

# The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing

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- This paper talks about the course of the wealth distribution and the consequent steady states in the presence of credit rationing.
- In Solow's model of capital accumulation, the equilibrium interest rate is determined by the marginal product of capital which is common across all agents.
- A consequence of this is the irrelevance of the wealth distribution: long-run capital stock, output and interest rates are all uniquely determined by savings behaviour independently of the initial wealth distribution.
- When credit markets are less than perfect, the situation may change: frictions in the credit market may lead to credit-rationing and upset the simple relationship between marginal product of capital and interest rates.

- It becomes possible that both high and low interest rates are self-sustaining. Higher interest rates induce a higher steady-state fraction of credit-constrained individuals, and therefore lower long-run capital accumulation.
- To each stationary interest rate, there is associated a unique stationary wealth distribution.
- Each of these stationary distributions is shown to be ergodic, so a poor dynasty has a positive probability of becoming rich in finite time and vice-versa: there is no inescapable poverty trap.

- However, the degree of wealth and income mobility do vary across steady states. Both upward and downward mobility are greater when the interest rate is lower.
- Steady states can also be ranked in terms of aggregate output: higher steady-state interest rates are associated with lower output and capital stock, because they involve a higher fraction of credit constrained individuals who invest and accumulate at inefficiently low levels.
- There exists a possible role for policy. A one-off lump sum manipulation of the wealth distribution or the interest rate might lead the economy to a different steady-state than would otherwise occur.

# The Model

- We consider a closed economy with an infinite, discrete time horizon  $t=0,1,2,\dots$  and a stationary population of infinitely-lived dynasties  $I=[0,1]$ .
- There are two goods, one labour good and one physical good that can serve both as a consumption good and a capital good.
- At each period  $t$  the state of the economy is described by the current distribution of wealth, represented by a distribution function  $G_t(w)$  where  $G_t(w)$  is the fraction of the population with current wealth below  $w$ .
- Aggregate wealth (also the average wealth)  $W_t$  is given by:

$$W_t = \int w dG_t(w)$$

# The Model

- At each period  $t$ , every dynasty  $i$  is endowed with one indivisible labour unit and an initial wealth  $w_{it}$ , and earns income by supplying labour and capital.
- The resulting income  $y_{it}$  is divided at the end of the period between consumption  $c_{it}$  and savings  $b_{it}$ , which constitutes the dynasty's initial wealth next period (i.e.  $w_{it+1} = b_{it}$ ).
- Agents are assumed to be risk neutral: they maximize total expected income minus the disutility of labour i.e.  $U=y-e$ , where  $e = 0$  or  $1$  is labour supply (effort).

# The Model

- In the same tradition as that of Solow, we assume that a fixed fraction  $s$  of total income is being saved ( $b_{it} = sy_{it}$ )
- One can think of each dynasty as maximising Cobb Douglas preferences defined directly over consumption and bequest.
- Each generation is maximizing

$$U = zc^{1-s}b^s - e$$

where  $z = (1 - s)^{s-1}s^{-s}$ . So that indirect utility for income is simply  $U = ye$ , and  $c = (1-s)y$ ,  $b = sy$ .

- We also assume that wealth can be stored costlessly, but that capital investments are sunk costs (i.e. a 100 per cent depreciation rate).

- The technology  $F(K,L)$  exhibits constant-returns-to-scale with respect to aggregate capital and labour inputs  $K$  and  $L$ .
- We study production at the individual level, viewing each agent as a prospective entrepreneur; the production function can be written  $f^*(k)=F(K/L,1)$  (with  $k = K/L$ ).
- The only difference with the usual neo-classical production is that we allow it to be stochastic at the individual level:  $f^*(k)$  can take different values depending on purely idiosyncratic shocks.



$$f^*(k) = \begin{cases} f(k) & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$$

if individual effort  $e=1$  and

$$f^*(k) = \begin{cases} f(k) & \text{with probability } q \\ 0 & \text{with probability } 1-q \end{cases}$$

if individual effort  $e=0$

- We assume  $0 < q < p < 1$  and
- Standard properties for  $f(k)$ :

$$f(0)=0, f' > 0, f'' < 0, f'(0) = \infty, f'(\infty) = 0$$

# Dynamics with First Best Credit

- In this section, we consider the case of first-best credit, which means that there is no moral hazard problem, ie, that lenders can make sure at no cost that borrowers don't shirk and do supply their unit of effort once the loan has been made .
- So in this section, all individuals always supply a high effort  $e=1$ , provided that this is indeed the first-best optimum.
- For any  $r \geq 0$ , we denote  $k(r)$  and  $y(r)$  (resp.  $k_0(r)$  and  $y_0(r)$ ) as the profit-maximising capital input and the corresponding profit when the interest rate is  $r$  and the entrepreneur takes effort  $e=1$  (resp.  $e=0$ ).

