# Contracting under Asymmetric Information and Imperfect Enforcement 

## 1. Credit and Default

- Two features of the credit market make it a fundamentally difficult institution to operate well.

1. It is often very difficult to monitor exactly what is being done with a loan. A loan may be taken for a productive reason, but may be used for other needs (such as immediate consumption).

This lack of knowledge about what a borrower is up to exactly with the money creates two problems.

- Adverse Selection: Borrowers come in different types (e.g., risky or safe, untrustworthy or reliable), and the lender does not know who is whom.
- Moral Hazard: The same borrower can behave in different ways depending on the terms that he is being offered. For instance, a loan may be deliberately channeled by him into a productive yet highly risky activity that may fail to pay off.

2. Borrowers typically have insufficient collateral, compared to what is needed to fully reassure the lender in the event of default.

- Formal credit agencies such as commercial banks often insist on adequate collateral before advancing a loan. For poor borrowers, however, this usually makes formal credit an infeasible option.
- It is not that they lack collateral to put up. But the collateral is often of a very specific kind.
- A farmer may have a small quantity of land that he is willing to mortgage. But a bank may not find this acceptable as collateral simply because the costs of selling the land in the event of a default is too high for the bank.
- A landless laborer may pledge his labour as collateral: he would work off the loan. But no bank will accept labour as collateral.
- Inadequate collateral means, more generally, that the borrower has limited liability on the loan that he is taking: in the event of a default, there is only so much you can do to punish a defaulter.
- Limited liability interacts with limited monitoring to create a set of problems that are specific to certain markets, and the credit market is one of them.
- From the point of view of the lender, the main issue is default. It is default (or the fear of it) that makes the credit market imperfect in its functioning, or even causes it to shut down altogether.
- Default can be of two kinds.

1. Involuntary default: This is the problem of "inability to repay": the borrower may be unable to repay, perhaps because of a project that went sour.

- The risk of involuntary default certainly rises with the ex-ante uncertainty surrounding the project, or with the effort that the borrower put into making the project work.

2. Voluntary or Strategic default: A situation in which the borrower can repay the loan, in principle, but simply does not find it in his interest to do so.

- This is especially pertinent in contexts where loan enforcement is weak, a common problem that plagues many developing countries.
- Internal courts of law are often weak or absent, and disgruntled lenders must take recourse to punitive measures that are often limited, such as the threat to advance no future loans.
- The less effective these threats, the more they constrain the operation of credit markets in the first place.


## An Example

- Projects $A$ and $B$, startup cost 100,000.
- Rates of return 15\% and 20\% (revenues 115,000 and 120,000).
- Bank rate of interest 10\%.
- If there is no uncertainty about the projects, we have perfect coincidence of interest between bank and borrower.
- Change $A$ to: 230,000 with probability $\frac{1}{2}$, and 0 with probability $\frac{1}{2}$. Expected return is just the same as before: 15\%.
- Assume limited liability (effectively same as limited collateral): if a project fails, the borrower cannot return any money to the lender; she simply declares bankruptcy.
- Assume both the bank and the borrower are risk-neutral.
- Now bank strictly prefers Project B. (Why?)
- But the borrower strictly prefers Project A! (Why?)
- What went wrong with the market here?
- There are two interacting problems: the lender cannot control where the money is going, and the borrower has limited liability.
- Borrower pays up if all goes well, but if the project fails, she does not repay anything.
- This creates an artificial tendency for a borrower to take on too much risk: she benefits from the project if it goes well, but is cushioned on the downside.
- The bank would like to prevent this risk from being taken. Often it cannot.
- The problem goes away if the borrower could somehow be made to repay the loan under every contingency. The bank won't care what the borrower did with the money, and the borrower would choose the project with the highest expected rate of return.
- But who can repay in all (or most) contingencies?
- They are the relatively rich borrowers, who can dig into their deep pockets to repay even if the project goes badly.
- We see here one important reason why banks discriminate against poor borrowers.
- Two interpretations of the example:
- The bank attracts type $A$ borrowers (adverse selection).
- The borrower diverts money to type $A$ projects (moral hazard).


