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EDUCATION POLICIES: EQUITY, EFFICIENCY AND VOTING EQUILIBRIUM*

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This paper investigates the effects of two specific forms of intervention in the market for education: an ability test for admission to university and a subsidy to tuition fees financed through general taxation. Both these measures enhance equality of opportunity, but their equity and efficiency effects are ambiguous. This ambiguity is reflected in the political economy equilibrium which would emerge as the result of voting on the level of the ability test and of the subsidy.

This paper studies the effects on individual choices of attending university and on the composition of the group of individuals who attend university of two specific education policies. These policies are:

- the imposition of an ability test for admission to university; and
- a uniform subsidy to university attendance financed by a proportional tax on (current) income.

These are both relevant in practice, in view of the current debate on higher education in various countries in the world. For example, the British government elected in 1997 has required university students to pay a fee, thereby reducing their subsidy from general taxation. With regard to (effective) admission thresholds, they exist in some university systems (e.g. in the United Kingdom and in the United States). In other countries (such as France and Italy), there are no restrictions for admission to university (with some exceptions), but their introduction is being debated (see Gary-Bobo and Trannoy (1998) for a theoretical analysis of various admission mechanisms from a French perspective).

We present a simple model of the individual choice of investment in education. In this model, households differ both with regard to the current (parental) income, and with regard to the children's future expected labour market income: households, therefore, differ both in the *ability to pay for* and in the *potential to benefit from* the investment in education. We show that, in the absence of government intervention, even in the extreme and unrealistic situation where households can borrow at the competitive market rate to finance their investment in education, there is a 'wealth' bias in the individual education decision: children from better-off households are over-represented

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among university students (Proposition 1 below).[1] According to Roemer (1998), opportunities are equalised when a person's expected earnings are a function only of her effort and not of her circumstances: in our model, circumstances are given by family background (parental income) and the expectation of earnings is taken over the possible realisations of innate ability. With this interpretation, equality of opportunity is equivalent to independence of expected earnings from parental income, and, therefore, our analysis implies that equality of opportunity does not emerge as a *laissez-faire* outcome.

De Fraja (1999) derives the optimal policy of a welfare maximising government, which can select an income related tuition fee. Here we take a more applied approach and study the two policy measures set out at the beginning. We show that both an ability test and a subsidy to tuition fees unambiguously increase equality of opportunity: students from high income households are less over-represented in the student population than in the absence of any intervention. When more general equity considerations are taken into account, the overall picture is less clear-cut: an admission test benefits better-off households with brighter children the most; and a tuition fee subsidy has the negative redistributive effect that income from worse off households (both in current and in future terms) is used to subsidise university attendance by better-off households. These policies also have ambiguous efficiency effects: an ability test makes the composition of the student body more efficient, but it also reduces the overall university attendance below the market level, which is already short of the efficient level; conversely, a subsidy may expand the university sector beyond the efficient size.

In line with the approach of the paper, we determine the level at which the policy variable is set, not as the solution to a government's maximisation problem, but as the outcome of a vote. The ambiguous equity effects of the policies are reflected in the voting behaviour. In choosing how to vote on a test, individuals are influenced exclusively by their children's ability, not everybody votes, and the median voter has income above the median. This is also true when voting on the extent of the subsidy: in this case, a partial 'end against the middle' phenomenon occurs (Epple and Romano, 1996*a*, *b*): better-off households would unambiguously like a lower subsidy, as would some of the worse-off households, those whose children are not very bright; only poor households with bright children would prefer an increase in the subsidy.

The paper is organised as follows: the model is introduced in Section 1, and Section 2 studies the individual households' decision process. Section 3 presents briefly the equilibrium in the absence of any government intervention. Sections 4 and 5 contain the main contributions of the paper, the analysis of the effects of ability tests and subsidies to tuition fees, respectively. Finally, Section 6 is a brief conclusion.

[1] De Fraja (1999) extends this result to the case in which households can also insure against labour market income risk at an actuarially fair price.

1. The Model

We use a simplified version[2] of De Fraja (1999). The economy comprises an infinite number of households, each constituted by a mother and a daughter. Households differ in two respects: the mother's after tax income, denoted by $Y \in [\underline{Y}, \overline{Y}]$, and the daughter's innate ability to earn income in the future, denoted by $\theta \in [\underline{\theta}, \bar{\theta}]$. Formally:

ASSUMPTION 1. *The number of households is normalised to 1. Income and utility are independently distributed in* $[\underline{Y}, \overline{Y}] \times [\underline{\theta}, \bar{\theta}]$. *The marginal distributions are denoted by* $H(Y)$, *with* $H'(Y) = h(Y)$, *and* $\Phi(\theta)$, *with* $\Phi'(\theta) = \phi(\theta)$.

Individuals derive utility from consumption. In each household, decisions are taken by the mother: she is altruistic, and she maximises a function given by the sum of her own and her daughter's utility from consumption, $u_m(c_m)$ and $u(c)$, respectively. u_m satisifies $u'_m(c_m) > 0$, $u''_m(c_m) \leqslant 0$; $u(c)$ is a von Neumann-Morgernstern utility function, not necessarily the same as u_m. $u(c)$ is defined for $c > 0$, and is assumed to satisfy, for every $c > 0$, $u'(c) > 0$, $u''(c) \leqslant 0$, $\lim_{c\to 0} u'(c) = +\infty$, and $d[-u''(c)/u'(c)]/dc < 0$. The last says that the daughter's utility function displays decreasing absolute risk aversion (this is plausible: Hirschleifer and Green, 1992, pp. 87–8).

