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Source: *The Quarterly Journal of Economics*, Vol. 108, No. 3 (Aug., 1993), pp. 619-652
Published by: The MIT Press
Stable URL: http://www.jstor.org/stable/2118403
Accessed: 11/03/2010 12:35

WORKINGS OF A CITY: LOCATION, EDUCATION, AND PRODUCTION*

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We model the links between residential choice, education, and productivity in a city composed of several communities. Local complementarities in human capital investment induce occupational segregation, although efficiency may require identical communities. Even when some asymmetry is optimal, equilibrium segregation can cause entire "ghettos" to drop out of the labor force. Underemployment is more extensive, the easier it is for high-skill workers to isolate themselves from others. When perfect segregation is feasible, individual incentives to pursue it are self-defeating and lead instead to a collapse of the productive sector.

INTRODUCTION

Every city, no matter how small, is in fact divided into two; one the city of the rich, the other the city of the poor [Plato, *The Republic*].

In most cities people with high-skill, high-wage jobs live in select areas or suburbs, and those with low-skill, low-wage jobs—or no job at all—reside in very different parts. Residential segregation is sustained primarily by differentials in the price of housing between the two types of communities.

This social polarization, which seems to have become more acute in recent years, is often deplored on grounds of creating inequity in educational opportunities (e.g., Reich [1991]). But there is also an implicit issue of efficiency: socioeconomic segregation is said to deprive some communities of the chance to acquire even modest levels of skills, and thus to adversely impact the quality of the labor force. Moreover, it is argued, the "urban problem" has, or will eventually have, negative repercussions on the standard of living of the high-skill class itself—although through which channels is rarely articulated.

This paper takes up these issues, by formalizing the links between residential choice, educational investment, and production in a city composed of several communities. We consider a model where identical agents choose whether to become high-skill workers, low-skill workers, or to remain outside the formal labor

*I wish to thank Olivier Blanchard, Patrick Bolton, Charles De Bartolome, Peter Diamond, Oliver Hart, John Heaton, Kiminori Matsuyama, James Rauch, Jerome Rothenberg, Jean Tirole, William Wheaton, and especially Julio Rotemberg for helpful comments. Financial support from the National Science Foundation is gratefully acknowledged. Any errors are my own.

The Quarterly Journal of Economics, August 1993

force.[1] At the same time they decide in which part of the city to live. The labor market is citywide, with high- and low-skill labor complementary factors in production. Education, however, is a local public good or club good. In each community the more agents invest in high skills, the easier it is to do so; also, but to a lesser degree, the easier it is to acquire low skills. This asymmetry makes agents who become high-skill workers willing to bid more for land in a more highly skilled community, and thus leads to segregation. This in turn affects the surplus generated by the city, through both the mix of skills chosen by agents and the cost at which the labor force is educated.

We show that stratification can be inefficient, even though each high-skill worker gains more than his low-skill counterpart from living in a better neighborhood. Nonetheless, the total cost of education may rise more in low-skill communities than it falls in high-skill communities. The problem can also be more severe: by attempting to coalesce into homogeneous areas, high-skill workers may turn other neighborhoods into unproductive "ghettos." Moreover, their attempt to secede turns out to be self-defeating, as it deprives them of complementary low-skill workers. We show that the extent to which a city "works" may be inversely related to the feasibility of segregation.

At the heart of the model lies the interplay of local and global interactions: community spillovers in education, and neoclassical complementarity in production, respectively. The presence of three occupations, endogenously chosen, is also important. While segregation could be obtained with only high- and low-skills agents (i.e., with a single level of human capital investment), most of the inefficiency results could not: they depend critically on how the cost of low skills is affected by stratification.

This paper brings together two strands of work: local public goods or club theory on the one hand, and the macroeconomic literature on human capital and income distribution on the other. With the former it shares a concern for the impact of community composition on the provision of public goods, particularly education. With the latter it shares the aim to explain endogenous, self-replicating distributions of skills and incomes.

The paper is particularly related to Berglas [1976], who

1. Our concern here is with the long run: the equilibria of our one-shot game correspond to the steady states of an overlapping-generations model. Also, by using a representative agent model, we can abstract from distributional issues and concentrate on efficiency.

introduced complementary skills into the Tiebout [1956] model; to Arnott and Rowse [1987], who compute optimal school composition for various forms of peer effects; and to De Bartolome [1990], who also studies inefficiencies in community composition resulting from peer effects in education.[2] We depart from this literature in two essential ways. The first one is through the general equilibrium nature of the model, which allows the interplay of local and global complementarities mentioned above. Second, we reject the standard assumption of an exogenously given distribution of agents with different abilities or tastes. Instead, the overall distribution of types (professional occupations and incomes) is determined in equilibrium, together with the composition of local communities.

This reflects the view that the distribution of "abilities" or "skills" in the (national) population should be explained rather than assumed, because of its central importance not only for local community composition, but also for macroeconomic productivity and growth. Moreover, empirical studies show that among the young's characteristics most relevant for their own and their peers' achievements, endogenous family attributes such as parents' education, profession, and income play a prominent role. The young's distribution of characteristics (the new input into the education process) must thus reflect the distribution of skills acquired by their parents (the previous output of the education process).

It is through this preoccupation that the paper relates to recent work on the macroeconomic implications of income distribution. This literature treats education as either a private investment [Loury, 1981; Galor and Zeira, 1993; Banerjee and Newman, 1993; Aghion and Bolton, 1991] or as a nationally provided public good, financed by taxes on which agents vote [Glomm and Ravikumar, 1992; Perotti, 1990; Fernandez and Rogerson, 1991; Saint-Paul and Verdier, 1991]. In both cases a central role is ascribed to capital market imperfections, which prevent poor agents or governments from borrowing to finance investment in skills. We stress here another essential feature of education, and of learning more generally: it is in many respects a local public good, significantly affected by group composition. As emphasized by Lucas [1988],

2. Other relevant references include Tiebout [1956], Brueckner and Lee [1989], Schwab and Oates [1990], and Scotchmer [1991]. The paper's spatial aspects also relate it to Krugman [1991] and Thomas [1990], who study the regional specialization of firms; the underlying externality is very different, however, and operates there through market size.

"Human capital accumulation is a *social* activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital." Lucas also identifies cities as important groups in this respect and suggests that land rents should reflect local externalities in human capital. Such is the case in our model, but rents are not sufficient to replace the missing markets. Because the imperfections that we consider do not reside in the financial system, they are also not amenable to simple measures such as government loans or transfers. Even educational subsidies might be poor substitutes for appropriate neighborhood composition. These points seem quite relevant to the debate on whether increased funding alone can improve the performance of the American education system, particularly in "disadvantaged" communities.

This paper also departs from the standard approach to nonconvexities in macroeconomic models, which focuses on a potential multiplicity of symmetric, Pareto-ranked equilibria. We explore instead the role of asymmetries, which lead to an endogenous partitioning of the set of agents, that has important aggregate implications. The emphasis on local interactions and stratification is shared with Montgomery [1990], who studies contagion in "underclass" behavior, and with Durlauf [1992], who shows how local externalities in education and income-based rules for community membership can create persistent pockets of poverty.

Section I presents and discusses the model. Section II examines the integrated city, a useful reference point. Sections III and IV show how segregation arises in a dual city, and how it affects human capital investment and efficiency. Section IV extends the argument to a large number of communities, and links the extent of segregation to that of productivity losses. Section V concludes. Proofs are gathered in the Appendix.

I. The Model

A. People

There is a continuum of identical individuals, with measure N. Each of them is endowed with one unit of labor, which can be used to participate in the production of a single, numeraire good. Each agent has the choice between three occupations:

(a) Remaining outside the (formal) labor force, e.g., being unemployed or engaging in home production. This yields a utility level v.

(b) Becoming a low-skill worker. This requires exerting an effort level C_L, but allows him to earn the low-skill wage w_L.

(c) Becoming a high-skill worker. This requires exerting effort $C_H > C_L$, but allows him to earn the high-skill wage $w_H > w_L$.

These alternatives will often be abbreviated as U, L, and H. High-skill workers can be thought of as managers and professionals; low-skill workers as employed in line production and clerical jobs. C_L and C_H would then represent the efforts required to complete high school, and to get into college, respectively. The determinants of w_H, w_L, C_H, and C_L are examined below. Utility is additively separable in income and effort:

$$U^i = w^i - C^i + \pi^i - r^i, \tag{1}$$

where w^i and C^i are the wage and effort level chosen by agent i, r^i the rent that he pays, and π^i any additional income that he might receive, from land ownership or home production. For an unemployed worker, $w^i = C^i = 0$, and $\pi^i = v$.

B. The City

These agents can either live in the city or remain outside—in the countryside or in other cities. This outside option yields the same utility v as remaining in the informal urban sector; this is a convenient but unessential normalization. The city is divided in two, say, by a river. In each community (East and West, or Center and Suburb) are $N/2$ housing units, each suitable for occupation by one individual or family.[3] Each unit belongs to a different landowner, who will receive a competitively determined rent. Landowners can be viewed as a separate group of agents who consume but do not work, and have utility given by (1). Equivalently, the N workers could each own a plot and receive its rent.

Whereas education takes place at the community level, production is a citywide activity. This interaction of local and global complementarities will be at the heart of the novel effects identified in this paper.

3. The model can be extended to allow for an upward-sloping supply curve for land in each community, or variable housing density. These margins *may* reduce the extent of segregation and its impact on efficiency, but only at the price of distortions in land utilization.

