Group-lending with sequential financing, contingent renewal and social capital

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This paper focuses on the dynamic aspects of group-lending, in particular sequential financing andcontingent renewal. We examine the efficacy of these two schemes in harnessing social capital.

1. In the Grameen Bank, for example, the groups have five members each. Loans are sequential in the sense that these are initially given to only two of the members (to be repaid over a period of 1 year). If they manage to pay the initial installments, then, after a month or so, another two borrowers receive loans and so on.

2. Contingent renewal of loans refers to the feature that in case of default by a group, no member of this group ever receives a loan in the future. Moreover, in case of repayment, there is repeat lending.

Here in this paper we consider groups of size 2.

- Sequential Lending – First one, then another.
- Contingent Renewal – First both, then both and so on.
- Sequential Lending with Contingent renewal – First one then both and so on.

<u>Concept of social capital</u>.

- Social capital may take the form of mutual help in times of distress (see Coate and Ravallion, 1993), mutual reliance in productive activities, status in the local community, etc.
- In case default by one borrower harms the other borrowers, such default may be penalized through a loss of this social capital. Social penalties may also take the form of a reduced level of cooperation, or even admonishment.

The Economic Environment.

- The market consists of many borrowers, such that **their mass is normalized to one** and none of the borrowers is an atom.
- The borrowers are heterogeneous, so that some borrowers (denoted the S type) have access to social capital, while the others (denoted the N type) do not. For an S type borrower, social penalty involves the withdrawal of this **social capital 's'(s is greater than zero)** whenever default by this borrower harms the other group-members.
- **Borrower *i* can invest in one of two projects, P_i^1 or P_i^2.**
- For every *i*, P_i^1 has **a verifiable income of H** and no non-verifiable income, whereas P_i^2 has no verifiable income and **a non-verifiable income of b**, where '0' is less than 'b' which is less than 'H'. Thus there is a moral hazard problem.
- **The sets of projects are different for different borrowers**. While the borrowers know the identity of their own projects, they do not know the identity of the other borrowers' projects. In every period, the borrowers consume all their income in that period.

- **All projects require an initial investment of 1 dollar**. Since none of the borrowers have any funds, they have to borrow the required 1 dollar from a bank. For every dollar loaned, **the amount to be repaid is r (≥1)**, where r is exogenously given.

The Setup (Recap)

- We build a simple infinite-horizon dynamic model based on social capital, moral hazard and endogenous group-formation.
- There are many borrowers, all of whom have access to two projects where the first one has a verifiable income, but no private benefit (non-verifiable), while the second one has a private benefit, but no verifiable income.
- The bank prefers the first project (when it can recoup its initial investment), while at least the N type borrowers prefer the second one.

- **Group Formation**.
- There is endogenous group-formation whereby, prior to the actual lending, the borrowers form groups of size two among themselves. The borrowers all know one another's types, but the bank does not.
- The key issue is whether there will be positive assortative matching or negative, i.e. whether group-formation will be homogenous or not.

What is positive/negative assortative matching?

What we want to do

We analyze the effect of social capital on

- Group formation (Who pairs up with whom and why?)
- The feasibility of financing by the bank.

We do this for each of the separate cases. Reiterating, cases are –

- Sequential Lending – First one, then another.
- Contingent Renewal – First both, then both and so on.
- Sequential Lending with Contingent renewal – First one then both and so on.

NOTE:

- The sets of projects are different for different borrowers. Nobody knows the identity of the other’s projects. Yet the returns from the projects are the same (?)
- <u>Side payments are possible</u>.

- A fraction $0 \leq \theta \leq 1$ of the borrowers has a social capital of s (≥ 0), whereas the other borrowers have no social capital. The borrowers with social capital are denoted by S, whereas the other borrowers are denoted by N.
- The social penalty involves a loss of this social capital. An S type borrower taking a group-loan is assumed to lose her social capital if she defaults and, moreover, this default affects the other group-member.
- NOTE: **An N type borrower has no social capital to lose if he defaults.**

- <u>**Assumptions**</u>

- **Assumption 1. H-r is less than b**.
  Suppose that a borrower has taken a loan of 1 dollar. If the borrower is of type N, then, given Assumption 1, she will prefer to invest in her second project. Further, we assume that the social capital s is not too small.
- **Assumption 2. H-r is greater than b-s**.
  Suppose some borrower of type S has taken a loan and that she will lose her social capital in case of default. In case she invests in her second project, she obtains a non-verifiable income of b, but loses her social capital, so that her net payoff is b-s. Given Assumption 2, the borrower will prefer to invest in her first project.

