aghion and Bolton (1997) or the ex-post moral hazard approach used by Holmstrom and Tirole (1997).

[14] The GNP accounting of a closed economy, of course, implies that saving by all residents is equal to investment by all residents, including not only the young but also the old. However, in this model, the old are never engaged in investment activity and consume all their income, so that their saving is zero. Hence, the equality of saving and investment by the young is indeed the equilibrium condition when the economy is in autarky. In what follows, we shall simply use domestic saving and domestic investment, without specifically mentioning "by the young."

to lending. Therefore, the equilibrium allocation necessarily involves credit rationing, where the fraction $1 - W(k_t)$ of young agents are denied credit. Those who are denied credit cannot entice potential lenders by raising the interest rate, because lenders would know that the borrowers would default at a higher rate.[15]

Since the measure of the young agents who start the project is equal to $W(k_t)$ and every one of them supplies R units of physical capital in period $t+1$,

$$k_{t+1} = RW(k_t). \tag{4}$$

Equation (4) completely describes the dynamics of capital formation in autarky. Note that, if $k_t < R$, $k_{t+1} = RW(k_t) < RW(R) < R$ from (A1). Therefore, $k_0 < R$ implies $k_t < R$ and $w_t = W(k_t) < 1$ for all $t > 0$, as has been assumed.

Notably, the dynamics of k, (4), is entirely independent of λ; the credit market imperfection has no effect on capital formation in the autarky case. This is because domestic investment is determined entirely by domestic saving. Any effect of the credit market imperfection is completely absorbed by interest rate movements. From (3), (4), and $R = R_t$, the equilibrium interest rate is given by

$$r_{t+1} = \begin{cases} \lambda Rf'(RW(k_t))/(1 - W(k_t)) & \text{if } k_t < K(\lambda), \\ Rf'(RW(k_t)) & \text{if } k_t \geq K(\lambda). \end{cases} \tag{5}$$

Note that a greater imperfection in the credit market (a smaller λ) manifests itself in the reduction of the interest rate.

Clearly, the result that the dynamics of capital formation in autarky is unaffected by the credit market imperfection is not a robust feature of the model. In particular, it critically depends on the fact that the aggregate supply of credit is inelastic. Nevertheless, this feature of the model makes the autarky case a useful benchmark for examining the effects of financial market globalization in the presence of the credit market imperfection. What is essential here is that the aggregate supply of credit is less elastic in autarky than in an open economy.

The dynamics of capital formation in autarky, given by (4), even though it is independent of λ, may still have multiple steady states. This feature of the overlapping generations model is well known (see, e.g., Azariadis (1993)) and it is a nuisance that has nothing to do with the credit market imperfection. To

[15] In the present model, credit rationing is an inevitable feature of equilibrium whenever the borrowing constraint is binding. This is, however, a mere artifact of the homogeneity of the agents. It can be shown that, in a more general setup that allows for heterogeneous agents, what is essential is the borrowing constraint, not credit rationing. See Section 7.1 and 7.2. See also Matsuyama (2001, Section 6).

avoid any unnecessary complications that arise from this feature of the overlapping generations model, we impose the following assumption:

$$\text{(A2)} \qquad W'(0) = \infty, \quad W''(k) < 0.$$

Many standard production functions imply (A2). For example, if $y = f(k) = A(k)^\alpha$ with $0 < \alpha < 1$, $W(k) = (1-\alpha)A(k)^\alpha$, which satisfies (A2).

As shown in Figure 1(a), (A1) and (A2) ensure that equation (4) has the unique steady state, $k^* = K^*(R) \in (0, R)$, defined implicitly by $k^* = RW(k^*)$, and for $k_0 \in (0, R)$, k_t converges monotonically to $k^* = K^*(R)$. The function, $K^*(R)$, is increasing and satisfies $K^*(0) = 0$ and $K^*(R^+) = R^+$. (Recall that R^+ was defined by $W(R^+) = 1$.) It is worth emphasizing that $K^*(R)$, the steady state level of k, is independent of λ, and $K(\lambda)$, the critical level of k, below which the borrowing constraint is binding, is independent of R. Therefore, the borrowing constraint may or may not be binding in the steady state.

To summarize, we provide the following proposition.

PROPOSITION 1: *In autarky, the dynamics of k is given by $k_{t+1} = RW(k_t)$, which is independent of λ, and converges monotonically to the unique steady state, $K^*(R)$, where $K^*(R)$ is increasing in R and satisfies $K^*(0) = 0$ and $K^*(R^+) = R^+$. If $K^*(R) < K(\lambda)$, the borrowing constraint is binding in the steady state. If $K^*(R) > K(\lambda)$, the profitability constraint is binding in the steady state.*

Figures 1(a) and (b) illustrate Proposition 1. The downward-sloping curve in Figure 1(b) is given by $K^*(R) = K(\lambda)$, which connects $(\lambda, R) = (0, R^+)$ and $(\lambda, R) = (1, 0)$. Below and left of this curve, the borrowing constraint is binding in the autarky steady state.

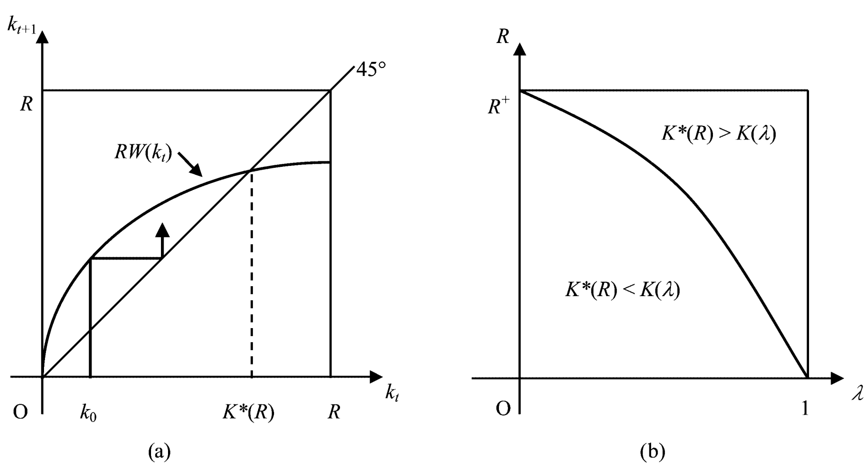

FIGURE 1.—Dynamics and parameter configuration—the autarky case.

5. THE SMALL OPEN ECONOMY

Let us now examine the small open economy case, which serves as a preliminary step for the analysis of the world economy in the presence of the international financial market.

The agents in the small open economy are allowed to trade intertemporally the final good with the rest of the world at exogenously given prices. In other words, international lending and borrowing is allowed. The interest rate, the intertemporal price of the final good, is exogenously given in the international financial market and assumed to be invariant over time: $r_{t+1} = r$.

In what follows, we will focus on the case $Rf'(R) < r$ for ease of exposition.[16] Then, the equilibrium condition is given by setting $R_t = R$ in (3), which can be further rewritten as

$$(6) \qquad k_{t+1} = \Psi(k_t) \equiv \begin{cases} \Phi\big(r(1 - W(k_t))/\lambda R\big) & \text{if } k_t < K(\lambda), \\ \Phi(r/R) & \text{if } k_t \geq K(\lambda), \end{cases}$$

where Φ is the inverse of f', which is a decreasing function and satisfies $\Phi(\infty) = 0$.