## Why Not Just Adjust the Interest Rate?

- One reaction to all this might be: just ignore questions of risk, let the borrower do what he wants, and simply adjust the interest rate so as to compensate the lender (in expected terms).
- That adjustment is often called the risk premium.
- Formally, if $p$ is the probability that a loan will be repaid, a moneylender's expected profit is $p(1+r) L$, where $L$ is loan size and $r$ the rate he charges.
- If the moneylender can get a rate of return $i$ elsewhere by safely investing his money,
the interest rate $r$ that will compensate him is given by

$$
p(1+r) L-(1+i) L=0
$$

that is,

$$
r=\frac{1+i}{p}-1
$$

- For instance, if the safe return gets the lender $5 \%$, and there is a 50-50 chance of default, then we have $r=110 \%$.
- This seemingly obvious solution is often wrong.
- The reason is that the interest rate premium itself affects borrower behaviour, and it may spark off a higher chance of default.
- Nothing gives us the right to assume that $p$ is exogenous. In fact, the higher is $r$, the lower might be $p$, a theme to which we return later.
- Interest rate adjustments can sometimes look like a dog chasing its own tail, and there may be no end-point to that chase.


## 2. Three Complications of Limited Liability

- Limited liability creates three major complications, each of which impede the credit market from functioning in a satisfactory way.

1. Adverse Selection: Each borrower comes attached with a project, and some projects are more risky than others. As the bank raises the interest rate, the project mix changes to one that is ever riskier.
2. Moral Hazard: Borrowers choose projects, and as the interest rate goes up they optimally choose projects that are more risky or they put in less effort to repay the loan.
3. Strategic Default: Borrowers might deliberately choose not to repay a loan that they are capable of repaying. The decision to repay will depend on the costs and benefits of default. Note yet again that the higher the interest rate, the higher the chances of default.

- In what follows, we look at each of these three situations in turn.


## 3. Risky Borrowers: The Problem of Adverse Selection

- Not all borrowers bear the same amount of risk. There are high-risk borrowers and there are low-risk borrowers.
- An entrepreneur looking to expand an existing business is a safer bet than an individual entering the world of business for the first time.
- A landless laborer in poor health is high risk.
- A farmer who owns a pump set or has access to assured irrigation carries lower risk than one who does not.
- A crop cultivated by a farmer may be more or less prone to the vagaries of the weather.
- So lending risk varies significantly from borrower to borrower.
- Some of this risk is correlated with characteristics of the borrower that are observable to the lender (such as landholdings or access to irrigation).
- However, a significant component may substantially depend on unobservables (farming skills, mental acumen in the face of a crisis, thriftiness, the quality of borrower-held assets, and so on).
- When the factors that make for risk are observable, the lender can select his clients or charge appropriately higher rates for the high-risk clients.
- However, to the extent that clients bear different risks that cannot be discerned by the lender, an additional dimension is added to credit market transactions:
- the interest rate now affects the mix of clients that are attracted (and hence, the average probability of default).
- This new dimension might give rise to a situation in which at prevailing rates, some people who want to obtain loans are unable to do so; however, lenders are unwilling to capitalize on the excess demand and raise interest rates for fear that they will end up attracting too many high-risk customers.
- Following Stiglitz and Weiss (1981), we use this idea as a starting point for an explanation of credit rationing.


### 3.1 Why Higher Interest Rates Attract Risky Types?

- Consider a project with a startup cost of $B$, that has to be borrowed, and that generates a (possibly risky) return $R$.
- There is limited liability: a borrower has access to some limited collateral $C<B$.
- If the rate of interest charged is $r$, then an individual repays if and only if

$$
R-(1+r) B \geq-C
$$

- So the borrower's return viewed as a function of $R$ and $r$ is given by

$$
\pi(R, r) \equiv \max \{R-(1+r) B,-C\}
$$

while the lender's return is

$$
\rho(R, r) \equiv \min \{R+C,(1+r) B\} .
$$

- Assume that the lender can seize both the total return $R$ as well as the collateral.
- The following figure shows the returns of borrower and lender.


Figure: Net Returns to Borrower and Lender

- Note that limited liability makes the borrower's return convex in $R$ and the lender's return concave in $R$. That brings us to a very important point:
- expected return to the borrower increases in riskiness of the project;
- opposite is true for the lender.
- The figure compares two projects each with just two values of $R$ :
- $a$ and $b$ with equal probability so that the mean return $m$ is midway between these two numbers;
- $a^{\prime}$ and $b^{\prime}$, with $a^{\prime}<a$ and $b^{\prime}>b$, and with the same expected value $m$.
- Note that, for the borrower, the expected return to the riskier project is higher, while the opposite is true for the lender.
- Hence a risky borrower will be willing to pay a higher interest rate than a safe one.
- If the lender raises the interest rate to compensate for default risk, he will also simultaneously be worsening the composition of the borrower pool, as the relatively safe borrowers drop out of the credit market.