Case 1: Group lending without sequential financing

Period 0. There is endogenous group-formation whereby the borrowers organize themselves into groups of two. Depending on the type of borrowers comprising the groups, these can be of three types, SS, NN and SN.

Optimal Sorting Principle. Borrowers from different groups cannot form a new group without making some member of the new group worse off.

Important: For every t≥1, there is a two-stage game.

Stage 1. The bank randomly selects one of the groups as the recipient and lends it **two dollars**, which are **divided equally among the two members** of the selected group. Note that the lending policy of the bank does not involve contingent renewal.

Stage 2. Both the borrowers then simultaneously invest 1 dollar into one of their two projects. **Why?**
Given the lending policy of the bank, once a group receives a loan, this group has zero probability of receiving a loan in the future. Hence, the members of this group are going to behave as if they are playing a one shot game.

If the i-th borrower invests in $\mathbf{P_i^1}$, she has a payoff of H-r; otherwise, she has a payoff of b.
Note that, given the lending policy, default by a borrower does not affect the expected income of the other borrower and hence does not attract the social penalty even if she is of type S.

The bank has a payoff of 2(r-1) in case both the borrowers invest in their $\mathbf{P_i^1}$, r-2 in case only one of the borrowers invests in her first project and the other borrower invests in her second project, and a payoff of -2 in case both the borrowers invest in their second projects.

Let V_{ij} denote the **expected** equilibrium payoff of a type i borrower at some period t≥1 if she forms a group with a type j borrower and the group receives the bank loan at this period.
Assuming that **<u>side payments are possible</u>**, there will be positive assortative matching if and only if the maximum, a type N borrower is willing to pay to a type S borrower, is strictly less than the minimum a type S borrower will need as compensation for having a type N partner, i.e.
$V_{SS} – V_{SN}$ is greater than $V_{NS} - V_{NN}$

Clearly, there will be negative assortative matching whenever $V_{SS} + V_{NN}$ is less than $V_{SN} + V_{NS}$.

Tie Breaking Rule

$V_{SS} - V_{SN} = V_{NS} - V_{NN}$

There is no strong justification for either positive or negative assortative matching. In general, we can expect that there will be x groups of type SN, where x ≤ min{, 1_h}, and the remaining borrowers will form groups with their own types.

However, for ease of exposition, we assume that in this case there will be negative assortative matching, i.e. x =min{ θ, 1- θ }

Stage 3. For any borrower, her payoff from investing in her first project is H-r, whereas her payoff from investing in her second project is b. Given Assumption 1, both the borrowers will invest in their second projects irrespective of their type. Thus

vSS = vSN = vNN = vNS = b

By backward induction to Stage 2:

Stage 2. Since the borrowers always invest in their second project, the bank's expected payoff at any period from making a loan is -2.

By backward induction to Stage 1:

Stage 1. Given Eq. (2), the tie-breaking rule implies that there will be negative assortative matching. Of course, the expected payoff of the bank is independent of the nature of the matching.

Hence, Conclusion: **Proposition 1. Group-lending without sequential financing is not feasible.**

<u>**Group-lending with (only) sequential financing**</u>

In this subsection, we examine a group-lending scheme with sequential financing, but <u>**no contingent renewal**</u>.

Note: So, one of the borrowers is randomly selected (with probability half) by the group as the recipient of the 1 dollar lent by the bank. If the loan is repaid, the bank lends a further 1 dollar to the group, which is allocated to the other borrower.

Thus, in every round, the members of the selected group receive loans in a staggered manner, but the selection of the recipient group is independent of history. We consider the following game.

<u>*Period 0*</u>. There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

NOTE: For every $t \geq 1$, there is <u>a three-stage game</u>. Previously it was only two.

