Reciprocity without Commitment

Stephen Coate and Martin Ravallion

Introduction

- Insurance markets are not found in primitive societies and many present day developing countries where households have no formal means of contract enforcement and little access to risk markets.

- Social insurance though, is still possible through “repeated interactions" in an environment with few informational asymmetries.

- There is considerable evidence of various forms of informal arrangements in village communities. (Gift giving, Reciprocal interest free credit, Shared Meals, Communal access to land, Sharing bullocks, Work-sharing arrangements etc.).

- The main risks covered by such arrangements are certain forms of crop damage, accidents or illness of productive family members or livestock and other mainly non-covariate income fluctuations. The common element in all these is: reciprocity, recipients at one date often become donors at another.

In this paper the authors:

- Characterize the 'best' informal insurance arrangement which can be sustained as a noncooperative equilibrium.
- Compare its properties with those of an arrangement which could, in principle, be achieved with binding contracts.
- Identify circumstances under which the divergence between them is the greatest.

The Basic Model

- Two risk-averse households who face intertemporally variable and independent income streams. So, at any given date they may have different incomes. Ex ante, they are similar i.e. they have same preferences and expected income.
- Let the households be A and B.
- In each period a household receives an income y^k (k=A,B) drawn from the set $\{y_1, y_2, \ldots, y_n\}$, where $y_1 < \ldots < y_n$.
- π_{ij} is the probability that household A receives an income y_i and B an income y_j. Thus, $\pi_{ij} = Prob\{(y^A, y^B) = (y_i, y_j)\}$. We only consider symmetric probabilities such that $\forall\ i, j \in \{1, 2, \ldots, n\}, \pi_{ij} = \pi_{ji} > 0$.

- The players have identical preferences defined over own income only and represented by the (per period) utility function $u(y)$. We assume that they are non-satiated and risk-averse, i.e., for all $y > 0, u'(y) > 0$ and $u''(y) < 0$.

- Each household has a utility discount rate r.

- We assume that households do not save.

- Since both households are risk averse and face uncertain income streams, there are potential gains from state-contingent transfers between them. In the absence of binding contracts, such arrangements can not be sustained in one period interactions. However, in repeated interactions they can be sustained.

- We consider the following repeated non-cooperative game:

  - In each period t, nature selects an income pair $y(t) = (y^A(t), y^B(t))$.
  - Observing $y(t)$ and knowing the history of the game, each household must choose a transfer to the other household.
  - The game is assumed to be infinitely repeated (with *dynasties* taken to be the players) and the equilibrium concept is that of subgame perfect equilibrium.

Informal Insurance Arrangements

- Aim is to characterize the insurance arrangements which can be implemented by the equlibria of the game.
- An arrangement specifies a net transfer between 2 players for each realized income pair.
- Only looking at pure insurance arrangements, i.e., transfers at any period depend only on incomes realized at that date.
- We define an Informal Insurance Arrangement to be an $n \times n$ matrix $\Theta = (\theta_{ij})$ where θ_{ij} denotes the net transfer from A to B when A gets an income y_i and B gets y_j.
- For feasibility, $\theta_{ij} \in [-y_j, y_i]$.

- Under the arrangement Θ, A's per period expected utility will be:

$$v^A(\Theta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{ij} u(y_i - \theta_{ij})$$

and B's per period expected utility will be:

$$v^B(\Theta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{ij} u(y_j + \theta_{ij})$$

- Each household's per capita expected utility in the absence of any kind of informal insurance is:

$$\overline{v} = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{ij} u(y_i) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{ij} u(y_j)$$

- An arrangement is implementable if there exist equlibrium strategies for the players which result in net transfers consistent with it.
- We consider an arrangement to be implementable if the difference between each household's expected utility under continued participation and the status quo (i.e., zero transfers) is always greater than the gain from the current defection, i.e. if it can be implemented using grim trigger strategy.
- Thus, an arrangement under is implementable using this strategy if and only if:

$$u(y_i) - u(y_i - \theta_{ij}) \leq (v^A(\Theta) - \overline{v})/r \text{ for all } (i, j) \quad \ldots (1)$$

and

$$u(y_j) - u(y_j + \theta_{ij}) \leq (v^B(\Theta) - \overline{v})/r \text{ for all } (i, j) \quad \ldots (2)$$

These are called the **Implementability Constraints**.