### 3.2 Adverse Selection and Credit Rationing

- Consider just two types of potential borrowers: safe and risky.
- Each needs $B$ to invest in a project.
- The safe type generates a sure return of $R(R>B)$.
- The risky type can obtain a still higher return $R^{\prime}>R$, but only with probability $p$. With probability $1-p$, his investment backfires and he gets a return of 0 .
- Assume (for simplicity) that the lender can freely set the interest rate without fear of losing his clients to competing lenders.
- Let us suppose that the lender has enough funds to lend to just one applicant.
- Should he raise his interest rate $r$ until one borrower drops out? Let us see.
- The net return to a safe borrower is $R-(1+r) B$, so he will want to borrow as long as $r \leq r_{1}=\frac{R}{B}-1$.
- The risky borrower's expected return is $p\left[R^{\prime}-(1+r) B\right]$; hence, the maximum rate he is willing to pay is $r_{2}=\frac{R^{\prime}}{B}-1$.
- Clearly, because $R^{\prime}>R$, we have $r_{2}>r_{1}$.
- If the lender charges $r_{1}$ or below, both borrowers will apply for the loan.
- If lender cannot tell them apart, he has to give the loan randomly to one of them.
- On the other hand, if a rate slightly higher than $r_{1}$ is charged, the first borrower drops out and excess demand for the loan disappears.
- The lender may then go all the way up to $r_{2}$ without fear of losing the second customer.
- The lender's choice is then really between the two interest rates $r_{1}$ and $r_{2}$.
- Which one should he charge?
- Suppose the lender charges $r_{2}$. His expected profit is then given by

$$
\pi_{2}=p\left(1+r_{2}\right) B-B .
$$

- If the lender charges $r_{1}$, he attracts each type of customer with probability $\frac{1}{2}$. His expected profit is then given by

$$
\pi_{1}=\frac{1}{2}\left[\left(1+r_{1}\right) B-B\right]+\frac{1}{2}\left[p\left(1+r_{1}\right) B-B\right]=\frac{1}{2}\left(1+r_{1}\right) B(1+p)-B
$$

- Under what condition will the lender be reluctant to charge the higher interest rate?
- This will happen when $\pi_{1}>\pi_{2}$.
- Using the expressions of $\pi_{1}$ and $\pi_{2}$, and substituting the values of $r_{1}$ and $r_{2}$ from above, we obtain the condition

$$
p<\frac{R}{2 R^{\prime}-R} .
$$

- This condition tells that if the high-risk type is "sufficiently" risky, then the lender will not raise his interest rate to $r_{2}$, thereby attracting the risky type.
- Instead, he will stick to the lower level $r_{1}$ and take the 50-50 chance of getting a safe customer.
- This will lead to credit rationing in equilibrium:
- out of two customers demanding a single available loan, only one will get it; the other will be disappointed.
- The price is not raised even in the face of excess demand.
- Raising the price would drive away the good borrower instead of the bad one, and the higher possible return cannot compensate for the lowered chance of repayment.


## 4. Moral Hazard and Credit Rationing

- Now we consider a case of moral hazard in the choice of effort to oversee the success of a project.
- If the distribution of returns from the project is affected by the borrower's actions, observability and monitoring will be a problem.
- Limited liability could then increase default risk by reducing the borrower's effort in avoiding low yield returns.
- Consider an indivisible project which requires funds of amount $L$ to be viable.
- Output is binary, taking values of either $Q$ (good harvest) or 0 (crop failure).
- The probability of a good harvest is $p(e)$, where $e$ is the effort level of the agent who oversees the project.
- Assume $p^{\prime}(e)>0$, and $p^{\prime \prime}(e)<0$ representing diminishing returns.
- Effort cost is given by $e$.
- All agents are risk neutral.


### 4.1 Benchmark: The First-Best

- As a benchmark, consider the problem of a self-financed farmer.
- If investment takes place, the effort level is chosen so as to

$$
\max _{e} p(e) \cdot Q-e-L
$$

- The optimum choice $e^{*}$ is described by the first-order condition:

$$
p^{\prime}(e)=\frac{1}{Q} .
$$

- This is the efficient or first-best level of effort, which forms the benchmark against which all subsequent results will be compared.