Equation (6) governs the dynamics of the small open economy. Unlike the autarky case, domestic investment is no longer equal to domestic saving. Instead, investment is determined entirely by the profitability and borrowing constraints. If the credit market were perfect ($\lambda = 1$ and $K(1) = 0$), the economy would immediately jump to $\Phi(r/R)$, from any initial condition. In the presence of the imperfection, this occurs only when the economy is at the higher level of development ($k_t \geq K(\lambda)$), where the profitability of the project is the only binding constraint. At the lower level of development ($k_t < K(\lambda)$), the borrowing constraint is binding, which creates the gap between the return to investment and the interest rate. In this range, the map is increasing in k_t. This is because a high domestic investment increases the wage income of domestic young agents, enabling them to accumulate more wealth, which alleviates the borrowing constraint and stimulates domestic investment. This effect is essentially the same as the credit multiplier effect identified by Bernanke and Gertler (1989) and others. In this range, the map is also increasing in $\lambda R/r$. In particular, a reduction in λ reduces k_{t+1}. In a small open economy, the interest rate is fixed in the international financial market. Therefore, greater imperfection has the effect of reducing domestic investment (and channeling more of the domestic saving into investment abroad). This differs significantly from

[16] If $Rf'(R) \geq r$, the dynamics is given by $k_{t+1} = \min\{R, \Psi(k_t)\}$, where $\Psi(k_t)$ is defined as in (6). Assuming $Rf'(R) < r$ ensures that not all the young invest, so that $k_{t+1} = \Psi(k_t) < R$, and hence the equilibrium is never at the corner. This restriction helps to reduce the notational burden significantly, but the result can be easily extended to the case where $Rf'(R) \geq r$ as well. This restriction can also be justified on the ground that, in the world economy version of the model developed later, the world interest rate prevailing in any steady state satisfies $Rf'(R) < r$.

the autarky case, where domestic investment is determined by domestic saving, and a reduction in λ reduces r_{t+1}, but has no effect on k_{t+1}.

The steady states of the small open economy are given by the fixed points of the map (6), satisfying $k = \Psi(k)$. The following lemma summarizes some properties of the set of fixed points. While elementary, they turn out to be quite useful, and will be evoked repeatedly in subsequent discussion.

LEMMA: (a) *Equation* (6) *has at least one steady state.*

(b) *Equation* (6) *has at most one steady state above* $K(\lambda)$. *If it exists, it is stable and equal to* $\Phi(r/R)$.

(c) *Equation* (6) *has at most two steady states below* $K(\lambda)$. *If there is only one*, k_L, *either it satisfies* $0 < k_L < \lambda R/r$ *and is stable, or*, $k_L = \lambda R/r$ *at which* Ψ *is tangent to the* 45° *line. If there are two*, k_L *and* k_M, *they satisfy* $0 < k_L < \lambda R/r < k_M < K(\lambda)$, *and* k_L *is stable and* k_M *is unstable.*

For the proof see the Appendix.

One immediate implication of the Lemma is that there are only three generic cases of the dynamics generated by (6). They are illustrated in Figures 2(a)–(c). In Figure 2(a), the unique fixed point, k_L, is located below $K(\lambda)$, to which k_t converges from any $k_0 \in (0, R)$. In Figure 2(c), the unique fixed point, $k_H = \Phi(r/R)$, is located above $K(\lambda)$, to which k_t converges from any $k_0 \in (0, R)$. In Figure 2(b), there are three fixed points; two stable steady states, k_L and k_H, are separated by the third (unstable) steady state, k_M, which is located between k_L and $K(\lambda)$, and k_t converges to k_L if $k_0 < k_M$ and to k_H if $k_0 > k_M$.[17]

The following proposition provides the exact condition for each of the three cases.

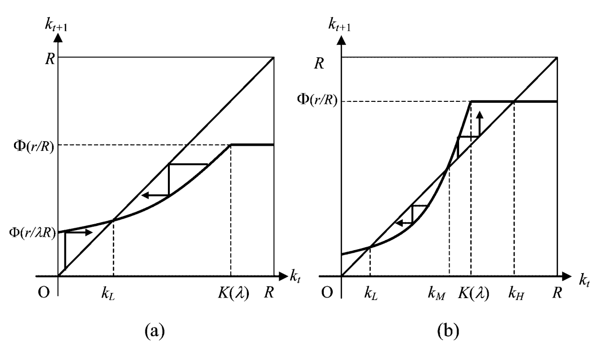

FIGURE 2.—Dynamics—the small open economy case.

[17]Figures 2(a)–(c) are drawn so that $\Psi'' > 0$ for $k < K(\lambda)$. This may or may not be true. Note that part (c) of the Lemma does not say that the map is convex in this range. It says that it cannot intersect the 45° line more than twice in this range.

PROPOSITION 2: *Let* $\lambda_c \in (0, 1)$ *be defined by* $f(K(\lambda_c)) = 1$. *Then*:

(a) *If* $Rf'(K(\lambda)) < r$, *there exists a unique steady state*, k_L; *it is stable and satisfies* $k_L < K(\lambda)$.

(b) *If* $Rf'(K(\lambda)) > r$, $f(\lambda R/r) < 1$, *and* $\lambda < \lambda_c$, *there exist three steady states*, k_L, k_M, *and* k_H. *They satisfy* $k_L < k_M < K(\lambda) < k_H$, *and* k_L *and* k_H *are stable and* k_M *is unstable*.

(c) *If* $Rf'(K(\lambda)) > r$ *and either* $f(\lambda R/r) > 1$ *or* $\lambda > \lambda_c$, *there exists a unique steady state*, k_H. *It is stable and satisfies* $k_H > K(\lambda)$.

For the proof see the Appendix.

Proposition 2 is illustrated by Figure 3. The conditions for Proposition 2(a), 2(b), and 2(c) are satisfied in Region A, B, and C, respectively. The outer limit of Region A is given by $Rf'(K(\lambda)) = r$, and the border between Regions B and C are given by $f(\lambda R/r) = 1$. These two downward-sloping curves meet tangentially at $\lambda = \lambda_c$.

Proposition 2 states that the dynamics of capital formation in the small open economy differ drastically from the autarky case. The difference is most significant when the world interest rate is such that the parameters lie in Region B, as illustrated by point P in Figure 3. In this case, an integration of this economy in the international financial market creates multiple steady states, as shown in Figure 2(b). Around k_M, investment is borrowing constrained, and the dynamics is unstable. If the integration occurs slightly below k_M, the economy experiences vicious circles of low-wealth/low-investment, and will gravitate toward the lower stable steady state, k_L, in which the borrowing constraint is binding. On the other hand, if the integration takes place slightly above k_M, the economy experiences virtuous circles of high-wealth/high-investment, and

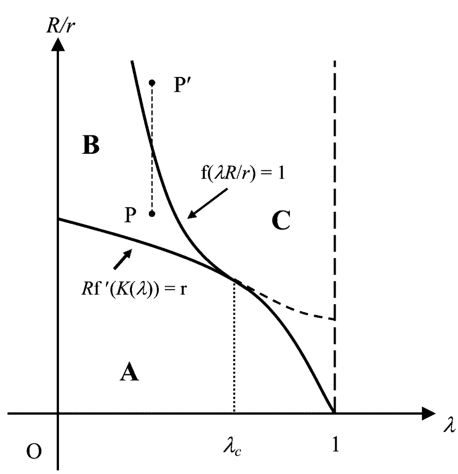

FIGURE 3.—Parameter configuration—the small open economy case.

eventually converges to the higher stable steady state, k_H, in which the borrowing constraint is no longer binding. This case thus suggests that the timing of the integration has significant permanent effects on capital formation.

This does not mean, however, that the integration would have negligible effects on capital formation in other cases. For example, suppose that the world interest rate is such that the parameters lie in Region C. In this case, the economy will eventually converge to the unique steady state, in which the borrowing constraint is not binding. This process could take a long time, however, because the economy must go through the "narrow corridor" between the map and the 45° line, as illustrated in Figure 2(c). More generally, as a comparison between the shapes of the two maps, $k_{t+1} = RW(k_t)$ and $k_{t+1} = \Psi(k_t)$, suggests, the integration would slow down the growth process of middle-income economies.

