RISK AND INSURANCE IN AN AGRICULTURAL ECONOMY

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Chapter 8

INTRODUCTION

- People who live in the rural areas of poor countries often must cope not only with severe poverty but with extremely *variable income*.
- Especially those dependent on agricultural income.
- Fluctuations in income can hamper livelihood of people even if their average income are high.

THREE MAIN GOALS

- Describing Pareto efficient allocation of risk within a community through formal insurance market or informal transfer mechanisms.
- Examine the use of inter-temporal consumption smoothing through saving and credit markets.
- Ex-ante mechanisms to ensure stable income stream.

GOAL 1 : COMPLETE RISK SHARING AND PARETO EFFICIENCY

- Households may share each others risk through institutional arrangements which approximate the pareto efficient allocation of risk.
- Incidence of random shocks to households income is common knowledge thus community level institutions don’t face the problems of *moral hazard and adverse selection.*

MODEL

- Assumption: A village economy with Pareto efficient allocation of risk but no access to credit markets and storage.
- Households, $i=1,2.....N$
- T periods
- S states of nature with known probability π_s
- $y_{is} > 0$ – i^{th} household's income in state 's'
- C_{ist} – consumption of i^{th} household is state s occurs in period t.

- Each household has a separable utility function f the form :

$$U_i = \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s u_i(c_{ist}) \qquad \ldots(1)$$

- u() is twice continuously differentiable , u'>0 , u''<0 and lim u'(x) = +∞ as x ⟶ 0
- A pareto efficient allocation of risk within the village can be found by maximizing the weighted sum of utilities of each of the N households, where the weight of household i in the pareto programme is λ_i

$$\underset{c_{iht}}{\text{Max}} \sum_{i=1}^{N} \lambda_i U_i \quad ...(2)$$

subject to the resources available in the village at each point in time in each state of nature:

$$\sum_{i=1}^{N} c_{ist} = \sum_{i=1}^{N} y_{ist} \; \forall s,t. \quad ...(3)$$

$$c_{ist} \geq 0 \; \forall i,s,t. \quad ...(4)$$

(4) is the non-negativity constraint, which will not bind if the village has any resources in each period along each possible history.

- F.O.C w.r.t c_{ist} and c_{jst} :

$$\frac{u'_i(c_{ist})}{u'_j(c_{jst})} = \frac{\lambda_j}{\lambda_i} \; \forall i, j, s, t. \qquad \text{...(5)}$$

- The equality extends across all N households in the village ,in any state at any point of time.
- The marginal utilities and therefore the consumption level of all the households in the village move together.
- Marginal utility of any household is a monotonically increasing function of the average marginal utility of households in the village in any state.

- Consumption of any household is a monotonically increasing function of the average village consumption.
- In a pareto-efficient allocation, then ,transient changes in income are fully pooled at the community level.
- There is no incentive for risk diversification at the household level.
- Only risk faced by a household is that faced by the community as a whole.

- Suppose that everyone in the village has an identical constant absolute risk aversion utility function

$$u_i(x) = -(1/\sigma)e^{-\sigma x}$$

- Applying this utility function to the FOC (5) and taking logs,

$$c_{ist} = c_{jst} + (1/\sigma)(\ln(\lambda_i) - \ln(\lambda_j)) \quad \ldots(6)$$

- As before the equality holds across all N households in the village at any point in time.

- If we sum across these N equalities

$$c_{ist} = \bar{c}_{st} + \frac{1}{\sigma}(\ln(\lambda_i) - \frac{1}{N}\sum_{j=1}^{N}\ln(\lambda_j)) \quad ...(7)$$

where,

$$\bar{c}_{st} = (1/N)\sum_{j=1}^{N} c_{jst}.$$

- So household consumption is equal to the average level of consumption in the village plus a time-invariant household fixed effect which depends upon the relative weight of the household in the pareto programme.

- Equation (7) implies that the change in a household’s consumption between any two periods is equal to the change in the average consumption between the two periods.
- Note: Household income does not appear in (7) After controlling for average consumption , a household’s consumption is unaffected by its own income.

