COMPETITION AND INCENTIVES WITH MOTIVATED AGENTS

By

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INTRODUCTION

- Importance of **mission** as opposed to profit as an organizational goal.
- Presence of **motivated agents** who subscribe to the mission.
- Role of **matching** the mission preferences of principals and agents in increasing **organizational efficiency** and in **economizing on the need for high-powered incentives.**

KEY FEATURES OF THE PAPER

This paper focuses on two key issues:

- The role that the **motivation** of an agent plays while structuring the optimal contracts.

- The role of **competition** between providers in determining the optimal contracts.

THE MODEL

- A 'firm' consists of a principal and an agent to carry out a project.
- Project's outcome can be 'high' (Y_H=1) or low (Y_L=0).
- Probability of outcome is 'e' which is also the effort supplied by agent. Effort is unobservable.
- Cost of effort is $c(e) = e^2/2$.
- Agent has no wealth. His minimum level of consumption is $\underline{w} \geq 0$.
- Principal does not face any binding wealth constraints.

- Autarky payoff of principal and agent is 0.
- Mapping from effort to outcome is the same for all projects.
- Agents identical in their ability to work on any type of project.
- Projects differ exclusively in terms of their "**mission**".
- Missions are exogenously given attributes of a project associated with a particular principal.

We consider a case where there are 3 types of principals, labelled i€ {0,1,2} and 3 types of agents, labelled j€ {0,1,2}. The economy is divided into a profit oriented sector (i=0) and mission-oriented sector (i € {1,2}).

- Types of principals and agents are perfectly observable.
- In case of success principal of type i receives payoff of $\pi_i>0$. In case of failure he receives 0.
- π_0 is entirely monetary. π_1, π_2 may have non-monetary component. We assume $\pi_1 = \pi_2 \equiv \hat{\pi}$.

- θ_{ij} denotes the j^{th} agent's non-monetary benefit when matched with i^{th} principal. For 0^{th} type agent, there is only monetary benefit. For j € {1,2}, if matched with principal of same type, they receive a benefit of $\bar{\theta}$, else they receive $\underline{\theta}$ provided i € {1,2}

Summarizing,

$$\theta_{ij} = \begin{cases} 0 & i = 0 \text{ and/or } j = 0 \\ \underline{\theta} & i \in \{1, 2\}, j \in \{1, 2\}, i \neq j \\ \bar{\theta} & i \in \{1, 2\}, j \in \{1, 2\}, i = j. \end{cases}$$

Therefore θ_{ij} is agent's **motivation**, type 1 and type 2 agents are motivated agents.

- A contract between the i^{th} principal and the j^{th} agent consists of two components: a fixed wage w_{ij} paid to the agent under all circumstances, and a bonus b_{ij} paid to the agent when project outcome is a success.

ANALYSIS OF THE MODEL

We analyse the model in two steps:

Firstly, we solve for the optimal contract for an exogenously given match between the principal of type i and agent of type j, taking the agent's reservation payoff $\bar{u}_j \geq 0$ to be exogenously given.

Secondly, we study matching of principals and agents where the reservation payoffs are endogenously determined.

ASSUMPTIONS

ASSUMPTION 1:

$$\max\{\pi_0, \hat{\pi} + \bar{\theta}\} < 1$$

to ensure interior solution for effort in all possible principal agent matches.

ASSUMPTION 2:

$$\frac{1}{4}[\min\{\pi_0, \hat{\pi}\}]^2 - \underline{w} > 0$$

a sufficient condition to ensure non-negative payoffs for both principal and agent.

OPTIMAL CONTRACTS

FIRST BEST CASE:

Effort is contractible. That effort level is chosen which maximises joint surplus (S).

Problem is

$$\max_{\{e_{ij}\}} \; S = (\pi_i + \theta_{ij})\, e_{ij} - e_{ij}^2/2$$

Result:

$$e_{ij} = \pi_i + \theta_{ij}$$

$$S = \tfrac{1}{2}\,(\pi_i + \theta_{ij})^2$$

SECOND BEST CASE:

Effort is non- contractible. Principal's optimal contracting problem under moral hazard solves

$$(1) \qquad \max_{\{b_{ij}, w_{ij}\}} u_{ij}^{p} = (\pi_i - b_{ij})e_{ij} - w_{ij}$$

subject to :

$$(2) \qquad b_{ij} + w_{ij} \geq \underline{w}, \qquad w_{ij} \geq \underline{w};$$

$$(3) \quad u_{ij}^{a} = e_{ij}(b_{ij} + \theta_{ij}) + w_{ij} - \tfrac{1}{2}e_{ij}^{2} \geq \bar{u}_j;$$

$$e_{ij} = \arg \max_{e_{ij} \in [0,1]} \left(e_{ij}(b_{ij} + \theta_{ij}) + w_{ij} - \tfrac{1}{2}e_{ij}^{2}\right)$$

$$(4) \qquad e_{ij} = b_{ij} + \theta_{ij}$$

Let $\bar{v}_{ij}$ denote the reservation payoff of the agent such that the principal makes zero profits.

Let $\underline{v}_{ij}$ denote the reservation payoff of the agent such that for $\bar{u}_j \geq \underline{v}_{ij}$ the agent's participation constraint binds.