$$\forall r \geq 0, pf'(k(r)) = 1 + r; y(r) = pf(k(r)) - (1 + r)k(r) \quad (1)$$

$$\forall r \geq 0, qf'(k_0(r)) = 1 + r; y_0(r) = qf(k_0(r)) - (1 + r)k_0(r) \quad (2)$$

- We then assume (A0) that, at least when the interest rate  $r=0$  it is first-best efficient to supply high effort and to make the corresponding high investment:

$$y(0) - 1 > y_0(0)$$

- This ensures that high effort ( $e=1$ ) is first-best efficient as long as the interest rate  $r$  is lower than some value  $r^*(q) > 0$  (i.e.  $y(r) - 1 > y_0(r)$  for  $r < r^*(q)$ ).
- To make sure that this will always be the case in the (long run of the) first-best economy, we then have to assume that the saving rate is high enough (Proposition 1).

# Dynamics with First Best Credit

- The essential implication of first-best credit is that the allocation of productive capital between agents and therefore the equilibrium interest rate are independent from the current dispersion of wealth levels.
- In the absence of borrowing constraints, everybody will make the optimum investment  $k(r)$  such that the current (gross) interest rate  $1+r$ , equals the (expected) marginal product of capital  $pf'(k(r))$  (so as to maximise expected income  $pf(k)-(1+r)k$ , irrespective of one's initial wealth  $w$  .
- Rich agents will lend capital to poor agents so as to equalize the marginal product of capital throughout the economy, over all production units.

# Dynamics with First Best Credit

- Thus aggregate capital demand is  $k(r)$  and since aggregate capital supply is equal to the average wealth  $W_t$ , the equilibrium interest rate at period  $t$ ,  $r_t$ , is given by:

$$k(r_t) = W_t$$

$$\text{i.e. } 1 + r_t = pf'(W_t).$$

- Thus, whatever the current wealth distribution  $G_t(w)$ , every agent will invest the average wealth  $W_t$ , so that individual (expected) income  $y_{it}(w_{it})$  as a function of initial wealth  $w_{it}$  is given by

$$y_{it}(w_{it}) = pf(W_t) - (1 + r_t)W_t + (1 + r_t)w_{it} \quad (3)$$

and aggregate income  $Y_t(G_t)$  is given by:

$$Y_t(G_t) = pf(W_t) \quad (4)$$

# Dynamics with First Best Credit

- Therefore with first-best credit, aggregate output depends only on aggregate wealth. This implies that we can track down the evolution of aggregate wealth and aggregate output without worrying about the way wealth and output are distributed: aggregate wealth at period  $t+1$ ,  $W_{t+1}$  is given by

$$W_{t+1} = sY_t = spf(W_t) \quad (5)$$

- The concavity of  $f$  together with equation (5) then implies that aggregate wealth  $W_t$ , will converge to a unique long-run aggregate wealth  $W_\infty^*$ , irrespective of initial aggregate wealth  $W_0$ . (and in particular irrespective of  $G_0(w)$ ),  $W_\infty^*$ , is given by

$$W_\infty^* = spf(W_\infty^*) \quad (6)$$

- This implies that the equilibrium interest rate,  $r_t$ , will converge globally to a unique long run interest rate rate  $r_\infty^*$  such that

$$1 + r_\infty^* = pf'(W_\infty^*)$$

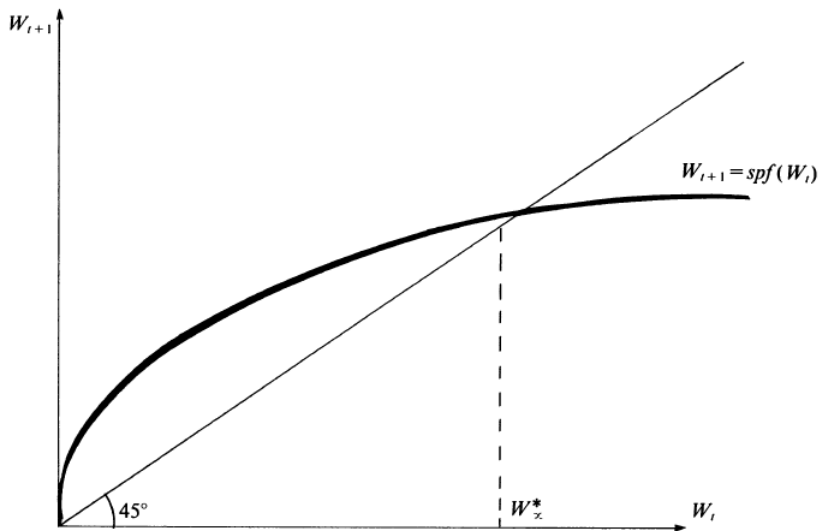


FIGURE 1  
Aggregate dynamics with first-best credit

- What about the long run wealth distribution:  $G_\infty(w)$  ?
- If individual income was deterministic (say, if  $p = 1$ ), all dynasties would converge to the average wealth level  $W_\infty^*$ .
- Since we assumed idiosyncratic shocks on individual investments, there will be some positive inequality in the long-run, but this inequality will be independent of initial inequality  $G_0(w)$ . This is so because  $r_\infty^*$  does not depend on initial inequality, and because for any given interest rate  $r$ , the wealth process follows a linear Markov process that converges globally toward a unique invariant distribution.