RESULT

- In a pareto-efficient allocation of risk within a community, households face only aggregate risk.
- Idiosyncratic income shocks are completely insured within the community.
- Within small regions, the incomes of households engaged in rain fed agriculture are likely to have high covariance, reducing the effectiveness of local risk-sharing arrangements.

PROBLEMS

- Second welfare theorem: Pareto efficient allocation of risk can be supported by a competitive equilibrium with complete contingent markets.
- Any risk pooling mechanism must overcome the information and enforcement problems associated with Insurance contracts.
- The insurer might be subject to either moral hazard or adverse selection (or both).
- Impractical to write contracts in enough detail and contract enforcement is difficult.

OTHER MECHANISMS THAT SUPPORT EFFICIENT RISK POOLING

- Interlinked, repeated personalized transactions between households provides an essential framework for economic activity in small communities.
- ‘Generalized Reciprocity’ – Those whose income is temporarily large provide gifts to those whose income temporarily is relatively small. Gifts are not necessarily reciprocated.

EXAMPLES OF SUCH COMMUNITIES

- Cashdan (1985) describes a system of gift exchange among the Basarwa farmer-herders in northern Botswana.
- Most of the Basarwa studied by Cashdan subsist by providing live-stock herding services to richer non-Basarwa cattle owners.
- In exchange, they receive milk from the cattle, some of the offspring and the opportunity to use the draught power of the animals to cultivate their own fields.

- As the herd size (and labour demand) of the wealthy cattle-owners fluctuate, the Basarwa workers find themselves forced to move to the cattle posts of new employers.
- Land is abundant and freely available for cultivation, but it takes time (2-3yrs) to clear and fence an optimally sized farm in new location.
- These unpredictable employment changes generate random variations in the income of the Basarwa households.
- The probability of moving to a new location in a given year is largely independent for different households.

- Income risk is idiosyncratic and can be addressed by local risk-sharing mechanisms.
- Within a given locality, households that have been resident for longer have relatively high incomes and they provide gifts of food to newer residents with smaller farms and incomes.
- The idiosyncratic risk which is insured, is certainly observable to all members.
- The problem of information assymetry seems unimportant w.r.t the source of risk.
- Ethnic identity and social costs of disengaging ensures generalized reciprocity.

IMPORTANCE OF RECIPROCAL CREDIT

- Platteau and Abraham (1987) studied the risk pooling system of fishermen in South India villages.
- These fishermen live close to the margin of subsistence and are engaged in a very risky activity.
- Little covariation in the incomes across households.
- Insurance is affected through frequent, very small, ’credit ‘ transactions within the village.

- Acceptance of loan by a fisherman implies that he will be concerned with the future economic fortune of his creditor.
- In case the creditor falls into distress, the borrower will not only have to return his debt immediately, but must also be ready to come to the help of his benefactor.
- If the debtor is in crisis, the creditor is expected to come to his rescue irrespective of whether or not he has cleared his first debt.
- These credit transactions serve to pool risk between borrowers and lenders in small community.

- There are a number of quantitative studies which aim to estimate the degree to which insurance systems in certain communities achieve a pareto efficent allocation of risk.

- Townsend (1994) and Ravallion and Chaudhuri (1997) examine consumption outcomes in the some Indian villages. Though there is considerable correlation in consumption of different households in the presence of idiosyncratic risk, they don’t achieve a pareto efficient allocation of risk.

- Deaton (1992*a) and Grimard (1997) examine* patterns of consumption to test the hypothesis of efficient risk-pooling within villages and ethnic groups, respectively, within Côte d'Ivoire. There is little evidence of any risk-pooling within villages, and somewhat stronger evidence of partial risk-pooling within ethnic groups. In neither case is full risk-pooling achieved.
- Udry (1994) rejects the hypothesis that Pareto-efficient risk-pooling is achieved in northern Nigerian villages using the specific mechanism of reciprocal credit transactions.
- In every case so far examined in the literature, the hypothesis of efficient risk pooling has been rejected

GOAL 2 : INTER-TEMPORAL CONSUMPTION SMOOTHING THROUGH SAVINGS AND CREDIT MARKETS

- A fully pareto efficient allocation of risk within local communities is rarely, if ever achieved. Some idiosyncratic variation generally remains uninsured.
- The complementary ex-post mechanisms for insulating consumption from the effects of income fluctuation is, consumption smoothing using saving and credit transactions.