Stages

Stage 1. The bank randomly selects a group and lends the selected group 1 dollar. Thus, as in the previous subsection, there is no contingent renewal.

Stage 2. One of the borrowers is randomly selected (with probability half) by the group as the recipient of the 1 dollar lent by the bank. (One can alternatively assume that this selection is done by the bank.) This borrower, say B_i, then decides whether to invest the 1 dollar in $\mathbf{P_i^1}$ or $\mathbf{P_i^2}$

Scenario of Default: If B_i invests in $\mathbf{P_i^2}$, then Bi defaults, there is no further loan by the bank and the game goes to the next period. Note that, in case of default by B_i, B_j does not obtain the loan at all. Hence, depending on its type, B_i obtains either b or b-s.

Scenario of Repayment: If Bi invests in $\mathbf{P_i^1}$, then there is a verifiable return of H, out of which the bank is repaid r and Bi obtains H-r.

NOTE: We assume that H-r is less than 1, so that this amount is not sufficient to finance the investment in the next stage.

<u>Stage 3</u>. This stage arises only if B_i had invested in $\mathbf{P_i^1}$ in stage 2. The bank lends a further 1 dollar to the group, which is allocated to the other borrower, B_j, who decides whether to invest it in $\mathbf{P_i^1}$ or $\mathbf{P_i^2}$.

Note that, in this case, default by B_j does not affect the payoff of B_i, the group-member who had received the loan earlier.

Hence, if this amount is invested in $\mathbf{P_i^2}$ then Bj obtains b and the bank obtains nothing. If it is invested in $\mathbf{P_i^1}$, then Bj obtains H-r and the bank obtains r.

Again, it is sufficient to restrict attention to one-shot games.

Let V_{ij} denote the expected equilibrium payoff of a type i borrower at some period $t \geq 1$, in case she forms a group with a type j borrower and this group obtains the loan at this period.
We next turn to solving this game. Consider $t \geq 1$.

Applying backward induction:

<u>Stage 3</u>. Both types of borrowers would invest in their second projects.

Stage 2. Given that borrowers of both types default in stage 3, in stage 2, S type borrowers will invest in their first projects (Assumption 2: page 10) and N type borrowers will invest in their second projects (Assumption 1: page 10). Hence

$$\hat{v}_{SS} = \frac{H - r + b}{2}, \hat{v}_{SN} = \frac{H - r}{2}, \hat{v}_{NN} = \frac{b}{2} \text{ and } \hat{v}_{NS} = b.$$

(To show.)

Stage 1.

NOTE:It is easy to see that, irrespective of the nature of the matching process, the expected per period payoff of the bank is

$$\theta r - 1 - \theta.$$

The expected payoff of the bank is independent of the exact nature of the matching.

This follows because the investment decision of a borrower does not depend on the nature of the group, but only on whether the borrower is the first recipient of the loan or not.

Given Eq. above, by Tie Breaking Rule, it is easy to see that group-formation would lead to negative assortative matching.

To conclude:

Proposition 2. Sequential financing is feasible if and only if $\theta r - 1 - \theta.$ is greater than zero.

Under sequential financing, default by the first recipient of the group-loan adversely affects her partner (who does not obtain any loan). Hence, for type S borrowers, the social capital is brought into play, so that they invest in their first projects. Thus, the moral hazard problem is resolved partially and group-lending **may** be feasible. Further, **note that group-lending may be feasible even if there is negative assortative matching.**

Remark 2. Consider the case where, in case the loan goes to a group of type SN, the S type borrower is the first recipient with probability α, 0≤ α ≤1. In this case, it is easy to see that

$$\hat{v}_{\mathrm{SS}} = \frac{H - r + b}{2}, \ \hat{v}_{\mathrm{SN}}(\alpha) = \alpha(H - r), \ \hat{v}_{\mathrm{NN}} = \frac{b}{2} \text{ and } \hat{v}_{\mathrm{NS}}(\alpha) = b.$$

Moreover, there is negative assortative matching if and only if α ≥1/2. Thus, somewhat surprisingly, positive assortative matching is more likely when the bargaining power of the S type agents is low, in the sense that α is small.

