Group-lending with sequential financing, contingent renewal and social capital

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Introduction

- This paper focuses on some of the dynamic aspects of micro-lending institutions, in particular those involving group-lending.
- We will analyze the efficacy of two dynamic schemes, namely sequential financing and contingent renewal, in harnessing social capital.

Introduction

Sequential financing: In the Grameen Bank, for example, the groups have five members each. Loans are sequential in the sense that these are initially given to only two of the members (to be repaid over a period of 1 year). If they manage to pay the initial installments, then, after a month or so, another two borrowers receive loans and so on.

Introduction

Contingent renewal: Contingent renewal of loans refers to the feature that in case of default by a group, no member of this group ever receives a loan in the future. Moreover, in case of repayment, there is repeat lending. In case of default, the original Grameen idea was not that group-members would have to pay for others, but rather that they would be cutoff from future loans.

Introduction

Social capital: Social capital may take the form of mutual help in times of distress, mutual reliance in productive activities, status in the local community, etc. In case default by one borrower harms the other borrowers, such default may be penalized through a loss of this social capital.

Economic Environment

- The market consists of many borrowers, such that their mass is normalized to one.
- Borrower i can invest in one of two projects, P_i^1 or P_i^2.
  - P_i^1 has a verifiable income of H and no non-verifiable income
  - P_i^2 has no verifiable income and a non-verifiable income of b, where $0 < b < H$.
- The sets of projects are different for different borrowers.
- In every period, the borrowers consume all their income in that period.

Economic Environment

- All projects require an initial investment of 1 dollar.
- None of the borrowers have any funds, they have to borrow the required 1 dollar from a bank.
- For every dollar loaned, the amount to be repaid is $r(\geq 1)$, where r is exogenously given.
- For the project to be profitable for the borrowers, it must be that $H > r$. For simplicity, we assume that $H \leq 2r$, so that $r < H \leq 2r$.

Economic Environment

- A fraction $0 \leq \theta \leq 1$ of the borrowers have a social capital of $s(> 0)$, whereas the other borrowers have no social capital.
- The borrowers with social capital are denoted by S, whereas the other borrowers are denoted by N.
- The social penalty involves a loss of this social capital. An S type borrower taking a group-loan is assumed to lose her social capital if she defaults and, moreover, this default affects the other group-member.
- The social penalty is anonymous in the sense that it is imposed irrespective of whether the default affects an S type or an N type borrower.
- The borrowers all know one anothers types, but the bank does not.

Economic Environment

- Time is discrete so that $t = 0, 1, 2, ...$
- Let $0 < \delta < 1$ denote the common discount factor of all the agents, the borrowers, as well as the bank.

Assumptions

1. $H - r < b$: Suppose that a borrower has taken a loan of 1 dollar. If the borrower is of type N, then she will prefer to invest in her second project.
2. $H - r > b - s$: Suppose some borrower of type S has taken a loan and that she will lose her social capital in case of default. In case she invests in her second project, she obtains a non-verifiable income of b, but loses her social capital, so that her net payoff is $b - s$. Thus, the borrower will prefer to invest in her first project.

Group-lending without sequential financing

Consider the following infinite horizon game.
Period 0. There is endogenous group-formation whereby the borrowers organize themselves into groups of two. Depending on the type of borrowers comprising the groups, these can be of three types, SS, NN and SN.

Group-lending without sequential financing

For every $t \geq 1$, there is a two-stage game.

- **Stage 1.** The bank randomly selects one of the groups as the recipient and lends it two dollars, which are divided equally among the two members of the selected group.
- **Stage 2.** Both the borrowers simultaneously invest 1 dollar into one of their two projects. If the i-th borrower invests in P_i^1, she has a payoff of $H - r$; otherwise, she has a payoff of b.

Given the lending policy, default by a borrower does not affect the expected income of the other borrower and hence does not attract the social penalty even if she is of type S.
The bank has a payoff of $2(r - 1)$ in case both the borrowers invest in their first projects, $r - 2$ in case only one of the borrowers invests in her first project and the other borrower invests in her second project, and a payoff of -2 in case both the borrowers invest in their second projects.

More definitions

- There is **positive assortative matching** if there are $\frac{\theta}{2}$ groups of type SS and $\frac{1-\theta}{2}$ groups of type NN.
- There is **negative assortative matching** if there are $min\{\theta, 1-\theta\}$ groups of type SN, $max\{\frac{1-2\theta}{2}, 0\}$ groups of type NN and $max\{\frac{2\theta-1}{2}, 0\}$ groups of type SS.