Let us now consider the effect of a change in the world interest rate on the capital formation of the small open economy. We focus on the case where the parameters lie in Region B, depicted by P in Figure 3, and the dynamics is hence illustrated by Figure 2(b). Suppose that the economy is trapped in k_L. A decline in the world interest rate, illustrated in Figure 3 as the vertical move from point P in Region B to point P′ in Region C eliminates k_L and the dynamics is now illustrated by Figure 2(c). The decline in the interest rate thus helps the economy to escape from the trap and to start a (perhaps long and slow) process of growth toward k_H. Furthermore, even a temporary decline in the interest rate could have similar steady state effects. Once the economy accumulates enough capital, the economy will not fall back to the trap, when the interest rate returns to the original level. Therefore, even a small, temporary decline in the interest rate could have a significant permanent effect.[18] Similarly, one could show that even a small, temporary rise in the world interest rate could lead to a permanent stagnation of the economy, if it is initially located at k_H in Figure 2(b).

One might be tempted to argue that Region B of Figure 3, which gives rise to the dynamics illustrated in Figure 2(b) with multiple stable steady states, can be used to explain endogenous inequality of nations. Imagine that there are two small open countries, called N and S, which share the same technology, the same demographic structure, etc. Furthermore, both countries are fully integrated into the international financial market and face the same world interest rate. The only difference is that the capital stock in N is equal to k_H and the capital stock in S is equal to k_L. The model does explain why this situation can persist, because both k_H and k_L are stable steady states of the dynamics, if the parameters lie in Region B of Figure 3.

While suggestive, this argument explains why it is *possible* that two otherwise identical countries perform differently, but does not say that it is *inevitable*.

[18]Of course, how small the decline can be in order to have the permanent effect depends on the distance between point P and the border between Regions B and C. Furthermore, the larger the decline, the shorter it can be to have the permanent effect.

Indeed, the situation in which the capital stocks are both equal to k_H in N and S and the situation in which they are both equal to k_L in N and S (as well as the situation in which it is equal to k_H in S and k_L in N) are also stable steady states under the same condition. The argument does not offer any reason why one should believe that the separation of the world economy into the rich and the poor is more plausible. In other words, the small open economy version of the model cannot impose any restriction on the equilibrium degree of inequality, because it takes into account no interaction between the dynamics of different countries.

To resolve this problem, therefore, one must move beyond the small open economy framework, and analyze the model from a global perspective. In the next section, the world economy version of the model is analyzed. This helps not only to endogenize the world interest rate, but also to address the issue of endogenous inequality in a more satisfactory manner.

Analyzing the model from a global perspective is also important for the policy analysis. From the perspective of an individual country, escaping from the poverty trap may appear simple. One might be tempted to argue that poor countries should temporarily cut their financial links or that foreign aid from rich countries should solve the problem. The global perspective will show, however, why these measures may not be able to eliminate the poverty trap.

6. THE WORLD ECONOMY

The world economy is made up of a continuum of inherently identical countries with unit mass. In the absence of the international financial market, this is merely a collection of autarky economies analyzed in Section 4. Hence one can immediately conclude that the world economy would converge to the symmetric steady state, in which each country holds $K^*(R)$ units of capital stock. In short, the world economy has a unique steady state, which is symmetric and globally stable.

In what follows, let us assume that all the countries are fully integrated in the international financial market, where each country faces the same interest rate. The world economy can hence be viewed as a collection of inherently identical small open economies of the type analyzed in Section 5. Since the world as a whole is a closed economy, the interest rate is now endogenously determined to equate world saving and world investment.

The presence of the international financial market does not change the fact that the state in which every country has capital stock equal to $K^*(R)$ is a steady state. However, it may change the stability property of the symmetric steady state. Furthermore, it may create stable steady states, which are not symmetric. We need to characterize the *entire* set of *stable* steady states of the world economy.

In any stable steady state of the world economy, each country must be at a stable steady state of the small open economy. As stated in the Lemma, there are at most two stable steady states in which each small open economy can be located. This means that a stable steady state of the world economy must be one of the following two types. The first type is the case of perfect equality. In such a steady state, all the countries have the same level of capital, k^*. The second type is the case of endogenous inequality. In such a steady state, the world economy is polarized into the rich and the poor, in which the poor (rich) countries have the same level of capital stock, given by k_L (k_H), which satisfies $k_L < K(\lambda) < k_H$. The next two subsections derive the condition for the existence of these two types of stable steady states. (The reader not interested in the derivation may want to skim through these sections and move onto Section 6.3, at least on the first reading.)

6.1. *The Symmetric Steady State*

Suppose that all countries have the same level of capital stock, k^*, in a steady state. Then, world saving is equal to $W(k^*)$. Since the world economy as a whole is closed, the measure of the young agents that invest in this steady state must be equal to $W(k^*)$. Since every one of them produces R units of capital, the steady state capital must satisfy $k^* = RW(k^*)$, or equivalently, $k^* = K^*(R)$. If $k^* = K^*(R) > K(\lambda)$, the borrowing constraint is not binding, hence the world interest rate in this steady state is $r = Rf'(K^*(R)) < Rf'(K(\lambda))$. This inequality can be rewritten as $\Phi(r/R) > K(\lambda)$, which is exactly the condition under which a small open economy has a stable steady state, $k_H = \Phi(r/R) = K^*(R) = k^*$. (See also Proposition 2(b) and (c).) This proves that $K^*(R) > K(\lambda)$ is the condition under which there exists a stable steady state in which all countries have the same level of capital stock, $k^* = K^*(R) > K(\lambda)$.

If $k^* = K^*(R) < K(\lambda)$, the borrowing constraint is binding, hence the world interest rate in this steady state is $r = \lambda Rf'(K^*(R))/[1 - W(K^*(R))]$. From (c) of the Lemma, $k^* = K^*(R) < K(\lambda)$ is a stable steady state for each small open economy, if and only if it satisfies $k^* = K^*(R) < \lambda R/r = [1 - W(K^*(R))]/f'(K^*(R))$. This condition can be rewritten to $K^*(R)f'(K^*(R)) + W(K^*(R)) = f(K^*(R)) < 1$. This proves that $K^*(R) < K(\lambda)$ and $f(K^*(R)) < 1$ are the conditions under which there exists a stable steady state in which all countries have the same level of capital stock, $k^* = K^*(R) < K(\lambda)$.

The above argument also shows that, if $K^*(R) < K(\lambda)$ and $f(K^*(R)) > 1$, a symmetric steady state, in which all countries have the same level of capital stock, is unstable. To see this, in such a steady state, the capital stock in each country must be equal to $k^* = K^*(R) < K(\lambda)$, which means that the borrowing constraint is binding. Therefore, the world interest rate is equal to $r = \lambda Rf'(K^*(R))/[1 - W(K^*(R))]$. When $f(K^*(R)) > 1$, this implies $k^* = K^*(R) > \lambda R/r$, which means that $k^* = k_M$ from Lemma part (c). Thus, it is unstable. Figure 4 illustrates this situation. Suppose that there is no international financial market at the beginning. Then, the dynamics of every country follows $k_{t+1} = RW(k_t)$, which converges to $K^*(R)$. In this steady state, the interest rates are equal across countries, even though there is no international

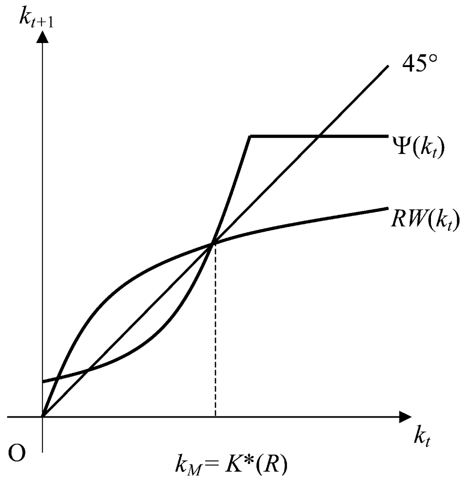

FIGURE 4.—The instability of the symmetric steady state under $K^*(R_c) < K^*(R) < K(\lambda)$.

lending and borrowing. If the international financial market is opened at this point, the dynamics of each country is now governed by $k_{t+1} = \Psi(k_t)$, which cuts the 45° line from below at $K^*(R)$. This situation is unstable, even though it is still a steady state.