GROUP LENDING

With Contingent Renewal and Sequential Financing

Contingent Renewal without Sequential Financing

Let us consider the following game

Period 0:

There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

For every $t \geq 1$, there is a two-stage game:
Stage 1:

At $t = 1$, the bank lends some randomly selected group 2 dollars. In case the recipient group at $t-1$ had repaid its loans, at t the bank makes a repeat loan to this group. In case the recipient group had defaulted at $t-1$, no member of this group ever obtains a loan. In that case, the bank lends 2 dollars to some randomly selected group.

Stage 2:
The borrowers simultaneously make their project choice.

V_{ij} be the expected equilibrium payoff of a type i borrower at period $t \geq 1$ if she forms a group with a type j borrower and the group receives the bank loan in period t.

Proposition 3.

(i) *If* $\delta \geq \frac{b-H+r}{b}$, *then the unique renegotiation-proof equilibrium involves borrowers of both types investing in their first projects at every period they obtain the loan.*

(ii) *If* $\delta < \frac{b-H+r}{b}$, *then the unique renegotiation-proof equilibrium involves all the borrowers investing in their second projects at every period they obtain the loan.*

Note :

- In case (i) holds, the incentive for S-type is even higher than (because of social capital), whereas for an N-type borrower the incentives are the same.
- In case of (ii), both the types invest in their second project and the presence of social capital does not upset the result. (Why?)

Given Proposition 3, it is easy to see that

$$V_{SS} = V_{SN} = V_{NN} = V_{NS} = \frac{H-r}{1-\delta}, \text{ if } \delta \geq \frac{b-H+r}{b},$$

$$V_{SS} = V_{SN} = V_{NN} = V_{NS} = b, \text{ otherwise.}$$

Proposition 4. *Group-lending with contingent renewal, but without sequential financing is feasible if and only if* $\delta \geq \frac{b-H+r}{b}$.[23]

Thus, for $\delta \geq \frac{b-H+r}{b}$, the first best outcome is implemented.[24] The argument clearly relies on the trigger strategy like aspect of contingent renewal. For $\delta < \frac{b-H+r}{b}$, however, all the borrowers invest in their second projects, so that contingent renewal fails to resolve the moral hazard problem.

Contingent Renewal with Sequential Financing

Consider the following game:

- *Period 0:*

There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

- For every $t \geq 1$, there is a three-stage game:

<u>Stage 1</u>:

At t =1, the bank lends some randomly selected group 1 dollars. For t>1, In case the first recipient at t-1 had repaid its loans, the bank lends 1 dollar to the second recipient. In case the recipient member had defaulted at t -1, no member of this group obtains a loan in this period or in the future. In that case, the bank choses to lend to some other group who had not defaulted earlier.

Stage 2:

One of the borrower is randomly selected(with probability half) as the recepient of the 1 dollar lent by the bank.

B_i's decision:

- $Invest\ in\ P_i^1$ $\quad payoff:\ H - r \quad for\ both\ the\ type$
  The game goes to the next stage.
- $Invest\ in\ P_i^2$ $\quad payoff:\ b \quad if\ type\ N$
  $\quad b - s \quad if\ type\ S$

Stage 3:

This stage comes only if borrower i choses to invest in project 1 in stage 2. The bank lends dollar 1 to the other borrower, who then decides to allocate this amount in one of the two projects.

Proposition 5.

(i) If $\delta \geq \frac{b-H+r}{b}$, then the unique renegotiation-proof equilibrium involves borrowers of both types investing in their first projects at every stage when they obtain the loan.

(ii) If $\delta < \frac{b-H+r}{b}$, then the unique renegotiation-proof equilibrium involves the S type borrowers investing in their first projects, and the N type borrower investing in their second projects at every stage when they obtain the loan.

Note:

- In this case, the incentive to invest in the first project is higher for the S-type compared to the case where there is **contingent renewal, but no sequential financing**. This is because in this case default by an S type borrower adversely affects her partner.

- Also, the incentive to invest in the first project in this case is higher for the S-type compared to the case where there is **sequential financing only.**

- Thus, the incentive to invest in the first project is higher in case both the schemes are used in conjunction.