$\bar{v}_{ij}$ and $\underline{v}_{ij}$ are positive real numbers under our assumption and $\underline{v}_{ij} < \bar{v}_{ij}$.

SOLUTION:

Form the Lagrangian

$$\mathcal{L} = (\pi_i - bij)(bij + \theta ij) - wij + \lambda\left[wij - \underline{w}\right] + \mu\left[\frac{(bij + \theta ij)^2}{2} + wij - \bar{u}j\right]$$

The FOCs are :

$$\mathcal{L}_b = \pi_i - \theta ij - 2bij + \mu(bij + \theta ij) = 0 \quad \text{—— (i)}$$

$$\mathcal{L}_w = -1 + \lambda + \mu = 0 \quad \text{—— (ii)}$$

$$\mathcal{L}_\lambda = wij - \underline{w} \quad \text{—— (iii)}$$

$$\mathcal{L}_\mu = \frac{(bij + \theta ij)^2}{2} + wij - \bar{u}j \quad \text{—— (iv)}$$

$$\lambda \mathcal{L}_\lambda = 0, \quad \mu \mathcal{L}_\mu = 0$$

CASE – I : $\lambda = 0, \quad \mu = 0$ (LLC, PC both do not bind)

From (ii)

$$\lambda + \mu = 1$$

So this is not possible.

CASE – II : $\lambda = 0, \quad \mu \geq 0$ (LLC does not bind, PC binds)

From (ii)

$$\mu = 1$$

From (i)

$$b_{ij} = \pi_i \quad ; \quad e_{ij} = \pi_i + \theta_{ij}$$

$$u_{ij}^p = -w_{ij} < 0$$

Not possible

CASE - III : $\mu = 0$, $\lambda \geq 0$ [PC does not bind, LLC binds]

From (i) & (iii)

$$b_{ij} = 0$$

$$w_{ij} = \underline{w}$$

$$e_{ij} = \theta_{ij}$$

Agent's payoff (u_{ij}^{a}) is $\frac{1}{2}\theta_{ij}^{2} + \underline{w} > \bar{u}_j$ or, $\frac{1}{2}\theta_{ij}^{2} > \bar{u}_j - \underline{w}$

Principal's payoff (u_{ij}^{p}) is $\pi_i \theta_{ij} - \underline{w}$

] if $\pi_i < \theta_{ij}$

$$b_{ij} = \frac{\pi_i - \theta_{ij}}{2}$$

$$w_{ij} = \underline{w}$$

$$e_{ij} = \frac{\pi_i + \theta_{ij}}{2}$$

Agent's payoff (u_{ij}^{a}) is $\frac{1}{8}(\pi_i + \theta_{ij})^2 + \underline{w} > \bar{u}_j$ or, $\frac{1}{8}(\pi_i + \theta_{ij})^2 > \bar{u}_j - \underline{w}$

Principal's payoff (u_{ij}^{p}) is $\frac{1}{4}(\pi_i + \theta_{ij})^2 - \underline{w}$

] if $\pi_i \geq \theta_{ij}$

CASE – IV : $\mu \geq 0$, $\lambda \geq 0$ [both PC and LLC bind]

From (iii) & (iv)

$$b_{ij} = \sqrt{2(\bar{u}_j - \underline{w})} - \theta_{ij}$$

$$w_{ij} = \underline{w}$$

$$e_{ij} = \sqrt{2(\bar{u}_j - \underline{w})}$$

Agent's payoff (u_{ij}^{a}) is $\bar{u}_j$

Principal's payoff (u_{ij}^{p}) is $\sqrt{2(\bar{u}_j - \underline{w})}\left[\pi_i + \theta_{ij} - \sqrt{2(\bar{u}_j - \underline{w})}\right] - \underline{w}$

Combining these results we get our first proposition.

PROPOSITION 1: *Suppose Assumptions 1 and 2 hold. An optimal contract* (b^*_{ij}, w^*_{ij}) *between a principal of type i and an agent of type j given a reservation payoff* $\bar{u}_j \in [0, \bar{v}_{ij}]$ *exists, and has the following features:*

(*a*) *The fixed wage is set at the subsistence level, i.e.,* $w^*_{ij} = \underline{w}$;

(*b*) *The bonus payment is characterized by*

$$b^*_{ij} = \begin{cases} \max\left\{0, \dfrac{\pi_i - \theta_{ij}}{2}\right\} & \text{if } \bar{u}_j \in [0, \underline{v}_{ij}] \\ \sqrt{2(\bar{u}_j - \underline{w})} - \theta_{ij} & \text{if } \bar{u}_j \in [\underline{v}_{ij}, \bar{v}_{ij}]; \end{cases}$$

(*c*) *The optimal effort level solves:* $e^*_{ij} = b^*_{ij} + \theta_{ij}$.