- The arrangement is **symmetric** if the net transfer from A to B when $(y^A, y^B) = (y_i, y_j)$ equals the net transfer from B to A when $(y^A, y^B) = (y_j, y_i)$ i.e., if $\theta_{ij} = -\theta_{ji}$. For a symmetric, Θ, $\theta_{ii} = 0$ for all i.

- A symmetric arrangement is completely characterized by the vector

$$\theta = (\theta_{21}; \theta_{31}, \theta_{32}; \ldots \theta_{n1}, \ldots, \theta_{nn-1})$$

- Each household's utility in terms of θ is given by:

$$v(\theta) = \sum_{i=1}^{n} \left[\sum_{j=1}^{i-1} \pi_{ij}(u(y_i - \theta_{ij}) + u(y_j + \theta_{ij})) + \pi_{ii} u(y_i) \right]$$

- Thus, $v^A(\Theta) = v^B(\Theta) = v(\theta)$.

- The implementability conditions thus reduce to (for a symmetric non-negative arrangement θ):

$$u(y_i) - u(y_i - \theta_{ij}) \leq (v(\theta) - \overline{v})/r \text{ for } i = 1, \ldots, n \text{ and}$$

$$j = 1, \ldots, i-1 \quad \ldots (3).$$

- Here, a non-negative arrangement is one where the net transfer from A to B is non-negative whenever A has greater income than B.

The Performance of Informal Insurance

- Comparison of the best possible implementable insurance arrangement with the first-best. We confine our attention to transfer arrangements which are symmetric and non-negative.

- $\hat{\theta}$, the **first-best** is defined as the set of state-contingent transfers which maximizes average expected utility allowing binding commitments. It is the solution to the unconstrained maximization of $v(\theta)$ with respect to θ. This gives us the condition:

$$\hat{\theta}_{ij} = (y_i - y_j)/2, \ i = 1, \dots, n \text{ and } j = 1, \dots, i-1$$

- Thus, the first-best involves full income pooling.

- The **best implementable contract**, θ^*, is the one which results in the highest average expected utility subject to the implementability constraints. We get θ^* by the constrained maximization of $v(\theta)$ subject to (3).

- Define $f(y, w)$ as:

$$u(y) - u(y - f) = w$$

- This can be thought of as the maximum amount of income that can be taken from a household with income y without inducing defection when the cost of defecting (in utility terms) is $w > 0$.

- f is increasing in both its arguments.

- Given θ, the maximal amount of income which can be taken from a household with income y without violating implementability is given by $f(y, [v(\theta) - \overline{v}]/r)$.

THEOREM

For all $i = 1, \ldots, n$ and $j = 1, \ldots, i-1$,

$$\theta^*_{ij} = min\{\hat{\theta}_{ij}, f(y_i, [v(\theta^*) - \overline{v}]/r)\} \qquad \ldots (4)$$

PROOF:

Let θ^*/θ_{ij} denote the vector θ^* with θ^*_{ij} replaced by θ_{ij}.

(i) To show, if $f(y_i, [v(\theta^*) - \overline{v}]/r) \geq \hat{\theta}_{ij}$, then $\theta^*_{ij} = \hat{\theta}_{ij}$.
Suppose that $\theta^*_{ij} \neq \hat{\theta}_{ij}$. Now consider the vector θ^*/θ_{ij}. Since, $\partial v/\partial\theta_{ij}$ is negative for all $\theta_{ij} \geq \hat{\theta}_{ij}$ and positive for all $\theta_{ij} \leq \hat{\theta}_{ij}$, we know that $v(\theta^*/\hat{\theta}_{ij}) > v(\theta^*)$. In addition since f is increasing in its second argument it follows that:

$$f(y_i, [v(\theta^*/\hat{\theta}_{ij}) - \overline{v}]/r) > f(y_i, [v(\theta^*) - \overline{v}]/r) \geq \hat{\theta}_{ij}$$

Thus, $\theta^*/\hat{\theta}_{ij}$ is implementable. Since, it also yields a higher level of expected utility, θ^* cannot be optimal. Contradiction.

(ii) To show, if $f(y_i, [v(\theta^*) - \overline{v}]/r) < \hat{\theta}_{ij}$, then $\theta^*_{ij} = f(y_i, [v(\theta^*) - \overline{v}]/r)$.