### 4.2 The Debt Overhang Problem

- Now consider a debt-financed farmer.
- Let $R=(1+i) L$ denote total debt repayment where $i$ is the interest rate.
- To introduce moral hazard, we assume that $e$ is not verifiable by third parties, hence not contractible.
- There is limited liability: the borrower faces no obligations in the event of a crop failure.
- However, we allow for some collateral. Let $w$ denote the value of the borrower's transferable wealth that can be put up as collateral.
- To make the problem interesting, assume $w<L$.
- The effort choice of a borrower facing a total debt $R$ is derived from:

$$
\max _{e} p(e) \cdot(Q-R)+(1-p(e)) \cdot(-w)-e .
$$

- Denote the optimal choice by $\hat{e}(R, w)$, defined by the following first-order condition:

$$
p^{\prime}(e)=\frac{1}{Q+w-R} .
$$

- $\hat{e}(R, w)$ is decreasing in $R$.
- A higher debt burden reduces the borrower's payoff in the good state, but not in the bad state, dampening incentive to apply effort.
- $\hat{e}(R, w)$ is increasing in $w$.
- A bigger collateral, imposes a stiffer penalty in the event of crop failure, thus stimulating the incentive to avoid such an outcome.
- The lender's profit is given by

$$
\pi=p(e) \cdot R+(1-p(e)) \cdot w-L
$$

- Compare the optimal effort choice of an indebted borrower $(\hat{e}(R, w))$ with the efficient effort level ( $e^{*}$ ).
- The f.o.c. for $e^{*}$ is $p^{\prime}(e)=\frac{1}{Q}$, while that for $\hat{e}(R, w)$ is $p^{\prime}(e)=\frac{1}{Q+w-R}$.
- Since $w<L$, by assumption, and $L<(1+i) L=R$, we have $w<R$.
- Also, $p^{\prime \prime}(e)<0$.
- So we can conclude that

$$
\hat{e}(R, w)<e^{*} .
$$

- Proposition 1. As long as the borrower does not have enough wealth to guarantee the full value of the loan, the effort choice will be less than the first-best.
- This is the debt overhang problem.
- An indebted borrower will always work less hard on his project than one who is self-financed.


### 4.3 Constrained Pareto Efficient Equilibrium

- To find the Pareto frontier of possible payoffs, we hold the lender's expected profit at any given level $\pi$, and maximize the borrower's utility, subject to incentive compatible choice of effort level.
- Since the Pareto frontier is constrained by the effort incentive problem, it is often called the "second-best frontier".
- In determining equilibrium choices, we will treat $\pi$ as given, and will later see the comparative static effects of increasing it.
- The special case where $\pi=0$ represents a perfectly competitive loan market with free entry.
- The variables to be determined in equilibrium are $R$ and $e$. They are determined jointly by
- the isoprofit curve: $\pi=p(e) \cdot R+(1-p(e)) \cdot w-L$, and the incentive curve: $p^{\prime}(e)=\frac{1}{Q+w-R}$.
- The locus described by the isoprofit curve is negatively sloped.
- If the borrower works harder, the risk of default is reduced, and $R$ must be lower to hold down the lender's profit at the same level.
- The locus described by the incentive curve also is negatively sloped.
- A reduced debt burden increases the incentive to work hard.
- Notice also that as we move downward along the incentive curve, the borrower's payoff is increasing.
- Lower debt ( $R$ ) increases borrower payoff for any given choice of effort, and hence also after adjusting for optimal choice.
$\Rightarrow$ If there are multiple intersections, only the lowest among these (the one associated with the lowest $R$ ) is compatible with Pareto efficiency.
- Figure 1 depicts a typical situation, point $E$ representing the equilibrium.


Figure 1: Equilibrium Debt and Effort in the Credit Market

### 4.4 Lender's Profit

- Examine the comparative static effect of higher lender's profit $(\pi)$.
- Figure 2 shows the effect of increasing $\pi$.
- The isoprofit curve shifts up.
- In the new resultant Pareto efficient equilibrium, the debt burden $(R)$ increases, and so does the interest rate (since the loan size is fixed), while the effort level falls.
- Proposition 2. Incentive-constrained Pareto efficient equilibria in which lenders obtain higher profits involve higher debt and interest rates, but lower levels of effort. Hence, these equilibria produce lower social surplus.
- Why higher rent extraction is associated with lower overall efficiency?
- Lenders earn more profit by increasing interest rate, which in itself is a pure transfer.
- However, a greater debt burden reduces the borrower's incentive to spend effort, increasing the chance of crop failure and creating a deadweight loss.