C. Production

All workers are employed in competitive firms that produce the numeraire good, with constant returns to scale. Sharing common labor and product markets is what makes communities part of the same metropolis, rather than mere disjointed cities. Technology is $F(H,L) = \theta f(H,L)$, with $\partial^2 f/\partial H^2 < 0$, $\partial^2 f/\partial L^2 < 0$, $\partial^2 f/\partial H\partial L > 0$, and θ a productivity parameter. The wages for high- and low-skill workers, $w_H \equiv \partial F(H,L)/\partial H$, $w_L \equiv \partial F(H,L)/\partial L$ and the wage differential $\Delta w(H,L) \equiv (w_H - w_L)(H,L)$ depend only on the citywide factor ratio H/L. Since acquiring high skills always takes greater effort (see below), $\Delta w(H,L)$ must be positive in any equilibrium. Therefore, H/L is always less than $\bar{\rho}$, defined as the unique solution to $w_H(\bar{\rho},1) = w_L(\bar{\rho},1)$; $\bar{\rho}$ is the ratio that maximizes the city's gross output, and is independent of θ.

D. Education as a Local Public Good

The first key assumption of the model is the presence of local human capital spillovers, which make education a "club good": in each community, the more agents acquire a high level of human capital, the easier it is to pursue any kind of education.[4] See Figure I.

ASSUMPTION A1. The costs of education $C_H(x)$ and $C_L(x)$ decrease with the fraction x of individuals in the community who invest in high skills.

(a) Examples. There are many plausible channels through which such local complementarities may operate. Since workers with high skills earn higher wages, the most obvious one is a fiscal externality. If schools must be financed from local resources, and if they provide a complementary input to individual effort, the return to studying will be higher in a richer community. The model's basic properties would thus remain unchanged if, instead of x, the argument of C_H and C_L was the community's per capita income.

There are also many channels that do not depend on imperfections in credit markets (and are therefore not amenable to simple policy measures such as government loans or income redistribution), but involve pure human capital externalities. The first one is peer effects in education. For instance, in a high school the more students who work hard with the aim of getting into college, the

4. As illustrated by the examples below, the relevant notion of "community" is the group or area within which this effect operates, and thus depends on the externality under consideration. We make the convenient assumption that each community extends over half the city; it will be relaxed in Section IV.

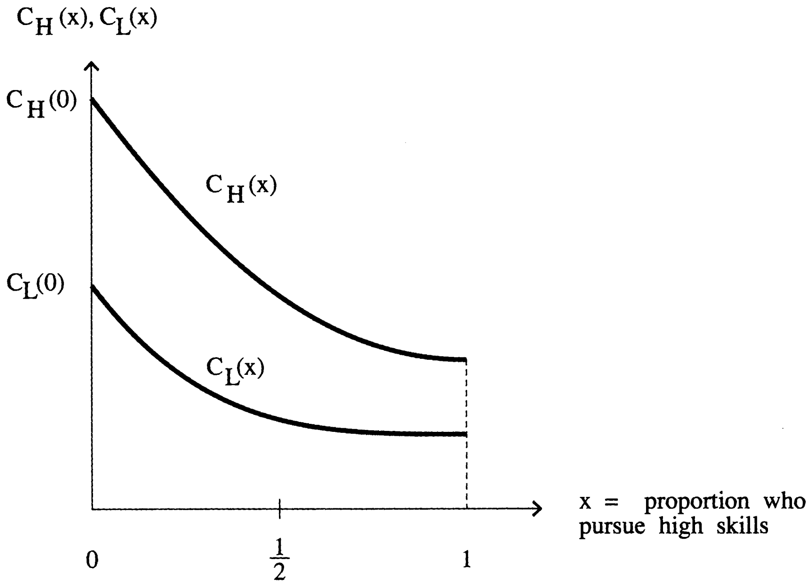


FIGURE I
The Costs of Skill Acquisition

less time teachers need to devote to discipline, and more generally the more studious the atmosphere. This makes it easier for any college-bound student, but also for one who aims only to graduate, to achieve his goal. An analogous effect arises in Banerjee and Besley's [1990] model of testing: a higher proportion of hard-workers makes grades more informative of individual ability, and thereby increases everyone's incentive to work hard.

Another example, now involving adults, is that of social networks: knowing an established worker—especially at the managerial level—whose recommendation could "get you in" decreases the cost of getting any type of job [Montgomery, 1991]. This person can also serve as a role model, demonstrating to the young the value of education [Streufert, 1991]. Finally, an alternative interpretation is that unemployed and low-skill workers, or some fraction of them, impose on their community negative externalities such as crime, drugs, and other disruptive influences.

(b) Empirical Evidence. A number of studies have found evidence of peer effects and local complementarities. Controlling for observable individual and family characteristics and for mate-

rial inputs, the literature on "educational production functions" finds a significant effect of class and school composition on a student's educational attainment [Summers and Wolfe, 1977; Henderson, Mieszkowski, and Sauvageau, 1978; Dynarski, Schwab, and Zampelli, 1989]. Crane [1991] finds that high-school dropout and teenage pregnancy rates are significantly affected by the proportion of adults holding managerial or professional jobs in the neighborhood. Case and Katz [1991] find contagion effects between neighboring youths for seven different outcomes: criminal conviction, drug use, single-parent status, unemployment, gang membership, alcohol use, and churchgoing.[5] Finally, Rauch [1991] finds evidence of local spillovers in human capital at the production stage: wages per efficiency unit of labor and land rents are both higher in metropolitan areas (SMSA's) where the average level of human capital is higher.

The second key feature of the model is that the spillover from high-skill workers affects the two types of investment *asymmetrically*. The following assumption accords well with most of the examples given earlier.

ASSUMPTION A2. Agents investing in high skills benefit more than those pursuing low skills from the presence of high-skill workers in their community: $\Delta C(x) \equiv C_H(x) - C_L(x)$ is positive and decreasing in x.

This sorting condition, reflected by a steeper slope in Figure I, is what leads agents to segregate.[6] Since those choosing high qualifications care more about the level of education around them, they will bid more for land in the more highly skilled community. We shall often refer to the slope differential $-\Delta C'$ as the "net" complementarity among high-skill workers. This is in contrast to the "gross" spillovers $-C'_H$ and $-C'_L$ that operate within the high-skill group, and from the high- to the low-skill group,

5. Borjas [1992] identifies a somewhat different effect: a male child's human capital is determined in part by the average level of human capital in his ethnic group. But, of course, ethnic groups often reside in distinct neighborhoods. Finally, one should note that it is still a debated issue whether all these studies truly capture group effects rather than some unobserved characteristic common to all member families; see Manski [1991].

6. Henderson, Mieskowski, and Sauvageau [1978] find that high- and low-ability grade school students benefit equally from peer group improvement. But all that is needed for segregation to occur is that $C'_L - C'_H$ be even slightly positive. Moreover, one can view A2 as representing the reduced form of other segregation-inducing effects, such as a differential sensitivity of high- and low-skill investments to educational expenditures, as in De Bartolome [1990].

respectively. Intuitively, stratification will arise in response to the net effect, no matter how small, whereas the city's surplus involves the gross effects, which can be quite large. When $C'_L = 0$, this distinction disappears, and with it the potential for segregation to be inefficient.[7] That is why it is important that agents choose among three occupations, H, L, and U, rather than just the first two.

We denote the total cost of educating a fraction x of a community's population to high skills, and the rest to low skills, as

$$\Phi(x) \equiv xC_H(x) + (1 - x)C_L(x). \tag{2}$$

Intuitively, one would expect $\Phi(x)$ to increase with x, at least past a certain level. But this will not be necessary for any of our results.

For some of the externalities that Assumptions A1 and A2 are meant to capture, particularly those related to education, the simultaneity of occupational choice, peer effects, and residential choice may seem unrealistic. It is mostly adults who determine the quality and resources of the communities in which children make their investments in education. Such intergenerational effects are nonetheless quite consistent with our model. As is intuitive, the equilibria of the simultaneous choice game studied here correspond to the steady states of an overlapping-generations model where adults choose location, recognizing that their children's education opportunities will be affected by community composition; see Benabou [1991].

E. Equilibrium Conditions

An equilibrium consists of wages, community compositions, and land rents that clear the labor and land markets: firms maximize profits, and no agent wants to change occupations or communities, or to leave the city. Denote the proportions of high-skill, low-skill, and unemployed workers in community $j = 1,2$ as $(x_j,y_j,1 - x_j - y_j)$ and the rent as r_j. The high-skill work force is then $H = N(x_1 + x_2)/2$, and the low-skill work force $L = N(y_1 + y_2)/$

7. In Miyao [1978] segregation (a homogeneous city) results from each group's either disliking the other ("negative intergroup externalities") or liking its own ("positive intragroup externalities"). Here, everyone likes high-skill workers, but others with high skills like them most; also, the distribution of types is endogenous. The welfare implications will be completely different.

2. In equilibrium one must have for each j:

Occupational choice:

$$(x_j,y_j) \in \underset{(x',y')}{\operatorname{argmax}} \{x'(w_H(H,L) - C_H(x_j)) + y'(w_L(H,L) - C_L(x_j)) + (1 - x' - y')v\}$$

Mobility:

$$r_j = \max\{w_H(H,L) - C_H(x_j) - v, w_L(H,L) - C_L(x_j) - v, 0\}.$$

The first condition requires all occupations chosen in community j to yield the same, maximal level of utility. The second one states that r_j is the maximum of the rents or surpluses which workers in each of the three occupations are willing to bid for living in community j. Equivalently, these are the rents that make them indifferent between living in community j and living outside the city, and hence also indifferent between communities. Note that if community j is not fully employed, or equivalently, is partially empty ($x_j + y_j < 1$), it generates no surplus: $r_j = 0$.

II. The Integrated City

We first study a city with a single community or sharing group. This is a natural starting point, where agents cannot isolate themselves from others. It also corresponds to the (unstable) symmetric equilibrium of a subdivided city. Using this benchmark to analyze the (stable) asymmetric equilibrium will bring to light most clearly the effects of mobility and stratification.