The mother allocates her income (which is given when she takes this decision) choosing how much to consume herself, how much to transfer to her daughter in monetary form, and how much to invest in the daughter's education. This is analytically equivalent to a single generation set-up where individuals have an initial endowment when young; in the context of education, the parent and child formulation seems closer to reality, and more natural in the light of the equality of opportunity debate: the mother chooses the daughter's education, and therefore the daughter's opportunity set depends on her mother's income. We simplify the model with the assumption that education can only take two values: either going to college/university, or not.

In addition to the transfer from her mother, a daughter receives income from her participation in the labour market. The amount of income is a random variable whose realisation depends on education and innate ability. If the daughter has invested in education, her labour market income is given by y_H with probability $P(\theta)$, and by $y_L < y_H$ with probability $1 - P(\theta)$. If an individual has not gone to university, then her income is given by y_H with probability $P_N(\theta)$, and by y_L with probability $1 - P_N(\theta)$.

ASSUMPTION 2. $P(\theta)$ *and* $P_N(\theta)$ *are continuous functions satisfying* $P'(\theta) > P'_N\theta$, *for every* $\theta \in [\underline{\theta}, \bar{\theta}]$.

The economic content of Assumption 2 is that education and ability are

[2] The main difference is the absence of labour market effort from the production technology. Here, as in Wigger and von Weiszäcker (1997), this implies that perfect insurance markets must also be ruled out, otherwise the market mechanism would in fact ensure equality of opportunity; see De Fraja (1999), especially Corollary 2, for a discussion.

complements: abler individuals benefit more from education. In a formulation where both education and ability vary continuously, this is equivalent to the second cross derivative of expected income being positive (see De Fraja (1998)). Note also that we are not implying any specific relationship between education and income risk. The variance of income is $P(\theta)[1-P(\theta)](y_H - y_L)^2$ for educated individuals, and $P_N(\theta)[1 - P_N(\theta)](y_H - y_L)^2$ for uneducated ones, and their difference is in general unsigned.[3]

We now introduce a deliberately unrealistic hypothesis on the functioning of capital markets: the monetary transfer is unconstrained. This implies that the mother can pay for her daughter's tuition fee, and even finance her own consumption, by borrowing against her daughter's future income at the market interest rate.[4] This 'stacks the deck' in favour of market provision; and yet, even in this extreme case, as Proposition 1 below shows, there remains a role for government intervention in the provision of education.

Going to university has a monetary cost $k > 0$. This is taken as given by households, and is determined endogenously in Section 3.

2. Households Decisions

The mother decides to undertake the investment in human capital on behalf of her daughter, if and only if the household utility for doing so, denoted by $U(\theta, Y, k)$, and given by

$$U(\theta, Y, k) = \max_t \{u_m(Y - t - k) + P(\theta)u(y_H + t) + [1 - P(\theta)]u(y_L + t)\}, \tag{1}$$

is at least equal to the utility obtained for not going to university, $U^N(\theta, Y)$:

$$U^N(\theta, Y) = \max_t \{u_m(Y - t) + P_N(\theta)u(y_H + t) + [1 - P_N(\theta)]u(y_L + t)\}. \tag{2}$$

Let $t(\theta, Y)$ and $t^N(\theta, Y)$ be the solutions to the above problems. To avoid unrewarding taxonomy, we make the following assumption.

ASSUMPTION 3. *For every* $Y \in [\underline{Y}, \overline{Y}]$, $U(\underline{\theta}, Y, k) < U^N(\underline{\theta}, Y)$ *and* $U(\bar{\theta}, Y, k) > U^N(\bar{\theta}, Y)$.

In words, whatever their parental income, the ablest children ($\theta = \bar{\theta}$) go to

[3] Machin (1996) presents some evidence suggesting that wage inequality is lower for educated individuals. In contrast, Pereira and Martins (2000) find that 'in eleven out of fifteen European countries, the dispersion in earnings increases with educational levels' and only 'in two [countries] earnings are less dispersed for higher educational levels' (p. 3). At any rate, these estimates are not necessarily an indication of the relationship between education and income risk. This is because the population variance is affected both by the idiosyncratic risk and by the composition of the sample: individuals in one group may all have a non-random income, but differ greatly in their income level.

[4] This extreme assumption captures the intention of the proponents of educational loans (Shell *et al.* (1968), or Barr (1991) for a more recent proposal). Such loans aim to remove the obstacle to finance education constituted by the impossibility of borrowing in the absence of collateral. Note also that the issue of default is irrelevant: the condition $\lim_{c\to 0} u'(c) = +\infty$ implies that the mother will always ensure that the daughter has sufficient funds to repay her debt.

university, and the least able ($\theta = \underline{\theta}$) do not. The following result describes the household decision with respect to university attendance.

PROPOSITION 1. *There exists a function defined in* $[\underline{Y}, \bar{Y}]$, *denoted by* $\theta^m(Y)$, *such that a household with income* Y *sends the daughter to university if and only if she has ability at least* $\theta^m(Y)$.