- Consider a household with no opportunity cost for cross-sectional risk-pooling, but with unlimited access to credit market.
- Household utility function is the same as in (1) dropping i,

$$U_t = E_t \sum_{\tau=t}^{T} \beta^{\tau-t} u(c_\tau) \qquad \text{... (1')}$$

U_t is the expected utility of the household over the remainder of its lifetime.

- Suppose that in any period, the household can borrow or lend on a credit market with a certain interest rate r_t

- Let the household's asset stock at the start of period t be A_t (positive when household is a lender, negative when it is a borrower)
- The household receives random income y_t and decides how to allocate its resources between consumption and net savings for the next period

$$A_{t+1} = (1 + r_t)(A_t + y_t - c_t) \qquad \text{...(8)}$$

- The household chooses consumption to maximize (1') subject to (8), non-negativity constraints on c , and the transversality condition $A_{T+1} >= 0$.
- Note: The household can be a debtor in any but the final period.
- The period t value function of the household's problem satisfies

$$V_t(A_t + y_t) = \underset{c_t}{\mathrm{Max}}\{u(c_t) + \beta E_t V_{t+1}[(1 + r_t)(A_t + y_t - c_t) + y_{t+1}]\}. \quad \ldots(9)$$

- Value of Current resources (asset + current income) = Maximized value of current consumption plus the discounted expected value of resources next period.
- Optimization and envelope conditions(Benvineste-Scheinkman) imply,

$$u'(c_t) = \beta(1 + r_t)E_t u'(c_{t+1}) \quad \text{...(10) (\textit{See Appendix 1})}$$

- Saving or lending decisions are made so that Marginal utility of current consumption = Discounted expected marginal utility of next period's consumption.

- If $\beta(1+r_t)=1$ for all t ,i.e yield on assets just offsets the subjective discount rate ,(10) simplifies to
  $u'(c_t) = E_t u'(c_{t+1})$.
  i.e Households make consumption plans such that expected consumption is constant.
- If we make the assumption that u is quadratic , then (10) becomes

$$c_t = E_t c_{t+1} \quad \text{...(11)}$$

- Since $A_{T+1} = 0$, the budget constraint (3') (with r_t constant at r) implies that the discounted value of income stream from t to T equals the value of the household's assets at t plus the discounted value of its income stream from t to T. (*See Appendix 2 equation 2*)

- Combine this result with (11) and let T go to infinity, we arrive at the Permanent Income Hypothesis (PIH) :

$$c_t = \frac{r}{1+r}\left(A_t + E_t \sum_{\tau=t}^{\infty} (1+r)^{-(\tau-t)} y_\tau\right)$$ ...(12) (*See Appendix 2*)

- Current consumption, therefore , is the annuity value of current assets + present value of expected stream of future income.
- If the PIH is valid, household consumption responds to random variations in household income depending on the information associated with the income shock.

- The change in consumption = Annuity value of the present value of the changes in expected stream of future income.
- If the income shock is transitory, and there is little or no change in the household’s expectations concerning its future income stream, the consumption will change little in response to the income shock.
- If income shock causes a large change in the household’s expectations concerning its future income stream, then the income shock will be seen as permanent and consumption will change dramatically in response to the income shock.