The principal's expected payoff can be summed up as:

Payoff of a principal of type i ($i = 0, 1, 2$) matched with an agent of type j ($j = 0, 1, 2$) is

$$
U_{ij}^{P}(\bar{u}_j) = \begin{cases}
\pi_i \theta_{ij} - \underline{w} & \text{for } \pi_i < \theta_{ij} \text{ and } \bar{u}_j - \underline{w} < \frac{1}{2}\theta_{ij}^2 \\[2ex]
\frac{1}{4}(\pi_i + \theta_{ij})^2 - \underline{w} & \text{for } \pi_i \geq \theta_{ij} \text{ and } \bar{u}_j - \underline{w} < \frac{1}{8}(\pi_i + \theta_{ij})^2 \\[2ex]
\sqrt{2(\bar{u}_j - \underline{w})}\left[\pi_i + \theta_{ij} - \sqrt{2(\bar{u}_j - \underline{w})}\right] - \underline{w} & \text{for, } \frac{1}{8}(\pi_i + \theta_{ij})^2 \leq (\bar{u}_j - \underline{w}) \leq \bar{\vartheta}_{ij}
\end{cases}
$$

EXPLANATION for PROPOSITION 1:

- Wage is set at the minimum, because agent is risk neutral, does not care about the spread of his income. Principal tries to maximise profit.
- Limited liability implies that the first best level of effort cannot be implemented.
- Principal faces a tradeoff between providing incentives and transferring surplus from the agent to himself.
- Agent's motivation plays an important role in determining the incentives, as motivation is a perfect substitute for b.

Thus the various possibilities can be classified into 3 cases:

CASE I – When agent is more motivated than principal and outside option is low,

$$b_{ij}^* = 0$$

CASE II – When principal is more motivated than agent and outside option is low,

$$b_{ij}^* = \frac{1}{2}(\pi_i - \theta_{ij})$$

CASE III – When outside option is high,

$$b_{ij}^* = \sqrt{2(\bar{u}_j - \underline{w})} - \theta_{ij}$$

Corollaries to Proposition 1:

COROLLARY 1: *In the profit-oriented sector* ($i = 0$), *the optimal contract is characterized by the following:*

(*a*) *The fixed wage is set at the subsistence level, i.e.,* $w^*_{0j} = \underline{w}$ ($j = 0, 1, 2$);
(*b*) *The bonus payment is characterized by*

$$b^*_{0j} = \begin{cases} \dfrac{\pi_0}{2} & \text{if } \bar{u}_j \in [0, \underline{v}_{0j}] \\ \sqrt{2(\bar{u}_j - \underline{w})} & \text{if } \bar{u}_j \in [\underline{v}_{0j}, \bar{v}_{0j}] \end{cases}$$

for $j = 0, 1, 2$;
(*c*) *The optimal effort level solves:* $e^*_{0j} = b^*_{0j}$ ($j = 0, 1, 2$).

COROLLARY 2: *Suppose that* $\bar{u}_0 = \bar{u}_1 = \bar{u}_2$. *Then, in the mission-oriented sector* ($i = 1, 2$), *effort is higher and the bonus payment lower if the agent's type is the same as that of the principal.*

COROLLARY 3: *Suppose that* $\bar{u}_0 = \bar{u}_1 = \bar{u}_2$. *Then, in the mission-oriented sector* ($i = 1, 2$) *bonus payments and effort are negatively correlated in a cross section of organizations.*

In the next section, we see the impact of competition among principals and agents, which results in the agent's reservation payoffs to be endogenously determined. This as we will see will help to increase efficiency.

COMPETITION

- In this section, we consider what happens when different sectors compete for agents.
- We look for allocation of principals and agents that are immune to a deviation in which any principal and agent can negotiate a contract that makes both of them strictly better off.
- Let n_i^P and n_j^A denote the number of principals of type i and the number of agents of type j.
- We assume $n_1^A = n_1^P$ and $n_2^A = n_2^P$ ie the number of agents and principal are equal in both the mission oriented sector. However, the population of principal and agents of type o(profit oriented sector) need not be balanced. We consider both cases $n_o^A > n_o^P$ *unemployment* and $n_o^A < n_o^P$ *full employment*.
- Till now we were taking outside option to be exogenously given now the assumption of *unemployment* and *full employment* endogenously generates the value of outside option.

PROPOSITION 2: *Consider a matching μ and associated optimal contracts (w_{ij}^*, b_{ij}^*) for $i = 0, 1, 2$ and $j = 0, 1, 2$. Then this matching is stable only if $\mu(p_i) = a_i$ for $i = 0, 1, 2$.*

- The above proposition claims that any stable matching must have agents matched with principal of the same type.
- Let z_j be the reservation payoff of an agent of type j(j=0,1,2). Then from the proof of proposition 1 the expected payoff of a principal of type i(i=0,1,2) when matched with an agent of type j(j=0,1,2) is given by

$$\Pi^*_{ij}(z_j) = \begin{cases} \pi_i\theta_{ij} - \underline{w} \\ \text{for } \pi_i < \theta_{ij} \\ \text{and } z_j - \underline{w} < \frac{1}{2}\theta^2_{ij} \\ \frac{(\pi_i + \theta_{ij})^2}{4} - \underline{w} \\ \text{for } \pi_i \geq \theta_{ij} \\ \text{and } z_j - \underline{w} < \frac{1}{8}(\pi_i + \theta_{ij})^2 \\ \sqrt{2(z_j - \underline{w})}(\pi_i + \theta_{ij} - \sqrt{2(z_j - \underline{w})}) - \underline{w} \\ \text{for } \frac{1}{8}(\pi_i + \theta_{ij})^2 \leq z_j - \underline{w} \leq \bar{v}^{\rho}_{ij}. \end{cases}$$