(i) $\theta^m(Y)$ *is horizontal if either* $u''_m(c_m) = 0$ *or* $u''(c) = 0$;
(ii) *if both* $u''_m(c_m) < 0$ *and* $u''(c) < 0$, *then* $\theta^m(Y)$ *is strictly decreasing.*

Proof. A mother is indifferent between sending her daughter to university or not if $U(\theta, Y, k) = U^N(\theta, Y)$, that is, if (we write u_X for $u[y_X + t(\cdot)]$ and u_X^N for $u[y_X + t^N(\cdot)]$, $X = H, L$):

$$u_m(Y - t - k) + Pu_H + (1 - P)u_L = u_m(Y - t_N) + P_N u_H^N + (1 - P_N)u_L^N. \tag{3}$$

Notice first of all that, whenever (3) holds, then $U_\theta(\theta, Y, k) > U_\theta^N(\theta, Y) > 0$: the assumption $P_\theta(\theta) > P_\theta^N(\theta)$ implies that the increase in the expected household income is greater for a household where the daughter goes to university; given that $U(\theta, Y, k) = U^N(\theta, Y)$, an increase in expected income implies an increase in expected utility, so that $U_\theta(\theta, Y, k) > U_\theta^N(\theta, Y)$. This and Assumption 3 imply that there is a unique value of $\theta \in (\underline{\theta}, \bar{\theta})$ which solves (3) for any given Y. This is the required $\theta^m(Y)$. Also note that $\theta > \theta^m(Y)$ implies that the LHS of (3) is strictly greater than the RHS, and vice versa. This establishes the existence of the locus $\theta^m(Y)$.

To complete the proof of the proposition, we need to determine the sign of $d\theta^m(Y)/dY$. Total differentiation of (3) gives:

$$[u'_m(Y - t - k) - u'_m(Y - t_N)]\,dY = -[U_\theta(\theta, Y, k) - U_\theta^N(\theta, Y)]\,d\theta, \tag{4}$$

noting that the terms in $dt/d\theta$, dt/dY, $dt_N/d\theta$, dt_N/dY all vanish by the envelope theorem. Since $U_\theta(\theta, Y, k) > U_\theta^N(\theta, Y)$, $\theta^m(Y)$ is a differentiable function, with

$$\frac{d\theta^m(Y)}{dY} = \frac{u'_m(Y - t - k) - u'_m(Y - t_N)}{-[U_\theta(\theta, Y, k) - U_\theta^N(\theta, Y)]}. \tag{5}$$

If $u''_m(c_m) = 0$, then, clearly, $d\theta^m(Y)/dY = 0$. If $u''(c) = 0$, then from the first order conditions of the two transfer problems, we have $u'_m(\cdot) = u'(\cdot) =$ constant, and therefore, again, $d\theta^m(Y)/dY = 0$. Now let $u''_m(c_m) < 0$ and $u''(c) < 0$.

LEMMA 1. *For every* $k > 0$, *and for every pair* (θ, Y) *satisfying* $\theta = \theta^m(Y)$, *the optimal transfer in the two cases of attendance and non-attendance at university satisfies:* $u'_m(Y - t - k) > u'_m(Y - t_N)$.

Proof. Note that, if a household is indifferent, then $P > P_N$. Rewrite (3) as

$$u_m(Y - t - k) + (P - P_N)u_H^* + (1 - P - P_N)u_L^* = u_m(Y - t_N) + u_L^*, \tag{6}$$

where $u_H^* = (Pu_H - P^N u_H^N)/(P - P^N)$, and $u_L^* = [(1 - P^N) u_L^N - (1 - P) u_L]/(P - P^N)$. According to (6), a household with income Y where the daughter has ability θ, is indifferent between paying $t + k$ for a prospect yielding utility u_H^* with probability $(P - P_N)$, and utility u_L^* with probability $(1 - P + P_N)$ and paying t_N for receiving utility u_L^* with certainty. The assumption of decreasing absolute risk aversion implies that a mother with income $Y + \epsilon$ strictly prefers the risky choice; therefore:

$$u_m(Y + \epsilon - t - k) + (P - P_N) u_H^* + (1 - P + P_H) u_L^* > u_m(Y + \epsilon - t_N) + u_L^*.$$

Now subtract (6) from the above to obtain:

$$u_m(Y + \epsilon - t - k) - u_m(Y - t - k) > u_m(Y + \epsilon - t_N) - u_m(Y - t_N),$$

divide through by ϵ, and take the limit as $\epsilon \rightarrow 0$, to establish the Lemma. □

By Lemma 1, the numerator in (5) is positive, and this completes the proof of Proposition 1. □

The locus $\theta^m(Y)$ is the *market indifference curve*, depicted as the solid line in Fig. 1 below. By Proposition 1, it is in general decreasing. This means that there exists a distributional bias in university attendance: individuals whose parents are better-off are more likely to attend university. In other words, the free market does not provide equality of opportunity, even when it is possible to borrow costlessly against future income.[5]

The intuition behind Proposition 1 is natural. If the daughter has decreasing absolute risk aversion, then the decision maker in the household, the mother, also has decreasing absolute risk aversion. Investment in education is risky, and, if the decision maker has decreasing absolute risk aversion, she is more willing to bear risk if her income is higher; in other words, she requires a lower expected return in order to opt for an investment of a given riskiness. And, as the investment in education of a less bright daughter has a lower return, it follows that a better-off mother is more willing to send a less bright daughter to university.[6] As Proposition 1.i illustrates, concavity of the utility functions is essential: if either the mother or the daughter have constant marginal utility of consumption, then the market ensures equality of opportunity: see De Fraja (1999), Corollary 3, and the related discussion.