Proposition 3 summarizes the above.

PROPOSITION 3: *Let* $R_c \in (0, R^+)$ *be defined by* $f(K^*(R_c)) = 1$. *Then*:

(a) *If* $K^*(R) < K(\lambda)$ *and* $R < R_c$, *the state in which all countries have* $k^* = K^*(R)$, *is a stable steady state of the world economy*.

(b) *If* $K^*(R) < K(\lambda)$ *and* $R > R_c$, *there exists no stable steady state in which all the countries have the same level of capital stock*.

(c) *If* $K^*(R) > K(\lambda)$, *the state in which all countries have* $k^* = K^*(R)$, *is a stable steady state of the world economy*.

Note R_c satisfies $K^*(R_c) = K(\lambda_c)$; it is well defined in $(0, R^+)$, since $f(K^*(0)) = 0 < 1 = W(R^+) < f(K^*(R^+))$ and $f(K^*(R))$ is strictly increasing and continuous in R.

Figure 5 illustrates the conditions in Proposition 3. In Regions A and AB, the condition in Proposition 3(a) is satisfied. In Region B, the condition in Proposition 3(b) is satisfied. In Regions BC and C, the condition in Proposition 3(c) is satisfied. The border between Regions AB and B is given by $f(K^*(R)) = 1$, i.e., $R = R_c$. The border between Regions B and BC (as well as the border between A and C) is given by $K^*(R) = K(\lambda)$. Note that, when the credit market imperfection is significant ($\lambda < \lambda_c$), the stability of the symmetric steady state requires that the productivity of the investment project, R, be either sufficiently high or sufficiently low. For an intermediate range of R, the condition in Proposition 3(b) holds and the symmetric steady state is unstable.

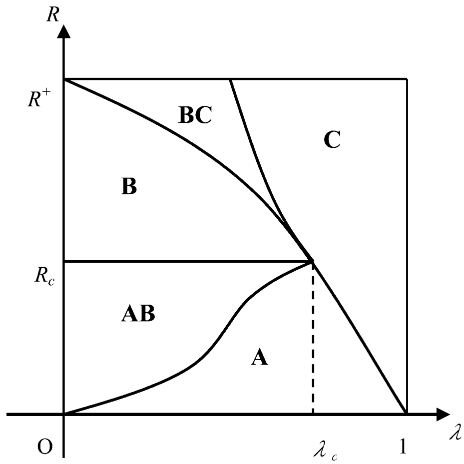

FIGURE 5.—Parameter configuration—the world economy case.

6.2. *The Asymmetric Steady States*

Suppose now that the world economy is a stable steady state, in which a fraction X of the countries have the capital stock equal to $k_L < K(\lambda)$, and a fraction $1-X$ of the countries have the capital stock equal to $k_H > K(\lambda)$. Since all the countries face the same world interest rate, k_L and k_H must satisfy $Rf'(k_H) = r = \lambda Rf'(k_L)/(1-W(k_L))$, or

$$f'(k_H) = \lambda f'(k_L)/(1-W(k_L)), \tag{7}$$

in addition to

$$k_L < K(\lambda) < k_H. \tag{8}$$

From Lemma part (b), $k_t = k_H$ is a stable steady state for each small open economy. From Lemma part (c), the stability of $k_t = k_L$ requires $k_L < \lambda R/r = [1-W(k_L)]/f'(k_L)$, which can be rewritten to $k_L f'(k_L) + W(k_L) = f(k_L) < 1$, or

$$k_L < K^*(R_c) = K(\lambda_c). \tag{9}$$

Since young agents in the fraction X of the countries earn $W(k_L)$ and those in the fraction $1-X$ earn $W(k_H)$, world saving is given by $XW(k_L) + (1-X)W(k_H)$, which is equal to world investment, which produces R units of capital per unit. Hence, total capital stock must satisfy

$$Xk_L + (1-X)k_H = XRW(k_L) + (1-X)RW(k_H). \tag{10}$$

A stable steady state with endogenous inequality exists if there are k_L and k_H that solve (7)–(10).

PROPOSITION 4: *Let* $R_c \in (0, R^+)$ *and* $\lambda_c \in (0,1)$ *be defined by* $f(K^*(R_c)) = f(K(\lambda_c)) = 1$. *The world economy has a continuum of stable steady states, in which a fraction* $X \in (X^-, X^+) \subset (0,1)$ *of the countries have capital stock,* $k_L < K(\lambda)$, *and a fraction* $1 - X$ *of the countries have capital stock equal to* $k_H > K(\lambda)$, *if and only if* $\lambda < \lambda_c$, $f'(K(\lambda)) > \lambda f'(K^*(R))/[1 - W(K^*(R))]$, *where* $R < R_c$, *and* $\lambda < f'(K^*(R))K(\lambda_c)$. *Furthermore,* $X^- > 0$ *if* $R > R_c$ *and* $X^+ < 1$ *if* $K^*(R) < K(\lambda)$.

For the proof see the Appendix.

The condition of Proposition 4 is satisfied in Regions AB, B, and BC of Figure 5. The border between A and AB is given by $f'(K(\lambda)) = \lambda f'(K^*(R))/[1 - W(K^*(R))]$ with $R < R_c$ and $\lambda < \lambda_c$. It is upward-sloping and connects $(\lambda, R) = (0, 0)$ and $(\lambda, R) = (\lambda_c, R_c)$. The border between BC and C is given by $f'(K^*(R))K(\lambda_c) = \lambda$. This curve is downward-sloping, and stays above $K^*(R) = K(\lambda)$ for $\lambda < \lambda_c$, and tangent to it at $(\lambda, R) = (\lambda_c, R_c)$.[19] Note that the existence of these asymmetric steady states requires that the credit market imperfection be significant ($\lambda < \lambda_c$), and that the productivity of the investment project, R, be neither too low nor too high.[20]

6.3. *The Effects of Financial Market Globalization: Discussion*

Having characterized all the stable steady states, we are now ready to discuss the effects of financial market globalization. In Regions A and C of Figure 5, there is a unique stable steady state, which is symmetric. In both cases, the model predicts no endogenous inequality across countries. In Region A, the investment is borrowing-constrained in each country; that is, all the countries are equally poor. In Region C, the borrowing constraint is not binding in any country; that is, all countries are equally rich. In Region B, there is no stable symmetric steady state. Even though there is a continuum of stable steady states, they all show that the long-run distribution of capital stock, and hence the distribution of income, wages, investment rates, etc., have two mass points. In other words, the model predicts symmetry-breaking in Region B, where the co-existence of rich and poor nations is an inevitable feature of the world economy. In Region AB, and Region BC, these two types of steady states co-exist.