EMPIRICAL RESULTS ON CONSUMPTION-SMOOTHING

- There is evidence from a variety of studies that households engage in a substantial degree of consumption smoothing.
- Paxson (1992) : The study uses deviations of rainfall from its average level to identify transitory income shocks affecting Thai rice farmers.
- The component of the income explained by variations in rainfall gives the transitory income in a period.
- Once this has been done, the propensity to save out of this transitory income can be calculated.
- The author uses these estimates to calculate the marginal propensity to save transitory income, and finds that these farmers save three-quarters to four-fifths of transitory income changes

- This is strong evidence of inter-temporal consumption smoothing.
- But the permanent income hypothesis can generally be rejected in all parts of the world.
- Microeconomic data reveals that even in countries such as Japan and the US, consumers are often liquidity constrained.
- Rural households in developing countries do not have access to perfect credit markets.

- Morduch (1992) finds evidence of borrowing constraints strongly affecting the behaviour of relatively poor households that seem to be liquidity constrained in a set of villages in semi-arid India.
- He also finds that households which are liquidity constrained engage in less risky activities than unconstrained households.
- Rosenzweig and Binswanger (1993) also find similar evidence: wealthier households, which are not liquidity constrained, invest in riskier activities with higher expected returns than poorer households.

- Deaton (1991) shows that even if households don’t have access to a credit market at all, they may still be able to achieve a high degree of inter-temporal consumption smoothing through the use of assets as buffer stocks.
- But as the household’s wealth falls to near zero, consumption may again become quite volatile.
- (Watts 1983, Ravallion 1997) It has been seen that famines often occur only after a succession of wealth failures, or after people’s savings are wiped out through other means.

- If the assets used to smoothen consumption are themselves used in the production process, then there can be important effects on future consumption from even temporary shocks to current income.

- Rosenzweig and Wolpin (1985) observe from a sample of rural households in India that bullocks are often purchased and sold to smooth consumption when income fluctuates. However, bullocks play an important role in the production process of these farm households.

- Therefore, if bullocks are sold in a period, the farm profit next period would be lower as a consequence of the loss of this productive asset.

- Udry (1995) shows that, as long as a household has stocks of an asset that is not used in production, this asset will be used to smooth consumption. However, once this asset is drawn down near zero, as for instance after a succession of bad harvests, then assets used in production maybe sold in order to smooth consumption.

- Rosenzweig and Wolpin (1985), for example, show that households subjected to two consecutive years of draught are 150 % more likely to sell land.

SUMMARY :TILL NOW

- When there is complete risk pooling, the household's consumption responds only to the average community consumption
- Holding community consumption constant, a shock to a household's own income, whether transitory or permanent, has no effect on household's consumption.
- No access to credit, no goods storage implies

  Community income = Community Consumption in each period.

  Even a transitory shock to community income causes the household to change its consumption.

- No risk pooling , with access to a perfect credit market, community income is irrelevant to the household's consumption decisions, but the consumption will vary with the changes in the household's permanent income.
- Both the Permanent Income Model and the Full Insurance Model imply that changes in household's income may have only a small correlation with the changes in household's consumption.
- This result would occur if the household is smoothing consumption over periods if the variation in its income are predominantly due to transitory shocks.

- Also both models imply that household consumption might be highly correlated with village consumption and uncorrelated with transitory shocks to household income.
- Examining the distribution of consumption of a cohort of people over time :
  In a pareto efficient allocation (7), this distribution will remain stable over time : all idiosyncratic risk has been insured against. If PIH is approximately true, the distribution of consumption will broaden (12) as different individuals over time receive different news concerning their future prospects.

- Deaton and Paxson provide a method to distinguish economies well characterized by inter-temporal consumption smoothing from those characterized by an approximately pareto efficient allocation of risk.
- The idea is to examine the patterns of a cohort of people over time. In a pareto-efficient allocation of risk, consumption of individuals will move together over time. Under consumption smoothing, it will tend to diverge.

COMBINING THE TWO MODELS

- Till now we considered insurance and inter-temporal smoothing mechanisms to be mutually exclusive.
- If both borrowing and lending in a perfect capital market and insurance within the village are possible, then household consumption will still depend only on average consumption within the village, but village consumption can deviate from village income.
- The village-level analogue to the PIH will imply that village consumption as a whole(and household consumption) will have little responsiveness to transitory shocks in village income.