Given Proposition 5, we have that

$$\hat{V}_{\mathrm{SS}} = \hat{V}_{\mathrm{SN}} = \hat{V}_{\mathrm{NN}} = \hat{V}_{\mathrm{NS}} = \frac{H-r}{1-\delta}, \text{ if } \delta \geq \frac{b-H+r}{b},$$

$$\hat{V}_{\mathrm{SS}} = \frac{H-r}{1-\delta}, \ \hat{V}_{\mathrm{SN}} = \frac{H-r}{2}, \ \hat{V}_{\mathrm{NN}} = \frac{b}{2}, \ \hat{V}_{\mathrm{NS}} = b, \text{ otherwise.}$$

If $\frac{b-H+r}{b+H-r} < \delta < \frac{b-H+r}{b}$, then there will be positive assortative matching and the expected payoff of the bank is

$$\frac{2\theta(r-1) - (1-\delta)(1-\theta)}{(1-\delta)[1-\delta(1-\theta)]}.$$ ~~(10)~~

Finally, if $\delta \leq \frac{b-H+r}{b+H-r}$, then there is negative assortative matching. Thus, the expected payoff of the bank is

$$\frac{2(2\theta-1)(r-1) + (1-\delta)(1-\theta)(r-3)}{(1-\delta)[1-2\delta(1-\theta)]}, \ \forall \theta \geq \frac{1}{2}, \tag{11}$$

$$\frac{\theta r - \theta - 1}{1-\delta}, \text{ otherwise.} \tag{12}$$

Proposition 6.

(i) *There is positive assortative matching if and only if* $\frac{b-H+r}{b+H-r}<\delta<\frac{b-H+r}{b}$.

(ii) *If* $\delta\geq\frac{H-r+b}{b}$, *then group-lending with both sequential financing and contingent renewal is feasible. For* $\delta<\frac{b-H+r}{b}$, *group-lending is feasible if and only if*

(a) $\frac{b-H+r}{b+H-r}<\delta<\frac{b-H+r}{b}$ *and* $2\theta(r-1)-(1-\delta)(1-\theta)\geq 0$, *or*

(b) $\delta<\frac{b-H+r}{b+H-r}$, $\theta\geq 1/2$ *and* $2(2\theta-1)(r-1)+(1-\delta)(1-\theta)(r-3)\geq 0$, *or*

(c) $\delta<\frac{b-H+r}{b+H-r}$, $\theta<1/2$ *and* $\theta r-\theta-1\geq 0$.[25]

From The Banks Perspective

If $\delta < \frac{b-H+r}{b}$,

- The lending policy ensures that S-type borrowers invest in their first projects, whereas N type borrowers invest in their second projects.

- In addition if there is positive assortative matching, which means that in case a NN type group obtains a loan, the first recipient will default and hence the sequential financing acts as a partial screening mechanism where the identity of the groups can be ascertained cheaply.

- We have seen that in this case, under contingent renewal (only), group lending was not feasible.

- Also, comparing to sequential financing (only), the combination of the results in a strictly better payoff to the bank if the proportion of the S type is more than half.

Non- Anonymous Social Penalty Function

Suppose the social penalty function in non-anonymous in the sense that it is imposed whenever a default by an S-type harms another S-type recipient.

Under Sequential Financing:

$$\hat{v}_{SS} = \frac{H - r + b}{2}, \ \hat{v}_{SN} = \frac{b}{2}, \ \hat{v}_{NN} = \frac{b}{2} \text{ and } \hat{v}_{NS} = \frac{b}{2}.$$

Under Contingent Renewal:
There is no change in the result.

Sequential Financing with Contingent Renewal and Non Anonymous Social Penalty function

$If\ \delta \geq \frac{b-H+r}{b}$,

Then the argument remains the same as earlier as both the types of borrowers find it profitable to invest in their first projects.