Suppose not. Then, by implementability it must be the case that $\theta^*_{ij} < f(y_i, [v(\theta^*) - \overline{v}]/r)$.

Choose $\tilde{\theta}_{ij} \in (\theta^*_{ij}, f(y_i, [v(\theta^*) - \overline{v}]/r))$ and consider the vector $\theta^*/\tilde{\theta}_{ij}$. Since, $\partial v/\partial \theta_{ij}$ is positive for all $\theta_{ij} < \hat{\theta}_{ij}$ and $\theta^*_{ij} < \tilde{\theta}_{ij}$, we know that $v(\theta^*/\tilde{\theta}_{ij}) > v(\theta^*)$.

By a similar argument as above it can be shown that $\theta^*/\tilde{\theta}_{ij}$ is implementable. This contradicts the optimality of θ^*.

Relation between θ^* and $\hat{\theta}$

- Under the first-best contract the size of the net transfer between households depends only on the difference between their incomes. The level of incomes is irrelevant. But, this is not the case for the optimal informal arrangement.

- **PROPOSITION 1**

  Let $y_i - y_j = y_g - y_h > 0$ and let $y_i > y_g$. If $\theta^*_{ij} < \hat{\theta}_{ij}$, then $\hat{\theta}_{gh} - \theta^*_{gh} > \hat{\theta}_{ij} - \theta^*_{ij}$.

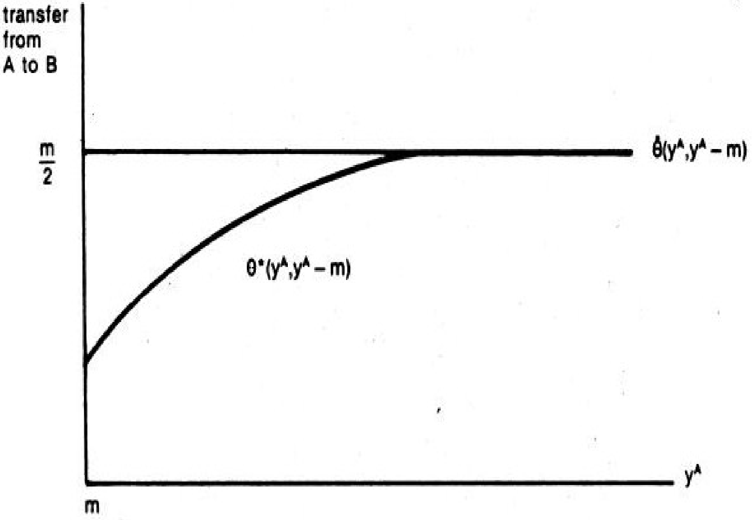
- **PROOF:**

  Follows from the previous theorem and the fact that f is increasing in y.

  We have, $\hat{\theta}_{ij} = \hat{\theta}_{gh}$ and
  $\theta^*_{ij} = f(y_i, [v(\theta^*) - \overline{v}]/r) > f(y_g, [v(\theta^*) - \overline{v}]/r) \geq \theta^*_{gh}$.

- If the informal insurance arrangement diverges from the first-best, then this divergence is greatest at dates with low income levels.

- At low income levels, the marginal utility of income is high and hence the incentives to defect are strong. Thus, if implementability constraint is already binding then it becomes even tighter at lower income levels.


transfer
from
A to B
m
2
θ̂(y^A,y^A – m)
θ*(y^A,y^A – m)
y^A
m

Effects of Changes in Current (Ex-Post) Income Inequality

- Suppose we fix household A's income and lower household B's income.
- Under the first-best arrangement A will always transfer one half of the income difference to B. So, the transfer increases in this case.
- What happens under the optimal informal insurance arrangement?
- **PROPOSITION 2**

  Let $y_i > y_j > y_h$. Then, if $\theta^*_{ij} < \hat{\theta}_{ij}$, $\theta^*_{ih} = \theta^*_{ij}$.

- **PROOF:**

  Since, $y_j > y_h$, we have $\hat{\theta}_{ih} > \hat{\theta}_{ij}$. It follows from the theorem that $\theta^*_{ih} = f(y_i, [v(\theta^*) - \overline{v}]/r)$.

- Once the implementability constraint bites, there is no scope for additional transfers no matter how low B's income falls.

- Therefore, post transfer income inequality will exist and will increase as the income divergence grows.