A. Equilibrium and Simple Dynamics

Assume that all agents are employed as either high- or low-skill workers, in proportions $\hat{x}$ and $1 - \hat{x}$. Since all face the same costs and rewards, they must all be indifferent between the two occupations:

$$\Delta w(\hat{x}, 1 - \hat{x}) = \Delta C(\hat{x}). \tag{3}$$

Multiple equilibria are not the focus of our interest. To ensure that (3) has a unique solution, we assume that the complementarity between H and L workers in production, which makes $\Delta w(x, 1 - x)$ decrease in x, dominates the (net) complementarity among H workers in education, which makes $-\Delta C(x)$ increase in x.[8] Such is

8. The precise conditions are A3 and A4 in the Appendix.

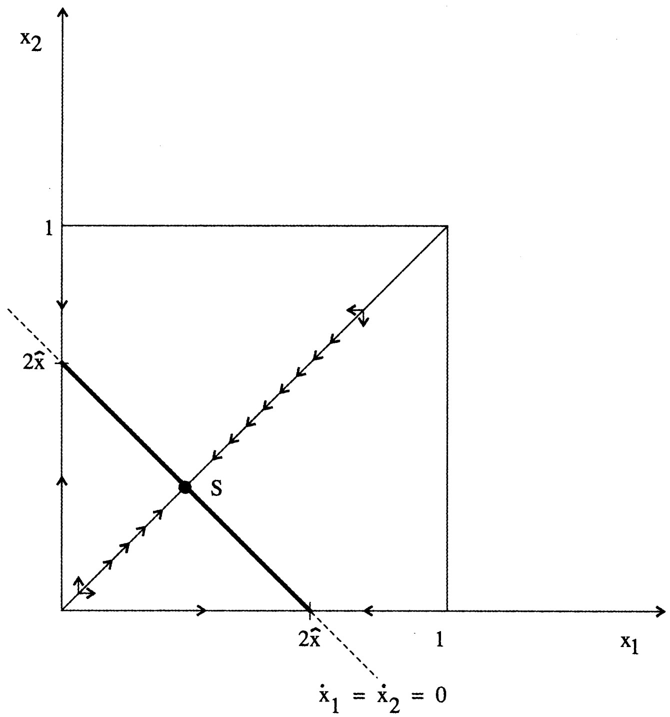


FIGURE IIa
Integrated City, or Dual City with Global Complementarities Only

the case if productivity is high enough, as wages are proportional to θ. It will be useful later on to think of the integrated city equilibrium as the outcome of a simple adjustment process:

$$\dot{x}_i = a[\Delta w(x,1 - x) - \Delta C(x)], \qquad i = 1, 2, \tag{4}$$

where $x \equiv (x_1 + x_2)/2$.[9] There are two geographically distinct neighborhoods, so as to allow comparison with the segregated city; but all interactions are citywide. The dynamics corresponding to (4) are drawn on Figure IIa. Any point along the $x_1 + x_2 = 2\hat{x}$ locus

9. Following Miyao [1978] and the rest of the literature, the adjustment processes in this paper are standard, myopic tatonnements. A dynamic rational expectations version of the model, with community compositions as state variables, would be needlessly complicated. Also, while stability arguments are convenient to focus on a single equilibrium, all the results could be restated in terms of how the *set* of equilibria is affected by self-selection.

is a stable equilibrium, and is completely equivalent to the symmetric allocation S, where $x_1 = x_2 = \hat{x}$.[10]

It remains to verify that agents find work preferable to unemployment, as has been assumed. In other words, an individual's decision to become educated and participate in production must generate a positive surplus:

$$\hat{r} \equiv w_L(\hat{x},1 - \hat{x}) - C_L(\hat{x}) - v > 0. \tag{5}$$

This net product increases with the level of productivity θ, both directly and through $\hat{x}$. We assume that θ is high enough for (5) to hold. Finally, the land market clears when workers are indifferent between living in or out of the city. This occurs when landowners appropriate all the surplus from production, leaving workers with their reservation utility.

PROPOSITION 1. There is a unique symmetric, or integrated, full employment equilibrium. A fraction $\hat{x}$ of agents acquire a high level of skills, where $\Delta w(\hat{x},1 - \hat{x}) = \Delta C(\hat{x})$. The land rent is $\hat{r}$.

At the other extreme from this full employment equilibrium, there may exist a trivial equilibrium where no one works, or equivalently where the city fails to materialize. In this case there also exists an unstable equilibrium with partial unemployment in-between.[11] Both these equilibria generate zero surplus or rents; they represent coordination failures in an integrated city, and have nothing to do with location. We shall essentially ignore them.

B. *Efficiency*

The integrated city equilibrium suffers from the standard underinvestment problem, as individuals acquiring high skills are not rewarded for lowering the education costs of others. Indeed, denoting aggregate surplus by $V(x) \equiv F(x,1 - x) - \Phi(x)$, we have

$$V'(\hat{x}) = -\hat{x}C_H'(\hat{x}) - (1 - \hat{x})C_L'(\hat{x}) > 0. \tag{6}$$

This inefficiency in the work force's composition does not describe anything new; nor is it related to location. The more interesting issue is whether it is improved or worsened by self-segregation.

10. Note also that if all interactions were local, i.e., if the two communities had separate labor markets (as in Berglas [1976]), they would operate as two different cities; the unique equilibrium would then be point S.

11. See the Appendix, following the proof of Proposition 1.

III. The Segregated City, with Full Employment

We now turn to the dual city. Our main purpose is less to explain stratification (which may arise from a variety of sources) than it is to elucidate how it shapes human capital decisions and the city's net product.

We focus in this section on the case where all agents choose to work. We first explain how those with high and low skills self-segregate, and how this in turn affects the occupational mix. We then show that stratification may be inefficient, even though each high-skill worker benefits more than his low-skill counterpart from living in a better educated community.

A. Simple Dynamics

Let us abstract for now from the land market as well as the possibility of unemployment, and consider again a simple process by which agents in each community respond to the incentive to switch from low to high skills:

$$\dot{x}_i = a[\Delta w(x_1 + x_2, 2 - x_1 - x_2) - \Delta C(x_i)] \tag{7}$$

for $0 < x_i < 1$, $i = 1,2$. Whereas the cost ΔC is locally determined, the payoff Δw is set at the citywide level. This has two implications. First, the only equilibrium where both communities are mixed is the symmetric, or integrated allocation: if $0 < x_2 \leq x_1 < 1$, then $\Delta C(x_1) = \Delta w(H,L) = \Delta C(x_2)$ requires that $x_1 = x_2 = \hat{x}$, by A2. Second, the slopes of the stationary loci are determined by the relative strength of *local and global interactions*:

$$\left.\frac{dx_2}{dx_1}\right|_{\dot{x}_1=0} = -1 + \frac{\Delta C'(x_1)}{\partial \Delta w/\partial x_2}, \quad \left.\frac{dx_2}{dx_1}\right|_{\dot{x}_2=0} = \left[-1 + \frac{\Delta C'(x_2)}{\partial \Delta w/\partial x_1}\right]^{-1} \tag{8}$$

The term -1 reflects the complementarity between H and L workers, or substitutability among H workers, in citywide production. When ΔC is constant, we are in the standard neoclassical case, and any (x_1,x_2) such that $\Delta w(x_1 + x_2, 2 - x_1 - x_2) = \Delta C$ is a stable equilibrium. Normalizing ΔC to its value $\Delta C(\hat{x})$ under integration, the dynamics are identical to those of Figure IIa.

The polar case, with only local complementarities in education, yields the usual increasing returns configuration. Let the wage differential Δw be a constant, which can be normalized to $\Delta w(\hat{x}, 1 - \hat{x})$. The $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ loci are then, respectively, vertical and horizontal, as illustrated in Figure IIb. Each community has two stable and one unstable equilibrium. For the city as a whole, the