[19]To see this, let $\Theta(\lambda) \equiv f'(K(\lambda))K(\lambda_c) - \lambda$. Then, $\Theta(\lambda_c) = f'(K(\lambda_c))K(\lambda_c) - \lambda_c = f'(K(\lambda_c))K(\lambda_c) - f(K(\lambda_c)) + (1 - \lambda_c) = (1 - \lambda_c) - W(K(\lambda_c)) = 0$, and $\Theta'(\lambda) \equiv f''(K(\lambda))K(\lambda_c)K'(\lambda) - 1 = K(\lambda_c)/K(\lambda) - 1 < 0$ for $\lambda < \lambda_c$, since $K'(\lambda) = 1/f''(K(\lambda))K(\lambda)$ by differentiating $W(K(\lambda)) = 1 - \lambda$. Therefore, $\Theta(\lambda) > \Theta(\lambda_c) = 0$ for $\lambda < \lambda_c$. Thus, $\lambda = f'(K^*(R))K(\lambda_c)$ implies $f'(K(\lambda))K(\lambda_c) > \lambda = f'(K^*(R))K(\lambda_c)$ or $K^*(R) > K(\lambda)$ for $\lambda < \lambda_c$. The tangency follows from $\Theta'(\lambda_c) = 0$.

[20]If we drop (A1) and allow R to be greater than R^+, the border between BC and C extends above R^+. Hence, these asymmetric steady states disappear not only when R is sufficiently low, but also when it is sufficiently high.

The prediction of the model is most stark when the parameters lie in Region B of Figure 5, the case of symmetry-breaking. See also Figure 4. In this case, $K^*(R) < K(\lambda)$ so that, in the absence of the international financial market, each country is in autarky and will converge to the same steady state, in which the borrowing constraint is binding. Despite that each country is borrowing-constrained, this symmetric steady state is stable. This is because the interest rates can adjust independently across countries, when different countries are hit by different shocks. In the presence of the international financial market, however, the symmetric steady state loses its stability. This is because integration forces interest rates in different countries to move together. In other words, all the agents must compete for the world saving in the international financial market; they all have to guarantee the same return, regardless of their locations. This puts the agents living in countries hit by worse shocks at a disadvantage compared to those living in countries hit by better shocks. This creates vicious circles of low-investment/low-wealth in the unlucky countries and virtuous circles of high-investment/high-wealth in the lucky countries. Only the asymmetric steady states are stable in Region B. That is to say, in any stable steady state, the world economy is polarized into the rich and the poor. This case thus captures the structualist view that the international financial market magnifies the inequality of nations and that a separation of the world economy into the rich and the poor is an inevitable feature of the International Economic Order. The rich accumulate enough capital that the borrowing constraint is no longer binding, while it is binding for the poor ($k_L < K(\lambda) < k_H$). One can also show that, from (A2) and (10), $k_L < K^*(R) < k_H$ in these steady states. That is to say, the rich countries become richer and the poor become poorer than in autarky. Furthermore, the world output in these steady states is strictly lower than in the symmetric steady state.[21] Therefore, this case offers theoretical support for the popular view that the international financial market is a mechanism through which rich countries become richer at the expense of poor countries and at the expense of the world economy as a whole.

When the world economy is polarized, the countries that became poor find themselves in the stable steady state with the binding borrowing constraint, k_L in Figure 2(b). From a perspective of an individual country, the problems of poor countries may seem easy to solve. It may appear that, in order to escape the poverty trap and to join the club of rich countries, all the government has to do is to cut its link to the international financial market temporarily. The global perspective, however, offers a different view. Such temporary isolationist policy cannot work when attempted by all countries. This is because, once

[21]To see this, consider the problem of maximizing the steady state world output subject to the steady state resource constraint: Maximize $\int_0^1 f(k(z))\,dz$, s.t. $\int_0^1 k(z)\,dz \le \int_0^1 RW(k(z))\,dz$, where $k(z)$ is the capital stock in country $z \in [0, 1]$. Since the feasibility set is convex and the objective function is symmetric and strictly quasi-concave, the solution is $k(z) = k^*$ for all $z \in [0, 1]$, where k^* satisfies $k^* = RW(k^*)$. That is, the world output is maximized when $k(z) = K^*(R)$ for all $z \in [0, 1]$.

the restriction is removed, a positive measure of countries must find themselves in the lower steady state. (Note that, in Region B, a fraction of the countries that become poor is bounded away from zero.) Similar points can be made for a joint attempt for the poor countries to cut their links to the rich countries and to unite among themselves to form a bloc. It is impossible for all of them to escape from the poverty trap because the same analysis would apply to the bloc newly formed. Nor would a one-time redistribution from the rich countries eliminate inequality. This is because $K^*(R) < K(\lambda)$ in Region B. That is, one of the reasons why the symmetric steady state is unstable is that there is not enough saving in the world economy to finance the investment required to make all the countries rich. As long as the parameters lie in Region B of Figure 5, some countries must be excluded from being rich, just as in a game of musical chairs.[22]

6.4. *The Effect of Technological Progress: An Application*

Throughout the discussion above, we have taken the integration of the financial markets as the sole exogenous change in the world economy, by keeping R fixed. Alternatively, one could examine the effects of an exogenous change in R, while taking the integration of the financial markets as given. If such a change passes the border of Region B, then the world economy experiences a symmetry-breaking bifurcation. For example, consider the following thought experiment, which arguably traces the evolution of the world economy. Suppose $\lambda < \lambda_c$ and R is sufficiently small so that the parameters lie in Region A. Then, let R increase gradually. Imagine that this exogenous technological progress is sufficiently slow that one could approximate the state of the world economy by a stable steady state. Initially, the world economy is in A, so that all the countries are equally poor and the borrowing constraint is binding in each country. Even when an increase in R pushes the world economy into Region AB, this situation does not change, because the symmetric steady state remains stable. This changes when a further increase in R makes $R > R_c$ and the world economy enters Region B. Then, the symmetry is broken and endogenous inequality begins to appear. Some, but not all, countries start growing rapidly. These countries become sufficiently rich and the borrowing constraint is no longer binding. The rest of the world is left behind. As R continues to rise, more and more countries start growing and catch up with the rich. Once R becomes big enough to push the world economy into Region C, then the catching up process is completed and symmetry

[22]When the parameters lie in Regions AB or BC, the world economy may find itself in an asymmetric steady state, in which case a one-time redistribution from the rich to the poor could eliminate inequality and move the world economy into the symmetric steady state.

is restored.[23] According to this thought experiment, the world economy experiences divergence first, and then convergence, a Kuznets inverted U-curve, because the endogenous components of inequality change as the parameter moves in the symmetry-breaking region. It should also be noted that this thought experiment suggests that the symmetry-breaking and the presence of stable asymmetric steady states are perfectly consistent with the evidence of "convergence" in the long run evolution of the world income distribution.

7. ALTERNATIVE SPECIFICATIONS

In the above model, many assumptions are made in order to simplify the analysis, to minimize the numbers of parameters, and to avoid distracting the reader's attention away from the main goal of the paper. Some of the results obviously depend on these simplifying assumptions, but the key result of the model, that financial market globalization may cause symmetry-breaking, is robust to many alternative specifications. To understand the robustness, note that symmetry-breaking occurs due to the following features of the model:

(i) For a fixed domestic interest rate, the domestic investment is an increasing function of the wealth held by the domestic entrepreneurs in the lower range.

(ii) Domestic investment increases the wealth held by domestic entrepreneurs (more than that of foreign entrepreneurs).

(iii) The domestic interest rate adjusts to balance domestic supply and domestic demand for credit in the absence of the international financial market, while it is linked to the foreign interest rate in the presence of the international financial market.

