Competition between Private and Public schools,Vouchers and Peer Group effect

Dennis Epple   Richard E.Romano

by
Silu Muduli
Indian Statistical Institute,Delhi

April 7,8, 2015

Mathematical Concepts

- Set theoretic concepts
- Single crossing property
- Lagrange multiplier
- Monopolistic Competition

Introduction

Theme of the paper

- A theoretical and computational model with tax-financed, tuition-free public schools and competitive, tuition-financed private schools is developed. Students differ by ability and income. Achievement depends on own ability and on peers' abilities.
- Equilibrium has a strict hierarchy of school qualities and two dimensional student sorting with stratification by ability and income. In private schools, high-ability, low-income students receive tuition discounts, while lowability, high-income students pay tuition premia. Tuition vouchers increase the relative size of the private sector and the extent of student sorting, and benefit high-ability students relative to low-ability students.

Introduction

A student in our model is then characterized by an ability and a household income (b, y),a draw from a continuous bivariate distribution.
A school's quality is determined by the mean ability of the student body θ, reflecting the model's peer-group effect.
Two key elements of the educational process.

- First, students differ in their abilities. Higher ability is assumed to increase a student's educational achievement and that of peers in the school attended.
- Second, households differ in their incomes, with higher income increasing the demand for educational achievement.

Introduction

Then we will have equilibrium with following characteristics:

- A hierarchy of school qualities will be present, with the set of (homogeneous) public schools having the lowest-ability peer group and a strict ability-group ranking of private schools.
- The equilibrium student bodies of schools correspond to a partition of the ability-income-type space of students with stratification by income and, in many cases, stratification by ability.

The Model

- Household income is denoted (y), and each household has a student of ability (b).
- The joint marginal distribution of ability and income in the population is denoted by $f(b, y)$ and is assumed to be continuous and positive on its support $S = (0, b_{max}] \times (0, y_{max}]$.
- All students attend a school since we assume that free public schooling is preferred to no schooling.
- Achievement $a = a(\theta, b)$,is a continuous and increasing function of student's ability and the mean ability of of the student body in the school attended,θ.
- Let y_t be after tax income and p tuition expenditure,the latter equal to zero if a public school attended.

The Model

The individuals have the following utility function U

$$U = U(y_t - p, a(\theta, b))$$

$$U_{y_t - p} > 0, U_a > 0, a_\theta > 0, a_b > 0$$

The achievement function captures the peer-group effect in our model. To maintain simplicity and highlight the role of peer groups, a school's quality is determined exclusively by the mean ability of its peer group.

The Model

- Single Crossing condition on Income (SCI)

$$\frac{\partial \left(\frac{\partial U / \partial \theta}{\partial U / \partial p} \right)}{\partial y_t} > 0$$

single crossing in (θ, p)space.

- Single Crossing condition on ability (SCA)

$$\frac{\partial \left(\frac{\partial U / \partial \theta}{\partial U / \partial y_t} \right)}{\partial b} > 0$$

single crossing in (θ, y_t) space.

Cost function

A school's costs depend only on the number of students it enrolls, since inputs vary only with size. All schools, public and private, have the simple cost function:

$$C(k) = V(k) + F$$

$$V' > 0, \;\; V'' > 0$$

Where k denote the number of attending students .
Let k^* denote the "efficient scale"

$$k^* = argmin\left[\frac{C(k)}{k}\right]$$

Public Schools

Main characteristics of public schools are:

- Publicsector schooling is financed by a proportional income tax,t , paid by all households, whether or not the household's child attends school in the public sector.
- In equilibrium every public school will have same θ.(Why?)

Private Schools

Main characteristics of private schools are:

- Private-sector schools maximize profits, and there is free entry and exit.
- Student types are observable, implying that tuition and admission can be conditioned on ability and income as competition permits.

Moving towards solution

- Let $i \in \{0, 1, 2, 3, ..., n\}$,set of schools. 0 stands for public school.
- Let $p_i(b, y)$ denote the tuition necessary to enter the school i,with $p_0(b, y) = 0 \;\; \forall(b, y)$.
- Let $\alpha_i(b, y) \in [0, 1]$ denote the proportion of type (b, y) in that school i admits.

Private school's profit maximisation problem

$$\max_{\theta_i, k_i, p_i(b,y), \alpha_i(b,y)} \pi_i \equiv \iint_S [p_i(b,y)\alpha_i(b,y)f(b,y)dbdy] - V(k_i) - F$$

subject to

$$\alpha_i(b,y) \in [0,1] \forall (b,y)$$

$$\forall (b,y) \;\; U(y_t - p_i(b,y), a(\theta_i, b)) \geq$$

$$\max_{j \in \{0,1,2,\ldots,n | j \neq i; \alpha_j > 0 \;\; is \;\; the \;\; optimal \;\; set \;\; of \;\; j\}} U(y_t - p_j(b,y), a(\theta_j, b))$$

$$k_i = \iint_S \alpha_i(b,y) f(b,y) dbdy$$

$$\theta_i = \frac{1}{k_i} \iint_S b\alpha_i(b,y) f(b,y) dbdy.$$

Equilibrium Conditions

UM(Utility Maximisation)

$$U^*(b,y) =$$

$$\max_{i \in \{0,1,2,...,n | \alpha_i > 0 \ \ is \ \ the \ \ optimal \ \ set \ \ of \ \ i\}} U(y_t - p_i(b,y), a(\theta_i, b))$$

πM(Profit Maximization):

$[\theta_i, k_i, p_i(b,y), \alpha_i(b,y)]$ satisfy above problem.$i \in \{1,2,...,n\}$.

