Group-lending with sequential financing, contingent renewal and social capital

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Introduction:

- The focus of this paper is dynamic aspects of micro-lending, namely "sequential lending" and "contingent renewal".
- Sequential financing refers to group-loans that are staggered within the same round, whereas contingent renewal implies that the selection of the recipient group is history dependent.
- The author has focused on the efficacy of these two dynamic schemes in harnessing social capital. Such social capital may take the form of mutual help in times of distress, mutual reliance in productive activities, status in the local community, etc. In case default by one borrower harms the other borrowers, such default may be penalized through a loss of this social capital. Social penalties may also take the form of a reduced level of cooperation, or even admonishment.

Introduction:

- Dynamic framework yields some interesting new insights, which cannot be replicated in a static framework: under certain circumstances, the lending bank can test for the type of a group by lending to just one of the members, thus screening out bad borrowers partially.
- Depending on parameter values, there may be either positive or negative assortative matching.
- Address the problem of collusion among borrowers. here the author has used the concept of “renegotiation proofness” – which I will not address “explicitly” keeping time constraint in mind.

The Economic Environment:

- The market consists of many borrowers, such that their mass is normalized to one and none of the borrowers is an atom.

  Choice of project: $P_i \in \{P_i^1, P_i^2\}$

  - P_i^1 with **observable return** H
  - P_i^2 with **unobservable return** b; where $0 < b < H$

- For each project $1 to be borrowed for investment at an interest rate r (>1): exogenously determined.
- Choice of project is private information.

Assumptions:

- $r < H \leq 2r$
- Two types of borrowers: $B_i \in \{S, N\}$;
  $\theta \in [0,1]$: fraction of S ;
  S has social capital s(>0); N does not have any social capital
- The social penalty involves a loss of social capital. An S type borrower taking a group-loan is assumed to lose her social capital if she defaults and, moreover, this default affects the other group-member. Thus, the social penalty is ***anonymous*** in the sense that it is imposed irrespective of whether the default affects an S type or an N type borrower. The borrowers all know one another's types, but the bank does not.
- H - r < b : ensures problem of moral hazard is not too small.
  Thus, N type borrowers will choose his/her second project.
- H – r > b – s : ensures social capital is not too small.
  Thus, S type borrowers will choose his/her first project.
- Time is discrete: t = 0, 1, 2, ...
- Let, 0 < δ < 1 denote the common discount factor of all the agents, the borrowers, as well as the bank.

Sequential Lending

Group-lending without Sequential Financing:

Consider the following infinite horizon game:

Period 0:

There is endogenous group-formation - the borrowers organize themselves into groups of two following the *optimal sorting* principle.

For every $t \geq 1$, there is a two-stage game:

Stage 1:

The bank randomly selects one of the groups as the recipient and lends it two dollars, which are divided equally among the two members of the selected group.

Stage 2:

The borrowers simultaneously make their project choice.

Group-lending without Sequential Financing:

Definitions:

- There is ***positive assortative matching*** if there are $\frac{\theta}{2}$ groups of type SS and $\frac{1-\theta}{2}$ groups of type NN.
- There is ***negative assortative matching*** if there are min{θ, 1-θ} groups of type SN, max $\{\frac{1-2\theta}{2}, 0\}$ groups of type NN and max $\{\frac{2\theta-1}{2}, 0\}$ groups of type SS.

We then describe our solution concept.

Group-lending without Sequential Financing:

- Given the lending policy of the bank, once a group receives a loan, this group has zero probability of receiving a loan in the future. Hence, the members of this group are going to behave as if they are playing a one shot game.
- v_{ij}: expected equilibrium payoff of a type i borrower at period t≥1 if she forms a group with a type j borrower and the group receives the bank loan at this period.
- There will be
  - $positive\ assortative\ matching\ if\ \ v_{SS} - \mathrm{v_{SN}} > \mathrm{v_{NS}} - \mathrm{v_{NN}}$
  - $negative\ assortative\ matching\ if\ v_{SS} - \mathrm{v_{SN}} < \mathrm{v_{NS}} - \mathrm{v_{NN}}$
  - $Tie\ breaking\ rule: negative\ assortative\ matching\ if\ v_{SS} - \mathrm{v_{SN}} = \mathrm{v_{NS}} - \mathrm{v_{NN}}$

Group-lending without Sequential Financing:

Consider some period t ≥ 1

Stage 3:

Payoff from investing in her first project: H-r

Payoff from investing in her second project: b.