$If\ \delta < \frac{b-H+r}{b}$,

Here an S-type borrower behaves like an N-type if paired with an N-type. Thus,

$$\hat{V}_{\mathrm{SS}} = \frac{H-r}{1-\delta},\ \hat{V}_{\mathrm{SN}} = \frac{b}{2},\ \hat{V}_{\mathrm{NN}} = \frac{b}{2} \text{ and } \hat{V}_{\mathrm{NS}} = \frac{b}{2}.$$

Proposition 7. *Suppose that* $\delta \leq \frac{b-2H+2r}{b}$ *and the social penalty function is non-anonymous. In case there is both sequential financing and contingent lending, the outcome involves negative assortative matching and, for* $\theta \leq 1/2$*, group-lending is not feasible. Whereas, if there is sequential financing alone, then there is positive assortative matching and, moreover, group-lending is feasible whenever* $\theta r - \theta - 1 \geq 0$.[27]

This means that schemes involving Contingent Renewal must be used with care,
Especially if the discount factor is small

THANK YOU

Sequential Financing & Group Lending

- Contingent Renewal without sequential financing

In proposition 3: the cutoff for δ is determined as :-

If any partner defaults, then one time payoff $= b$

If no one defaults, total discounted payoff $= \frac{H-r}{1-\delta}$

So players would chose to default if

$$b \geq \frac{H-r}{1-\delta}$$

$$\Rightarrow \boxed{\delta \leq \frac{b-H+r}{b}}$$

- Contingent Renewal with Sequential Financing (Proposition 5)

We know

If $\delta \geq \frac{b-H+r}{b}$ $\quad V_{SS} = V_{SN} = V_{NN} = V_{NS} = \frac{H-r}{1-\delta}$ ($\because$ Matching does not

If $\delta < \frac{b-H+r}{b}$ $\quad V_{SS} = \frac{H-r}{1-\delta}$ $\quad V_{SN} = \frac{H-r}{2}$ $\quad V_{NN} = \frac{b}{2}$ $\quad V_{NS} = b$

Now if $\delta < \frac{b-H+r}{b}$

there will be positive Assortive Matching if the side payment that S-type would demand for pairing with ~~a~~ a N-type is greater than what N-type person is willing to pay i.e.

$V_{SS} - V_{SN} > V_{NS} - V_{NN}$

i.e when

$$\frac{H-r}{1-\delta} - \frac{H-r}{2} > b - \frac{b}{2}$$

$$\Rightarrow \frac{(H-r)(2-1+\delta)}{\cancel{2}(1-\delta)} > \frac{b}{\cancel{2}}$$

$$\Rightarrow 2(H-r) - (1-\delta)(H-r) > b(1-\delta)$$

$$\Rightarrow 2(H-r) > (1-\delta)(H-r+b)$$

$$\Rightarrow \frac{2H-2r}{H-r+b} > 1-\delta$$

$$\Rightarrow \delta > 1 - \frac{2H-2r}{H-r+b}$$

$$\Rightarrow \boxed{\delta > \frac{b-H+r}{b+H-r}}$$

implies positive assortive matching

If $\frac{b-H+r}{b+H-r} < \delta < \frac{b-H+r}{b}$

- To find the expected payoff of the bank in case of positive Assortive Matching

Total No of SS type grps $= \theta$

Total " " NN type grps $= 1-\theta$

Prob that bank picks up a SS-type grp (randomly) $= \frac{\theta}{\theta+1-\theta} = \theta$

Prob that " " " a NN-type grp $= 1-\theta$

Payoff to bank if loan is given to a SS type $= 2(r-1)$
(∵ Only the first recepient pays back)

Payoff to bank if loan is given to NN type grp = -1

(∵ First recepient doesn't pay back and thus the bank doesn't lend to the second Recepient)

∴ Let S be the expected payoff to the bank

So,

$$S = \underbrace{\theta \times \frac{2(r-1)}{1-\delta}}_{\substack{\text{If SS grp is}\\ \text{Selected, bank}\\ \text{keeps getting}\\ 2(r-1)\text{ in every}\\ \text{period.}}} + \underbrace{(1-\theta)\left[-1+\delta S\right]}_{\substack{\text{If NN grp is selected, bank loses }-1\\ \text{and then picks up another}\\ \text{group (randomly).}}}$$

$$S = \frac{2\theta(r-1)}{1-\delta} - (1-\theta) + \delta(1-\theta)S$$

$$S(1-\delta(1-\theta)) = \frac{2\theta(r-1) - (1-\delta)(1-\theta)}{1-\delta}$$

$$\Rightarrow \boxed{S = \frac{2\theta(r-1) - (1-\delta)(1-\theta)}{(1-\delta)(1-\delta(1-\theta))}}$$

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If $\delta < \frac{b-H+r}{b+H-r}$

- To find the expected payoff to bank in case of negative Assortive Matching.