3. The Market Equilibrium

We now determine the total number of students who attend university in absence of any intervention. To this end, let $D(k)$ be the demand for university

[5] Relaxing this assumption, for example by having a strictly positive interest rate for borrowing, would create a kink at income level Y_0 satisfying $t[\theta^m(Y_0), Y_0] = 0$, with a clockwise rotation for $Y < Y_0$.

[6] Proposition 1 holds irrespectively of the relationship between the income variance for educated and uneducated individuals. The point here is that attendance at university is a *risky* investment in the sense of involving the *certain* sacrifice of the cost of attendance, in exchange for the benefit of a *random* expected higher labour market income, and this is where attitude towards risk matters.

education, as a function of the tuition fee. This is given by (see Apostol, 1974, p. 396–8):

$$D(k) = \int_{\underline{Y}}^{\bar{Y}} \int_{\theta^m(Y)}^{\bar{\theta}} \phi(\theta)\, h(Y)\, d\theta\, dY. \tag{8}$$

D is a function of k because k affects the position of the market indifference curve, $\theta^m(Y)$. Clearly $D'(k) < 0$ (see De Fraja, 1999).

We assume that university education is supplied by a competitive industry which is subject to aggregate decreasing returns to scale in the number of students.[7] Formally:

ASSUMPTION 4. *Let n be the total number of student attending university. The total cost of the university system is given by $C(n)$, satisfying $C'(n) > 0$, $C''(n) \geqslant 0$, and* $\lim_{n\to 0} C'(n) = 0$, $\lim_{n\to 1} C'(n) = +\infty$.

The market equilibrium is now simply the intersection of demand and supply, that is, the simultaneous solution in k and n of:

$$D(k) = n, \qquad k = C'(n). \tag{9}$$

Since D is decreasing and $C'(n)$ increasing, there is a unique solution to (9). Let it be denoted by n^m and k^m. Note that $n^m \in (0, 1)$ because of the last part of Assumption 4. The assumption of decreasing returns to scale implies that the sector makes strictly positive profits. If these may not be distributed in monetary form, they could be used internally by the institutions, for example by financing research.

To end this section, it is useful to determine the benchmark constituted by the maximisation of the rate of return of the university sector, given by the difference between the total *income* gain and the total cost.

PROPOSITION 2. *The rate of return of the university sector is maximised if an individual attends university if and only if her ability is θ^* or higher, where θ^* is defined by:*

$$[P(\theta^*) - P_N(\theta^*)]\, y_H + [1 - P(\theta^*) + P_N(\theta^*)]\, y_L = c'[1 - F(\theta^*)]. \tag{10}$$

Proof. The maximand is $\int_{\theta^*}^{\bar{\theta}} \{[P(\theta) - P_N(\theta)]\, y_H + [1 - P(\theta) + P_N(\theta)]\, y_L\} f(\theta)\, d\theta - c'[1 - F(\theta^*)]$, and (10) follows. □

That is, production efficiency requires equality of opportunity.

[7] Of course, in practice there are increasing returns to scale for a sufficiently small enrolment. Assumption 4 is equivalent to assuming that the minimum efficient size of the university system is quite small, and that, in practice, an increase in the number of students increases the average cost.

4. An Ability Test for Access to University

The market mechanism does not ensure equality of opportunity (Proposition 1), nor a productively efficient outcome (Proposition 2). The government may therefore wish to intervene in this sector. In the companion paper we study the second best policy of a utilitarian government who designs the complete schedule of income related tuition fees. Here we consider a more limited approach, and study the effects of two specific forms of intervention.

In this section, we assume that the government can require that all students admitted to university have ability at or above a certain level, $\theta^f \in [\underline{\theta}, \bar{\theta}]$. We begin by determining each household preferred level of the ability test. It is immediate to establish the following.

PROPOSITION 3. *The preferred ability threshold for admittance to university of a household whose daughter has ability θ is $\theta^f = \theta$.*

Proof. Consider a household where the daughter has ability θ: it cannot benefit from an ability level above θ, as this would simply restrict choice. If $\theta^f < \theta$ the household would benefit from a increase to $\theta^f + \epsilon < \theta$, as this would determine a fall in the total number of students, and therefore decrease the cost of attendance to university, without preventing the daughter from attending. □

We now determine the level at which the test is set as a result of a majority voting process. We assume that only the mothers vote, and that a mother votes for a test level if and only if the imposition of a test level may affect the household utility (alternatively, and equivalently, a mother indifferent between two policies could vote for either policy with equal probability). Voters are aware of the future consequence of their vote.

We begin by determining the demand for university places in the presence of a test. Let $\theta^{m^{-1}}$ denote the inverse function of θ^m, so that $\theta^{m^{-1}}(\theta_0)$ is the income level such that the household with this income and daughter's ability θ_0 is indifferent between going and not going to university.

LEMMA 2. *Let $D^T(k, \theta_0)$ be the demand function when the tuition fee is k and the threshold abiltity is θ_0. Then*

$$D^T(k, \theta_0) = \int_{\underline{Y}}^{\bar{Y}} \int_{\theta^m(Y)}^{\bar{\theta}} \phi(\theta)\, h(Y)\, d\theta\, dY - \int_{\theta^{m^{-1}}(\theta_0)}^{\bar{Y}} \int_{\theta^m(Y)}^{\theta_0} \phi(\theta)\, h(Y)\, d\theta\, dY.$$

Proof. The first term is the number of households who apply to university (this number is unaffected by the test) from which the second term (the individuals who do not pass the test) must be subtracted: only households with income above $\theta^{m^{-1}}(\theta_0)$ are affected by the test, and at income level Y, these are the households with ability between $\theta^m(Y)$ and θ_0. □

The next results determines the households who are affected by a test, and who therefore vote.