- Household consumption will change only in response to variations in the permanent income of the village.
- If consumption smoothing is possible through either or both of these ex-post avenues, then risk averse households will act in some other aspects as if they were risk-neutral.

MODEL

- Assumptions
  -Pareto efficient allocation of risk within a community.
  -Production is possible.
  -Inelastic supply of labour.
  -Current output must be invested in order to produce next year.
  -Consumption and income now depend on the history of the past realized states (investment creates a link across periods)

- Household i's income in the state s of period t after a history of states through period t – 1(h_{t-1})

$$y_{ist}(h_{t-1}) = g_i(s, k_{i,t-1}(h_{t-1})) \quad \text{...(13)}$$

- g() is the production function , $\partial g_i/\partial k_i>0$ and $\partial^2 g_i/\partial^2 k_i<0$
- The capital invested in i's farm in period t-1 (in order to produce output in period t) depends on the history of states realized upto and including period t-1.

- Resource constraint (3) must be modified

$$\sum_{i=1}^{N} k_{it}(h_{t-1}) = \sum_{i=1}^{N} [y_{ist}(h_{t-1}) - c_{ist}(h_{t-1})] \quad \text{...(3')}$$

- It reflects the commitment of current resources for future production.
- FOCs: Investments made in period t-1 for production in period t satisfy

$$\sum_{s} \lambda_{s}(h_{t-1}) \frac{\partial g_{i}(s,k)}{\partial k_{i,t-1}(h_{t-1})} = \sum_{s} \lambda_{s}(h_{t-1}) \frac{\partial g_{j}(s,k)}{\partial k_{j,t-1}(h_{t-1})} \quad \text{...(14)}$$

(*See Appendix 3*)

- $\lambda_s(h_{t-1})$- lagrange multiplier corresponding to the resource constraint (3') in state s of period t after history t-1.
- $\lambda_s(h_{t-1})$ is the increment in the value of the pareto programme resulting from an increase in resources in state s of period t.
- So (14) implies Marginal value of investment in period t-1 (weighted over S states which might occur in period t) is equated across households. Investment, therefore, is determined entirely by consideration of productive efficiency .

- Differences in risk aversion or wealth levels across households have no effect on the allocation of investment in a Pareto-efficient allocation.
- Assume $y_{is}(h_{t-1}) = \theta_s g_i (k_{i,t-1}(h_{t-1}))$

  Production function is characterized by a simple multiplicative factor.
- (14) becomes

$$\frac{\partial g_i(k)}{\partial k_{i,t-1}(h_{t-1})} = \frac{\partial g_j(k)}{\partial k_{j,t-1}(h_{t-1})} \quad \ldots(14')$$ (*See Appendix 4*)

Marginal product of Investment is equated across all households.

- Households in poor, risky agrarian environments engage in both cross-sectional risk-pooling and consumption-smoothing over time.
- The information and enforcement difficulties associated with both insurance and credit transactions frustrate households' efforts to insulate their consumption from income shocks.
- Given the lack of access to complete and smoothly operating insurance and credit markets, households devote substantial resources to stabilizing the incoming stream of income in order to protect themselves from the dire consequences of substantial income fluctuations.

GOAL 3 :EX-ANTE MEANS OF REDUCING INCOME FLUCTUATIONS

- Two kinds of mechanisms to counter risk – ex ante mechanisms and ex post mechanisms.
- Ex post mechanisms here refer to measures taken to reduce variation in consumption over periods after the realization of a period’s income.
- Ex ante mechanisms, on the other hand, are measures that are taken to reduce variation in consumption over periods before the realization of a period’s income.

- They might spread their family members across space through migration or marriage in order to reduce the variance of aggregate household income.
- They might adopt contractual agreements such as sharecropping which reduce variance in income.
- Any of these ex ante mechanisms might be costly in the sense that they might reduce expectation of income also, along with its variance.

- In an agricultural economy, these may take several forms.
- For example, adoption of low-yielding but rapidly maturing varieties of crops minimize the probability that rainfall shortages will cause crop failure.
- Planting multiple crops on widely dispersed fields.
- Households might work in a diverse range of activities rather than in a single profit-maximizing business in order to diversify some of the income risk.