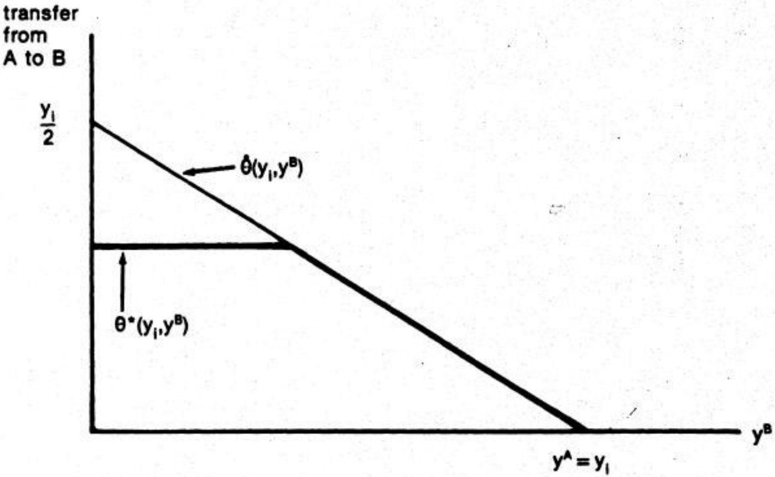

transfer
from
A to B
$\frac{y_l}{2}$
$\hat{\theta}(y_l, y^B)$
$\theta^*(y_l, y^B)$
y^B
$y^A = y_l$

Fig. 2

Effects of Changes in Current (Ex-Post) Income Inequality

- Suppose, now we fix household B's income and increase household A's income.

- Under the first-best contract A will still transfer one half of the income difference to B. So, the transfer increases in this case.

- What happens under the optimal informal arrangement?

- **PROPOSITION 3**

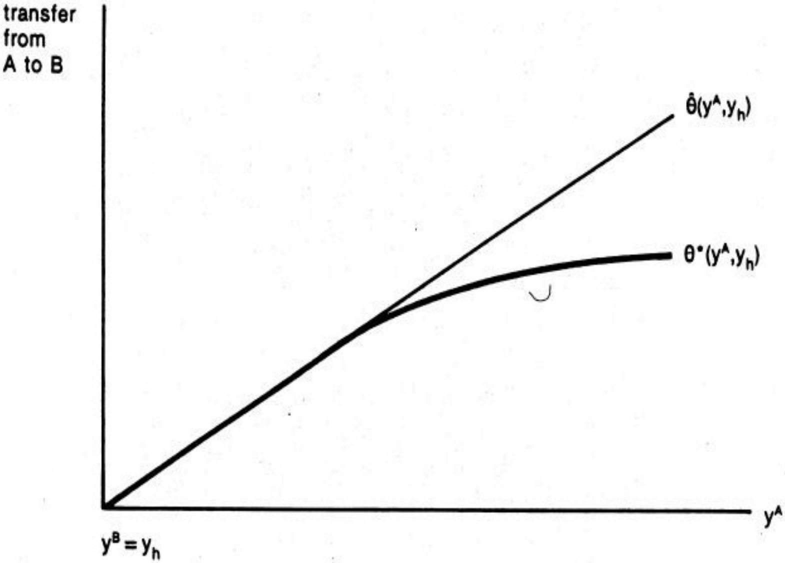
  Let $y_i > y_g > y_h$ and suppose that $1/u'(y)$ is concave. If $\theta^* \neq 0$ and $\theta^*_{gh} < \hat{\theta}_{gh}$, then $\hat{\theta}_{ih} - \theta^*_{ih} > \hat{\theta}_{gh} - \theta^*_{gh}$.