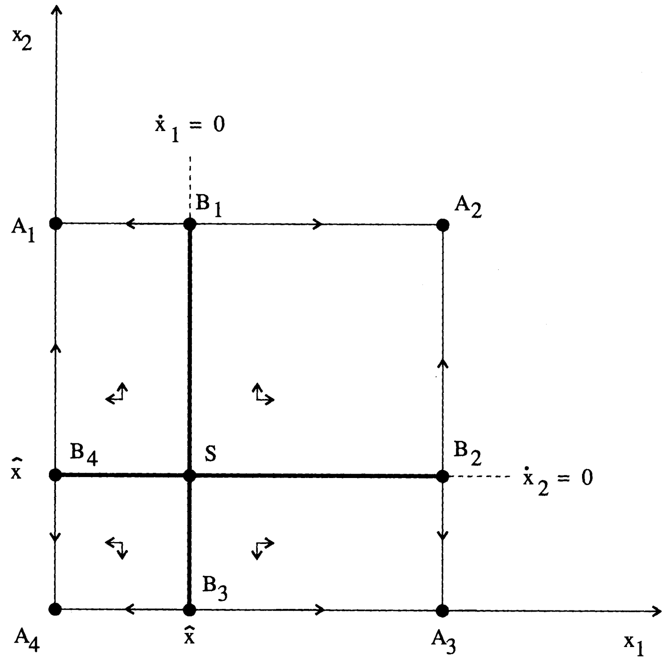


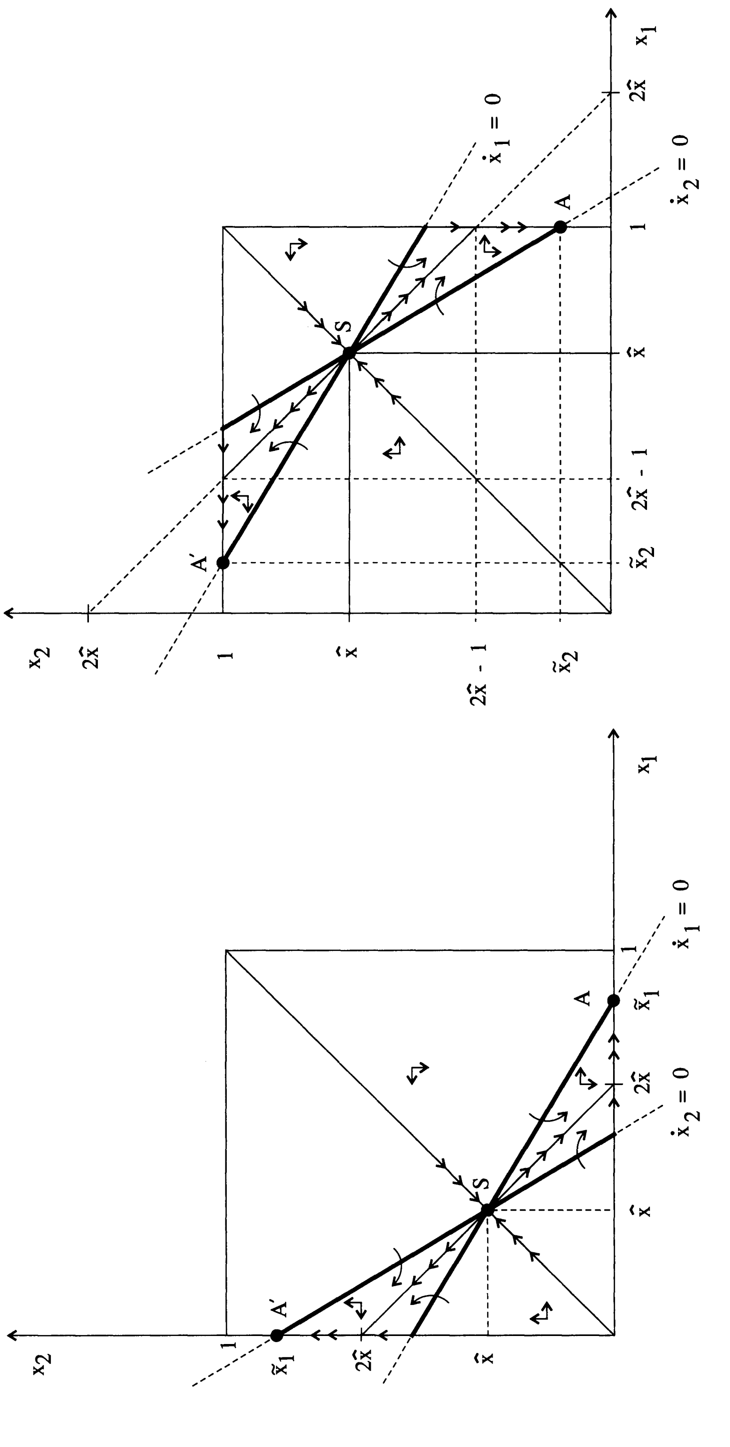
FIGURE IIb
Dual City, with Local Complementarities Only

symmetric allocation $(\hat{x},\hat{x})$ is now an unstable equilibrium; the four stable equilibria correspond to extremal community compositions.

The interplay of community-level and citywide complementarities embodied in (7) leads to an interesting combination of the two previous cases. The resulting dynamics are illustrated in Figure III. These reflect the assumption, which we shall make from here on, that the (net) local interaction is relatively weak compared with the global one:

$$(9) \quad |\Delta C'(x_i)| < \left| \frac{\partial \Delta w(x_1 + x_2, 2 - x_1 - x_2)}{\partial x_1} \right| = \frac{\theta \partial^2 f}{HL} \cdot \frac{(H,L)}{\partial H \partial L}, \qquad \text{for all } x_1, x_2.$$

This ensures that both stationary loci slope down, precluding the kind of multiplicity depicted on Figure IIb. Yet even the *slightest amount* of local complementarity $\Delta C' < 0$ leads to maximal



IIIa: The case $\hat{x} < \frac{1}{2}$

IIIb: The case $\hat{x} > \frac{1}{2}$

FIGURE III
Dual City, with Local and Global Complementarities

stratification as the only stable equilibrium, because it makes the $\dot{x}_2 = 0$ locus steeper than the $\dot{x}_1 = 0$ locus.

The symmetric equilibrium S is now occupationally stable but locationally unstable, in the following sense. The stable path, converging to S, lies along the first diagonal; thus, citywide, symmetric perturbations to the skill mix do not persist. Conversely, the unstable path lies along the direction $(1,-1)$, which corresponds to H and L individuals trading places, with total numbers unchanged.[12] Identifying allocations that are mirror images of one another, we shall always take community 1 to be better educated: $x_1 \geq x_2$.

B. Stratification, Land Rents, and the Skill Mix

We now examine how segregation is sustained in the land market, and how it affects the distribution of skills acquired by the labor force. The equilibrium corresponding to the symmetric allocation S is illustrated in the left panel of Figure IV. Since communities are identical, so are rents, and there is no reason to move. But as soon as x_1 becomes slightly larger than x_2, the West side becomes more attractive to all agents; its rent increases, with the land going to the highest bidders. Those are clearly the individuals investing in high skills, since they value living in community 1 more than the others: $C_H(x_2) - C_H(x_1) > C_L(x_2) - C_L(x_1)$. So additional high-skill workers move West, making that area even more attractive and driving $\Delta r \equiv r_1 - r_2$ up farther; low-skill workers migrate in the opposite direction.

This stratification continues until one of the two communities becomes homogeneous: in Figure III the unstable path intersects either the $x_2 = 0$ or the $x_1 = 1$ boundary, depending on whether $\hat{x}$ is above or below $1/2$. In the first case, all high-skill workers are able to regroup on the West side. In the second, some of them have to live on the East side, with the low-skill workers. But neither situation is yet an equilibrium, because segregation has altered the incentives for occupational choice.

When $\hat{x} < 1/2$, the concentration of human capital into community 1 lowers the net cost $\Delta C(x_1)$, allowing yet more agents to pursue high qualifications. In equilibrium, there are $\tilde{x}_1 > 2\hat{x}$ such workers; see panel a of Figures III and IV. Finally, the rent differential $r_1 - r_2$ ensures that low-skill workers in both communities are equally well off.

12. The stable and unstable roots are, respectively, $-a[2(-\partial\Delta w/\partial x_1) + \Delta C')] < 0$ along (1,1) and $-a \cdot \Delta C' > 0$ along $(1,-1)$.

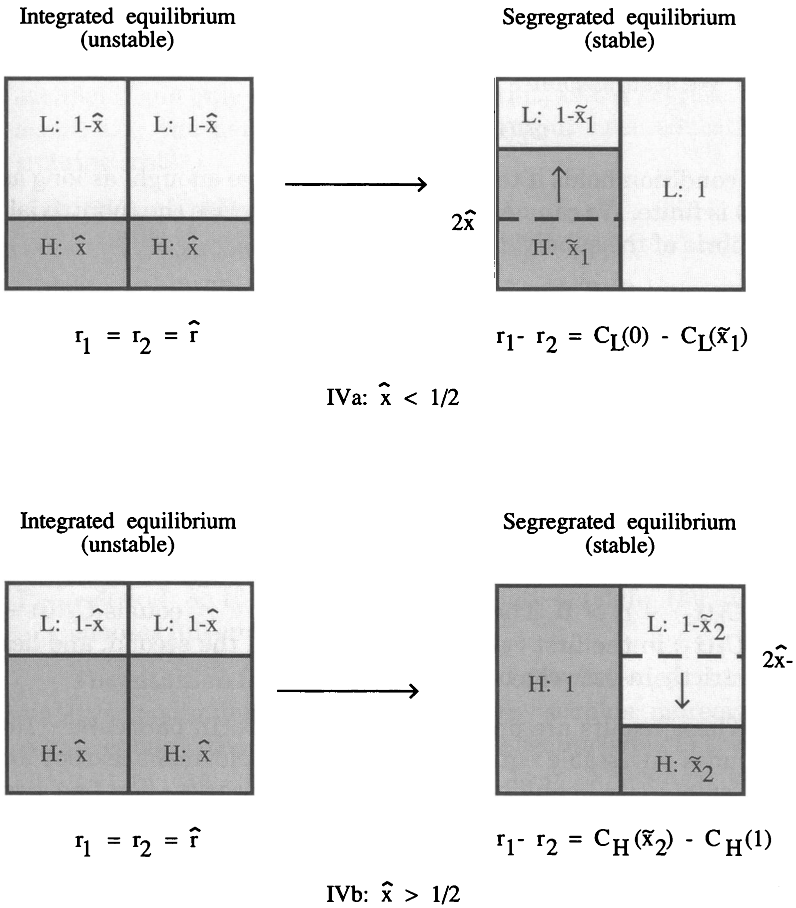


FIGURE IV
Full Employment Equilibrium

When $\hat{x} > 1/2$, the marginal agent is in community 2, which has experienced a "brain drain." The resulting rise in $\Delta C(x_2)$ causes a decline in high skills investment, whose total level falls to $1 + \tilde{x}_2 < 2\hat{x}$; see panel b of Figures III and IV. Since some H workers reside in each community, $r_1 - r_2$ must now compensate for the differential cost of high-skill investment.

C. Equilibrium

It remains to verify that work in the formal sector is preferred to unemployment or self-employment. For this to be the case, the effort required to acquire low skills must not be too high, even in a community deprived of the benefits conferred by workers with high skills. We assume that

$$(10) \qquad w_L(\hat{x},2 - \hat{x}) > C_L(0) + v.$$

This condition holds if technology is productive enough, as long as $C_L(0)$ is finite. We can now completely characterize the (nontrivial) equilibria of the subdivided city.

