Opting out of publicly provided services: A majority voting result

Introduction

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- Examples include education, public transportation, healthcare etc.
- We analyze a model in which households are differentiated by income and have the option of choosing between publicly provided services and private services.

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- Government collects taxes from all individuals at a constant rate τ.

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- The public expenditures are converted into quality of service according to

$$Q = \frac{\tau Y}{N} \; \textit{if} \; \tau > 0 \; \textit{and} \; N > 0$$

$$Q = 0 \;\; \textit{if} \; \tau = 0 \; \textit{and} \; N = 0$$

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- No agent can choose the publicly provided services and supplement it with some private services.

Model

- Let the indirect utility of agent i with income y_i who chooses to obtain the services from public sector be denoted by $V^u(\tau, y_i, Y, N)$ and $V^r(\tau, y_i)$ denote i's indirect utility if he chooses private services.

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- Agent i will choose publicly provided services over private services if and only if $V^u(\tau, y_i, Y, N) \geq V^r(\tau, y_i)$
- The equilibrium fraction of agents choosing publicly provided services, N^* must solve

$$N = \mu\{i : V^u(\tau, y_i, Y, N) \geq V^r(\tau, y_i)\} \tag{1}$$

where $\mu\{.\}$ is the probability measure associated with the distribution function $F(.)$.

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- A majority voting equilibrium is a pair $\{\tau^*, N^*\}$ which satisfies
(i) given τ^*, the solution to equation (1) is N^* and

  (ii) there does not exist another pair $\{\tau^{'}, N^{'}\}$ such that
a) given $\tau^{'}$, $N^{'}$ solves equation (1) and
b) $\tau^{'}$ is preferred over τ^* by more than half the population.

Majority voting equilibrium

- **Proposition 1**. Assume that $U(.)$ is homothetic and that $lim_{c->\infty} U_c(c, e) = 0$ for all $e > 0$. Given $\tau \in (0, 1)$, $N \in (0, 1]$ and $Y \in \mathbb{R}_{++}$, there exists a unique $\hat{y} > 0$ such that $V^u(\tau, y_i, Y, N) \geq V^r(\tau, y_i)$ if and only if $y_i \leq \hat{y}$.

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- **Lemma 1**. (i) For $N \in (0, 1)$, $\hat{y}$ is decreasing in N. (ii) For $\tau \in (0, 1)$, $\hat{y}$ is increasing in τ. (iii) For $\tau \in (0, 1)$, $\hat{y}$ is increasing in τ.

Majority voting equilibrium

- The fraction of agents choosing publicly provided services N^* solves

$$N = F(\hat{y}(\tau, Y, N)) \tag{2}$$

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- **Proposition 2**. For all $\tau \in (0,1)$ and $Y \in \mathbb{R}_{++}$, there exists a unique $N^* \in (0,1)$ which solves equation (2).

Majority voting equilibrium

- We now endogenize the tax rate through majority voting. The most preferred tax rate for an individual with income y is given by

$$\tau^* = argmax \ V(\tau, y)$$

subject to $\tau \in [0, 1]$
where $V(\tau, y) = max\{V^u(\tau, y, N(\tau)), V^r(\tau, y)\}$ and $N(\tau)$ is the solution to (1).

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  where $V(\tau, y) = max\{V^u(\tau, y, N(\tau)), V^r(\tau, y)\}$ and $N(\tau)$ is the solution to (1).
- If preferences over tax rates are not single peaked, a majority equilibrium may not exist.

Majority voting equilibrium

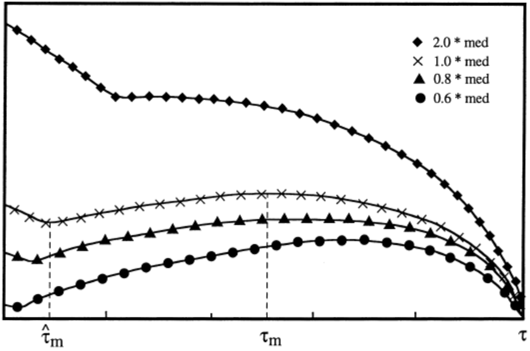

Fig. 2 Preferences over tax rates

Majority voting equilibrium

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- At $\hat{\tau}(y)$, an agent with income y is indifferent between public and private services.

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Majority voting equilibrium

- **Lemma 5**. Let N_m be the public school enrollment evaluated at the tax rate τ_m i.e., $N_m = N(\tau_m)$.

  (i) If $V^r(0, y_m) < V^u(\tau_m, y_m, N_m)$ then $V^r(0, y) < V^u(\tau_m, y, N_m)$ for all $y < y_m$.
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- **Proposition 3**. If $V^r(0, y_m) < V^u(\tau_m, y_m, N_m)$, then the pair $\{\tau_m, N_m\}$ is a majority voting equilibrium.