Figure 2: Effect of an Increase in Lender's Profit

- Consider two extreme cases.
- The case of $\pi=0$ represents perfect competition.
- This situation generates the highest level of effort among all.
- Notice, however, that since the debt burden $(R)$ still exceeds $w$, effort will nevertheless be less than first-best.
- This tells us that the source of the inefficiency is not so much monopolistic distortion created by the lender's market power (although that certainly exacerbates the problem), but the agency problem itself, and the distortion in incentives created by limited liability.
- While the borrower shares in capital gains, he bears no part of the capital losses (beyond the collateral posted). Working with other people's money is not the same as working with one's own.
- The other extreme case is that of monopoly.
- In this case, the value of $\pi$ is maximized from among all feasible and incentive compatible alternatives.
- The monopolistic lender will choose the point on the incentive curve that attains the highest isoprofit curve. The condition is the standard one of tangency between the two curves.
- This provides a ceiling on the interest rate, or debt level $(\bar{R})$, and the lender will not find it profitable to raise it above this level.
- In more competitive conditions, this ceiling will still apply.
- If, in a competitive credit market, there is excess demand for funds at $\bar{R}$, the interest rate will not rise to clear the market.
- We have an exact counterpart of Stiglitz-Weiss type of credit rationing (rationing of borrowers) in the presence of moral hazard rather than adverse selection.


## Credit Rationing: Intuition

- The model illustrates the trade-off between extraction of rents and the provision of incentives to induce a good harvest.
- Higher interest rates cause the problem of debt overhang - a highly indebted farmer has very little stake in ensuring a good harvest, since the large loan repayments this outcome induces imply that he captures only a small portion of the returns from the harvest.
- Keeping this in mind, lenders will be reluctant to raise interest rates beyond some level.
- As in the adverse selection theory, the interest rate may not rise enough to guarantee that all loan applicants secure credit, in times when loanable funds are limited.


## Policy Implications

- That borrower-friendly equilibria are more efficient has broad policy implications.
- Any change which reduces interest rates, or improves the bargaining power of the borrower will enhance effort and productivity.
- The latter involves institutional changes, such as a reallocation of property rights over relevant productive assets from lenders to borrowers, or an improvement in the latter's outside options.
- Note, however, that such policy interventions cannot result in improvements for both agents - since equilibrium contracts are by definition constrained Pareto-efficient.
$\Rightarrow$ Such policy interventions must make some agents in the economy worse off.
- Despite the fact that the gainers (borrowers) could potentially compensate the losers, such compensations cannot actually be paid, owing to the wealth constraints of the borrowers.
- Accordingly such policies will tend to be resisted by the losers, and may not actually be adopted.


### 4.5 Borrower's Collateral

- Consider the role of collateral $(w)$ in the credit market.
- Figure 3 captures the effect of an increase in $w$ on equilibrium interest rate and effort.
- The incentive curve moves up and to the right: for each stipulated value of $R$, the borrower now has more to lose (he has more collateral to give up) and so will put in more effort in order to avoid that outcome.
- The isoprofit curve shifts down: for any effort level $e$, since the return in the bad state is higher due to more collateral, the return in the good state, i.e., the interest charged, must be lower to keep profits the same.
- The joint effect of these changes is that $R$ falls - and so does the interest rate - and borrower effort goes up.
- Proposition 3. An increase in the size of collateral, $w$, leads to a fall in the equilibrium interest rate and debt, and an increase in the effort level. For a fixed $\pi$, the borrower's expected income increases; hence, the utility possibility frontier shifts outwards.


Figure 3: Effect of Higher Borrower Wealth (Collateral)

- The intuition is fairly simple.
- Ceteris paribus, a bigger collateral increases the incentive to put in effort, since failure is now more costly to the borrower.
- If lender's profits are to be preserved at the same level, the interest rate must fall, because there is lower default risk.
- This causes less debt overhang, further reinforcing the effect on incentives.
- Higher effort levels increase the total surplus. But since lender's expected profits are held constant, borrowers must get more in net terms.
- These results illustrate how interest rate dispersion might arise, even in competitive credit markets.
- In the presence of default risk and moral hazard, the interest rate will be closely tied to borrower characteristics such as wealth or ability to post collateral.
- Wealthier borrowers pose less risk for two reasons: these loans have better guarantees in case of default, plus lower default risk arising from better incentives.
- Hence, wealthier borrowers have access to cheaper credit.
- Arbitrage opportunities are illusory - the isoprofit line restricts lenders to the same profit level for different types of borrowers.
- The functioning of the credit market may exacerbate already existing inequalities.
- Those with lower wealth are doubly cursed: they not only face lower consumption potential from asset liquidation, but also lower income earning potential, owing to costlier (or restricted) access to credit.
- The reason is that the poor cannot credibly commit to refrain from morally hazardous behaviour as effectively as the rich.
- This process of magnification of past inequalities through the operation of the credit market may cause the persistence of poverty (Galor and Zeira, 1993, for example).