PROPOSITION 2. There is a unique stable equilibrium, which involves maximal concentration of highly educated workers.
(a) If $\hat{x} < 1/2$, all high-skill workers reside in community 1, and their total number is increased by mobility: $\tilde{x}_1 > 2\hat{x}$.
(b) If $\hat{x} > 1/2$, all low-skill workers reside in community 2, and the high-skill labor force is decreased by mobility: $\tilde{x}_1 + \tilde{x}_2 = 1 + \tilde{x}_2 < 2\hat{x}$.
(c) When $\hat{x}$ is close to $1/2$, there is complete segregation: $\tilde{x}_1 = 1$, $\tilde{x}_2 = 0$.
Rents are $r_1 = w_H(H,L) - C_H(x_1) - v > 0$ and $r_2 = w_L(H,L) - C_L(x_2) - v > 0$. The rent differential $r_1 - r_2$ equals $C_L(0) - C_L(\tilde{x}_1)$ in the first case, $C_H(\tilde{x}_2) - C_H(1)$ in the second, and lies strictly in-between in the third.

These results are proved in the Appendix. In particular, (10) precludes any stable equilibrium with unemployment, except for the "empty city" equilibrium where no one works. The first two cases correspond to the two panels of Figure IV. The last one is also intuitive: acquiring high skills is very easy when almost everyone else is doing it, but very difficult when no one else is doing it.

D. Efficiency

We now examine the efficiency properties of the market outcome. We start with a simple example and then examine the general principles at work.

Consider a Leontief technology, requiring both types of labor in unit proportions. The only issue is the cost of training this labor force. In an integrated city it is $C_H(1/2) + C_L(1/2)$ (times $N/2$); with H and L workers segregated, it becomes $C_H(1) + C_L(0)$. This arrangement is less efficient if $C_L(0) - C_L(1/2) > C_H(1/2) - C_H(1)$, which means that $C_L(x)$ is steeper at lower values of x than $C_H(x)$ at high values; see Figure I. In economic terms, this represents decreasing social returns to the concentration of high skills, which agents fail to internalize as they segregate in response to the differential $(C'_H - C'_L)(x)$ in private returns.

We now turn to the general analysis. An allocation is Pareto optimal if and only if it maximizes the utility of the representative agent, i.e., the per capita surplus of output over education or training costs:

$$(11)\quad 2V(x_1,y_1,x_2,y_2) \equiv F(x_1 + x_2, y_1 + y_2) - x_1C_H(x_1) - y_1C_L(x_1) - x_2C_H(x_2) - y_2C_L(x_2) + v(2 - x_1 - y_1 - x_2 - y_2)$$

over x_i, y_i and $1 - x_i - y_i$ in $[0,1]$, $i = 1,2$. We can also write

$$V - v \equiv \frac{1}{2}\sum_j x_j(w_H(H,L) - C_H(x_j) - v) + y_j(w_L(H,L) - C_L(x_j) - v)$$

so that $V - v$ is the sum of all land rents. We focus on the case where the planner wants the whole labor force to work.

PROPOSITION 3. If productivity θ is high enough, the planner chooses full employment.

The intuition for the (nontrivial) proof is that a less than fully employed community does not produce any surplus; moreover, if θ is high enough, the surplus from the $N/2$ agents living in the other community can be shown to be less than $N\hat{r}$, which can be generated by a fully employed city. The planner's problem now simplifies to choosing the skill mix of each community, and her objective function becomes

$$(12)\quad 2V(x_1,x_2) \equiv F(x_1 + x_2, 2 - x_1 - x_2) - \Phi(x_1) - \Phi(x_2).$$

The problem is separable into finding the geographical allocation that minimizes the cost of achieving any given proportion $H/L = x/(1 - x)$, and then finding the value of this ratio that maximizes surplus:

$$\max_{0 \le x \le 1}\{F(x,1 - x) - \min\{\Phi(x_1) + \Phi(x_2)\,|\,x_1 + x_2 = 2x\}\}.$$

Whether it is optimal to group high-skill workers together or to spread them evenly depends on whether education costs $\Phi(x)$ are concave or convex. This in turn reflects a tradeoff between two effects. On the one hand, there is the *asymmetry* that makes high-skill workers care more about community composition. On the other, there may be *decreasing returns* to the agglomeration of high skills. To see this, consider what happens when an H worker from community 2 and an L worker from community 1 trade places. Total output is unchanged, but total cost rises by

$$(13)\quad \Phi'(x_1) - \Phi'(x_2) = [C_L(x_2) - C_L(x_1)] - [C_H(x_2) - C_H(x_1)] + [x_1C_H'(x_1) + (1 - x_1)C_L'(x_1)] - [x_2C_H'(x_2) + (1 - x_2)C_L'(x_2)].$$

The first two terms represent the effort savings to each worker from living in community 1. Since the difference is negative, the trade is mutually beneficial, allowing the high-skill worker to "buy out" the other. The last two terms (in absolute value) are the external benefit to community 1 and loss to community 2, which are ignored in private migration decisions. Their net impact tends to be positive if the gains $-C'_H$ and $-C'_L$ from a marginal high-skill neighbor are larger when high skills are scarce (at a low x_2) than when they are already abundant (at a high x_1); see Figure I. This last effect is dominant, and stratification drives up the total cost of education, if

$$(14)\quad \Phi''(x) > 0, \qquad \text{or} \quad xC_H'''(x) + (1 - x)C_L'''(x) > -2(C_H - C_L)'(x) \quad \text{for all } x,$$

meaning that the curvature in local interactions dominates their asymmetry. The costs of educating low-skill workers play a key role here. If C_L is constant, then concentrating all high-skill workers together, when feasible ($\hat{x} \leq 1/2$), is always efficient: $\Phi'(x_1) < \Phi'(0)$.

Henderson, Mieszkowski, and Sauvageau [1978] find significant concavity in the effects of mean class ability on a student's educational achievement. Dynarski, Schwab, and Zampelli [1989] find that greater income dispersion, ceteris paribus, raises a school district's average test performance. Crane [1991] finds that contagion effects fostering high school dropouts and teenage pregnancies become prevalent once the proportion of professional adults in the neighborhood falls below a critical threshold. The case where the concentration of workers with similar levels of human capital involves social diseconomies is therefore of particular interest.

PROPOSITION 4. If productivity is high enough and education costs $\Phi(x) = xC_H(x) + (1 - x)C_L(x)$ are convex, the planner's problem has a unique solution, which is symmetric. In each community a fraction x^* of individuals acquire high skills, where

$$(15)\quad (w_H - w_L)(x^*,1 - x^*) - (C_H - C_L)(x^*) = x^*C_H'(x^*) + (1 - x^*)C_L'(x^*) < 0.$$

The rest of the population acquires low skills; so all are employed.

High-skill workers are evenly spread to minimize total costs, and their total number chosen so that the net social value of training an additional one is zero. We can now summarize how self-segregation affects welfare.

(a) Its impact on the underinvestment problem is beneficial in communities where high-skill workers concentrate, but adverse in those that they desert. In our simple model only one of these margins is operative at a time, depending on $\hat{x} \gtrless 1/2$.

(b) It lowers the total cost of education in the high-skill community, but raises it in the low-skill community. When Φ is convex, total costs rise, creating another inefficiency on top of the underinvestment problem. When Φ is concave, total costs fall, improving welfare.

When Φ is convex but $\hat{x} < 1/2$, these two effects go in opposite directions. The segregated equilibrium of Figure IIIa is dominated by a symmetric allocation, $x_1 = x_2 = \tilde{x}_1/2$, but it could still be more efficient than the symmetric equilibrium: $\tilde{r}_1 + \tilde{r}_2 \gtrless 2\hat{r}$. This implies that imposing integration (say, of adults) without at the same time subsidizing high skills (for children) may reduce total surplus.

IV. THE SEGREGATED CITY: GHETTO AND SELF-DEFEATING SECESSION

We now turn to a case where stratification has even more drastic consequences. Instead of merely affecting the skill mix and education cost of the labor force, it causes part of the city to drop out of the productive sector. Moreover, the attempt by the high-skill group to secede is self-defeating, in a sense specified below.

We again start from the symmetric equilibrium, assuming for now that $\hat{x} < 1/2$; see Figure V. As the high-skill workers of community 2 (say, the Center) move to community 1 (say, the Suburb), it becomes more difficult for those left behind to maintain even low skills. In the previous section, (10) ensured that this

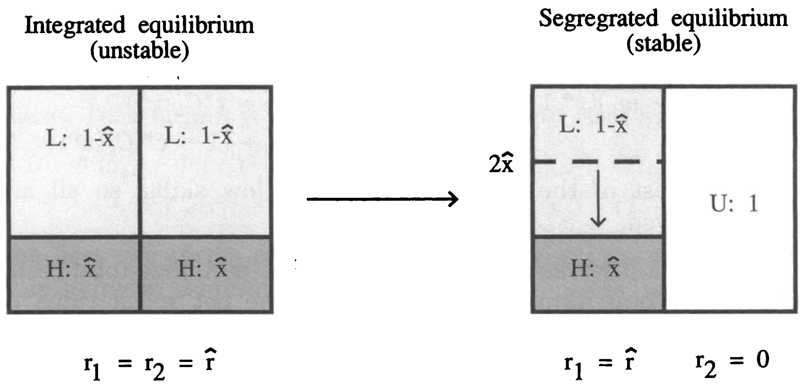


FIGURE V
Unemployment Equilibrium

investment still remained profitable. We now replace (10) by

$$(16) \qquad w_L(\bar{\rho},1) < C_L(0) + v.$$

Recall that $\bar{\rho}$ is an upper bound on the factor ratio H/L in any equilibrium, so that $w_L(\bar{\rho},1) = w_H(\bar{\rho},1)$ is an upper bound on the low-skill wage. This condition therefore means that in a community totally deprived of high-skill workers, the return to low-skill education is negative. As a result, the exodus of the high-skill group leads agents in community 2 to remain unskilled and to drop out of the labor force. The city center becomes an unproductive "ghetto."