PROPOSITION 4. *There exist a locus $\theta^T(Y)$ such that households where the daughter has ability $\theta \geqslant \theta^T(Y)$ vote in favour of a test $\theta^f = \theta$. If $u''_m(c_m) < 0$ and $u''(c) < 0$, then this locus is strictly decreasing, and satisfies $\theta^T(\overline{Y}) = \theta^m(\overline{Y})$ and $\theta^T(Y) < \theta^m(Y)$ for $Y < \overline{Y}$.*

Fig. 1 depicts the market indifference curve, $\theta^m(Y)$ (solid), the curve $\theta^T(Y)$ (dashed), which we can define as the 'voting indifference curve', and the threshold test, the horizontal solid line θ^f.

Proof. A mother votes if and only if she would send her daughter to university were her preferred test level chosen. We begin by determining the locus of points such that a mother is indifferent between voting and not voting. This is the sought function $\theta^T(Y)$, the 'voting indifference curve'.

Consider first the function $K(\theta_0)$. This is unit cost which would result if the admission level were set at θ_0, and is determined with the procedure of Section 3. $K(\theta_0)$ is the solution in k of:

$$D^T(k, \theta_0) = n \qquad k = C'(n). \tag{11}$$

The function $K(\theta_0)$ is decreasing: a tougher test implies fewer students at university, and hence a lower marginal cost of tuition. Now, the 'voting indifference curve' is simply the locus of points representing combinations of Y and θ satisfying:

$$U[\theta, Y, K(\theta)] = U^N(\theta, Y). \tag{12}$$

Exactly as in (3), except for the fact that now the mother takes k as a function of θ. Total differentiation of (12) gives:

$$[U_\theta(\cdot) + U_k(\cdot)K'(\theta) - U^N_\theta(\cdot)]\,d\theta = [U^N_Y(\cdot) - U_Y(\cdot)]\,dY,$$

and therefore the slope of the voting indifference curve is given by:

$$\frac{d\theta^T}{dY} = \frac{U^N_Y(\cdot) - U_Y(\cdot)}{U_\theta(\cdot) - U^N_\theta(\cdot) + U_k(\cdot)K'(\theta)}.$$

The numerator is negative by Lemma 1, and the denominator is positive as $U_\theta(\cdot) > U^N_\theta(\cdot)$ and $U_k(\cdot) < 0$ and $K'(\theta) < 0$.

Next, consider any point Y_0 where this voting indifference curve intersects the market indifference curve. At these points the market indifference curve is steeper:

$$\left|\frac{d\theta^m}{dY}\right|\Bigg|_{Y=Y_0} = \frac{-U^N(Y_0) + U_Y(\cdot)}{U_\theta(\cdot) - U^N_\theta(\cdot)} > \frac{-U^N(Y_0) + U_Y(\cdot)}{U_\theta(\cdot) - U^N_\theta(\cdot) + U_k(\cdot)K'(\theta)} = \left|\frac{d\theta^T}{dY}\right|\Bigg|_{Y=Y_0}.$$

Therefore there can be at most one such intersection, and the market indifference curve is above (below) the voting indifference curve for $Y < Y_0$ (for $Y > Y_0$). Finally note that these two curves do in fact intersect at $Y = \overline{Y}$, because the household characterised by the pair $(\overline{Y}, \theta^m(\overline{Y}))$ is indifferent between voting and not voting when the test is set at $\theta^m(\bar{Y})$. This establishes the Proposition. □

Fig. 1 illustrates the distributional gains and losses of the introduction of an ability test for admittance to university. Area A represents high income-high ability households. These households gain: they still send the daughter to university, but, because the test reduces the number of students, the cost of attending university is lower, and the household's income, after paying for the tuition fee, is higher than without the test.[8] Households in the small area above the locus $\theta = \theta^f$ and in between the loci $\theta^T(Y)$ and $\theta^m(Y)$ (area E) also gain from the introduction of the test, but for a different reason: without the test they find university too expensive, but the cost reduction brought about by the test is sufficient for them to be willing to pay for tuition. Households in area B have relatively high income and middling daughter's ability. These are the households who are made worse-off by the test, as they no longer attend university: not because of cost, which they were willing to pay even at the higher, pre-test, level, but because they do not pass the admission test. All other households are as well-off as in the absence of the test. Note however that some households vote for a test level but the level determined by the majority voting is too high for them. These are the household whose ability is between $\theta^T(Y)$ and $\theta^m(Y)$ (households in area C).