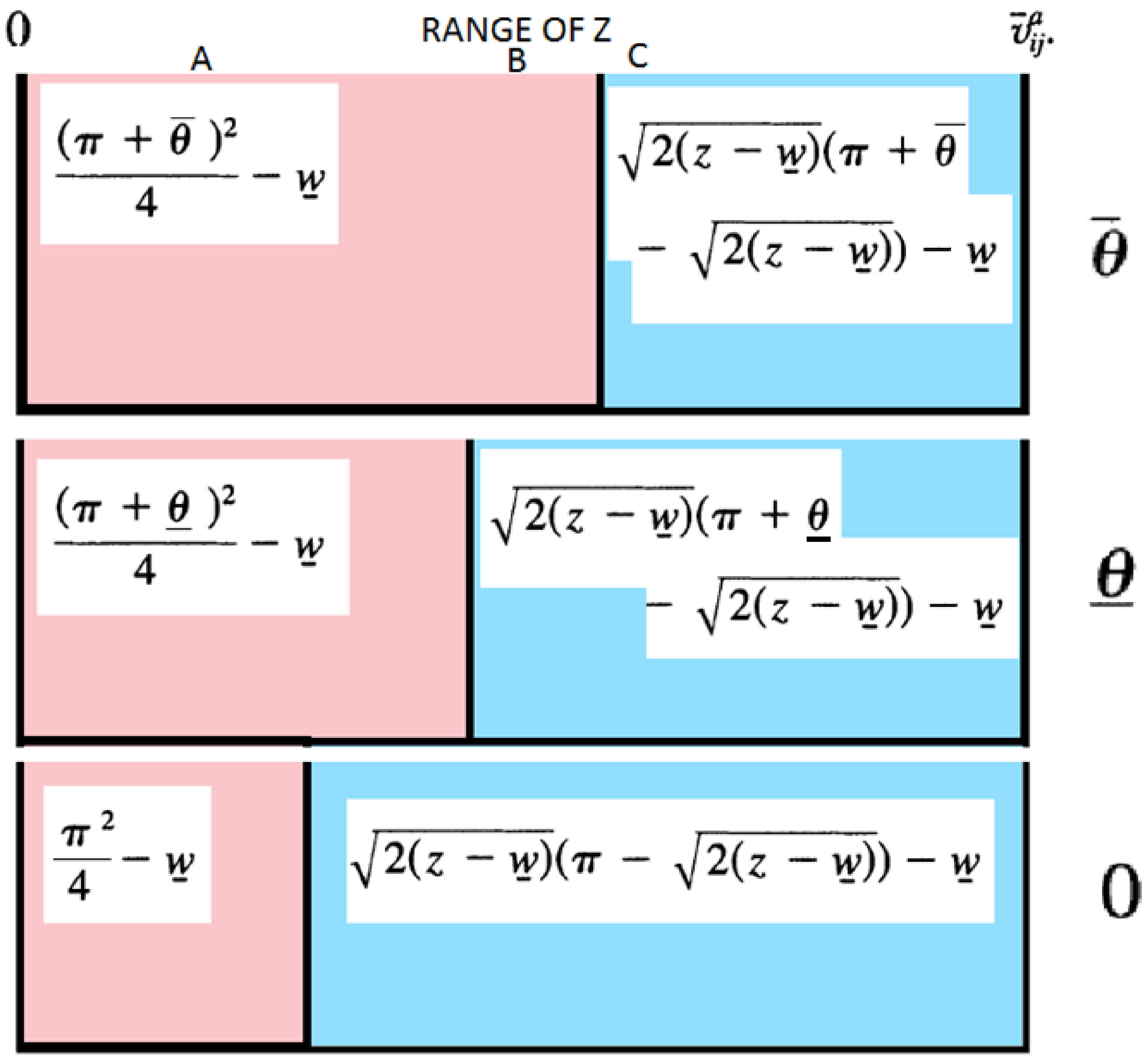
- From the proof of proposition 1 $\pi_{ij}^*(z_j)$ ie the pricipals expected payoff is (weakly) decreasing in z_j for all i=0,1,2 and j=0,1,2. This simply means as an agents outside option increases the expected pay off of the principal decreases.

- We know that :