But this is not the end of the story. Since the number of low-skill workers has declined, their wages rise. This reduces the incentive for individuals in community 1 to become highly skilled workers; hence less of them do so, until the labor market clears. As the labor market is now reduced to community 1, constant returns to scale imply that in equilibrium the factor ratio must be the same as in an integrated city: $H/L = \hat{x}/(1 - \hat{x})$; see Figure V. The end result of stratification is a halving of production and surplus.

PROPOSITION 5. When $\hat{x} \leq 1/2$, there is a unique stable equilibrium. Self-segregation reduces the productive labor force *by half,* with its skill composition unchanged: $x_1 = 1 - y_1 = \hat{x}; x_2 = y_2 = 0$. Rents are $r_1 = \hat{r}$, and $r_2 = 0$.

When $\hat{x} > 1/2$, this same allocation remains a stable equilibrium. The only other possible stable equilibrium is the full-employment allocation of Proposition 2(a): $x_1 = 1; x_2 = \tilde{x}_2$. It is an equilibrium if $\tilde{x}_2$ is high enough so that $w_L(1 + \tilde{x}_2, 1 - \tilde{x}_2) \geq C_L(\tilde{x}_2) + v$.

The first result stands in sharp contrast to Proposition 2(a). Where the concentration of high-skill workers led to a desirable increase in H/L, with $H + L$ unchanged, it now leads to a reduction in both H and L, with H/L unchanged.

The intuition is simple. Each high-skill worker who moves away from community 2 contributes to driving up the cost, hence reducing the supply, of a complementary input: low-skill labor. This in turn reduces the demand for high-skill workers. This "self-defeating flight" arises from the two key features that distinguish our model from previous ones: the interplay of local and global complementarities, and the endogenous distribution of types. We shall come back to it in the next section.

The second case in Proposition 5 is also quite intuitive. Consider the allocation described in Figure IVb: $\tilde{x}_1 = 1$, $\tilde{x}_2$ solving $\Delta w(1 + \tilde{x}_2, 1 - \tilde{x}_2) = \Delta C(\tilde{x}_2)$. Full employment is sustainable only if $\tilde{x}_2$ is high enough for low-skill investment to be profitable in community 2.[13] If not, some agents switch to inactivity. By reducing the wage gap, this induces others to switch from high to low skills, thereby driving up the costs of any type of education. The unraveling continues until the city Center ends up completely unemployed, as in Figure V.

The efficiency implications of Proposition 5 are clear; we add only two remarks. First, in contrast to the full-employment case, H always declines, aggravating the underinvestment problem. Second, we have placed no restriction on the convexity of education costs $\Phi(x)$. Thus, self-selection can severely damage efficiency even when the optimum calls for some segregation. When education costs are concave, a planner would implement an asymmetric allocation; but unlike the market, she would ensure that community 2 residents have sufficient incentives to remain in the labor force.

V. The Extent of Mixing and the Extent of Production

The preceding sections compared the equilibria of an integrated and of a dual city. The assumption of two communities was convenient but somewhat arbitrary. Moreover, whether or not the

13. As usual, between this equilibrium and that with $x_2 = y_2 = 0$, there is then an unstable one where community 2 is partially employed.

whole high-skill labor force of the integrated equilibrium could regroup into a single community ($\hat{x} \gtrless 1/2$) played an important role in shaping the segregated equilibrium.

To show that the basic insights are quite robust, we derive in this section a related result that holds for any number of communities and is invariant to the value of $\hat{x}$. But its primary interest is to make strikingly clear that the degree to which a city "works," in both senses of the term, may be inversely related to the feasibility of segregation.

Let the city be divided into m communities or neighborhoods, of equal size N/m. The parameter m measures how effectively groups of agents can segregate themselves from others. It may reflect technological constraints such as a minimum efficient community size resulting from fixed costs, or institutional ones such as school districting or zoning laws.

We start on Figure VIa from the symmetric, full-employment equilibrium, with $\hat{x}N/m$ high-skill workers in each neighborhood. As usual, it is unstable since high-skill workers will try to regroup

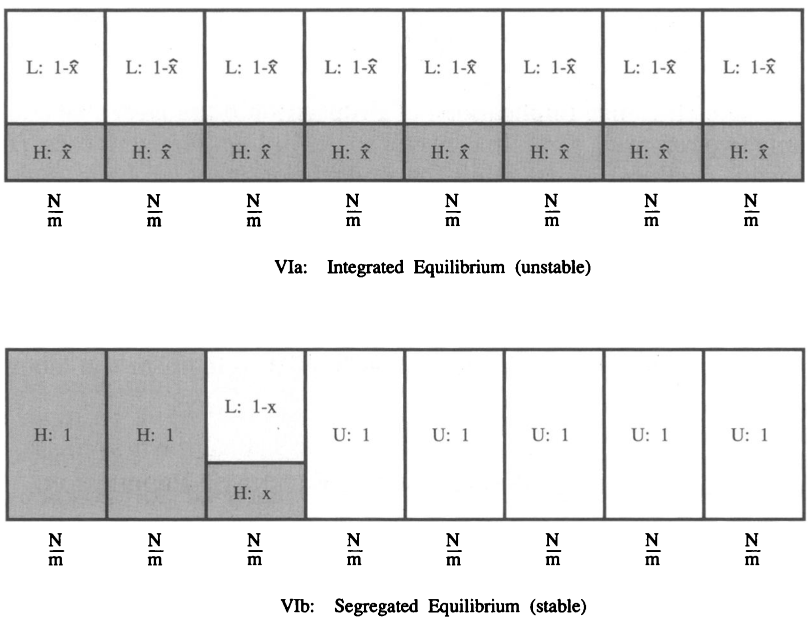


FIGURE VI
Segregation as a Limit on Production in a City with Many Communities

into homogeneous communities. Clearly, a stable equilibrium can have at most one mixed community, that is, with a proportion $0 < x_j < 1$ of high-skill workers. Moreover, maintaining assumption (16), any community without high skills is idle, no matter what happens elsewhere. As a result, there is at most one community containing low-skill workers, namely the mixed one, and at most $\max\{\bar{\rho} - 1, 0\}$ communities containing only high-skill workers; see Figure VIb. Thus, under (16) we have the following proposition.

PROPOSITION 6. As the ability to segregate, measured by m, increases, the per capital productive labor force, output, and surplus in any stable equilibrium remain bounded by

$$
(17) \qquad \begin{array}{ll} \dfrac{L}{N} < \dfrac{1}{m}, & \dfrac{H}{N} < \dfrac{\bar{\rho}}{m}, \\[2ex] \dfrac{F(H,L)}{N} < \dfrac{F(\bar{\rho},1)}{m} & \dfrac{V-v}{N} < \dfrac{F(\bar{\rho},1) - C_L(1)}{m}. \end{array}
$$

The "productivity meltdown" identified in the preceding section thus becomes worse, the easier it is for those seeking to become high-skill workers to isolate themselves from their low skill-counterparts. Their individual incentives to secede are self-defeating, preventing most of them in equilibrium from achieving the occupation they seek. In the limit where perfect segregation is feasible, its pursuit leads to a *total collapse* of the productive sector. As usual, this is a steady-state outcome, which may be reached only over the course of several generations.[14]

This striking result has a simple intuition. When everyone can "walk away" from helping to train low-skill workers, no one will provide that public good. Production would then have to be carried out with high-skill workers only. But this cannot be an equilibrium, even when it is technologically feasible: any agent in a high-skill community would want to switch to low skills, which command a higher wage and have a lower cost $C_L(1) < C_H(1)$. And if some did switch, their high-skill neighbors would secede, driving up the cost of low skills, and so on.

The results of Proposition 5 show most clearly the destructive potential of residential self-segregation in the presence of externalities in human capital investment. They formalize Bradbury,

14. The result is completely opposite to that of Montgomery [1990a]. There, peer effects only distort otherwise efficient decisions by ex ante heterogeneous individuals; self-sorting eliminates them and restores the first-best.

Downs, and Small's [1980] discussion of how city-suburban mobility may generate self-aggravating forces leading to very inferior outcomes. One particularly interesting interpretation, which seems consistent with the fate of industrial American cities, is that of a *failure to modernize*: the city is unable to adapt to new, superior technologies that require greater or more up-to-date skills from production workers. Similarly, in a less developed country a modern industrial sector fails to develop and replace agriculture and other traditional activities.

These results are also quite robust. They do not require that the planner want complete integration, or even full employment. The planner can always generate the per capita surplus $\hat{r} > 0$ corresponding to the integrated city, whereas the laissez-faire surplus becomes arbitrarily small. Acquiring low skills without any high-skill workers need not be impossible, just costly enough. Last but not least, the results do not require either type of labor to be essential to production, but only that the elasticity of substitution σ be finite. For a given m, of course, the size and composition of the sustainable productive sector depend on σ. For instance, if $F(H,L) = \theta[\alpha H^{1-1/\sigma} + (1 - \alpha)L^{1-1/\sigma}]^{\sigma/(\sigma-1)}$, then $\bar{\rho} = (\alpha/(1 - \alpha))^{\sigma}$. If $\alpha/(1 - \alpha) > 1$, maximal employment is higher, and more skewed toward high-skill workers, the higher is σ. But if $\alpha/(1 - \alpha) < 1$, greater substitutability actually contributes to shrinking production.