- Previously we had discussed ex post mechanisms to counter risk, namely, insurance and consumption smoothing over time.
- When these ex post mechanisms for mitigating the adverse consequences of income fluctuations fail, risk averse households invest in ex ante means of reducing income fluctuations.

MODEL

- Suppose that households face a liquidity constraint such that in any period, the following holds :

$$A_t + y_t - c_t \geq 0.$$

- In addition, suppose that farmers face a portfolio choice between two activities, one of which is more risky than the other.

- In particular, let period t income be determined by the realization of a zero mean independent and identically distributed shock ε_t and the previous period portfolio choice x_{t-1} so that

$$y_t = y(x_{t-1}, \in).$$

- $\partial y_t / \partial \in_t > 0,$ and the portfolio choice is such that $\partial y_t / \partial x_{t-1} > 0 \text{ if } \in_t > 0 \text{ and } \partial y_t / \partial x_{t-1} < 0 \text{ if } \in_t < 0.$
- This means that in good times choosing more of the risky activity increases output, while in bad times choosing more of the risky activity reduces output.

- Since x is costless, if the household is maximizing expected income it will choose x_{t-1} such that $E_{t-1}\ \partial y_t / \partial x_{t-1} = 0.$

- The period t value function of the household now satisfies the following :

$$
\begin{aligned}
V_t(A_t + y_t) \quad &= \mathrm{Max}\{u(c_t) + \beta E_t V_{t+1}[(1 + r_t)(A_t + y_t - c_t) \\
&\quad + y(x_t, \in_{t+1})] + \lambda_t(A_t + y_t - c_t)]
\end{aligned}
$$

where λ_t is the lagrange multiplier corresponding to the liquidity constraint in period t.

- Consumption in period *t* will be chosen to satisfy the following:

$$u'(c_t) = E_t \beta (1 + r_t) V'_{t+1} [(1 + r_t)(A_t + y_t - c_t) + y(x_t, \in_{t+1})] + \lambda_t,$$

(*See Appendix 5*)

with complementary slackness between λ_t and $(A_t+y_t-c_t)$

- Now, by the envelope property, x_{t-1} will satisfy

$$E_{t-1} \frac{\mathrm{d}V_t(\cdot)}{\mathrm{d}x_{t-1}} = E_{t-1} U'(c_t) \frac{\partial y}{\partial x_{t-1}} = 0.$$

(*See Appendix 6*)

- Substituting for $u'(c_t)$ we have the following :

$$E_{t-1}[\beta(1+r)V'_{t+1}(\cdot)+\lambda_t]\frac{\partial y}{\partial x_{t-1}}=0.$$

- If λ_t =0 in all states of period so that the individual knows that the liquidity constraint will not bind in period t, then x_{t-1} is chosen so that

$$E_{t-1}V'_{t+1}(\cdot)\frac{\partial y}{\partial x_{t-1}}=0.$$

- On the other hand, if $\lambda_t > 0$ for some states of period t, then x_{t-1} is chosen so that

$$\beta(1+r)E_{t-1}V'_{t+1}(\cdot)\frac{\partial y}{\partial x_{t-1}} = -E_{t-1}\lambda_t\frac{\partial y}{\partial x_{t-1}} > 0,$$

- Where the latter inequality holds because the liquidity constraints bind in low income states of period t.

- It is to be expected that poorer households are more likely to be subject to binding liquidity constraints. These households, therefore, will choose a more conservative portfolio of activities than richer households.

- Therefore, they will have lower expected returns and lower variance of income as compared to wealthier households.