## 5. Strategic Default

- Results similar to those in the previous sections can also arise from costly contract enforcement, where the principal problem faced by lenders is in preventing wilful default ex post by borrowers who do in fact possess the means to repay their loans.
- Most credit contracts in developing world are not enforced by courts, but instead by social norms of reciprocal and third-party sanctions.
- Contracts have to be self-enforcing, where repayment of loans rely on the selfinterest of borrowers, given the future consequences of a default.
- In this respect the problem is akin to that of sovereign debt where lender countries and international courts do not have the means of enforcing loan repayments by borrowing countries.
- Defaults are sought to be deterred solely by the threat of cutting the borrower off from future access to credit.
- Empirical and historical accounts of trade and credit in countries lacking a developed system of legal institutions amply document the role of such reputational mechanism.
- Theoretical models have shown how such enforcement problems can also give rise to the phenomena like
- adverse incentive effects of raises in interest rates,
- credit rationing,
- long term relationships,
- the role of social networks.
- To understand these issues, we turn our attention to the problem of voluntary or strategic default.
- In the absence of usual enforcement mechanisms (courts, collateral, etc.), compliance must be achieved through the use of dynamic incentives, that is, from the threat of losing access credit in the future.
- We use a simple infinite horizon repeated lending-borrowing game to illustrate such a mechanism, and derive its implications for rationing and efficiency in the credit market.
- Since bankruptcy and involuntary default are not the focus here, we remove any source of production uncertainty.
- Each period, the borrower has access to a production technology which produces output $F(L)$, where $L$ is the value of inputs purchases and applied.
- We assume $F^{\prime}(\cdot)>0$, and $F^{\prime \prime}(\cdot)<0$.
- Suppose production takes the length of one period, and let $r$ be the bank rate of interest (opportunity cost of funds).
- To set the benchmark, consider the case of a self-financed farmer.
- The optimum investment $L^{*}$ is given from the solution to

$$
\max _{L} F(L)-(1+r) L,
$$

which yields the first-order condition

$$
F^{\prime}\left(L^{*}\right)=1+r .
$$

- Next we turn to debt-financed farmers.
- For simplicity, we assume that such farmers do not accumulate any savings and have to rely on the credit market to finance investment needs every period.
- Borrowers live for an infinite number of periods, and discount the future by a discount factor $0<\delta<1$.


### 5.1 Partial Equilibrium: Single Lender

- We first solve a partial equilibrium exercise where there is a single borrower and a single lender.
- We focus on a stationary subgame perfect equilibrium, where
- the lender offers a loan contract $\{L, R=(1+i) L\}$ every period, and
- follows the trigger strategy of never offering a loan in case of default.
- The defaulting borrower still has an outside option that yields a payoff $v$ every period.
- For now, we treat $v$ as exogenous.
- Later, we show $v$ can be "rationalized" as the value arising in a general equilibrium model with many borrowers and lenders.


### 5.1.1 The No-Default Constraint

- We can calculate the incentive for the borrower to remain in the relationship.
- If he defaults on the loan, he gets to keep $F(L)$ the current period (fully) and from the following period he gets $v$ every period, so his total payoff from defaulting is given by

$$
F(L)+\delta v+\delta^{2} v+\ldots=F(L)+\frac{\delta v}{1-\delta}
$$

- If he sticks to the terms of the contract, he gets

$$
[F(L)-R] \cdot\left(1+\delta+\delta^{2}+\ldots\right)=\frac{F(L)-R}{1-\delta}
$$

- Hence the no-default constraint is given by

$$
\frac{F(L)-R}{1-\delta} \geq F(L)+\frac{\delta v}{1-\delta}
$$

- Rearranging, we get the condition

$$
R \leq \delta[F(L)-v]
$$

### 5.1.2 Pareto Frontier of Stationary Equilibria

- As with all repeated games, there are many equilibria.
- We characterize the Pareto frontier of all stationary equilibria, in which the same loan contract is offered at all dates.
- In order to generate the Pareto frontier, we must maximize the borrower's per period net income, while satisfying his no-default constraint and holding the lender's profit at some fixed level $z$. Mathematically,

$$
\max _{\{L, R\}} F(L)-R,
$$

subject to the constraints

$$
R \leq \delta[F(L)-v]
$$

and

$$
z=R-(1+r) L .
$$

- The nature of the solution is illustrated in Figure 4.
- The boundary of the no-default constraint (the incentive constraint) is the positively sloped, concave curve with slope $\delta F^{\prime}(L)$, while the lender's profit constraint is represented by a straight line with slope $1+r$.
- The points of intersection $A$ and $B$ are where both constraints bind.
- Clearly, the line segment AB represents the feasible set.
- The borrower's indifference curves are rising, concave curves with slope $F^{\prime}(L)$, indifference curves to the right representing higher borrower payoff.
- If these indifference curves attain tangency at some point on $A B$, it is the solution to the problem, and has the property: $L=L^{*}$, and $R=(1+r) L^{*}+z$.
- This will happen if the strategic default condition is "not binding".
- If the strategic default condition is binding, the solution must be at the corner B .