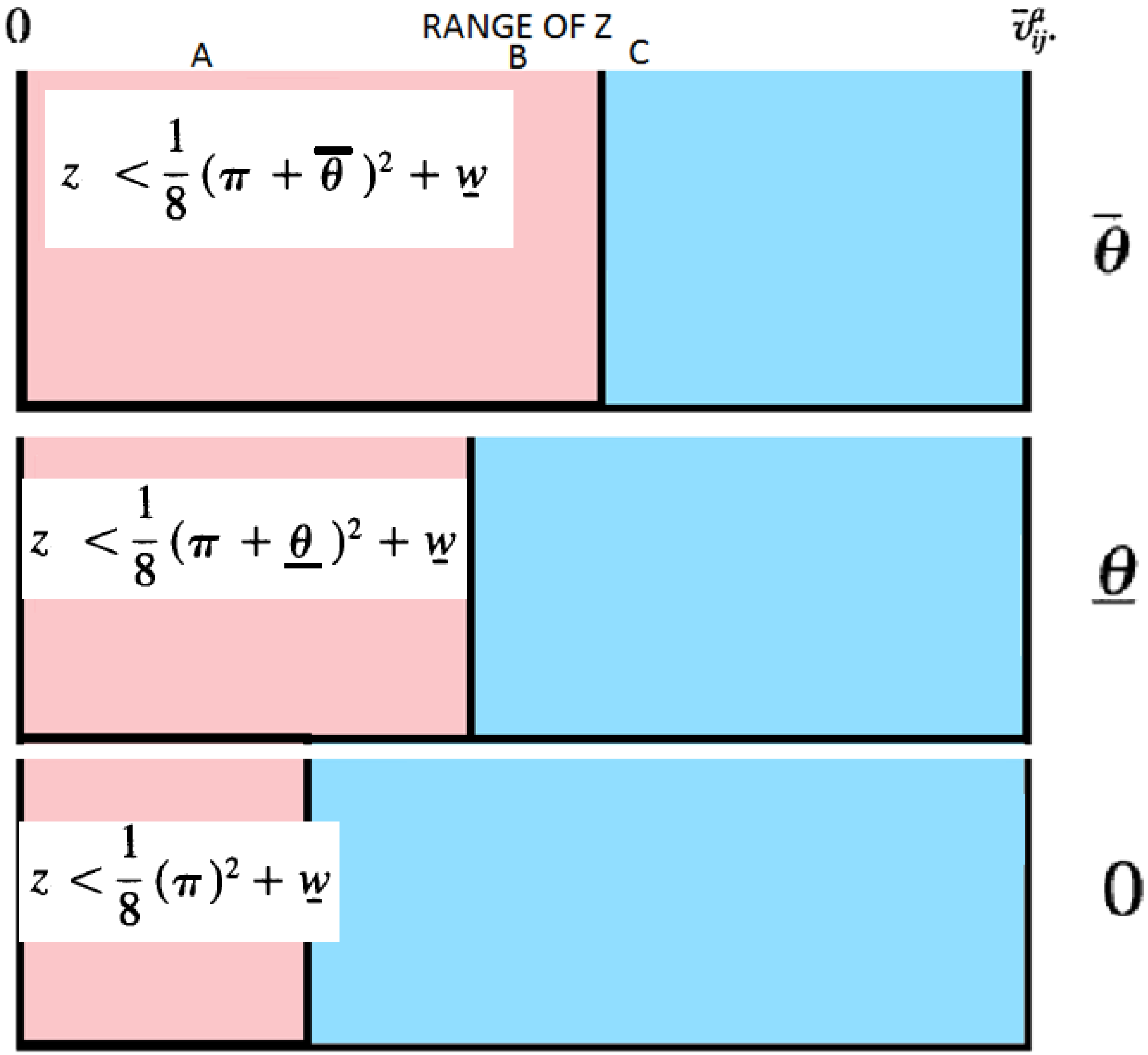
$$\theta_{ij} = \begin{cases} 0 & i = 0 \text{ and/or } j = 0 \\ \underline{\theta} & i \in \{1, 2\}, j \in \{1, 2\}, i \neq j \\ \bar{\theta} & i \in \{1, 2\}, j \in \{1, 2\}, i = j. \end{cases}$$

Therefore $\pi_{00}^*(z) = \pi_{01}^*(z) = \pi_{02}^*(z)$, i.e. for a given level of outside option the expected profit of a type 0 principal remain the same for all types of agent.

- Now let us consider the motivated principals i=1,2. He will always be better off hiring the agent of same type. Let us consider the diagram, the three boxes represents his profits by hiring the three type of agent j=0,1,2 and i =1 over the entire range of z (outside option). In the pink region LLC binds PC doesn't bind and in the blue region both LLC and PC binds.


0
RANGE OF Z
A
B
C
$\bar{v}^{\ell}_{ij\cdot}$
$\frac{(\pi+\bar{\theta})^2}{4} - \underline{w}$
$\sqrt{2(z-\underline{w})}(\pi+\bar{\theta} - \sqrt{2(z-\underline{w})}) - \underline{w}$
$\bar{\theta}$
$\frac{(\pi+\underline{\theta})^2}{4} - \underline{w}$
$\sqrt{2(z-\underline{w})}(\pi+\underline{\theta} - \sqrt{2(z-\underline{w})}) - \underline{w}$
$\underline{\theta}$
$\frac{\pi^2}{4} - \underline{w}$
$\sqrt{2(z-\underline{w})}(\pi - \sqrt{2(z-\underline{w})}) - \underline{w}$
0

- Consider the point z=A at this point profit from hiring the same type of agent is the maximum.
- Now the not so direct case is that of point B, at point B the type 1 principal if matched with the type 1 agent will get the payoff in the pink region whereas when he is matched with the type 0 or type 2 agent he will get the payoff in the blue region. Any point in the pink region is better than any point in the blue region for the three cases therefore by transitivity point B yields the maximum payoff for principal of type 1 when he is matched with type 1.
- Similarly for point C he is the most well off when he is matched with type 1.
- Thus the motivated agents will always want to be paired with the same type agents as it yields the maximum payoff.
- Similarly now let us analyze the agents payoff when the outside option is the same for all types of agents.