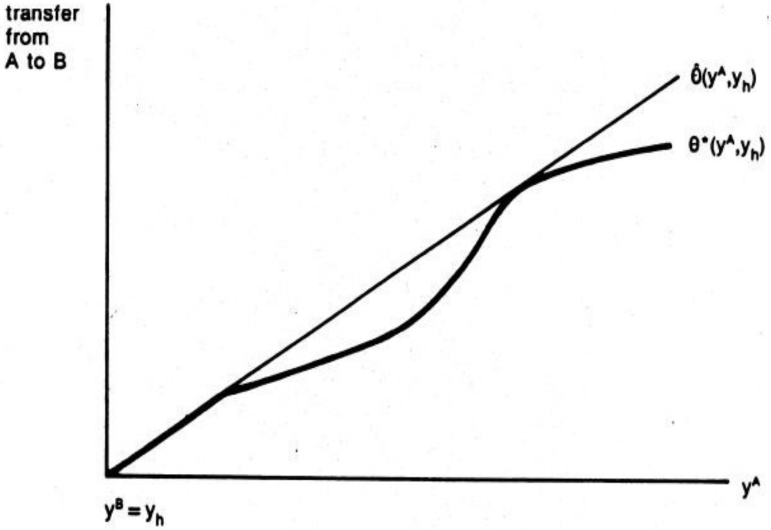
- **PROOF:**

  The proof is given in the Appendix of the Paper.

- If the utility function satisfies the above property and if the informal arrangement differs from the first-best, then this divergence will increase as household A's income increases, holding B's constant.

- If the utility function does not satisfy this property then this difference can close after some point. As income difference grows, the first-best transfer increases which makes implementability more difficult. But, on the other hand, diminishing marginal utility of income implies that the utility cost of a given transfer decreases as income increases.


transfer
from
A to B
$\hat{\theta}(y^A, y_h)$
$\theta^*(y^A, y_h)$
y^A
$y^B = y_h$


transfer
from
A to B
θ̂(y^A,y_h)
θ*(y^A,y_h)
y^A
y^B = y_h

Comparative Static Properties

- The effect of some exogenous variables of the model on the optimal informal insurance arrangement.

- We look at the impact of changes in households' discount rates and the probability distribution over incomes.

Discount Rate

- Risk-sharing institutions are less likely to form if the households are highly impatient, since the benefits from being in such an arrangement are enjoyed in the future.

- Thus, we would expect the divergence between the first-best transfer and the optimal informal transfer at any income level to increase as the discount rate rises.

Let $\theta^*(r)$ denote the optimal implementable contract when the discount rate is r.

- **PROPOSITION 4**

  Let $r^0 < r^1$ and suppose that $\theta^*(r^1) \neq 0$. Then $\theta^*_{ij}(r^1) \leq \theta^*_{ij}(r^0)$, with the inequality holding strictly if $\theta^*_{ij}(r^1) < \hat{\theta}_{ij}$.

- **PROOF:**

  $\theta^*(r^1)$ is implementable when the discount rate is r^0 and hence $v(\theta^*(r^0)) \geq v(\theta^*(r^1))$. Therefore, $[v(\theta^*(r^0)) - \overline{v}]/r^0$ exceeds $[v(\theta^*(r^1)) - \overline{v}]/r^1$. Using the theorem and the fact that f is increasing in its second argument the result follows.

- The discount rate reflects the households' assessment of the probability of playing the game in the future. Thus, we can interpret the game as: if the households do not expect to be playing the game for long the divergence between the optimal arrangement and the first-best arrangement will be large.

- Therefore, as traditional societies become more mobile, so that future generations are less likely to be in close contact, the moral economy will tend to perform less well.

- Also, if the income draws are more frequent the discount rate will be lower.

Probability distribution

- We expect the informal insurance to perform poorly if it was unlikely that the households would earn different incomes.
- In such a case the potential risk-sharing gains would be small and therefore there would be less incentive not to defect.
- Thus, we can conjecture that the divergences between the optimal informal arrangement and the first-best will be greater when the participants face more covariate income streams.

  Let $\theta^*(\pi)$ denote the optimal arrangement associated with the probability distribution π.

- **PROPOSITION 5**

Let π^0 and π^1 be two probability distributions such that:

(i) $\sum_{j=1}^{n} (\pi_{ij}^1 - \pi_{ij}^0) = 0$ and
(ii) $\pi_{ij}^1 < \pi_{ij}^0$ for all $i \neq j$.

If $\theta^*(\pi^1) \neq 0$, then $\theta_{ij}^*(\pi^1) \leq \theta_{ij}^*(\pi^0)$, with the inequality holding strictly if $\theta_{ij}^*(\pi^1) < \hat{\theta}_{ij}$.

- **PROOF:**

The proof is given in the Appendix of the Paper.

Conclusion

- Community wide participation in various risk-sharing practices is a common feature of traditional rural societies.
- These strategies can be reasonable and sustainable but may not be particularly good risk-sharing institutions. Their performance will generally be lower relative to the first-best risk sharing.
- The model analysed suggests that such a divergence in performance will be larger in many situations where insurance is badly needed such as dates when incomes are generally low or those when few incomes are low, generating high inequality.