Figure 4: Optimal Solution to the Enforcement Problem

- Let $\hat{L}(v, z)$ be the value of $L$ at B , and let $\tilde{L}(v, z)$ denote the solution to the problem above (the corresponding value of $R$ is given by $\tilde{R}(v, z)=(1+r) \tilde{L}(v, z)+z$.
- The preceding discussion leads to the conclusion:

$$
\tilde{L}(v, z)=\min \left\{L^{*}, \hat{L}(v, z)\right\} .
$$

- Note that if the second argument applies above (that is, the solution is at the corner B), credit rationing will arise:
- credit limits are placed on borrowers (below first-best levels), that is, $\tilde{L}(v, z)<L^{*}$.
- We will show in a moment that this is possible.


### 5.1.3 Lender's Profit

- We analyze the effect of a parametric shift in $z$, lender's profit.
- Figure 5 shows that if $z$ increases, the isoprofit line shifts up and the point $B$ moves to the left, that is, $\hat{L}(v, z)$ is decreasing in $z$.
- if this is indeed the solution, then the equilibrium volume of credit is reduced and rationing becomes more acute.
- Notice that the interest rate also rises, as indicated by the fact that the ray connecting point $B$ to the origin becomes steeper.
- If the solution is interior $\left(L^{*}\right)$ to begin with, a small increase in $z$ will raise the interest rate, but will leave the loan size unaffected.


Figure 5: Effect of an Increase in Lender's Profit

### 5.1.4 Borrower's Outside Option

- Figure 6 illustrates the effect of increasing the borrower's outside option $v$.
- The curve representing the boundary of the incentive constraint undergoes a parallel downward shift, moving the corner point B to the left.
- The effect on loan sizes and interest rates is similar to the case of increasing lender's profit.
- If $\hat{L}=L^{*}$ to begin with, nothing changes (since $v$ affects only the no-default constraint, which is not binding).
- If $\tilde{L}=\hat{L}$, on the other hand, increasing $v$ has the implication that the equilibrium loan size falls and the interest rate rises.


Figure 6: Effect of an Increase in Borrower's Outside Option

### 5.1.5 Can Credit Rationing Arise in Equilibrium?

- The answer is yes, and to see this note that if the value of $z$ (given $v$ ) or $v$ (given $z$ ) is too high, the constrained maximization problem does not have a solution,
- since the isoprofit line will lie everywhere above the boundary of the no-default constraint.
- The borderline case is one where the two are tangent, i.e., when the points $A$ and $B$ converge to each other so that the feasible set of the problem becomes singleton.
- The solution must then be this single feasible point.
- Tangency implies that $\delta F^{\prime}(\tilde{L})=1+r$, implying $\tilde{L}<L^{*}$ since $\delta<1$ and $F$ is concave.
$\Rightarrow$ There is credit rationing if $z$ (or $v$ ) is sufficiently high.
- Since the solution is continuous in $z$ (or $v$ ), and given the comparative static properties of the corner solution, it follows that there will be credit rationing if either $z$ or $v$ (given the other) is above a critical value.
- These observations are summarized in the following proposition.

Proposition 4. There is credit rationing if $z$, the lender's profit (given $v$ ), or $v$, the borrower's outside option (given $z$ ), is above some threshold value. If rationing is present, a further increase in the lender's profit, or the borrower's outside option, leads to further rationing (i.e., a reduction in the volume of credit) as well as a rise in the interest rate.

- Equilibria which give more profit to the lender involve lower overall efficiency, because credit rationing is more severe in such equilibria.
- Increased bargaining power of lenders thus reduce productivity, echoing a similar result in the previous model involving involuntary default.