Given Assumption 1, both the borrowers will invest in their second projects

Thus

$$v_{SS} = \mathrm{v_{SN}} = \mathrm{v_{NS}} = \mathrm{v_{NN}} = \mathrm{b}$$

Stage 2:

The bank's expected payoff at any period from making a loan is -2.

Stage 1:

The tie-breaking rule implies that there will be negative assortative matching. Of course, the expected payoff of the bank is independent of the nature of the matching.

Group-lending without Sequential Financing:

Summarizing the above discussion, we obtain our first proposition.

<u>Proposition 1:</u> Group-lending without sequential financing is not feasible.

<u>Remark 1</u>:

It is clear this analysis goes through even if $H>2r$.

Group-lending with Sequential Financing:

In every round, the members of the selected group receives loans in a staggered manner, but the selection of the recipient group is independent of history.

Consider the following game:

Period 0:

There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

For every $t \geq 1$, there is a two-stage game:

Stage 1:

The bank randomly selects one of the *groups* and lends it one dollar.

Group-lending with Sequential Financing:

Stage 2:

One of the *borrowers* is randomly selected as the recipient of the 1 dollar lent by the bank.

B_i's decision:

- $Invest\ in\ P_i^1$ $\quad$ $payoff:\ H - r\ \ for\ both\ the\ type$

  The game goes to the next stage.
- $Invest\ in\ P_i^2$ $\quad$ $payoff:\ b\quad if\ type\ N$

  $b - s\ \ if\ type\ S$

Assumption:

H-r <1, so that this amount is not sufficient to finance the investment in the next stage.

Group-lending with Sequential Financing:

Stage 3:

This stage arises only if B_i had invested in P_i^1 in stage 2.
The bank lends a further 1 dollar to the group, which is allocated to the other borrower, B_j.

B_j's decision:

- $Invest\ in\ P_i^1 \quad payoff: \ H - r \quad for\ both\ the\ type$
- $Invest\ in\ P_i^2 \quad payoff: b \quad for\ both\ the\ type$

Group-lending with Sequential Financing:

As in the previous subsection, for t ≥1, it is sufficient to restrict attention to one-shot games.

Stage 3:

Both types of borrowers would invest in their second projects.

Stage 2:

Given that borrowers of both types default in stage 3, in stage 2, S type borrowers will invest in their first projects and N type borrowers will invest in their second projects. Hence,

$$\hat{v}_{SS} = \frac{H - r + b}{2}, \qquad \hat{v}_{SN} = \frac{H - r}{2}, \qquad \hat{v}_{NN} = \frac{b}{2}, \qquad \hat{v}_{NS} = b$$

Group-lending with Sequential Financing:

Stage 1:

Irrespective of the nature of the matching process, the expected per period payoff of the bank is

$$\theta r - 1 - \theta.$$

Period 0:

Group-formation would lead to *negative assortative* matching.

Group-lending with Sequential Financing:

***Proposition 2:* Sequential financing is feasible if and only if**

$$\theta r - 1 - \theta \geq 0$$

Default by the first recipient of the group-loan adversely affects her partner. Hence, for type S borrowers, the social capital is brought into play, so that they invest in their first projects. Thus, the moral hazard problem is resolved partially and group-lending may be feasible, even if there is negative assortative matching.

Group-lending with Sequential Financing:

Remark 2:

Consider the case where, in case the loan goes to a group of type SN, the S type borrower is the first recipient with probability α, 0 ≤ α ≤ 1. In this case,

$$\hat{v}_{SS} = \frac{H - r + b}{2}, \hat{v}_{SN}(\alpha) = \alpha(H - r), \qquad \hat{v}_{NN} = \frac{b}{2}, \qquad \hat{v}_{NS}(\alpha) = b$$

There is negative assortative matching if and only if α≥0.5. Thus, positive assortative matching is more likely when the 'bargaining power' of the S type agents is low, in the sense that α is small.