As long as these features of the model are maintained, alternative specifications would not eliminate the key result, although they would considerably complicate the analysis. This section gives brief sketches of how the analysis needs to be modified when alternative specifications are used.

7.1. *Heterogeneous Agents and Wealth Inequality Within Each Country*

The basic model assumes that agents are homogeneous. They are equally productive as entrepreneurs. Their labor endowments are identical, which means that there is no wealth inequality across young agents within each country. The latter, in particular, may lead one to conjecture that the symmetry-breaking case would disappear if there were enough wealth inequality within each country to allow for the possibility that some young agents in the

[23]Figure 5 seems to suggest that symmetry could not be restored for a small λ. However, if we drop (A1) and let R be greater than R^+, then a sufficiently large R pushes the world economy into Region C for any $\lambda < \lambda_c$.

poor countries may be rich. This section shows that such a conjecture is false, by extending the model to allow the agents to differ in their endowment.[24]

Let $G(z)$ denote the cumulative distribution of the labor endowment of young agents, z, with its density function, $g(z) = G'(z) > 0$, and with its mean being equal to one. Thus, $G(z)$ presents the fraction of the agents whose wealth is less than zw_t at the end of period t. In autarky, the domestic interest rate adjusts so as to make domestic investment determined by domestic saving. The investment is made by the $W(k_t)$ richest young agents, i.e., the agents with $z \geq G^{-1}(1 - W(k_t))$, and (4) continues to govern the dynamics in autarky, regardless of whether the borrowing constraint is binding or not. Consider now the small open economy case. If $Rf'(k_{t+1}) > r$, all the young agents are willing to invest, but only those agents who are rich enough to satisfy the borrowing constraint,

$$(2') \qquad \lambda Rf'(k_{t+1}) \geq r_{t+1}(1 - zW(k_t)),$$

can borrow and invest. Thus, domestic investment is equal to $1 - G([1 - \lambda Rf'(k_{t+1})/r]/W(k_t))$. Thus, k_{t+1} is given by the unique solution of $k_{t+1} = R[1 - G([1 - \lambda Rf'(k_{t+1})/r]/W(k_t))]$, as long as it satisfies $Rf'(k_{t+1}) > r$. By denoting this unique solution by $\Psi(k_t; \lambda, R, r)$, the dynamics of the small open economy can be expressed by

$$(6') \qquad k_{t+1} = \begin{cases} \Psi(k_t; \lambda, R, r) & \text{if } k_t < K(\lambda, R, r), \\ \Phi(r/R) & \text{if } k_t \geq K(\lambda, R, r), \end{cases}$$

where $K(\lambda, R, r)$ is defined uniquely by the K that solves $\Phi(r/R) = R[1 - G((1 - \lambda)/W(K))]$. It is easy to verify that (6) is a limit case of (6′), as $G(z) \to 0$ for $z < 1$ and $G(z) \to 1$ for $z \geq 1$. Note that (6′) has many of the key features of (6). For $k_t < K(\lambda, R, r)$, $Rf'(k_{t+1}) > r$, so that the profitability constraint is not binding. What determines domestic investment is the borrowing constraint, which is binding for the marginal agent, i.e., the agent with $z = [1 - \lambda Rf'(k_{t+1})/r]/W(k_t)$. In this range, the map is increasing in k_t, R, and λ/r, because an increase in these variables allows the agent with lower endowments to satisfy the borrowing constraint. For $k_t \geq K(\lambda, R, r)$, $Rf'(k_{t+1}) = r$, so that the profitability constraint determines domestic investment. In this range, the map is flat. Note that these key features of the map (6′) would not disappear even if there were a few, very rich young agents in each country (i.e., even if G has a thin, but long upper tail). There are two notable differences between (6) and (6′). First, the threshold level of k_t below which the borrowing constraint determines the domestic investment is no longer independent of R or r.

[24]Matsuyama (2001, Section 6) discusses an extension in which the agents differ in productivity, R.

Second, the map (6′) may have more than one stable intersection with the 45° line below $K(\lambda, R, r)$.

In the world economy case, it is straightforward to show that stable asymmetric steady states exist whenever the symmetric steady state is unstable. Thus, the condition for symmetry-breaking can be derived by finding the condition under which the slope of the map (6′) is less than one when evaluated at the symmetric steady state, where $k_t = K^*(R)$ and $r = r^*$, where r^* is the unique solution to $W(K^*(R)) = 1 - G([1 - \lambda R f'(K^*(R))/r^*]/W(K^*(R)))$. A complete characterization of asymmetric stable states is hopelessly complicated. This is because there may be more than two stable steady states of the small open economy, which dramatically increases the number of types of steady states for the world economy. If there are m stable steady states for the small open economy, $2^m - 1 - m$ different types of the stable asymmetric steady states for the world economy need to be distinguished, and only $m!/(m-2)!2!$ of them are characterized by a two-point distribution.

7.2. *Allowing Agents to Run More than One Project*

It has been assumed so far that the young agent can run at most one, indivisible investment project. That is, the project technology of each young agent may be written as $y(i) = 0$ for $0 \leq i < 1$ and $y(i) = R$ for $i \geq 1$. The assumption that each young agent runs at most one project is reasonable, when capital is interpreted as human capital and the project is interpreted as education, such as going to college. Nevertheless, one might think that the result in the previous section may depend critically on this assumption. One's intuition might say that symmetry-breaking would not happen if a few rich agents in the poor countries were allowed to run as many projects as they want. If so, one's intuition is faulty. If the rich agents in the poor countries were allowed to run multiple projects, they would expand their operations until their borrowing constraint would become binding. Therefore, at the margin, domestic investment is still constrained by domestic wealth in the poor countries.

To see this formally, let us now assume that the project technology of each young agent is given by $y(i) = 0$ for $0 \leq i < 1$ and $y(i) = Ri$ for $i \geq 1$. The agent is still subject to the minimum investment requirement of one, but once this requirement is satisfied, the project technology generates physical capital at the rate equal to R per unit of investment. If the young agent runs the project at the scale, $i_t \geq 1$, the project revenue is $Ri_t f'(k_{t+1})$, only λ fraction of which is pledgeable to the creditor. Thus, the borrowing constraint of each agent with z units of endowment can be written as

$$\lambda R i_t f'(k_{t+1}) \geq r_{t+1}(i_t - zW(k_t)), \quad i_t \geq 1. \tag{2''}$$

In autarky, the domestic interest rate still adjusts so as to make domestic investment equal to domestic saving, $W(k_t)$. Thus, (4) continues to govern the dynamics in autarky. Consider now the small open economy case.

If $Rf'(k_{t+1}) > r$, the profitability constraint is not binding. All the young agents are willing to invest as much as possible, which means that they invest until they all face the binding borrowing constraint. In other words, (2′) holds with equality as long as $i_t \geq 1$. The investment by a young agent with z is thus equal to $i_t(z) = 0$ for $z < z_t$; and $i_t(z) = z/z_t$ for $z \geq z_t$, for $z_t \equiv [1 - \lambda Rf'(k_{t+1})/r]/W(k_t)$. The aggregate domestic investment is hence equal to $H(z_t) \equiv \int_{z_t}^{\infty} (z/z_t)\, dG(z)$. Thus, k_{t+1} is given by the unique solution of $k_{t+1} = RH(z_t)$, where $z_t \equiv [1 - \lambda Rf'(k_{t+1})/r]/W(k_t)$, as long as it satisfies $Rf'(k_{t+1}) > r$. By denoting this unique solution by $\Psi(k_t; \lambda, R, r)$, the dynamics of the small open economy can be expressed by

$$(6'') \qquad k_{t+1} = \begin{cases} \Psi(k_t; \lambda, R, r) & \text{if } k_t < K(\lambda, R, r), \\ \Phi(r/R) & \text{if } k_t \geq K(\lambda, R, r), \end{cases}$$

where $K(\lambda, R, r)$ is now defined uniquely by the K that solves $\Phi(r/R) = RH((1 - \lambda)/W(K))$.