Group-lending without sequential financing

Given that the lending policy of the bank doesn't involve contingent renewal, so members of all groups are going to behave as if they are playing a one shot game.

- Let v_{ij} denote the expected equilibrium payoff of a type i borrower at some period $t \leq 1$ if she forms a group with a type j borrower and the group receives the bank loan at this period.
- Assuming that side payments are possible, there will be positive assortative matching if and only if the maximum, a type N borrower is willing to pay to a type S borrower, is strictly less than the minimum a type S borrower will need as compensation for having a type N partner, i.e. $v_{SS} - v_{SN} > v_{NS} - v_{NN}$.
- There will be negative assortative matching whenever $v_{SS} - v_{SN} < v_{NS} - v_{NN}$. For ease of exposition, we assume that there will be negative assortative matching if $v_{SS} - v_{SN} = v_{NS} - v_{NN}$.

Group-lending without sequential financing

- **Stage 3.** For any borrower, her payoff from investing in her first project is $H - r$, whereas her payoff from investing in her second project is b. Given Assumption 1, both the borrowers will invest in their second projects irrespective of their type. Thus

$$v_{SS} = v_{SN} = v_{NN} = v_{NS} = b$$

- **Stage 2.** Since the borrowers always invest in their second project, the banks expected payoff at any period from making a loan is -2.
- **Stage 1.** Given above equation, the tie-breaking rule implies that there will be negative assortative matching. Of course, the expected payoff of the bank is independent of the nature of the matching.

Proposition 1

Group-lending without sequential financing is not feasible
In this case, default by a borrower does not affect her partners payoff and hence, even for an S type, does not attract the social penalty. Thus, given the parameter restrictions, the borrowers always invest in their second projects, so that lending is not feasible.
Remark : It is clear that our analysis goes through even if $H > 2r$.

Group-lending with sequential financing

In every round, the members of the selected group receives loans in a staggered manner, but the selection of the recipient group is independent of history.

Consider the following game

Period 0. There is endogenous group-formation whereby the borrowers organize themselves into groups of two.

For every $t \geq 1$, there is a three-stage game.

- **Stage 1.** The bank randomly selects a group and lends the selected group 1 dollar.

Group-lending with sequential financing

- **Stage 2.** One of the borrowers is randomly selected (with probability half) by the group as the recipient of the 1 dollar lent by the bank. This borrower, say B_i, then decides whether to invest the 1 dollar in P_i^1 or P_i^2. If B_i invests in P_i^2 , then B_i defaults, there is no further loan by the bank and the game goes to the next period. Note that, in case of default by B_i, B_j does not obtain the loan at all. Hence, depending on its type, B_i obtains either b or $b-s$. If B_i invests in P_i^1, then there is a verifiable return of H, out of which the bank is repaid r and B_i obtains $H-r$. We assume that $H-r<1$, so that this amount is not sufficient to finance the investment in the next stage.

Group-lending with sequential financing

- **Stage 3.** This stage arises only if B_i had invested in P_i^1 in stage 2. The bank lends a further 1 dollar to the group, which is allocated to the other borrower, B_j, who decides whether to invest it in P_j^1 or P_j^2. Note that, in this case, default by B_j does not affect the payoff of B_i, the group-member who had received the loan earlier. Hence, if this amount is invested in P_j^2, then B_j obtains b and the bank obtains nothing. If its invested in P_j^1, then B_j obtains $H - r$ and the bank obtains r.

Group-lending with sequential financing

Solution of the game:

- **Stage 3.** Both types of borrowers would invest in their second projects.
- **Stage 2.** Given that borrowers of both types default in stage 3, in stage 2, S type borrowers will invest in their first projects (Assumption 2) and N type borrowers will invest in their second projects (Assumption 1). Hence $v_{SS} = \frac{H-r+b}{2}$, $v_{SN} = \frac{H-r}{2}$, $v_{NN} = \frac{b}{2}$ and $v_{NS} = b$
- **Stage 1.** The expected per period payoff of the bank is $\theta r - 1 - \theta$.

The investment decision of a borrower does not depend on the nature of the group, but only on whether the borrower is the first recipient of the loan or not.

Period 0. Group-formation would lead to negative assortative matching.