THANK YOU

Reciprocity Without Commitment
S.Coate and M.Ravallion
Technical Appendix

PROOF OF PROPOSITION 3

In the proof given on pg 21 of the paper the authors use the fact that $f_{11} \leq 0$.
To show this, define:

$$h(y) = 1/u'(y)$$

Now,

$$h''(y) = -\frac{[u'(y)]^2 u'''(y) - 2u'(y)[u''(y)]^2}{[u'(y)]^4}.$$

If,

$$h''(y) < 0, \text{ then } u'(y)u'''(y) > 2[u''(y)]^2.$$

$$\Rightarrow u'''(y) > 2\frac{[u''(y)]^2}{u'(y)}$$

$$\Rightarrow u'''(y) > 0 \quad \ldots (1)$$

We know that, $u(y) - u(y - f) = w$.
Therefore,

$$u'(y) - [u'(y - f)](1 - f_1) = 0 \qquad \ldots (2)$$

And,

$$u''(y) - [u''(y - f)](1 - f_1)^2 + [u'(y - f)]f_{11} = 0$$

$$\Rightarrow f_{11} = \frac{[u''(y - f)](1 - f_1)^2 - u''(y)}{u'(y - f)} \qquad \ldots (3)$$

Using equation (2),

$$(1-f_1) = \frac{u'(y)}{u'(y-f)}$$

Since,

$$u'(y) > 0 \text{ and } f_1 \geq 0, \text{ we have, } 0 < 1 - f_1 \leq 1$$

Using this and equation (1),

$$u''(y) > u''(y-f) \geq u''(y-f)(1-f_1)^2$$

Therefore, $f_{11} < 0$ (Using equation (3)).

PROOF OF PROPOSITION 4

We have $r^0 < r^1$. Thus,

$$\frac{v(\theta^*(r^1)) - \overline{v}}{r^1} < \frac{v(\theta^*(r^1)) - \overline{v}}{r^0}$$

Therefore,

$$f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^0) > f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^1) \geq \theta^*_{ij}(r^1).$$

Since, the maximum amount of income that can be taken from a household with income y_i without inducing defection when the discount rate is r^0 and the arrangement is $\theta^*(r^1)$ is atleast $\theta^*_{ij}(r^1)$. Thus, $\theta^*(r^1)$ is implementable when the discount rate is r^0.

Now, $f(y_i, [v(\theta^*(r^0)) - \overline{v}]/r^0) \geq f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^0)$ and f is increasing in its second argument

$$\Rightarrow v(\theta^*(r^0)) \geq v(\theta^*(r^1))$$

$$\Rightarrow \frac{v(\theta^*(r^0)) - \overline{v}}{r^0} > \frac{v(\theta^*(r^1)) - \overline{v}}{r^1}$$

$$\Rightarrow f(y_i, [v(\theta^*(r^0)) - \overline{v}]/r^0) > f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^1)$$

Using this and the Theorem,

$$\theta^*_{ij}(r^1) \leq f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^1) < f(y_i, [v(\theta^*(r^0)) - \overline{v}]/r^0).$$

If,

$$\theta^*_{ij}(r^1) = \hat{\theta}_{ij} \leq f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^1) < f(y_i, [v(\theta^*(r^0)) - \overline{v}]/r^0)$$

Then,

$$\theta^*_{ij}(r^1) = \hat{\theta}_{ij} = \theta^*_{ij}(r^0).$$

And if,

$$\theta^*_{ij}(r^1) < \hat{\theta}_{ij}, \text{ then } \theta^*_{ij}(r^1) = f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^1) < \hat{\theta}_{ij}$$

Now, if

$$\theta^*_{ij}(r^0) = \hat{\theta}_{ij} \text{ then } \theta^*_{ij}(r^1) < \theta^*_{ij}(r^0)$$

and if,

$$\theta^*_{ij}(r^0) = f(y_i, [v(\theta^*(r^0)) - \overline{v}]/r^0)$$

Then,

$$\theta^*_{ij}(r^1) = f(y_i, [v(\theta^*(r^1)) - \overline{v}]/r^1) < f(y_i, [v(\theta^*(r^0)) - \overline{v}]/r^0) = \theta^*_{ij}(r^0).$$