Again, (6″) shares many common features with (6) and with (6′). For $k_t < K(\lambda, R, r)$, $Rf'(k_{t+1}) > r$, so that the profitability constraint is not binding. What determines domestic investment is the borrowing constraint, which is binding for all agents. In this range, the map is increasing in k_t, R, and λ/r. For $k_t \geq K(\lambda, R, r)$, $Rf'(k_{t+1}) = r$, so that the profitability constraint determines domestic investment. In this range, the map is flat. As in (6′), but unlike (6), the threshold level of k_t below which the borrowing constraint determines domestic investment depends not only on λ but also on R and r, and the map (6″) may have more than one stable intersection with the 45° line below $K(\lambda, R, r)$.

As in Section 7.1, the condition for the symmetry-breaking in the world economy is equivalent to the condition under which the slope of the map (6″) is less than one when evaluated at the symmetric steady state, that is, at $k_t = K^*(R)$ and $r = r^*$, where r^* is now given by the unique solution to $K^*(R) = RH(z^*)$, where $z^* = [1 - \lambda Rf'(K(R))/r^*]/W(K^*(R))$. As in Section 7.1, a complete characterization of asymmetric stable states is hopelessly complicated, because there may be more than two stable steady states of the small open economy, which dramatically increases the number of types of steady states for the world economy.

7.3. *Factor Market Integration*

In this paper, it is assumed that physical capital is nontradeable, and that there is no foreign direct investment (i.e., an agent can start an investment project only in his/her own country). What is essential is the presence of a home bias in the investment demand spillovers. That is to say, a higher aggregate investment by the agents from one country increases the wealth of the agents from the same country more than that of the agents from other countries. Such a home bias creates a larger credit multiplier within the same country than across countries. As long as some impediments to factor movements exist, a home bias arises naturally. Even if factor movements are completely free, a home bias may still exist. For example, the investment project run by an agent from one country may create more demand for the endowment held by the younger agents from the same country than the endowment held by the others, because of the differences in languages, business cultures, etc. In such a setting, a mechanism similar to those discussed above could cause symmetry-breaking, even if all the factors and all the endowments are costlessly tradeable. Such alternative specifications, however, complicate the analysis substantially, because a three-step analysis of autarky, small open economy, and the world economy cases is possible only when domestic investment does not change the value of the endowment abroad.

Needless to say, a larger home bias would make symmetry-breaking more likely.[25] Thus, one important implication of the symmetry-breaking mechanism based on credit market imperfection is that the effects of globalization differ depending on whether it takes place in financial markets or in factor markets.

8. CONCLUDING REMARKS

Globalization is a highly divisive issue and its proponents and opponents hardly communicate with each other.[26] Globalization is also a multifaceted process. The aim of this paper is modest and limited. It addresses only one aspect of globalization, the integration of financial markets, and attempts to reconcile the seemingly contradictory views of the world by building a simple theoretical framework. The model is based on the standard neoclassical overlapping generations model, modified only to incorporate credit market imperfection. Within this framework, the necessary and sufficient condition for symmetry-breaking was derived, i.e., the condition under which financial market globalization magnifies the inequality of nations. This enabled us to put some of the arguments made by the opponents of financial market globalization under logical scrutiny. One major advantage of the model presented here is its tractability. It may be modified to address many issues in macroeconomics of credit market imperfection. See, for example, Matsuyama (2002b).

Some limitations of the above analysis should be pointed out. First, the effects of financial market globalization were examined by comparing the two extreme cases, autarky and full financial market integration. It would be more satisfactory to introduce some parameters (say, financial transaction costs, the Tobin tax, etc.) that may be interpreted as a measure of financial market globalization. Second, the model assumes that globalization has no effect on the

[25] In the limit, where all the home biases disappear completely, symmetry-breaking cannot occur. However, in such a perfectly integrated, frictionless world, the very notion of the "country" would lose its meaning.

[26] See, for example, Dollar and Kraay (2002) and Scott (2001), two recent articles in *Foreign Affairs* and a large number of comments published in subsequent issues.

degree of credit market imperfection. This assumption may be justified as a benchmark case, because it is not obvious in which direction globalization might affect the operation of credit markets.[27] Yet, the reader should keep in mind that the results of this paper are conditional on this assumption. Third, the model does not allow for sustainable growth of the world economy as a whole. It would be interesting to examine the condition under which endogenous inequality of nations occurs in a growing global economy. This would require the model to be extended in such a way that the minimum investment requirement for the project would increase with the growth of the world economy. Fourth, the model has only one type of capital good and one final goods industry. In a model with many capital goods or final goods industries, which differ in the minimum investment requirements or in the degree of credit market imperfection, poor countries may find comparative advantages in the sectors with less stringent borrowing constraints. With trade in final goods, a change in the terms of trade may amplify or mitigate the mechanism identified in this paper. It would be interesting to investigate how financial market globalization affects the interactive process between cross-country patterns of development and the industrial structures of the economies.

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APPENDIX

Proof of the Lemma

Part (a) of the Lemma follows from Brouwer's fixed point theorem and because Ψ is a continuous map on $[0, R]$ into itself. Part (b) follows from the fact that the map, Ψ, is constant above $K(\lambda)$ and equal to $\Phi(r/R)$. To prove part (c), first differentiate (6), which yields $\Psi'(k_t) = k_t[f''(k_t)/f''(\Psi(k_t))](r/\lambda R)$ for $k_t < K(\lambda)$. By setting $k_t = \Psi(k_t) = k$, the slope of the map at a steady state, $k < K(\lambda)$, is equal to $\Psi'(k) = k(r/\lambda R)$, which is increasing in k. Also, $\Psi(0) = \Phi(r/\lambda R) > 0$. Therefore, at the smallest steady state, $0 < k_L < K(\lambda)$, if there is one, either Ψ is tangent to the 45° line (i.e., $\Psi'(k_L) = k_L(r/\lambda R) = 1$ or $k_L = \lambda R/r$), in which case it is the only intersection below $K(\lambda)$, or Ψ cuts the 45° line from above (i.e., $\Psi'(k_L) = k_L(r/\lambda R) < 1$ or $k_L < \lambda R/r$), in which case it is stable. At the second smallest steady state, k_M, if it exists, Ψ cuts the 45° line from below (i.e., $\Psi'(k_M) = k_M(r/\lambda R) > 1$, or $k_M > \lambda R/r$) and hence it is unstable,

[27]On one hand, one might argue that the lower the cost of international financial transactions, the easier it would be for borrowers to take the money and run, and the harder it would be for lenders to catch those who defaulted. If so, globalization has the effect of reducing the efficiency of credit markets. On the other hand, one might also argue that the globalization and resulting competition for the world saving provide a greater incentive for an individual country to improve its corporate governance. If so, globalization may have the effect of enhancing the efficiency of credit markets. See Ando and Yanagawa (2002), who extended a small country version of the present model to allow the local government to choose λ. See, however, Tirole (2002b, Section 3.2, Application #4), who argues that the government's ability to choose λ ex post could undermine the credit-worthiness of domestic borrowers.

which also implies that Ψ cannot cut the 45° line from above between k_M and $K(\lambda)$, ruling out the existence of a third steady state below $K(\lambda)$. This completes the proof of part (c) and the Lemma.

Proof of Proposition 2

The proof consists of four steps.