Zπ(Zero Profit):

$$\pi_i = 0 \ \ i \in \{1,2,...,n\}$$

PSP(Public sector policy):

$$p_0(b,y) = 0 \forall (b,y)$$

$$\alpha_0(b,y) \in [0,1] \forall (b,y)$$

$$k_0 = \iint_S \alpha_0(b,y) f(b,y) dbdy$$

$$\theta_0 = \frac{1}{k_0} \iint_S b\alpha_0(b,y) f(b,y) dbdy.$$

MC(Market clearance):

$$\sum_{i=0}^{n} \alpha_i(b, y) = 1 \;\; \forall (b, y)$$

Using UM ,the first-order conditions for private schools optimisation problem we have the following solutions

Solution to the private schools problem

1

$$U(y_t - p_i^*, a(\theta, b)) = U^*(b, y) \;\; \forall (b, y)$$

2.$\forall (b, y)$

$$\alpha_i(b, y) = \begin{cases} 0 & \text{if } p_i^*(b, y, \theta_i) < V'(k_i) + \eta_i(\theta_i - b) \\ \in [0, 1] & \text{if } p_i^*(b, y, \theta_i) = V'(k_i) + \eta_i(\theta_i - b) \\ 1 & \text{if } p_i^*(b, y, \theta_i) > V'(k_i) + \eta_i(\theta_i - b) \end{cases}$$

$$where$$

$\eta_i = \frac{1}{k_i} \iint_S \left[\frac{\partial p_i^*(b, y, \theta_i)}{\partial \theta_i} \alpha_i(b, y) f(b, y) dbdy \right]$

Effective marginal cost

$$MC_i(b) = V'(k_i) + \eta_i(\theta_i - b)$$

Interpretation:

- The term $\eta_i(\theta_i - b)$ may be thought of as the marginal cost of admission operating via peer-group externality in school i.
- η_i equals to per student revenue change in school$i$ deriving from achange in θ_i

Properties of Equilibrium

We now turn to the properties of equilibrium, assuming it exists.

Proposition-I

A strict hierarchy of school qualities results, with the public sector having the lowest-ability peer group:

$\theta_n > \theta_{n-1} > ... > \theta_2 > \theta_1 > \theta_0$

Proof:

If $\theta_i = \theta_j$,let's consider school i

$\rightarrow$ expel student (b_1, y_1) and admit student (b_2, y_2) such that $b_2 > b_1, y_2 > y_1$ with $y_2 - y_1 > b_2 - b_1$.

$\rightarrow \theta_i$ increases without affecting the cost $C(k_i)$.

$\rightarrow$ Using SCI we can charge more to this individual and school can make profit.

This proposition leads to "diagonal stratification".

Properties of equilibrium

Before we going to proposition-II we will look after this division of S.
Admission space of school i:

$$A_i = \{(b, y) \in S | \alpha_i(b, y) > 0 \;\; is \;\; optimal\} \; \forall i \in \{0, 1, 2, ..., n\}$$

$$If \;\; A_i \cap A_j \neq \phi \;\;\; for \;\; some \;\; i \neq j \;\; then \;\; \mu(A_i \cap A_j) = 0$$

$$and \;\; \cup_{i=0}^{n} A_i = S \;\; (Why?)$$

Properties of equilibrium

Proposition-II

(i)*On a boundary locus between school* i *and* j *,*
$p_i = MC_i(b) \ \ and \ \ p_j = MC_j(b)$*;pricing on the boundary loci is strictly according to ability in private schools.*

(ii)$p_i(b, y) > MC_i(b)$ *for off boundary students who attend private school* i *;pricing off boundary loci depends on income in private schools.*

(III)*Every student attends a school that would maximize utility if all schools instead set* p_i *equal to equilibrium* MC_i *for all students.The allocation is as though effective marginal cost pricing prevails in private schools.*

Properties of equilibrium

- Students are indifferent to attending the schools sharing the locus.
- Private schools then have no power to price discriminate with respect to income on boundary loci.
- Prices are, however, adjusted to differing abilities because private schools internalize the peer-group effect.
- Tuition to private school i decreases with ability at rate η_i along its boundary loci, reflecting the value of peer-group improvements of the school's student body.

Properties of Equilibrium

- Near a boundary in a school's admission space, a student's preference for the school attended would be slight under effective marginal-cost pricing, so that the admitting school can capture little rent.
- The number and sizes of private schools then determine their power to price discriminate over income.
- All private schools have student bodies less than k^* by a similar argument to that in more standard monopolistically competitive equilibria.
- Zero profits then implies a scale below k^*. If we let k^* decline, then private schools become more numerous and less differentiated (have closer $\theta's$), and incomerelated price discrimination declines.

Stratification

Stratification by Income(SBI).
This hold if for any two households having students of the same ability one household's choice for a higher -θ school implies it has weakly higher income than the other household.
Stratification by Ability(SBA)
This is present if,holding income fixed, the household that chooses a higher-θ school must have a student of weakly higher ability

Stratification

Proposition-III

(i)*SBI characterizes equilibrium.*

(ii) *If preferences satisfy weak single crossing in ability (W-SCB) and* $\eta_1 \leq \eta_2 \leq ... \leq \eta_n$ *,then SBA also characterizes equilibrium.*

Properties of Equilibrium

Pareto efficiency conditions:

- (1)a student allocation that internalizes the peer-group externality given the number of schools.
- (2)Entry as long as aggregate household net willingness to pay for an allocation with one more school exceeds the change in all schools' costs.

Efficiency

Proposition-IV

(i) *The allocation in a fully private equilibrium is (Pareto) efficient given the number of schools; the equilibrium number of schools is not, however, generally efficient*.

(ii) *The public-private-sector equilibrium has neither an efficient number of schools, nor an efficient student allocation given the number of schools.*