Proposition 2

Sequential financing is feasible if and only if $\theta r - 1 - \theta \geq 0$
Under sequential financing, default by the first recipient of the group-loan adversely affects her partner (who does not obtain any loan). Hence, for type S borrowers, the social capital is brought into play, so that they invest in their first projects. Thus, the moral hazard problem is resolved partially and group-lending may be feasible.

Contingent renewal without sequential financing

Consider a game where the selection of the recipient group is history dependent, but in any round, all members of the recipient group receive loans simultaneously.

Period 0. The borrowers endogenously form groups of size two. For every $t \geq 1$, there is a two-stage game with the following sequence of actions.

- **Stage 1.** At $t = 1$, the bank lends some randomly selected group 2 dollars. Next consider $t > 1$. In case the recipient group at $t - 1$ had repaid its loans, at t the bank makes a repeat loan to this group. In case the recipient group had defaulted at $t - 1$, no member of this group ever obtains a loan, either at t or in the future. In that case, the bank lends 2 dollars to some randomly selected group (among those who had not defaulted earlier). Thus, there is contingent renewal.
- **Stage 2.** The borrowers simultaneously make their project choice.

Proposition 3

- **If $\delta \geq \frac{b-H+r}{b}$, then equilibrium involves borrowers of both types investing in their first projects at every period they obtain the loan.**
- **If $\delta < \frac{b-H+r}{b}$, then the equilibrium involves all the borrowers investing in their second projects at every period they obtain the loan.**

Proposition 3

Consider $\delta \geq \frac{b-H+r}{b}$. It is clear that, even under individual lending, contingent renewal would lead a borrower to invest in her first project whenever she obtains the loan. The same result goes through under group-lending also, since, for an S type borrower, the incentive to invest in her first project is higher (because of social capital), whereas for an N type borrower the incentives are the same.

Proposition 3

Next consider $\delta < \frac{b-H+r}{b}$. Under individual lending with contingent renewal, any borrower would invest in her second project whenever she gets the loan.

Let us consider group-lending. Why does not the presence of social capital upset this result?

Suppose the loan goes to the group B_iB_j, where B_j is of type S. Let the borrowers coordinate on the outcome where both invest in their second projects. Given that B_i is investing in her second project, she will not obtain any more loans in the future anyway. Hence, her payoff is not adversely affected even if B_j defaults, so that such default does not attract the social penalty. Given that $\delta > \frac{b-H+r}{b}$, this strategy payoff dominates any other subgame perfect equilibria. Consequently, the borrowers coordinate on this outcome.

$v_{SS} = v_{SN} = v_{NN} = v_{NS} = \frac{H-r}{1-\delta}$, if $\delta \geq \frac{b-H+r}{b}$,
$v_{SS} = v_{SN} = v_{NN} = v_{NS} = b$, otherwise.

Contingent renewal without sequential financing

In case $\delta \geq \frac{b-H+r}{b}$, the borrowers always invest in their first projects and the bank has a per period payoff of $2(r-1) > 0$. If, however, $\delta < \frac{b-H+r}{b}$, then the borrowers always invest in their second projects, so that the bank makes a loss.

Proposition 4

Group-lending with contingent renewal, but without sequential financing is feasible if and only if $\delta \geq \frac{b-H+r}{b}$.
For $\delta < \frac{b-H+r}{b}$, however, all the borrowers invest in their second projects, so that contingent renewal fails to resolve the moral hazard problem. The presence of social capital does not affect the performance of contingent renewal schemes.

Contingent renewal with sequential financing

Consider the following game.

In period 0, the borrowers endogenously form groups of size two.

For every $t \geq 1$, there is a three-stage game with the following sequence of actions.

- **Stage 1.** At $t = 1$, the bank lends some randomly selected group 1 dollars. Consider $t > 1$. In case the recipient group at $t - 1$ had repaid its loans, the bank gives the group 1 dollar in this period. In case the recipient group at $t - 1$ had defaulted, no member of this group ever obtains a loan in this period or in the future.
- **Stage 2.** One of the borrowers is randomly selected (with probability half) as the recipient of the 1 dollar lent by the bank. This borrower, say B_i, then decides whether to invest the 1 dollar in P_i^1 or P_i^2. If B_i invests in P_i^2, then, depending on her type, B_i obtains either b or $b - s$, and the bank obtains nothing. In that case, there is no further loan in this period and the game moves to the next period. If B_i invests in P_i^1, then the bank is repaid r, B_i obtains $H - r$ and the game goes to the next stage.