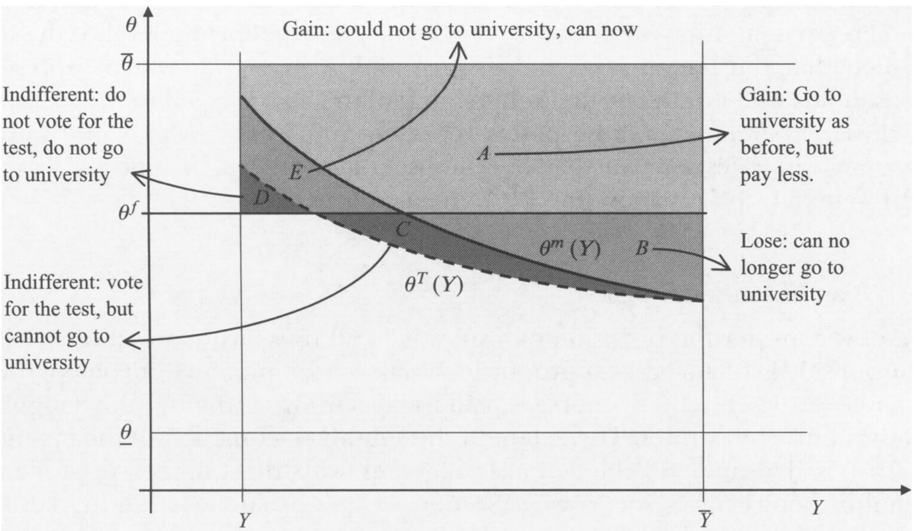

Fig. 1. *Gainers and Losers From Voting on a Test*

[8] This utility gain would be enhanced if the introduction of a test also increased the *value* of attending university, either because the presence of fewer graduates increases their rent in the labour market, or because the higher average ability makes university attendance more productive, for example, via a peer group effect or by allowing more advanced teaching.

Fig. 1 illustrates that the introduction of a test unambiguously improves equality of opportunity: the locus of points separating households who send their daughter to university from those who do not (it can no longer be called an indifference curve, as households on the horizontal portion of the curve strictly prefer attendance) becomes flatter (note that it might happen that students sufficiently bright to go to university still do not go because of cost: these are the households in area D in Fig. 1). The equity effects of the introduction of a test, however, are ambiguous: a test is most beneficial to high income households, who go to university at a lower cost than before the test, and therefore the distribution of utilities in society may become less desirable from the equity point of view.

The introduction of a test has two contrasting effects on efficiency (measured by the gap with the benchmark identified in Proposition 2). On the one hand, fewer students attend university than with no test, and, since too few students attend university in the absence of a test in the first place, this is a negative effect on efficiency. However, there is an efficiency enhancing effect because the students' average ability (and hence their future income) is higher: brighter students go to university who did not go before, and less bright students are no longer allowed to attend. Again, the overall result is ambiguous: which effect prevails depends on the functional forms considered.

Finally, since every mother who votes votes for the strictest standard which allows the daughter to be admitted to university (Proposition 3), preferences are single peaked, and the median voter theorem applies. The majority voting equilibrium is given by the level of the test at which half the voters would like to increase it, and the other half to reduce it. The equilibrium level of the test is such that the measure of the areas A and E in Fig. 1, which represent households who would favour a toughening of the test, is equal to the measure of the areas B and C, whose points represent households who would rather have an easier test (recall that only households with an income-ability pair represented by points above the $\theta^T(Y)$ locus vote).

5. A Tax Financed Subsidy

We now consider the provision of a subsidy to all those who attend university. The subsidy is financed by a proportional tax on the mothers' income, levied at a rate $\tau \in [0, 1]$; this of course is paid irrespectively of whether the daughter attends university or not. The extent of the subsidy is a function of the tax rate, $s(\tau)$: it is determined endogenously together with the number of students attending university, exactly as in Section 4. The position of the market indifference curve is now affected by the tax rate, and we therefore denote it by $\theta^{m\tau}(Y, \tau)$. It is characterised by the condition:

$$U[\theta, (1-\tau)Y, k - s(\tau)] = U^N[\theta, (1-\tau)Y], \tag{14}$$

which is the analogous to (3), and is again a negatively sloped curve. We begin by establishing the relationship between the tax rate and the level of the subsidy.

PROPOSITION 5. $s'(\tau) > 0$: *an increase in the tax rate increases the individual subsidy.*

Proof. In equilibrium, the level of the subsidy is given by the solution of the following system of equations in s, k and n:

$$D(k-s) = n, \qquad ns = \tau\hat{Y}, \qquad k = C'(n),$$

where $\hat{Y} = \int_{\underline{Y}}^{\overline{Y}} Yh(Y)\,dY$ is the total (and average) income. Let $N(k)$ be the supply function, ie, the inverse of $C'(n)$; substitute $n = N(k)$ in the first two equations, totally differentiate them with respect to s, k, and τ, to obtain:

$$\begin{bmatrix} -D'(k-s) & D'(k-s) - N'(k) \\ 1 & \tau\hat{Y}/[N(k)]^2 \end{bmatrix} \begin{bmatrix} ds \\ dk \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{Y}\,d\tau/N(k) \end{bmatrix}.$$

Let $\Delta = N'(k) - D'(k-s)\{1 + \tau\hat{Y}/[N(k)]^2\} > 0$ (because $N'(k) > 0$ and $D'(k-s) < 0$). Then we have:

$$\frac{ds}{d\tau} = \frac{N'(k) - D'(k-s)}{\Delta}\frac{\hat{Y}}{N(k)} > 0. \tag{15}$$

We may also note that the marginal cost of university provision, k, increases with the tax rate: $dk/d\tau = -\hat{Y}D'(k-s)/[\Delta N(k)] > 0$. This is obvious, as the number of students increases; also note that the fee paid by the students (or their parents) decreases: $d(k-s)/d\tau = [-N'(k)]/N(k)(\hat{Y}/\Delta) < 0$. □