Contingent Renewal And sequential financing

Contingent Renewal without sequential financing

Selection of the recipient group is history dependent, but in any round, all members of the recipient group receive loans simultaneously.

Consider the following game:

Period 0:

There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

For every $t \geq 1$, there is a two-stage game:

Stage 1:

At t =1, the bank lends some randomly selected group 2 dollars. In case the recipient group at t-1 had repaid its loans, at t the bank makes a repeat loan to this group. In case the recipient group had defaulted at t -1, no member of this group ever obtains a loan. In that case, the bank lends 2 dollars to some randomly selected group, of borrowers who have not defaulted earlier.

Contingent Renewal without sequential financing

Stage 2:

The borrowers simultaneously make their project choice.

V_{ij}: expected equilibrium payoff of a type i borrower at period $t \geq 1$ if she forms a group with a type j borrower and the group receives the bank loan in period t.

Contingent Renewal without sequential financing

Consider some subgame B_iB_j. Note that, in any subgame perfect

equilibrium, if, in period T, B_i invests in her second project, then so must borrower B_j (Assumption 1).

The present discounted value of the borrowers' payoff from investing in first project for the first t periods and then to deviate:

$$\frac{(1-\partial^T)(H-r)}{1-\partial}+\partial^T b$$

Contingent Renewal without sequential financing

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And, the discounted value of the borrower's payoff from investing in first project always is $\frac{H-r}{1-\partial}$

Proposition 3.

(i) If $\partial \geq \frac{b-H+r}{b}$*, then the borrowers of both types invest in their first projects at every period they obtain the loan.*

(ii) If $\partial < \frac{b-H+r}{b}$*, then all the borrowers invest in their second projects at every period they obtain the loan.*

Contingent Renewal without sequential financing

From proposition 3:

$$If\ \partial \geq \frac{b-H+r}{b}$$

$$\boldsymbol{V_{SS} = V_{SN} = V_{NN} = V_{NS} = \frac{H-r}{1-\partial}}$$

$$\boldsymbol{And\ bank's\ payoff = 2(r-1)}$$

$$If\ \partial < \frac{b-H+r}{b}$$

$$\boldsymbol{V_{SS} = V_{SN} = V_{NN} = V_{NS} = b}$$

$$\boldsymbol{And\ bank's\ payoff = -2}$$

Contingent Renewal without sequential financing

***<u>Proposition 4:</u>* Group-lending with contingent renewal , but without sequential financing is feasible if and only if**

$$\partial \geq \frac{b - H + r}{b}$$

Contingent Renewal with sequential financing

Consider the following game:

Period 0:

There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

For every $t \geq 1$, there is a three-stage game:

Stage 1:

At t =1, the bank lends some randomly selected group 2 dollars. In case the recipient group at t-1 had repaid its loans, at t the bank makes a repeat loan to this group. In case the recipient group had defaulted at t -1, no member of this group ever obtains a loan. In that case, the bank lends 2 dollars to some randomly selected group.

Contingent Renewal with sequential financing

Stage 2:

One of the borrowers is randomly selected as the recipient of the 1 dollar lent by the bank.

B_i's decision:

- $Invest\ in\ P_i^1 \qquad payoff:\ H - r \quad for\ both\ the\ type$
  The game goes to the next stage.
- $Invest\ in\ P_i^2 \qquad payoff:\ b \qquad if\ type\ N$
  $b - s \quad if\ type\ S$

Contingent Renewal with sequential financing

Stage 3:

This stage arises only if B_i had invested in P_i^1 in stage 2.
The bank lends a further 1 dollar to the group, which is allocated to the other borrower, B_j.

B_j's decision:

- $Invest\ in\ P_i^1$ $\quad payoff: \ H - r \quad for\ both\ the\ type$
- $Invest\ in\ P_i^2$ $\quad payoff: \ b \quad if type\ N$

$$b - s \quad if\ type\ S$$

Contingent Renewal with sequential financing

Proposition 5.