CONCLUSION

- In a small community where there is perfect information ,no savings and no credit market cross sectional risk pooling can help in obtaining pareto efficient allocation of risk
- If we consider the no risk pooling case with savings and credit markets we obtained the permanent income hypothesis
- When we combined the above two ex-post mechanism of risk we showed that households will invest such that the marginal product of investment is equated across households
- When households use ex-ante mechanisms to counter risk then poor households whose liquidity constraints are binding will choose activities that will reduce the variance of their income and will also earn lower expected return

THANK YOU

APPENDIX

1) LET $A_t + y_t = a$

Dropping time subscripts and assuming stationarity $(V_t(\cdot) = V_{t+1}(\cdot))$,

$$V(a) = \max_c u(c) + \beta \cdot E V[\underbrace{(1+r)(a-c)}_{a'} + y']$$

FOC

$$\{c\} : \quad u'(c) = \beta \cdot E V'(a')(1+r)$$

$$\Rightarrow \quad E[V'(a')] = \frac{u'(c)}{\beta(1+r)} \quad \text{——— } (1)$$

B.S [Benviniste - Scheinkman]

$$\{a\} \quad V'(a) = \beta \cdot E[V'(a')](1+r)$$

$$\therefore \quad E[V'(a')] = \frac{V'(a)}{\beta(1+r)} \quad \text{——— } (2)$$

Equating (1) and (2)

$$\frac{V'(a)}{\beta(1+r)} = \frac{u'(c)}{\beta(1+r)}$$

$$\text{or} \quad V'(a) = u'(c) \quad \text{——— } (3)$$

This gives us

$$E[V'(a')] = E[u'(c')]$$

From (1), we get

$$\frac{u'(c)}{\beta(1+r)} = E[u'(c')] \quad \Rightarrow \quad u'(c_t) = \beta(1+r) E_t[u'(c_{t+1})]$$

2)

$$A_{T+1} = (1+r)[A_T + y_T - c_T)$$

But $A_{T+1} = 0$

$$\therefore \; (1+r)[A_T + y_T - c_T] = 0$$

$$\Rightarrow \quad A_T + y_T - c_T = 0 \qquad \text{— (1)}$$

Now

$$A_T = (1+r)[A_{T-1} + y_{T-1} - c_{T-1}]$$

$$A_{T-1} = (1+r)[A_{T-2} + y_{T-2} - c_{T-2}]$$

$$\therefore \; (1+r)(A_{T-1}) + (1+r)y_{T-1} - (1+r)c_{T-1} + y_T - c_T = 0$$

$$(1+r)[(1+r)A_{T-2} + (1+r)y_{T-2} - (1+r)c_{T-2}] + [y_T + (1+r)y_{T-1}] - [c_T + (1+r)c_{T-1}] = 0$$

or $$(1+r)^2 A_{T-2} + [y_T + (1+r)y_{T-1} + (1+r)^2 y_{T-2}] - [c_T + (1+r)c_{T-1} + (1+r)^2 c_{T-2}] = 0$$

or $$(1+r)^{T-t}A_t + \sum_{\tau=t}^{T}(1+r)^{T-\tau}y_\tau = \sum_{\tau=t}^{T}(1+r)^{T-\tau}c_\tau$$

or $$A_t + \sum_{\tau=t}^{T}\frac{(1+r)^{T-\tau}}{(1+r)^{T-\tau}}y_\tau = \sum_{\tau=t}^{T}\frac{(1+r)^{T-\tau}}{(1+r)^{T-\tau}}c_\tau \qquad \text{— (2)}$$

Taking exp. & $T \to \infty$

$$A_t + E_t\left[\sum_{\tau=t}^{\infty} (1+r)^{-(\tau-t)} y_\tau\right]$$

$$= \sum_{\tau=t}^{\infty} (1+r)^{-\tau-t} E_t(c_\tau)$$

$$= c_{\cancel{\tau}}\left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots\right]$$

(Using law of iterative expectations)

$$= \frac{1+r}{r} c_\tau.$$

Hence the result.