0
RANGE OF Z
A
B
C
$\bar{v}^{e}_{ij}$.
$z < \frac{1}{8}(\pi + \bar{\theta})^2 + \underline{w}$
$\bar{\theta}$
$z < \frac{1}{8}(\pi + \underline{\theta})^2 + \underline{w}$
$\underline{\theta}$
$z < \frac{1}{8}(\pi)^2 + \underline{w}$
0

- Here also we can see at z=A the agent of type 1 gets the maximum payoff when he is matched with the principal of type 1.
- For point B when he is matched with the principal of type 1 he gets a payoff of amount

$$\frac{1}{8}(\pi + \overline{\theta})^2 + \underline{w}$$

Which is greater than A as PC doesn't bind. But when he is matched with a principal of different type he gets the payoff exactly B, thus he is better off being matched with the principal of same type.
- So both the motivated principal and agent is the most well off when they are paired with the same type so we have assortative matching.

Full employment

- Now we analyze the case of full employment ie $n_0^A < n_0^P$. Number of principals is greater than the number of agents in the profit oriented sector.
- $n_1^A = n_1^P$ and $n_2^A = n_2^P$ ie the number of principals and agents in the motivated sector are the same.
- Now as the number of principals in the profit sector are greater than the number of agents the principals compete among themselves for agents. This competition drives down their expected profit to zero.
- So with expected profit equal to zero for the principal we can calculate expected payoff for the agents in the profit sector, which comes out to be

$$\hat{u} = \frac{1}{8}(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}})^2 + \underline{w}.$$

- Therefore $\hat{u}$ is the payoff that an agent of any type who is matched with a principal of type 0 receives when the principals expected payoff is zero. Accordingly this is the relevant reservation payoff of all agents under full employment.
- So earlier when we were finding out the optimal contracts in proposition 1 we were taking the outside option to be given, but with the introduction of competition the outside option is being endogenously generated. In the case of full employment the outside option is equal to $\hat{u}$. With this change let us find out the optimal contracts.
- Before that let us state two things:

$$\xi \equiv \max\{\bar{\theta}, \hat{\pi}\} + \bar{\theta}.$$

ASSUMPTION 3:

$$\bar{\theta} + \hat{\pi} \geq \pi_0 .$$

- Assumption 3 basically states that the surplus generated in the motivated sector with a motivated agent is greater than the surplus generated in the profit sector.

- Now let us derive the optimal contracts.
- We have already shown that for the same value of the outside option we will have assortative matching, so the optimal contracts will include the contract between the similar type principal and agents. So basically we will get

$$\left\langle b^*_{11}, w^*_{11} \right\rangle$$

$$\left\langle b^*_{22}, w^*_{22} \right\rangle \qquad \left\langle b^*_{00}, w^*_{00} \right\rangle$$

- LLC will always bind, we have already proved that therefore
  a) The fixed wage is set at the subsistence level,

$$w^*_{jj} = \underline{w} \text{ } for \text{ } j = 0, 1, 2;$$

- Now let us denote the bonus payments:

  b) As we have already said that due to competition in the profit sector the expected payoff should be zero, with the expected payoff set at zero for the principal the bonus comes out to be

$$b_{00}^{*} = \frac{\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}}{2};$$

Using the bonus and wage rate of the profit sector we calculate the outside option u^ which is nothing but the agents payoff in the profit sector, any agent at any time can join the profit sector and earn this payoff thus u^ is the default ouside option.

Now we look at the motivated sector, from proposition 1 we have

We already know from Corollary I that:

$$w_{00} = \underline{w}$$

$$e_{00} = b_{00}$$

Thus, to calculate the value of b_{00} such that $u_{00}^{P} = 0$ is

$$e_{00}(\pi_0 - b_{00}) - w_{00} = 0$$

or, $$b_{00}(\pi_0 - b_{00}) - \underline{w} = 0$$ [Substituting the values from above]

or, $$b_{00}^2 - b_{00}\pi_0 + \underline{w} = 0$$

By Shridharacharya's formula we get,

$$b_{00} = \frac{\pi_0 \pm \sqrt{\pi_0^2 - 4w}}{2}$$

We take the higher value of b_{00} as that induces more effort,

Thus, $$b_{00} = \frac{\pi_0 + \sqrt{\pi_0^2 - 4w}}{2}$$

Again from Corollary I

$$b_{00} = \sqrt{2(\bar{u}_0 - \underline{w})}$$

Equating this equation with the previous one we can calculate the agent's reservation payoff $(\hat{u})$ for which principal's expected payoff is 0.

$$\sqrt{2(\hat{u} - \underline{w})} = \frac{\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}}{2}$$

$$2(\hat{u} - \underline{w}) = \frac{\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right)^2}{4}$$

$$\hat{u} = \frac{1}{8}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right)^2 + \underline{w}$$

$$b_{ij}^* = \begin{cases} \max\left\{0, \dfrac{\pi_i - \theta_{ij}}{2}\right\} & \text{if } \bar{u}_j \in [0, \underline{v}_{ij}] \\ \sqrt{2(\bar{u}_j - \underline{w})} - \theta_{ij} & \text{if } \bar{u}_j \in [\underline{v}_{ij}, \bar{v}_{ij}]; \end{cases}$$

The outside option is being endogenously generated and is equal to u^, now we analyze the contract structure of agent 1 and principal 1.