In practice, cities contain more than one occupationally mixed community. What matters then is the relative measure of mixed to fully homogeneous areas; this is really how $1/m$ should be interpreted. Note that individual agents seeking to achieve the best education at minimal cost will always try to achieve maximal segregation, i.e., to increase m.

V. Extensions and Related Issues

This paper has shown how the tendency for different social classes to segregate in response to local externalities or public goods influences the way in which a city works, or fails to work. We explained how stratification affects the costs of educating the labor force and the mix of skills that it acquires. We also showed how it can create ghettos, and even bring about a complete collapse of the city's productive capacity. The model is simple enough, and the basic ideas sufficiently general, to allow for many extensions.

One can think of a country as, essentially, a collection of cities plus an agricultural sector. Since the stable city equilibrium is unique and gives labor its reservation utility, the national outcome simply replicates it, with any local variations in technology or urban structure reflected in land rents. The model then shows that countries with identical endowments but different degrees of social stratification can have very unequal levels of productivity. The natural next step is to endogenize the technologies and institutions that constrain individuals' ability to isolate themselves from others.

Another country-level application of the model becomes readily apparent if the East and West sides of the city are relabeled as East and West Germany: the whole analysis can be directly transposed to the effects of migration on both regional disparities and national income.

Whatever the relevant notion of community, collective action will often interact with the individual mobility and education decisions on which we have focused. For instance, landowners or developers should compete for high-skill agents, who bring with them higher land values. Decentralized taxes and subsidies may nonetheless fail to restore the social optimum, especially if it is asymmetric: there may be no pure strategy equilibrium, because by offering a little more to the highly skilled, a community can attract a large number of them. Moreover, it neglects any impact this might have on the rest of the labor force. These issues will be explored in future work. More generally, extending the model in the direction of the local public goods and political economy literature should prove fruitful.

We have abstracted from several other features of the real world. One is heterogeneity in abilities, tastes, and endowments. As mentioned earlier, this omission is intentional, to show that one need not appeal to significant innate differences between people to explain how neighborhood effects shape the labor force, or even the existence of unemployed ghettos. Nor are imperfections in financial markets necessary: ours is a *representative agent* model, where initial resources play no role. The other omission is dynamics, as we have focused on long-run outcomes: the equilibria of our static game correspond to the steady states of an overlapping-generations model, where community spillovers operate from adults to the young.

In reality, initial conditions and transition paths do matter, as shocks to a family's human or financial wealth seem to have long-lasting effects. In particular, if residential choice is hampered by inherited characteristics such as race [Loury, 1977] or wealth constraints [Durlauf, 1992], the long-run distribution of skills and income may be path-dependent. Moreover, the idea that the manner in which agents coalesce at the local level has a powerful influence on human capital accumulation and aggregate productivity is clearly relevant to growth, where dynamics are of the essence. We pursue this link in Benabou [1992], and show that stratification often has opposite effects on growth in the short and in the long run. This presents society with an interesting intertemporal trade-off.

APPENDIX

Technical Conditions

Throughout the paper we maintain two assumptions that ensure that both kinds of labor are supplied, and that a city where agents cannot segregate has a unique full-employment equilibrium. Both hold if θ is high enough.

ASSUMPTION A3. $(w_H - w_L)(x,1 - x) - (C_H - C_L)(x)$ is decreasing in x.

ASSUMPTION A4.

$$\lim_{x \to 0} [(w_H - w_L)(x,1 - x) - (C_H - C_L)(x)] > 0$$

$$\lim_{x \to 1} [(w_H - w_L)(x,1 - x)] < \min\{\Phi'(1),0\}.$$

Note that the second part of A4 is stricter than a simple analog of the first one. This will ensure that (i) even a planner would not choose $H = 1$: see the proof of Proposition 3; (ii) the equation $\partial f(\overline{\rho},1)/\partial H = \partial f(\overline{\rho},1)/\partial L$ has a finite solution, which then bounds H/L in any equilibrium.

Proof of Proposition 1 and Related Results

Conditions A3 and A4 immediately imply the existence of a unique full-employment equilibrium (Proposition 1). A zero-employment equilibrium (z.e.e.) occurs when the absence of high-skill workers makes both factors' opportunity cost too high for firms to employ them profitably. Formally, for any pair of wages (ω_H,ω_L), let $\rho(\omega_H,\omega_L)$ denote firms' cost-minimizing factor ratio, and $\lambda(\omega_H,\omega_L)$ the corresponding marginal cost; the z.e.e. exists if and only if $\lambda(C_H(0) + v, C_L(0) + v) > 1$. In this case there is also a unique partial-employment equilibrium (p.e.e.), with $x, y, x + y$ in $(0,1)$ solving

$$\psi(x,y) \equiv \begin{bmatrix} w_H(x,y) - C_H(x) - v \\ w_L(x,y) - C_L(x) - v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\text{or} \quad \begin{bmatrix} \rho(C_H(x) + v, C_L(x) + v) = x/y \\ \lambda(C_H(x) + v, C_L(x) + v) = 1 \end{bmatrix};$$

the equivalence is by definition of ρ and λ. Indeed, for the full-employment equilibrium, by (3) and (5): $\lambda(C_H(\hat{x}) + v, C_L(\hat{x}) + v) < \lambda(w_H(\hat{x},1 - \hat{x}), w_L(\hat{x},1 - \hat{x})) \equiv 1$. So there is a unique $\tilde{x} \in (0,\hat{x})$ such that $\lambda(C_H(\tilde{x}) + v, C_L(\tilde{x}) + v) = 1$. Moreover, $\tilde{y} \equiv \tilde{x}/\rho(C_H(\tilde{x}) + v, C_L(\tilde{x}) + v) < 1 - \tilde{x}$, or else $\Delta w(\tilde{x},1 - \tilde{x}) \geq \Delta w(\tilde{x}, \tilde{y}) = \Delta w(\rho(C_H(\tilde{x}) + v, C_L(\tilde{x}) + v),1) \equiv \Delta C(\tilde{x})$, so $\tilde{x} \geq \hat{x}$ by (A3), a contradiction; hence the result. Finally, while the z.e.e. is stable (being defined by a strict inequality), the p.e.e. is saddlepoint-unstable: the Jacobian of ψ has determinant $C_L' \partial^2 F/\partial H \partial L - C_H' \partial^2 F/\partial L^2 < 0$.

Proof of Proposition 2

We first show that the symmetric allocation is unstable. Indeed, it is defined as the zero of the function,

$$\xi(x_1,x_2) \equiv \begin{bmatrix} \Delta w(x_1 + x_2, 2 - x_1 - x_2) - \Delta C_H(x_1) \\ \Delta w(x_1 + x_2, 2 - x_1 - x_2) - \Delta C_L(x_2) \end{bmatrix},$$

whose Jacobian at $(\hat{x},\hat{x})$ admits $-\Delta C'(\hat{x}) > 0$ as an eigenvalue associated with the eigenvector $(1,-1)$. Allowing for different adjustment speeds a_1 and a_2 on x_1 and x_2 leads to similar results: there is always one positive and one negative root, as long as $\Delta C' < 0$.

We now turn to segregated equilibria. Note that (9) means that $\psi(x,x') \equiv \Delta w(x + x', 2 - x - x') - \Delta C(x)$ is decreasing in x, for all x'.

1. Full-employment equilibria. We showed that the only such equilibrium with $x_1 > 0$, $x_2 > 0$ is the symmetric, unstable allocation. Therefore, in a stable equilibrium one of the following must hold:

(i) Only community 1 is mixed: $0 = x_2 < x_1 < 1$. Agents there must be indifferent between skills, so $\Delta w(x_1, 2 - x_1) = \Delta C(x_1)$, or $\psi(x_1,0) = 0$. The rent differential must make L workers indifferent between the two communities, so $r_1 - r_2 = C_L(0) - C_L(x_1)$.

(ii) Only community 2 is mixed: $0 < x_2 < x_1 = 1$. Agents in that community must be indifferent between the two skills, so $\Delta w(1 + x_2, 1 - x_2) = \Delta C(x_2)$, or $\psi(x_2,1) = 0$. Now it is H workers who must be indifferent between communities, so $r_1 - r_2 = C_H(x_2) - C_H(1)$.

(iii) Complete segregation: $x_1 = 1, x_2 = 0$. Labor market equilibrium then requires that $\psi(1,0) = \Delta w(1,0) - \Delta C(1) \geq 0 \geq \Delta w(1,0) - \Delta C(0) = \psi(0,1)$. Residential equilibrium requires that $r_1 - r_2 = \Delta w(1,0) + C_L(0) - C_H(1)$.

Finally, rents must leave all agents with utility v: $r_2 = w_L(x_1 + x_2, 2 - x_1 + x_2) - C_L(x_2) - v$, $r_1 = w_H(x_1 + x_2, 2 - x_1 + x_2) - C_H(x_1)$. In each case the properties of (x_1, x_2) with respect to ψ ensure consistency with the condition on $r_1 - r_2$.

Having characterized potential full-employment equilibria, we now show existence and uniqueness.

Case 1. $\Delta w(1) \leq \Delta C(1/2)$; i.e., $\hat{x} \leq 1/2$. Note that $\psi(2\hat{x},0) = \Delta w(2\hat{x}, 2 - 2\hat{x}) - \Delta C(2\hat{x}) = \Delta C(\hat{x}) - \Delta C(2\hat{x}) > 0$. First, since $\Delta w(1) < \Delta C(0)$, $0 > \psi(0,1) > \psi(x,1)$ for all x; hence there can be no equilibrium of type (ii). Two cases are possible:

(a) If $\Delta C(1) \leq \Delta w(1) < \Delta C(1/2)$, then $\psi(x,0) > \psi(1,0) > 0$ for all $x < 1$, and the unique equilibrium is of type (iii), with $\psi(1,0) > \psi(0,1)$.