- **Stage 3.** The bank lends a further 1 dollar to the group, which is allocated to the other borrower, B_j, who decides whether to invest it in P_j^1 or P_j^2.. If she invests in P_j^2, then, depending on her type, B_j obtains either b, or $b - s$, and the bank obtains nothing. If she invests in P_j^1, then the bank is repaid r and B_j obtains $H - r$.

Proposition 5

- **If $\delta \geq \frac{b-H+r}{b}$, then equilibrium involves borrowers of both types investing in their first projects at every period they obtain the loan.**
- **If $\delta < \frac{b-H+r}{b}$, then the equilibrium involves the S type borrowers investing in their first projects, and the N type borrower investing in their second projects at every stage when they obtain the loan..**

Proposition 5 Vs. Proposition 3

Critically, in this case, the S type borrowers invest in their first projects even if $\delta < \frac{b-H+r}{b}$. Thus, for the S types, the incentive to invest in their first projects is greater compared to the case where there is contingent renewal, but no sequential financing. This is because in this case default by an S type borrower adversely affects her partner (which it does not under contingent renewal alone if her partner is also defaulting). In case the S type borrower is the first recipient, her partner receives no loan in this period, as well as in the future. Whereas if she is the second recipient, her partner obtains no loan in the future. Hence, any default by an S type borrower attracts the social penalty.

Proposition 5 Vs. Proposition 2

In case there is sequential financing alone, an S type invests in her first project if she is the first recipient, but not otherwise. Thus, the incentive to invest in the first projects is higher in case both the schemes are used in conjunction.

Proposition 5

$v_{SS} = v_{SN} = v_{NN} = v_{NS} = \frac{H-r}{1-\delta}$, if $\delta \geq \frac{b-H+r}{b}$,
$v_{SS} = \frac{H-r}{1-\delta}$, $v_{SN} = \frac{H-r}{2}$, $v_{NN} = \frac{b}{2}$ and $v_{NS} = b$, otherwise.
There is going to be positive assortative matching if and only if $\frac{b-H+r}{b+H-r} < \delta < \frac{b-H+r}{b}$.

Bank's expected payoff

In case $\delta \geq \frac{b-H+r}{b}$, the borrowers always invest in their first projects and the bank has a per period payoff of $2(r-1) > 0$.
If $\frac{b-H+r}{b+H-r} < \delta < \frac{b-H+r}{b}$, then there will be positive assortative matching and the expected payoff of the bank is

$$\frac{2\theta(r-1) - (1-\delta)(1-\theta)}{(1-\delta)[1-\delta(1-\theta)]}$$

If $\delta \leq \frac{b-H+r}{b+H-r}$, then there there is negative assortative matching. Thus, the expected payoff of the bank is

$$\frac{2(2\theta-1)(r-1) + (1-\delta)(1-\theta)(r-3)}{(1-\delta)[1-2\delta(1-\theta)]}$$

, $\forall \theta \geq \frac{1}{2}$

Bank's expected payoff

$$\frac{\theta r - \theta - 1}{1 - \delta}$$

, otherwise

Proposition 6

1. There is positive assortative matching if and only if $\frac{b-H+r}{b+H-r} < \delta < \frac{b-H+r}{b}$.
2. If $\delta \geq \frac{b-H+r}{b}$, then group-lending with both sequential financing and contingent renewal is feasible. For $\delta < \frac{b-H+r}{b}$, group-lending is feasible if and only if
   - $\frac{b-H+r}{b+H-r} < \delta < \frac{b-H+r}{b}$ and $2\theta(r-1) - (1-\delta)(1-\theta) \geq 0$, or
   - $\delta \leq \frac{b-H+r}{b+H-r}$, $\theta \geq \frac{1}{2}$ and $2(2\theta-1)(r-1) + (1-\delta)(1-\theta)(r-3) \geq 0$, or
   - $\delta \leq \frac{b-H+r}{b+H-r}$, $\theta < \frac{1}{2}$ and $\theta r - \theta - 1 \geq 0$

Proposition 6

1: The intuition is as follows. For $\delta < \frac{b-H+r}{b}$, the lending policy ensures that S type borrowers invest in their first projects, whereas N type borrowers invest in their second projects. If, in addition, $\delta > \frac{b-H+r}{b+H-r}$, then contingent renewal ensures that SS type groups are very profitable, leading to positive assortative matching. Thus, in case an NN type group obtains the loan, the first recipient will default and the other N type borrower will not get a loan at all. Thus, sequential financing acts as a partial screening mechanism whereby the identity of the good and bad groups can be ascertained relatively cheaply.
In the presence of sequential financing, contingent renewal has a dual role. Not only does it promote positive assortative matching, it also increases the incentive to invest in the first projects. This is interesting since, for $\delta < \frac{b-H+r}{b}$, contingent renewal by itself fails to solve the moral hazard problem.