Optimal contract:

$$b_{11}^* = \begin{cases} \max\left\{0, \dfrac{\hat{\pi} - \bar{\theta}}{2}\right\} & \text{if } \hat{u} \in [0, \underline{v}_{ij}] \\ \sqrt{2(\hat{u} - \underline{w})} - \bar{\theta} & \text{if } \hat{u} \in [\underline{v}_{ij}, \bar{v}_{ij}]; \end{cases}$$

This can also be written as

$$b_{11}^* = \tfrac{1}{2} \max\{\xi, \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\} - \bar{\theta}$$

We know $\mathcal{E}_S = \max\{\hat{\pi}, \bar{\theta}\} + \bar{\theta}$

$$b_{11}^* = \frac{1}{2}\max\left[\mathcal{E}_S, \ \pi_0 + \sqrt{\pi_0^2 - 4\underline{\omega}}\right] - \bar{\theta}$$

CASE 1: $\bar{\theta} > \hat{\pi}$

$\mathcal{E}_S = 2\bar{\theta}$

$$\therefore \ b_{11}^* = \frac{1}{2}\max\left[2\bar{\theta}, \ \pi_0 + \sqrt{\pi_0^2 - 4\underline{\omega}}\right] - \bar{\theta}$$

CASE 1(a): $2\bar{\theta} > \pi_0 + \sqrt{\pi_0^2 - 4\underline{\omega}}$

Multiplying by $\frac{1}{8}$ and adding $\underline{\omega}$ to both sides, after squaring the two sides we get

$$\frac{1}{8}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{\omega}}\right)^2 + \underline{\omega} < \frac{1}{2}\bar{\theta}^2 + \underline{\omega}$$

or, $$\hat{u} < \frac{1}{2}\bar{\theta}^2 + \underline{\omega} \qquad \text{(i)}$$

Thus $b_{11}^* = 0$ ....

CASE 1(b): $2\bar{\theta} < \pi_0 + \sqrt{\pi_0^2 - 4\underline{\omega}}$

By similar calculation we get, $\hat{u} > \frac{1}{2}\bar{\theta}^2 + \underline{\omega}$

Thus, $b_{11}^* = \frac{1}{2}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right) - \bar{\theta}$

$$= \sqrt{2 \times \frac{1}{8}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right)^2} - \bar{\theta}$$

$$= \sqrt{2\left[\frac{1}{8}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right) + \underline{w} - \underline{w}\right]} - \bar{\theta}$$

$$= \sqrt{2(\hat{u} - \underline{w})} - \bar{\theta} \quad \text{[Substituting by } \hat{u}\text{]}$$

. . . . (ii)

CASE – II : $\bar{\theta} < \hat{\pi}$

$$\xi = \hat{\pi} + \bar{\theta}$$

$$b_{11}^* = \frac{1}{2}\max\left\{\hat{\pi} + \bar{\theta},\ \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right\} - \bar{\theta}$$

Case – II (a) : $\hat{\pi} + \bar{\theta} > \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}$

By similar calculation,

$$\frac{1}{8}(\hat{\pi} + \bar{\theta})^2 + \underline{w} > \hat{u}$$

and $b_{11}^* = \frac{1}{2}(\hat{\pi} + \bar{\theta}) - \bar{\theta} = \frac{1}{2}(\hat{\pi} - \bar{\theta})$ . . . . . (iii)

CASE II (b) : $\hat{\pi} + \bar{\theta} < \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}$

$$\hat{u} > \frac{1}{8}(\hat{\pi} + \bar{\theta})^2 + \underline{w}$$

and $$b_{11}^* = \frac{1}{2}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right) - \bar{\theta}$$

$$= \sqrt{2(\hat{u} - \underline{w})} - \bar{\theta}$$ [As shown earlier]

. . . . (iv)

By, (i), (ii), (iii), (iv), combining the results, we get,

$$b_{11}^* = \frac{1}{2}\max\left\{ \mathcal{E}_s,\ \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}} \right\} - \bar{\theta}$$

can also be rewritten as

$$b_{11}^* = \begin{cases} \max\left[0, \dfrac{\hat{\pi} - \bar{\theta}}{2}\right] & \text{if } \hat{u} \in [0, \underline{v_{ij}}] \\ \sqrt{2(\hat{u} - \underline{w})} - \bar{\theta} & \text{if } \hat{u} \in [\underline{v_{ij}}, \bar{v}_{ij}] \end{cases}$$

Similarly for agent 2 we have:

$$b_{22}^* = \frac{1}{2}\max\{\xi, \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\} - \bar{\theta}$$

Further from proposition 1 we have,

The optimal effort level solves: $e_{ij}^* = b_{ij}^* + \theta_{ij}$.

Thus putting all these together we have,

PROPOSITION 3: *Suppose that* $n_0^a < n_0^p$ *(full employment in the profit-oriented sector). Then the following matching* μ *is stable:* $\mu(a_j) = p_j$ *for* $j = 0, 1, 2$ *and the associated optimal contracts have the following features:*

(*a*) *The fixed wage is set at the subsistence level, i.e.,* $w^*_{jj} = \underline{w}$ *for* $j = 0, 1, 2$;

(*b*) *The bonus payment in the mission-oriented sector is:*

$$b^*_{11} = b^*_{22} = \frac{1}{2}\max\{\xi, \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\} - \bar{\theta}$$

and the bonus payment in the profit-oriented sector is:

$$b^*_{00} = \frac{\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}}{2};$$

(*c*) *The optimal effort level solves:* $e^*_{jj} = b^*_{jj} + \bar{\theta}$ *for* $j = 1, 2$ *and* $e^*_{00} = b^*_{00}$.