(i) If $\partial \geq \frac{b-H+r}{b}$***, then the unique renegotiation-proof equilibrium involves borrowers of both types investing in their first projects at every period they obtain the loan.***

(ii) If $\partial < \frac{b-H+r}{b}$***, then the unique renegotiation-proof equilibrium involves the S type borrowers investing in their first projects, and the N type borrowers investing in their second projects at every period they obtain the loan.***

Contingent Renewal with sequential financing

From proposition 5:

$$\boldsymbol{If}\ \partial \geq \frac{b-H+r}{b}$$

$$\widehat{\boldsymbol{V}}_{\boldsymbol{SS}} = \widehat{\boldsymbol{V}}_{\boldsymbol{SN}} = \widehat{\boldsymbol{V}}_{\boldsymbol{NN}} = \widehat{\boldsymbol{V}}_{\boldsymbol{NS}} = \frac{\boldsymbol{H}-\boldsymbol{r}}{\mathbf{1}-\boldsymbol{\partial}}$$

$$\boldsymbol{If}\ \partial < \frac{b-H+r}{b}$$

$$\widehat{\boldsymbol{V}}_{\boldsymbol{SS}} = \frac{\boldsymbol{H}-\boldsymbol{r}}{\mathbf{1}-\boldsymbol{\partial}}, \widehat{\boldsymbol{V}}_{\boldsymbol{SN}} = \frac{\boldsymbol{H}-\boldsymbol{r}}{\mathbf{2}},$$

$$\widehat{\boldsymbol{V}}_{\boldsymbol{NN}} = \frac{\boldsymbol{b}}{\mathbf{2}}, \widehat{\boldsymbol{V}}_{\boldsymbol{NS}} = \boldsymbol{b}$$

Contingent Renewal with sequential financing

- $If\ \frac{b-H+r}{b} \leq \partial, the\ borrowers\ invest\ in\ their\ first\ projects,$
$$Bank's\ per\ period payoff = 2(r-1)$$

- $If\ \frac{b-H+r}{b+H-r} < \partial < \frac{b-H+r}{b}$, ***there will be positive assortative matching***

Bank's expected payoff $= \frac{2\theta(r-1)-(1-\partial)(1-\theta)}{(1-\partial)[1-\partial(1-\theta)]}$

  - $If \partial \leq \frac{b-H+r}{b+H-r}, there\ will\ be\ negative\ assortative\ matching$

$$Bank's\ expected\ payoff = \frac{2(2\theta-1)(r-1)+(1-\partial)(1-\theta)(r-3)}{(1-\partial)[1-2\partial(1-\theta)]}, \forall\theta \geq 0.5$$

$$= \frac{\theta r-\theta-1}{1-\partial}, \qquad otherwise$$

Contingent Renewal with sequential financing

Propostion 6:

i) ***There is positive assortative matching iff*** $\frac{b-H+r}{b+H-r} < \partial < \frac{b-H+r}{b}$

ii) ***If*** $\frac{b-H+r}{b} \leq \partial$***, then group lending with sequential financing and contingent renewal is feasible.***

For $\partial < \frac{b-H+r}{b+H-r}$***, group lending is feasible iff***

a. $\frac{b-H+r}{b=h-r} < \partial < \frac{b-H+r}{b}$ ***and*** $2\theta(r-1) - (1-\partial)(1-\theta) \geq 0$

b. $\partial < \frac{b-H+r}{b+H-r}, \theta \geq 0.5$ *and* $2(2\theta-1)(r-1) + (1-\partial)(1-\theta)(r-3) \geq 0$ *or,*

c. $\partial < \frac{b-H+r}{b+H-r}, \theta < 0.5$ *and* $\theta r - \theta - 1 \geq 0$

What I have skipped:

- Renegotiation proofness – explicitly I am not discussing it.
- Discussion regarding the robustness of the model.

  Relaxation of some assumptions like

  i)non-anonymous social penalty function

  ii)implication of endogenously determined interest rate. etc

Conclusion:

- Focus of this paper: dynamic aspect of group lending.
- It has been shown that - under the appropriate parameter configurations, there is positive assortative matching, so that the bank can test whether a group is good or bad relatively cheaply, i.e. without lending to all its members, thus leading to a partial screening out of bad borrowers.
- In contrast to most of the literature, there may be negative assortative matching if the discount factor is sufficiently small.
- Under appropriate parameter configuration group lending would be feasible.