In general, there is a trade-off: on the one hand, with a fixed total income, an increase in the tax rate necessarily increases the total budget available for subsidies; on the other hand, it may also increase the number of recipients, so that the change in the per capita subsidy could be ambiguous. By Proposition 5, the first effect is stronger.[9]

Proposition 5 implies that for some income levels, the market indifference curve is lower as a consequence of a marginal change in the tax rate. It is difficult, in a general model, to obtain more definite conclusions on the effects of changes in tax rate. To gain some intuition, totally differentiate (14), using the fact that $U_Y(\theta, Y, k) = -U_k(\theta, Y, k)$, and rearrange, to obtain:

$$\frac{dY}{d\tau} = \frac{1}{1-\tau}\left\{Y - \frac{u'_m[Y - t(\cdot) - k + s(\tau)]}{u'_m[Y - t(\cdot) - k + s(\tau)] - u'_m[Y - t^N(\cdot)]}s'(\tau)\right\}.$$

Therefore, the market indifference curve shifts down (up) for values of Y such that the term in the curly bracket is negative (positive). It follows that if the mother's utility function is sufficiently regular, that is if the coefficient of $s'(\tau)$ does not vary 'too much' with income, then the market indifference curve rotates anticlockwise around Y^m as τ increases, where Y^m is defined as

[9] This need not be the case: Epple and Romano's model has some similarities with the present set up, but the analogue of Proposition 5 does not necessarily hold in their model (1996*b*, p. 305).

$$Y^m = (u'_m[Y^m - t(\cdot) - k + s(\tau)]/\{u'_m[Y^m - t(\cdot) - k + s(\tau)] - u'_m[Y^m - t^N(\cdot)]\}s'(\tau)).$$

This rotation makes the market indifference curve flatter, implying that an increase in the tax rate enhances equality of opportunity in access to university education.

As before, we devote the rest of this section to the study of the political economy equilibrium. While, in general, the determination of the voting equilibrium with a two dimensional population is complex, here it simplifies considerably (Couffinhal *et al.* (1999) study a similar voting game).

To avoid corner solutions, we consider the case $s''(\tau) < 0$. The sign of $s''(\tau)$ depends in general on the interplay between demand and cost, and more precise conditions are not very illuminating.

PROPOSITION 6. *Let $s''(\tau) < 0$. There exists a decreasing curve, denoted by $\theta^\tau(Y)$, which satisfies*

$$\theta^\tau(Y) < \theta^m(Y) \qquad \text{for } Y < \hat{Y}/n_m,$$
$$\theta^\tau(Y) = \theta^m(Y) \qquad \text{for } Y \geqslant \hat{Y}/n_m.$$

This locus is such that households whose ability is above $\theta^\tau(Y)$ and whose income is below $\hat{Y}/n_m$ prefer a positive tax rate, which depends only on their income, and satisfies $Y = s'(\tau)$. All other households prefer $\tau = 0$.

Proof. Consider a household with income Y and daughter's ability θ. If it could choose the tax rate, it would solve

$$\max_\tau U[(1-\tau)Y, \theta, k - s(\tau)] \qquad \tau \in [0, 1]. \tag{17}$$

Recall that $U(\theta, Y, k)$ is the utility of household with income Y, daughter's ability θ, when the cost of tuition is k. The first order condition for (17) is:

$$-u'_m(\cdot)[Y - s'(\tau)] + \{\cdot\}\frac{\partial t}{\partial \tau} = 0.$$

The term in $\{\cdot\}$ is 0 by the envelope theorem and the optimality of the choice of t, and therefore the first order condition reduces to $Y - s'(\tau) = 0$. The second order condition is $-u''_m(\cdot)\{\cdot\}[Y - s'(\tau)] + u'_m(\cdot)s''(\tau) < 0$. At an interior stationary point, $[Y - s'(\tau)] = 0$, and therefore $s''(\tau) < 0$ ensures that the second order condition holds at a stationary point. From (15) note that $s'(0) = \hat{Y}/n_m$. Thus, if $Y < \hat{Y}/n_m$, then the maximand in (17) is increasing at $\tau = 0$, and therefore an interior maximum exists (because $\tau = 1$ is never a solution to (17)). If $Y > \hat{Y}/n_m$, then there is no interior maximum and the solution to (17) is $\tau = 0$. Let $\tilde{\tau}(Y)$ be the solution to (17) note also that, if $\tilde{\tau}(Y) > 0$, $d\tilde{\tau}/dY = s''(\tau) < 0$. Therefore if $Y < \hat{Y}/n_m$, the $\tilde{\tau}(Y)$ is strictly positive, and if $Y \geqslant \hat{Y}/n_m$, then $\tilde{\tau}(Y) = 0$.

The household utility is $U\{\theta, [1 - \tilde{\tau}(Y)]Y, k - s[\tilde{\tau}(Y)]\}$, which, for $\tilde{\tau}(Y) > 0$, is strictly greater than $U(\theta, Y, k)$. This implies that for given Y, the value of θ which ensures indifference when the preferred tax rate is selected, $\theta^{\tau}(Y)$, namely the solution to $U\{\theta, [1 - \tilde{\tau}(Y)]Y, k - s[\tilde{\tau}(Y)]\} = U^N(\theta, Y)$, is below $\theta^m(Y)$, the solution to $U(\theta, Y, k) = U_N(\theta, Y)$. Finally note that $\theta^{\tau}(Y)$ is decreasing for the same reason $\theta^m(Y)$ is. Note also that, if $\tilde{\tau}(Y) > 0$, $d\tau/dY = s''(\tau) < 0$. □