UNEMPLOYMENT

- Now we analyze the case of full employment ie $n_0^A > n_0^P$. Number of agents is greater than the number of principals in the profit oriented sector.
- $n_1^A = n_1^P$ and $n_2^A = n_2^P$ ie the number of principals and agents in the motivated sector are the same.
- Now as the number of agents are much higher than the number of principals, the agents compete among themselves, and drive down their expected payoff to zero. The participation constraint of an agent become,

$$e_{ij}(b_{ij} + \theta_{ij}) + w_{ij} - \frac{1}{2}e_{ij}^2 \geq 0 \ ;$$

The rest of the analysis is pretty similar to the last analysis and we obtain;

- Now let us derive the optimal contracts.
- We have already shown that for the same value of the outside option we will have assortative matching, so the optimal contracts will include the contract between the similar type principal and agents. So basically we will get

$$\left\langle b^*_{11}, w^*_{11} \right\rangle$$

$$\left\langle b^*_{22}, w^*_{22} \right\rangle \qquad \left\langle b^*_{00}, w^*_{00} \right\rangle$$

- LLC will always bind, we have already proved that therefore
  c) The fixed wage is set at the subsistence level,

$$w^*_{jj} = \underline{w} \text{ for } j = 0, 1, 2;$$

- Now let us derive the bonus payments:
  a) As we have already said that due to competition in the profit sector the outside option should be zero, with the outside option set at zero for the agent the bonus comes out to be

$$b^*_{00} = \frac{\pi_0}{2}$$

  - Outside option u^=0; which is nothing but the agents payoff in the profit sector, any agent at any time can join the profit sector and earn this payoff thus u^ which is the default outside option.
  - Now we look at the motivated sector, from proposition 1 we have

$$b_{ij}^{*} = \begin{cases} \max\left\{0, \dfrac{\pi_i - \theta_{ij}}{2}\right\} & \text{if } \bar{u}_j \in [0, \underline{v}_{ij}] \\ \sqrt{2(\bar{u}_j - \underline{w})} - \theta_{ij} & \text{if } \bar{u}_j \in [\underline{v}_{ij}, \bar{v}_{ij}]; \end{cases}$$

In this case, reservation payoff of agent is 0.

From corollary I, $w_{00} = \underline{w}$, $e_{00} = b_{00}$

Substituting these values, the principal tries to maximize his expected payoff u_{00}^{p} with respect to b_{00} and calculate the optimal b_{00}.

Thus, $\max\limits_{\{b_{00}\}} u_{00}^{p} = e_{00}(\pi_0 - b_{00}) - \underline{w}$

$= \pi_0 b_{00} - b_{00}^2 - \underline{w}$

FOC: $b_{00}^{*} = \dfrac{\pi_0}{2}$

The outside option is being endogenously generated and is equal to 0, now we analyze the contract structure of agent 1 and principal 1.

$$b_{11}^{*} = \max\left\{0, \frac{\hat{\pi} - \bar{\theta}}{2}\right\}$$

This can also be written as

$$b_{11}^{*} = \frac{\xi}{2} - \bar{\theta}$$

Further from proposition 1 we have,

The optimal effort level solves: $e_{ij}^{*} = b_{ij}^{*} + \theta_{ij}$.

$$b_{11}^* = b_{22}^* = \frac{1}{2}\mathcal{E}_S - \bar{\theta}$$

CASE I : when $\bar{\theta} > \hat{\pi}$

$$\mathcal{E}_S = 2\bar{\theta}$$

$$b_{11}^* = 0 \quad \ldots\ldots \quad (i)$$

CASE II : when $\bar{\theta} < \hat{\pi}$

$$\mathcal{E}_S = \hat{\pi} + \bar{\theta}$$

$$b_{11}^* = \frac{1}{2}\left[\hat{\pi} + \bar{\theta}\right] - \bar{\theta}$$

$$= \frac{1}{2}\left(\hat{\pi} - \bar{\theta}\right) \quad \ldots\ldots \quad (ii)$$

From (i) and (ii), we see that $b_{11}^* = \frac{\mathcal{E}_S}{2} - \bar{\theta}$ is equivalent to $b_{11}^* = \max\left[0, \frac{\hat{\pi} - \bar{\theta}}{2}\right]$, which we got from Proposition I, when PC was not binding and $u_4^* = 0$

PROPOSITION 4: *Suppose that* $n_0^a > n_0^p$ *(unemployment in the profit-oriented sector). Then the following matching* μ *is stable:* $\mu(a_j) = p_j$ *for* $j = 0, 1, 2$ *and the associated optimal contracts have the following features:*

(*a*) *The fixed wage is set at the subsistence level, i.e.,* $w^*_{jj} = \underline{w}$ *for* $j = 0, 1, 2$;

(*b*) *The bonus payment in the mission-oriented sector is:*

$$b^*_{11} = b^*_{22} = \frac{\xi}{2} - \bar{\theta}$$

and the bonus payment in the profit-oriented sector is:

$$b^*_{00} = \frac{\pi_0}{2};$$

(*c*) *The optimal effort level solves:* $e^*_{jj} = b^*_{jj} + \bar{\theta}$ *for* $j = 1, 2$ *and* $e^*_{00} = b^*_{00}$.