(b) If $\Delta w(1) < \Delta C(1)$, then $\psi(1,0) < 0 < \psi(2\hat{x},0)$. Hence there is a unique $\tilde{x}_1 \in (2\hat{x},1)$ such that $\psi(\tilde{x}_1,0) = 0$, defining a unique equilibrium, of type (i).

Case 2. $\Delta w(1) > \Delta C(1/2)$; i.e., $\hat{x} > 1/2$. Note that $\psi(2\hat{x} - 1,1) = \Delta w(2\hat{x}, 2 - 2\hat{x}) - \Delta C(2\hat{x} - 1) = \Delta C(\hat{x}) - \Delta C(2\hat{x} - 1) < 0$. First, since $\Delta w(1) > \Delta C(1)$, $\psi(x,0) \geq \psi(1,0) > 0$ for all x; hence there can be no equilibrium of type (i). Two cases are possible:

(a) If $\Delta C(0) \geq \Delta w(1) > \Delta C(1/2)$, then $\psi(x,1) \leq \psi(0,1) \leq 0$ for all x, and the unique equilibrium is of type (iii), with $\psi(1,0) > \psi(0,1)$.

(b) If $\Delta w(1) > \Delta C(0)$, then $\psi(0,1) > 0 > \psi(2\hat{x} - 1,1)$; hence there is a unique $\tilde{x}_2 \in (0, 2\hat{x} - 1)$ such that $\psi(\tilde{x}_2,1) = 0$. It defines the unique equilibrium, which is of type (ii).

2. *Unemployment equilibria.* We now show that no such equilibrium can exist and be stable, except possibly for one where nobody works. Given Assumptions A1 and A2, high-skill workers will always outbid low-skill workers, and the latter will always outbid unemployed agents, for any available land in community 1. Hence stability of equilibrium requires one of the following three cases.

Case 1. High-skill workers fill up community 1 completely, i.e., $x_1 = 1$; then $H/L = (1 + x_2)/y_2 > 1 > \hat{x}/(2 - \hat{x})$, so $w_L(H,L) > w_L(\hat{x},2 - \hat{x}) > C_L(0) + v \geq C_L(x_2) + v$ by (10). Thus, the unemployed in community 1 would rather acquire low skills, a contradiction.

Case 2. High-skill workers are in community 1 only, and low-skill agents are in both. Community 1 can then have no unemployed agents; so $0 < x_1 = 1 - y_1 < 1, 0 = x_2 < y_2 \leq 1$. But then $\rho \equiv x_1/(1 - x_1 + y_2) \in (x_1/(2 - x_1), x_1/(1 - x_1))$, so $\Delta w(x_1,1 - x_1) < \Delta C(x_1)$, implying that $x_1 > \hat{x}$. Therefore, $w_L(\rho,1) > w_L(x_1,2 - x_1) > w_L(\hat{x},2 - \hat{x}) > C_L(0) + v$, yielding the same contradiction.

Case 3. Both high- and low-skill workers are in community 1 only: $0 < x_1 < x_1 + y_1 \leq 1, x_2 = y_2 = 0$. But then the labor market reduces to community 1, so the results shown with Proposition 1 for the integrated city imply that the only possible stable equilibrium has $x_1 = 1 - y_1 = \hat{x}$. But since $w_L(\hat{x},1 - \hat{x}) > C_L(0) + v$, we are led to the same contradiction.

Q.E.D.

Proof of Proposition 3

Recall first that the planner can always get $\hat{r} > 0$ per capita, by implementing the symmetric equilibrium. Note also from (3), which defines $\hat{x}$, that as θ increases to $+\infty$, $\hat{x}/(1 - \hat{x})$ increases to a limit of $\bar{\rho}$.

In any allocation where $x_j + y_j < 1$, the first-order conditions for (6) show that $w_H - C_H(x_j) = v + x_jC'_H(x_j) + (1 - x_j)C'_L(x_j) < v = w_L - C_L(x_j)$. So unemployment in community 1 implies that $V < 0$ (recall that $x_1 \geq x_2$) and cannot be optimal. Suppose now that there is unemployment in community 2 only. We shall denote $\rho \equiv H/L = (x_1 + x_2)/(1 - x_1 + y_2)$. From (6) and the associated first-order conditions, we have

$$2V \leqslant x_1(w_H(\rho,1) - C_H(x_1) - v) + y_1(w_L(\rho,1) - C_L(x_1) - v) + v.$$

Case 1. If $x_1 = 1$, then $\rho > 1$, so $V < (w_H(\rho,1) - C_H(x_1) - v)/2 < (w_H(1,1) - C_H(1) - v)/2$. But under constant returns to scale, and by definition of $\bar{\rho}$:

$$(w_H(1,1) + w_L(1,1)/2 = F(1/2,1/2) \leq F(\bar{\rho},1 - \bar{\rho}) = w_H(\bar{\rho},1) = w_L(\bar{\rho},1).$$

So,

$$\hat{r} - V > w_L(\hat{x},1-\hat{x}) - w_L(\bar{\rho},1)$$
$$+ \frac{w_L(1,1) + C_L(1) - 2C_H(\hat{x}) - v}{2} = \theta\left[\frac{\partial f(\hat{x},1-\hat{x})}{\partial L} - \frac{\partial f(\bar{\rho},1)}{\partial L} + \frac{1}{2}\frac{\partial f(1,1)}{\partial L}\right] + \frac{C_H(1) - 2C_L(\hat{x}) - v}{2}.$$

Now, as $\theta \to +\infty$, the right-hand side becomes equivalent to $(\theta/2)\,\partial f(1,1)/\partial L$, and hence tends to $+\infty$. So for θ large enough, the allocation cannot be optimal.

Case 2. If $x_1 < 1$, then $\Delta w = \Delta C(x_1) + x_1 C_H'(x_1) + (1 - x_1)C_L'(x_1) \equiv \Phi'(x_1)$. Therefore, $W - v < (w_L(\rho,1) - C_L(x_1) - v)/2 < (w_L(\rho,1) - C_L(1) - v)/2$, so that

$$2(\hat{r} - W) > \theta\left[2\frac{\partial f(\hat{x},1-\hat{x})}{\partial L} - \frac{\partial f(\rho,1)}{\partial L}\right] + C_L(1) - 2C_L(\hat{x}) - v.$$

So for θ large enough, the optimality of V requires that $\partial f(\rho,1)/\partial L > 2\partial f(\hat{x},1-\hat{x})/\partial L \approx 2\partial f(\bar{\rho},1)/\partial L$. This in turn requires that $\rho > \bar{\rho}$, hence $x_1 > \bar{\rho}/(2+\bar{\rho})$ since $\rho = (x_1 + x_2)/(1 - x_1 + y_2) \le 2x_1/(1 - x_1)$. But now for θ large enough, $\Delta w = \theta[\partial f(\rho,1)/\partial H - \partial f(\rho,1)/\partial L] = \Phi'(x_1)$ requires that $\partial f(\rho,1)/\partial H - \partial f(\rho,1)/\partial L$ be close to zero (as long as Φ' is bounded on $(\bar{\rho}/(2+\bar{\rho}),1]$, which we shall assume), i.e., that ρ be close to $\bar{\rho}$. This contradicts $\partial f(\rho,1)/\partial L > 2\partial f(\bar{\rho},1)/\partial L$.

Q.E.D.

Proof of Proposition 4

With Φ convex, the planner's problem reduces to maximizing the strictly convex function $V(x)/2 = F(x,1-x) - \Phi(x)$ over x in $[0,1]$. Since $V'(x) = \Delta w(x,1-x) - \Phi'(x) = \Delta w(x,1-x) - \Delta C(x) - xC_H'(x) - (1-x)C_L'(x)$, the boundary conditions A3 and A4 ensure a unique, interior solution.

Q.E.D.

Proof of Proposition 5

Note first that under (10), if $x_j = 0$ in any community j, then $y_j = 0$. If there is a full-employment equilibrium, it clearly must be the one described in Proposition 2. When $\hat{x} < 1/2$, this requires that $x_2 = 0$, hence $y_2 = 0$ by (10), a contradiction. When $\hat{x} > 1/2$, it is defined by the unique solution to $\Delta w(1 + \tilde{x}_2, 1 - \tilde{x}_2) = \Delta C(\tilde{x}_2)$, with $0 < \tilde{x}_2 < 2\hat{x} - 1$. This is indeed an equilibrium if $w_L(1 + \tilde{x}_2, 1 - \tilde{x}_2) > C_L(\tilde{x}_2) + v$. If not, there is no full-employment equilibrium. Next, consider equilibria with unemployment.

(a) If $x_2 = 0$, then $y_2 = 0$, and only community 1 operates. Under constant returns to scale, its equilibria are those of an integrated city, scaled down to half-size. We thus know that the only stable ones involve either shutdown, or full employment according to $x_1 = 1 - y_1 = \hat{x}$.

(b) If $x_2 > 0$, residential stability requires that $x_1 = 1$. Then (x_2, y_2) must solve

$$\psi(x_2,y_2) \equiv \begin{bmatrix} w_H(1 + x_2,y_2) - C_H(x_2) - v \\ w_L(1 + x_2,y_2) - C_L(x_2) - v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The Jacobian $D\psi(x_2,y_2)$ has determinant $C_H' \cdot (-\partial^2F/\partial L^2) + C_L'(-\partial^2F/\partial H\partial L) < 0$. This implies that at any point of intersection of the curves $w_H(1 + x,y) - C_H(x) = v$ and $w_L(1 + x,y) - C_L(x) = v$, the latter has a higher slope, so there is at most one such solution. Moreover, it must be saddlepoint-unstable.

Q.E.D.

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