Proposition 6

For $\delta \leq \frac{b-H+r}{b+H-r}$, given that the discount factor is small, SS type groups are not very attractive, so that the outcome involves negative assortative matching. Further, given that, in this case the partial screening effect does not operate, the expected payoff of the bank is lower compared to what it would have been under positive assortative matching.

$\delta \leq \frac{b-H+r}{b+H-r}$: Let us consider sequential financing by itself. For $\theta < \frac{1}{2}$, the banks payoff in this case is the same as that when sequential financing and contingent renewal are used together. For $\theta \geq \frac{1}{2}$, however, a combination of sequential financing and contingent renewal payoff dominates sequential financing by itself. This is because, for $\theta \geq \frac{1}{2}$, there will be some SS type groups even under negative assortative matching. Since the S type borrowers have a greater incentive to invest whenever sequential financing and contingent renewal are used in conjunction.

A non-anonymous social penalty function

Social penalty is imposed whenever default by an S type borrower harms other S type borrowers, but not otherwise.

Sequential financing

The previous analysis goes through for SS or NN type groups. However, given that the social penalty is non-anonymous, an S type borrower would behave like an N type, if her partner is an N type. Hence
$v_{SS} = \frac{H-r+b}{2}$, $v_{SN} = \frac{b}{2}$, $v_{NN} = \frac{b}{2}$ and $v_{NS} = \frac{b}{2}$
Thus, there is positive assortative matching. The expected per period payoff of the bank, however, is the same as that under the anonymous social penalty function, i.e. $\theta r - \theta - 1$. In this case group-lending would not have been feasible without positive assortative matching.

Contingent lending

For Propositions 3 and 4, the argument does not depend on the presence, and thus on the nature, of the social penalty. Thus, they go through in this case also.

Sequential financing with contingent lending

For $\delta \geq \frac{b-H+r}{b}$, the argument is not affected. For $\delta < \frac{b-H+r}{b}$ also goes through whenever the borrowers are members of SS or NN type groups. However, given the social penalty function, an S type borrower would behave as an N type if she has an N type partner.
$v_{SS} = \frac{H-r}{1-\delta}$, $v_{SN} = \frac{b}{2}$, $v_{NN} = \frac{b}{2}$ and $v_{NS} = \frac{b}{2}$
Hence, there is positive assortative matching if and only if $\frac{b-H+r}{b} > \delta > \frac{b-2H+2r}{b}$, with the expected payoff of the bank being given by

$$\frac{2\theta(r-1) - (1-\delta)(1-\theta)}{(1-\delta)[1-\delta(1-\theta)]}$$

.

Proposition 7

Suppose that $\delta \leq \frac{b-2H+2r}{b}$ and the social penalty function is non-anonymous. In case there is both sequential financing and contingent lending, the outcome involves negative assortative matching and, for $\theta \leq \frac{1}{2}$, group-lending is not feasible. Whereas, if there is sequential financing alone, then there is positive assortative matching and, moreover, group lending is feasible whenever $\theta r - \theta - 1 \geq 0$.

Proposition 7

When both the schemes are used in conjunction, S type borrowers invest in their second projects whenever they have N type partners (since the social capital is non-anonymous). Hence, for $\theta \leq \frac{1}{2}$, lending is not feasible. Whereas if there is sequential financing alone, then positive assortative matching implies that lending is feasible whenever $\theta r - \theta - 1 \geq 0$. Proposition 7 suggests that schemes involving contingent renewal needs to be used with care, especially if the discount factor is small.

Conclusion

- This paper focused on some dynamic aspects of Group lending namelysequential financing and contingent renewal.
- It is shown that, under the appropriate parameter configurations, there is positive assortative matching, so that the bank can test whether a group is good or bad relatively cheaply, i.e. without lending to all its members, thus leading to a partial screening out of bad borrowers.
- Contingent renewal by itself may lead to collusion, thus failing to harness the social capital. Hence, it can resolve the moral hazard problem if and only if the discount factor is relatively large.
- In case the social penalty is non-anonymous and the discount factor is relatively small, sequential financing by itself may be feasible, whereas a combination of sequential financing and contingent renewal may not be.

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