### 5.2 General Equilibrium: Multiple Lenders

- An obvious shortcoming of the model so far is that the outside option $v$ has been assumed exogenous.
- In a competitive setting with multiple lenders - which fits descriptions of informal credit in many developing countries - a defaulting borrower can switch to a different lender.
- If there is a good deal of information flow within the lending community, the defaulting borrower could face social sanctions (as opposed to merely individual sanction from the past lender), thus restoring the discipline.
- The strength and reliability of such information networks could vary from one context to another, and is a factor that needs to be taken into account.
- Accordingly, the strength of such networks can be treated as a parameter of the model.
- Suppose that following a default, the existing credit relationship is terminated.
- The borrower can then approach a new lender, who checks on the borrower's past and uncovers the default with probability $p$ (i.i.d. across periods).
- If uncovered, the lender refuses the loan, and the borrower remains unmatched in that period.
- In the following period, the borrower approaches yet another lender, whereupon the same story repeats itself.
- If the lender fails to uncover the default, the borrower enters into a new credit relationship with the lender.
- Given the assumption of a symmetric (and stationary) equilibrium, the borrower the same contract $(L, R)$ as with previous lenders with payoff denoted by $\omega$.
- Then $v$, the expected value of the outside option, is given by

$$
v=p \delta v+(1-p) \omega=\left[\frac{1-p}{1-\delta p}\right] \omega .
$$

- Then we can write

$$
v=(1-\rho) \omega,
$$

where $\rho \equiv \frac{p(1-\delta)}{1-\delta p}$ can be viewed as the scarring factor.

- If $p$ gets very close to one, so that a default is always recognized, then the scarring factor converges to one as well.
- For any $p$ strictly between zero and one, the scarring factor goes to zero as $\delta$ goes to unity, or if $p$ itself goes to zero.
- For the endogenous determination of $v$, we utilize our analysis of the partial equilibrium model to construct a function $\phi(v ; z)$ whose fixed point gives the equilibrium in this more general setting.
- Consider a given $z$ and any arbitrary value of $v$ for which the constrained maximization problem (described in the partial equilibrium model) has a solution.
- The borrower's per period payoff (on the equilibrium path) is given by

$$
\omega(v, z)=F(L)-R=(1-\delta)\left[F(L)+\frac{\delta v}{1-\delta}\right]=(1-\delta) F(\tilde{L}(v, z))+\delta v
$$

treating that the no-default constraint is binding.

- If he defaults, his expected per period payoff thereafter is $(1-\rho) \cdot \omega(v, z)$.
- The original $v$ is "rationalized" if this latter value coincides with $v$,
- that is, the defaulting borrower's continuation payoff is precisely what he can expect to get from the market itself after termination by his current lender.
- We define the following function:

$$
\phi(v ; z)=(1-\rho) \cdot \omega(v, z),
$$

and note that, given $z$, any fixed point of $\phi$ (with respect to $v$ ) denotes an equilibrium.

- Proposition 4 tells us that an exogenous increase in either $v$ or $z$ leads to a smaller loan size and higher interest rate, which adversely affects borrower payoffs.
$\Rightarrow \phi(v, z)$ is decreasing in both its arguments.
- Further, if $v$ is higher than some threshold $\bar{v}(z)$, the problem has no solution, and the value of $\phi(v, z)$ can be taken to be 0 in that case.
- Take $z$ as given. Figure 7 shows the nature of the function $\phi$ :
- it is downward sloping, with a downward jump at $\bar{v}$.
- There is a unique fixed point - $v^{*}$ in the diagram - if there is an intersection with 45 degree line before the point of discontinuity.
- Otherwise, no symmetric equilibrium exists.
- It can be shown that if either the probability of detection $p$ is high enough, or borrowers are sufficiently patient, an equilibrium usually exists.


Figure 7: The Function $\phi(v, z)$

- Finally, we wish to check whether equilibria that provide higher profits to the lender create more credit rationing and reduce efficiency.
- First, observe that a rise in $z$ shifts the $\phi$ function downwards, implying that the equilibrium value of $v$ must fall (see Figure 8).
- Next, noting that in equilibrium $\phi(v, z)=v$, we can write

$$
v=(1-\rho)[(1-\delta) F(\tilde{L})+\delta v]
$$

which, on rearrangement, yields

$$
v=\frac{(1-\rho)(1-\delta) F(\tilde{L})}{1-\delta(1-\rho)}
$$

where $\tilde{L}$ denotes the equilibrium loan.

- This establishes that in equilibrium, $v$ and $\tilde{L}$ are positively related.
- Since $v$ falls due to a parametric increase in lender's profit $z$, so does $\tilde{L}$.


Figure 8: Effect of an Increase in Lender's Profit

## 6. References

- Sections 4 and 5 of this note is based on

1. Ghosh, Parikshit, Dilip Mookherjee and Debraj Ray (2001), "Credit Rationing in Developing Countries: An Overview of the Theory", Chapter 11, Readings in the Theory of Economic Development, edited by Dilip Mookherjee and Debraj Ray, London: Blackwell.