Example

- Suppose agent i's preferences are represented by

$$U(c_i, q_i) = \frac{1}{1-\sigma}\{c_i^{1-\sigma} + q_i^{1-\sigma}\}, \ \sigma \in (0,1)$$

, and income distribution is Dagum. That is

$$F(y) = \{1 + \lambda y^{-\alpha}\}^{-\beta}, \ \alpha > 0, \ \beta > 0, \ \textit{and} \ \lambda > 0$$

Example

- $$V^u(\tau, y_i, Y, N) = \frac{1}{1-\sigma}\left[(1-\tau)^{1-\sigma} y_i^{1-\sigma} + \left(\frac{\tau Y}{N}\right)^{1-\sigma}\right]$$

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- $$V^r(\tau, y_i) = \frac{2^\sigma}{1-\sigma}(1-\tau)^{1-\sigma} y_i^{1-\sigma}$$
- The critical income is given by
$$\hat{y} = (2^\sigma - 1)^{\frac{1}{\sigma-1}}\left[\frac{\tau Y}{(1-\tau)N}\right]$$

Example

- Equilibrium N^* solves

$$N = F\left((2^\sigma - 1)^{\frac{1}{\sigma-1}}\left[\frac{\tau Y}{(1-\tau)N}\right]\right)$$

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- The critical tax rate of an individual with income y must solve

$$\frac{\tau Y}{(1-\tau)N(\tau)} = y\{2^\sigma - 1\}^{\frac{1}{1-\sigma}}$$

which is increasing in y

Concluding remarks

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- We saw a model where each individual had the choice of opting out of the publicly provided services.
- Taxes on individuals' income determine the quality of publicly provided services.
- Although the preferences over tax rates are not single peaked, we saw that a majority equilibrium does exist and the decisive voter is the agent with median income.

① $\max \; H\left(c((1-\tau)y),\, e((1-\tau)y)\right)$ (P)

subject to $c + e = (1-\tau)y$

where $H(\cdot)$ is homogeneous of degree one

i.e $H(\lambda a, \lambda b) = \lambda H(a,b) \quad \forall \lambda \in \mathbb{R}$.

let c^*, e^* solve (P).

Claim: $H(c^*, e^*)$ is linear in y

Proof: $\mathcal{L} \equiv H(c,e) - \lambda\left[c + e - (1-\tau)y\right]$

First order conditions for a maximum

$$\frac{\partial \mathcal{L}}{\partial c} \equiv \frac{\partial H}{\partial c} - \lambda = 0 \quad \text{i.e} \quad \left.\frac{\partial H}{\partial c}\right|_{c=c^*} = \lambda \quad \text{—①}$$

$$\frac{\partial \mathcal{L}}{\partial e} \equiv \frac{\partial H}{\partial e} - \lambda = 0 \quad \text{i.e} \quad \left.\frac{\partial H}{\partial e}\right|_{e=e^*} = \lambda \quad \text{-②}$$

For a homogeneous function f of degree n we know that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf \quad \forall x, y$

$$\therefore \quad c\frac{\partial H}{\partial c} + e\frac{\partial H}{\partial e} = (1)H \quad \text{is true } \forall c, e$$

$$c^*\left.\frac{\partial H}{\partial c}\right|_{c=c^*} + e^*\left.\frac{\partial H}{\partial e}\right|_{e=e^*} = H(c^*, e^*)$$

$$(c^* + e^*)\lambda = H(c^*, e^*)$$

$$\Rightarrow H(c^*, e^*) = \lambda(1-\tau)y$$

i.e $H\left(c^*((1-\tau)y),\, e^*((1-\tau)y)\right) = \lambda(1-\tau)y$

□

② The critical tax rate $\hat{\tau}(y)$ is non-decreasing in y.

Proof: $\hat{\tau}(y)$ solves $V^r(\hat{\tau}, y) = V^u(\hat{\tau}, y, N(\hat{\tau}))$
where $V^r(.)$ is the utility derived from private service and $V^u(.)$ is the utility from public service.

Let $\hat{\tau}$ & $\hat{\tau}'$ be the critical tax rates for agents with income y and y' respectively.
Without loss of generality assume that $y' > y$. If $\hat{\tau}' \geq \hat{\tau}$ then there is nothing to prove. So suppose that $\hat{\tau}' < \hat{\tau}$.

For the agent with income y' we have

$$V^u(\hat{\tau}', y', N(\hat{\tau}')) = V^r(\hat{\tau}', y')$$

$$\Leftrightarrow \quad U\left((1-\hat{\tau}')y', \frac{\hat{\tau}'Y}{N(\hat{\tau}')}\right) = U\left(c^*((1-\hat{\tau}')y'), e^*((1-\hat{\tau}')y')\right)$$

$$\Leftrightarrow \quad H\left((1-\hat{\tau}')y', \frac{\hat{\tau}'Y}{N(\hat{\tau}')}\right) = H\left(c^*((1-\hat{\tau}')y'), e^*((1-\hat{\tau}')y')\right)$$

where H is homogeneous of degree one

$$\Leftrightarrow \quad H\left(1, \frac{\hat{\tau}'Y}{N(\hat{\tau}')(1-\hat{\tau}')y'}\right) = H(s_c, s_e) \quad \text{——} \quad ③$$

where s_c and s_e are the (after-tax) income shares of consumption and quality of education.

Since $\hat{\tau}' < \hat{\tau}$, for the agent with income y we must have

$$V^u(\hat{\tau}', y, N(\hat{\tau}')) < V^u(\hat{\tau}, y, N(\hat{\tau}))$$
$$= V^r(\hat{\tau}, y)$$
$$< V^r(\hat{\tau}', y)$$

This implies $H\left(1, \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y}\right) < H(s_c, s_e)$

From ③ $H(s_c, s_e) = H\left(1, \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y'}\right)$

$\therefore$ we should have

$$H\left(1, \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y}\right) < H\left(1, \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y'}\right)$$

However this is a contradiction since H is increasing in both of its arguments.

$$\left[y' > y \Rightarrow \frac{1}{y} > \frac{1}{y'} \Rightarrow \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y} > \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y'}\right.$$
$$\left. \Rightarrow H\left(1, \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y}\right) > H\left(1, \frac{\hat{\tau}' Y}{N(\hat{\tau}')(1-\hat{\tau}')y'}\right)\right]$$

$\therefore \hat{\tau}' \geq \hat{\tau}$

i.e critical tax rate $\hat{\tau}(y)$ is non-decreasing in y.