We may define the curve $\theta^{\tau}(Y)$ as the 'best tax indifference curve'. It is the locus of points representing income-ability combinations such that the mother would be indifferent between sending her daughter to university or not if her preferred rate of tax were chosen. It is depicted as the dotted line in Fig. 2. The dashed line is $\theta^{m\tau}(Y, \tau)$: the market indifference curve when the tax rate is τ. This curve crosses the best tax indifference curve at point a. At this point, Y is the income level of the household whose preferred tax level is τ^*, and who is indifferent between going to university or not at tax rate τ^*; it follows that (14) is above the 'best tax indifference curve' $\theta^{\tau}(Y)$ for $Y \leqslant s'(\tau^*)$ and below otherwise, and may or may not cross the original market indifference curve in the absence of tax, $\theta^m(Y)$.

As the figure illustrates, when the tax rate is τ, households can be classified in five groups, according to their preference. The first group are the households whose income is so high that they never vote for a positive tax rate; these are the households in area A. Note also that if $\hat{Y}/n_m > \overline{Y}$, there is no area A. The second group is area B. These are households where the daughter is not sufficiently bright to make it worthwhile to go to university, even if they could choose their preferred tax rate; since they do not benefit from the subsidy they also vote for a zero tax rate. The households in area C have a strictly positive preferred tax rate, but it is lower than τ: they would favour a marginal reduction in the tax rate. Households in area D go to university and the tax

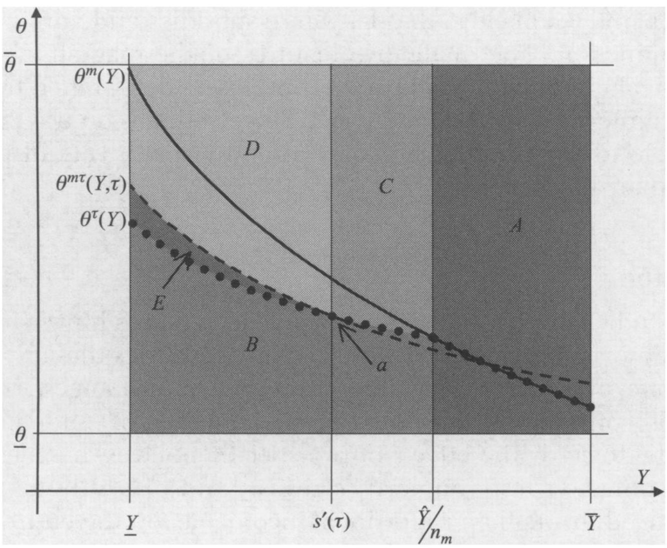

Fig. 2. *Gainers and Losers From Voting on a Tax Rate*

rate τ is lower than their preferred tax rate. They would like an increase in the tax (and subsidy) rate. Finally consider area E. Households in this area would like a higher tax rate than τ; however, when the tax rate is τ, they do not send their daughter to university, and therefore they would prefer a lower tax rate.

As with an admission test, the distributional consequences of a subsidy are ambiguous: high ability-low income households definitely benefit; however, low ability-low income households (in area B) lose out. Moreover, as all low ability individuals are made worse off, the policy is clearly output regressive in the sense of Arrow (1971). With regard to efficiency, while the student mix improves, it may happen that too many students choose to go to university.

Finally to determine the political economy equilibrium, note that, as the proof of Proposition 6 shows, for $\tilde{\tau}(Y) > 0$, we have $d\tau/dY = s''(\tau) < 0$: if a mother sends the daughter to university, then she prefers a lower tax rate the higher her income. This corresponds to the situation described in Epple and Romano (1996*a* p. 69, and 1996*b*, p. 306), where the income elasticity of demand for education is exceeded by the (absolute value of the) price elasticity (slope declining with income). In particular, this implies that preferences are singled peaked, and hence a voting equilibrium exists. To determine it, note that households characterised by points in areas C, A, B, and E prefer a reduction in the tax rate; only households in area D would prefer an increase. The equilibrium is such that the two groups of households are equal in size.

Note that the equilibrium tax rate, τ^*, is the preferred tax rate of a household whose income is above the median income. This is because there is a partial 'end against the middle' phenomenon: some of the low income households (those whose daughters are not very bright) ally themselves with higher income households to block any proposed tax increases from the rest of the lower income households, those with the bright daughter.

We do not study explicitly a model where subsidies and admission tests are used in conjunction. The qualitative features of the diagrammatical analysis would not be altered, the overall gainers and losers determined by the relative level at which the two instruments are set. Since voting is on two dimensions, it is not possible to use the median voter to argue that a certain policy pair is more likely than another.

6. Conclusion.

This paper studies the effects of specific policies towards university education. The motivation is eminently practical; in many countries the university system has undergone, or is undergoing, important structural changes, which, in most cases, entails considerable increases in the number of students. We use a stylised model to study the effects of two specific policies, namely, the institution of an admission test (numerus clausus), and of a subsidy to university tuition, financed through a proportional income tax on the entire population. We identify the beneficiaries and the losers from these policies, and the type of concensus they would receive.

We find that the distributional consequences of the two reforms considered are ambiguous. We also find that the political consensus on these reforms would not follow income (class) lines: voting on the test level does not depend on income, whereas voting on the level of subsidy determines a partial 'ends against the middle' phenomenon: high income households and some of the low income households are against tax increases when the rest of the low income households are in favour.

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