Here we have to consider two subcases :-

<u>Subcase I</u>: When $\theta > 1/2$ i.e. there are more S-type ppl in the population.

All the N-type ppl would form a group with S-type first and few S-type ppl would be left who would then form a group among themselves.

$\therefore$ Total No of SN-type grps = Total No of N-type ppl $= 1-\theta$

Total No of SS type ppl left $= 1 - 2(1-\theta)$

$\therefore$ Total No. of SS type grps $= \dfrac{1-2(1-\theta)}{2} = \theta - 1/2$

Probability that bank choses a SN type grp $= \dfrac{1-\theta}{1-\theta+\theta-1/2} = 2(1-\theta)$

Prob that bank choses a SS type grp $= \dfrac{\theta - 1/2}{1-\theta+\theta-1/2} = 2\theta - 1$

For a given period,

Payoff to bank if loan is given to SN type =

With prob 1/2 loan goes to S type first

$\therefore$ Payoff $= r-2$

With prob 1/2, loan goes to N type first

$\therefore$ Payoff $= -1$

Per period Payoff to the bank if loan is given to SS type grp $= 2(r-1)$

As both of them pays back

$\therefore$ Net Expected payoff to the bank is :-

$$S = \underbrace{(2\theta-1) \times \frac{2(r-1)}{1-\delta}}_{\substack{\text{loan going} \\ \text{to SS grp in} \\ \text{each period}}} + \underbrace{(1-\theta)\left[-1+\delta S\right]}_{\substack{\text{Loan going to} \\ \text{SN grp with N-type} \\ \text{as first recepient} \\ \text{and bank choses} \\ \text{another grp} \\ \text{(same cycle repeated)}}} + \underbrace{(1-\theta)\left[(r-2)+\delta S\right]}_{\substack{\text{loan going to SN} \\ \text{grp with S-type as} \\ \text{first recepient} \\ \text{and then bank} \\ \text{chosing another grp} \\ \text{(same cycle repeated)}}}$$

So

$$S = (2\theta-1) \times \frac{2(r-1)}{1-\delta} - (1-\theta) + (1-\theta)\delta S + (1-\theta)(r-2) + \delta S(1-\theta)$$

$$S\left(1 - 2\delta(1-\theta)\right] = \frac{(2\theta-1) \times 2(r-1)}{1-\delta} + (1-\theta)(r-3)$$

$$\boxed{S = \frac{(2\theta-1)2(r-1) + (1-\delta)(1-\theta)(r-3)}{(1-\delta)\left[1-2\delta(1-\theta)\right]}}$$

<u>Subcase II</u> : When $\theta < 1/2$

In this case, there will surplus of NN type ppl

Total No of SN grps $= \theta$

Total No of NN grps $= \frac{1-2\theta}{2} = 1/2 - \theta$

Prob that bank choses SN grp $= 2\theta$

" " " " NN grp $= 1-2\theta$

(5)

If SN grp is chosen, payoff to bank is

With prob 1/2 S type is the first recepient

$r-2$

With prob 1/2 N-type is the first recepient

-1

If NN grp is chosen, payoff to the bank is -1.

After 1st period, bank choses another grp in each case here (cycle repeats)

$\therefore$

$$S = \theta[r-2+\delta S] + \theta[-1+\delta S] + (1-2\theta)(-1+\delta S)$$

$$\Rightarrow S = \theta[r-2+\delta S] + (1-\theta)[-1+\delta S]$$

$$\Rightarrow S = \theta(r-2) + \theta\cancel{\delta S}\ \cancel{\theta} - (1-\theta) + \delta S - \theta\cancel{\delta S}$$

$$\Rightarrow (1-\delta)S = \theta(r-2) - 1 + \theta$$

$$\Rightarrow S = \frac{\theta r - 2\theta - 1 + \theta}{1-\delta}$$

$$\Rightarrow \boxed{S = \frac{\theta r - 1 - \theta}{1-\delta}}$$