3)
$$\max_{k_{ist}(h_{t-1}),\, c_{ist}(h_{t-1})} \sum_{i=1}^{N} \lambda_i \left\{ \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s u_i(c_{ist}(h_{t-1})) \right\}$$

Subject to,

$$\sum_{i=1}^{N} k_{ist}(h_{t-1}) = \sum_{i=1}^{N} \left[g_i(s, k_{ist-1}(h_{t-1})) - c_{ist}(h_{t-1}) \right]$$

Setting the Lagrangean,

$$\mathcal{L} = \sum_{i=1}^{N} \lambda_i \left\{ \sum_{t=1}^{T} \beta^t \cdot \sum_{s=1}^{S} \pi_s u_i(c_{ist}(h_{t-1})) \right\}$$

$$+ \underset{(t \times s)}{\sum_t \sum_s} \mu_s(h_{t-1}) \left[\sum_{i=1}^{N} g_i(s, k_{ist-1}(h_{t-1})) - \sum_{i=1}^{N} c_{ist}(h_{t-1}) - \sum_{i=1}^{N} k_{ist}(h_{t-1}) \right]$$

F.O.C

$\{c_{ist}(h_{t-1})\}$: $\lambda_i \beta^t \pi_s u_i'(c_{ist}(h_{t-1})) = \mu_s(h_{t-1})$

(This will give the result of complete risk pooling in section 2).

$\{k_{i,t-1}(h_{t-1})\}$: $$\sum_s \mu_s(h_{t-1}) \frac{\partial g_i(s,k)}{\partial k_{i,t-1}(h_{t-1})} = \mu_s(h_{t-2}) \quad \text{— (1)}$$

(Similar result for j^{th} household.)

Equating i^{th} and j^{th} household we get the result.

Note: λ_s replaced by μ_s., μ_s is the Lagrange multiplier.

4) When,

$$y_{ist} = \theta_s \, g(k_{i,t-1}(h_{t-1}))$$

Using result in appendix 3, we get,

$$\frac{\partial g_i(k_i)}{\partial k_i} \cdot \sum_s \mu_s(h_{t-1}) \cdot \theta_s = \frac{\partial g_j(k_j)}{\partial k_j} \cdot \sum_s \mu_s(h_{t-1}) \cdot \theta_s .$$

The result follows.

5)

$$V_t(A_t+y_t) = \text{Max}\left\{ u(c_t) + \beta.\, E_t.\, V_{t+1}\left[(1+r_t)(A_t+y_t-c_t)+y(x_t,\epsilon_{t+1})\right] + \lambda_t(A_t+y_t-c_t) \right\}.$$

F.O.C. w.r.t. c_t gives us,

$$u'(c_t) - \beta.(1+r_t).\,E_t.\,V_{t+1}(\cdot) - \lambda_t = 0,$$

or, $$u'(c_t) = E_t.\beta.(1+r_t).\,V'_{t+1}\left[(1+r_t)(A_t+y_t-c_t)+y(x_t,\epsilon_{t+1})\right] + \lambda_t.$$

6) At optimality household will choose that x_{t-1} such that the expected value function at 't' sitting at time t-1 is maximised i.e.,

$$E_{t-1} \cdot \frac{dV_t(\cdot)}{dx_{t-1}} = 0. \quad \text{——} \quad (1)$$

Now,

$$\frac{dV_t(\cdot)}{dx_{t-1}} = \left.\frac{\partial V_t(\cdot)}{\partial x_{t-1}}\right|_{c_t = c_t^*} \quad \text{[from envelope theorem].}$$

Now, $y_t = y(x_{t-1}, \varepsilon_t)$.

$$\therefore \frac{dV_t(\cdot)}{dx_{t-1}} = \frac{\partial . V_t(\cdot)}{\partial y_t} \cdot \frac{\partial y_t}{\partial x_{t-1}} \quad \text{[chain rule]}$$

$$= V'(\cdot) \cdot \frac{\partial y}{\partial x_{t-1}}$$

$$= u'(c_t) \cdot \frac{\partial y}{\partial x_{t-1}} \quad \text{[From result obtained in appendix 1 equation (3), excluding constraint].}$$

Substituting in (1) we have,

$$E_{t-1} \cdot \frac{dV_t(\cdot)}{dx_{t-1}} = E_{t-1} \cdot u'(c_t) \cdot \frac{\partial y}{\partial x_{t-1}} = 0.$$