Step 1. Since $f(K(\lambda))$ is strictly decreasing and continuous in λ and $f(K(1)) = f(0) = 0 < 1 = W(R^+) < f(R^+) = f(K(0))$, $\lambda_c \in (0, 1)$ is well defined and $f(K(\lambda)) > (<)\ 1$ if and only if $\lambda < (>)\ \lambda_c$.

Step 2. Consider the nongeneric case of $Rf'(K(\lambda)) = r$. Then, $K(\lambda) = \Phi(r/R)$ and hence $K(\lambda)$ is a fixed point of the map, Ψ. Because $f(K(\lambda)) - 1 = K(\lambda)f'(K(\lambda)) + W(K(\lambda)) - 1 = K(\lambda)r/R - \lambda = \lambda[\lim_{k\uparrow K(\lambda)} \Psi'(k) - 1]$, the left derivative of the map at $K(\lambda)$ is greater (less) than one if and only if $f(K(\lambda)) > (<)\ 1$ or $\lambda < (>)\ \lambda_c$. These properties are illustrated in Figure A.1 for $\lambda < \lambda_c$ and Figure A.2 for $\lambda \geq \lambda_c$. Note that, from the Lemma, Ψ has another intersection, $0 < k_L < K(\lambda)$, in Figure A.1, and has no other intersection in Figure A.2.

Step 3. Consider the case where $Rf'(K(\lambda)) < r$. This case can be studied by reducing R, starting from the case, $Rf'(K(\lambda)) = r$, while fixing λ and r. This change is captured by a downward shift of the map, Ψ, in Figures A.1 and A.2. Clearly, with any downward shift, Ψ has the unique stable fixed point, which satisfies $k_L < K(\lambda)$. This proves Proposition 2(a).

Step 4. Consider the case where $Rf'(K(\lambda)) > r$, which can be studied by increasing R, starting from the case, $Rf'(K(\lambda)) = r$, while fixing λ and r. This change is captured by an upward shift of the map Ψ in Figures A.1 and A.2. In Figure A.2, i.e., if $f(K(\lambda)) \leq 1$, Ψ has the stable unique fixed point, $k_H = \Phi(r/R) > K(\lambda)$, after any upward shift. In Figure A.1, i.e., if $f(K(\lambda)) > 1$, there is a critical value of R, R', such that, if $r/f'(K(\lambda)) < R < R'$, there are three fixed points, $k_L < k_M < K(\lambda) < k_H$, and, if $R > R'$, there is the unique fixed point, $k_H = \Phi(r/R) > K(\lambda)$. In the borderline case, $R = R'$, Ψ is tangent to the 45° line below $K(\lambda)$. From part (c) of the Lemma, the value of k at the tangency is equal to $\lambda R'/r$, and hence $\Psi(\lambda R'/r) = \lambda R'/r$, which can be rewritten as $(\lambda R'/r)f'(\lambda R'/r) = 1 - W(\lambda R'/r)$, or $f(\lambda R'/r) = 1$. Thus, $f(\lambda R/r) < 1$ implies the three fixed points and $f(\lambda R/r) > 1$ implies the unique steady state, $k_H = \Phi(r/R) > K(\lambda)$. This proves Proposition 2(b) and 2(c).

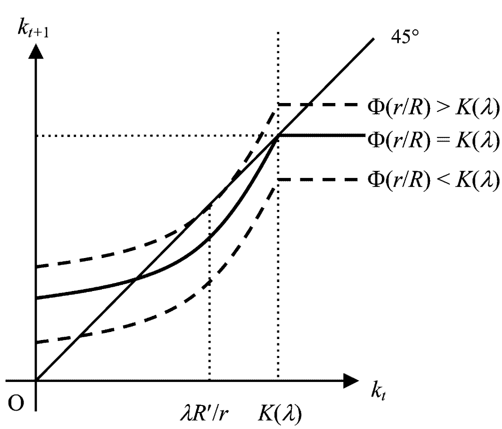

FIGURE A.1.—$\lambda < \lambda_c$.

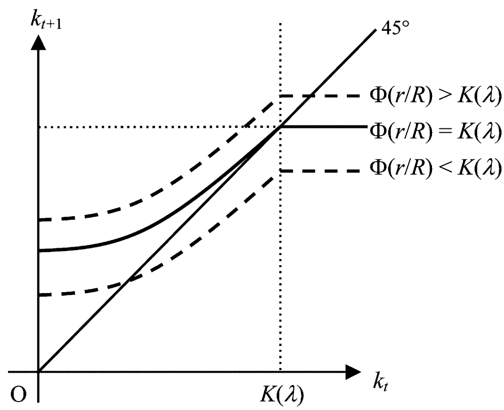

FIGURE A.2.—$\lambda > \lambda_c$.

Proof of Proposition 4

First, note that (7) defines k_H as a function of k_L. Differentiating (7) shows that this function, denoted by $k_H = \phi(k_L)$, is increasing if and only if $f(k_L) < 1$ or equivalently $k_L < K^*(R_c) = K(\lambda_c)$. Furthermore, it satisfies $\phi(0) = 0$ and $\phi(K(\lambda)) = K(\lambda)$. If $\lambda \geq \lambda_c$, $K(\lambda) \leq K(\lambda_c)$ and hence $k_L < K(\lambda)$ implies $k_H = \phi(k_L) < \phi(K(\lambda)) = K(\lambda)$, which violates (8). If $\lambda < \lambda_c$, the set of (k_L, k_M) that satisfies (7), (8), and (9) is nonempty, and is illustrated by the solid curve in Figure A.3.

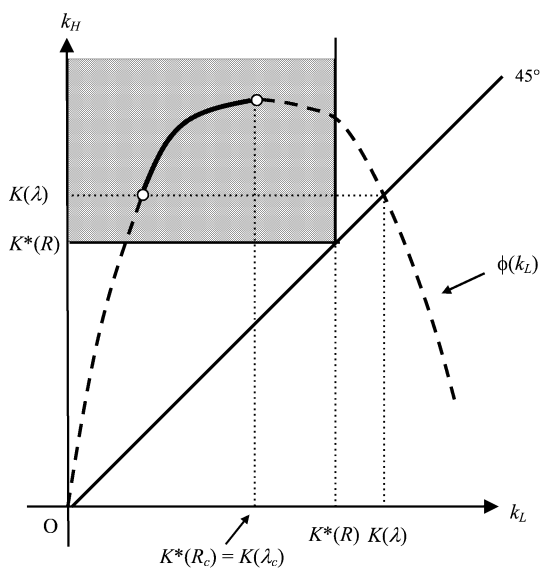

FIGURE A.3.

Second, (A.2) and (8) imply that (10) has a solution, $X \in (X^-, X^+) \subset (0,1)$, if and only if $k_L < K^*(R) < k_H$. This condition is illustrated by the shaded area in Figure A.3. Therefore, a stable steady state, (k_L, k_H, X), exists if and only if the solid curve, the segment of $k_H = \phi(k_L)$ satisfying (8) and (9), overlaps with the shaded area, or equivalently, if and only if $K(\lambda) < \phi(K^*(R))$ and $\phi(K(\lambda_c)) = \phi(K^*(R_c)) > K^*(R)$. The first condition can be rewritten to $f'(K(\lambda)) > \lambda f'(K^*(R))/[1 - W(K^*(R))]$, where $R < R_c$ and the second to $\lambda < f'(K^*(R))K(\lambda_c) = f'(K^*(R))K^*(R_c)$.

That $X^- > 0$ requires that the upper-right end of the solid curve be strictly inside the shaded area, or equivalently $R > R_c$. Similarly, $X^+ < 1$ if and only if the lower-left end of the solid curve is strictly inside the shaded area, or equivalently, $K(\lambda) > K^*(R)$.

This completes the proof.

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