- One can see that by looking at the transitional equations:

$$w_{it+1}(w_{it}) = \begin{cases} s[f(k(r)) + (1+r)(w - k(r))/p] & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$$

- Because of risk neutrality, all agents are actually indifferent between all divisions of their total expected income between the lucky and the unlucky states of nature.
- These particular transition functions are of no consequence for the dynamics with first best credit. The uniqueness of the long run interest rate and distribution would hold with any transition function belonging to the agents' indifference curves.

- The concavity of the individual transition functions given by the above equation implies that there can be no trap. i.e. that one can communicate between (any neighbourhood of) any two possible long-run wealth levels with positive probability in a finite time.

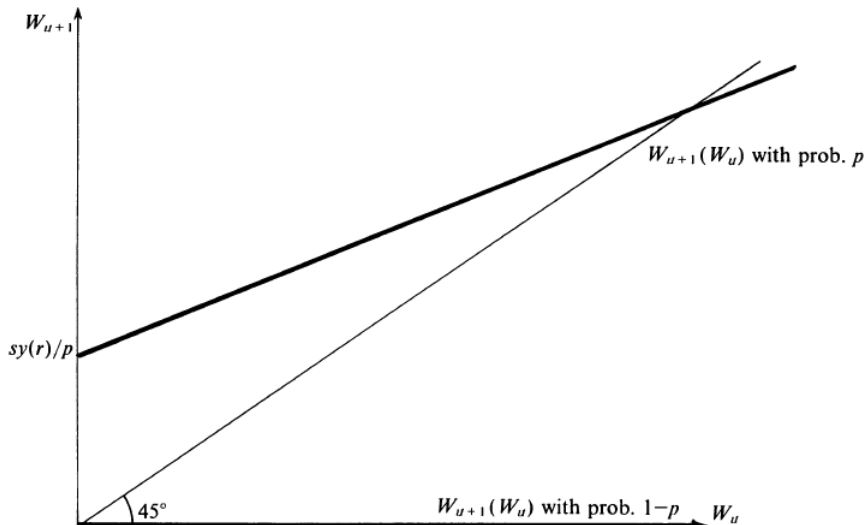


FIGURE 2

Individual transitions with first-best credit

- Thus the wealth process is globally ergodic and the distribution  $G_t(w)$  converges to the unique invariant distribution  $G_\infty^*$  associated with the interest rate  $r_\infty^*$ . We summarize these properties with the following proposition:

## Proposition 1

(A0) implies that there exists  $s_0 = s_0(q)$  such that if  $s > s_0$ , there exist unique levels of long-run aggregate wealth  $W_\infty^*$ , aggregate output  $Y_\infty^*$ , the interest rate  $r_\infty^*$  and inequality  $G_\infty^*$  toward which  $W_t, Y_t, r_t$  and  $G_t(w)$  converge as  $t$  goes to  $\infty$ , irrespective of the initial wealth distribution  $G_0(w)$ .

# Credit Rationing

- We choose to model credit-rationing as arising from a moral-hazard problem.
- Now we assume that individual labour supply ( $e=0$  or  $1$ ) is no longer observable, so that lenders must check beforehand whether borrowers have adequate incentives to supply their unit of effort.
- Assume that the current interest rate is  $r > 0$ , and consider an agent whose initial wealth  $w$  is below the optimum investment  $k(r)$  associated to  $r$ .
- Assume also that  $r < r^*(q)$  so that it is indeed first-best optimal to supply high effort ( $e=1$ ) and to make the high investment  $k(r)$ . (If  $r > r^*(q)$ , the all prefer to make the low investment  $k_0(r)$  and to supply low effort ( $e=0$ ) and everyone can obtain sufficient credit, since borrowers cannot reduce their effort further and indulge in moral hazard).

# Credit Rationing

- Since lenders cannot directly observe the agents effort supply, they can provide proper incentives only by offering a financial contract specifying repayments  $(d_f, d_s)$  depending on whether the project fails (output =0) or succeeds (output = $f(k)$ ), in exchange for investing  $k(r)-w$ .
- We assume perfect competition between lenders, so that whenever a contract yielding non-negative expected profits does exist it will be offered, and only zero- profit contracts will be traded in equilibrium.
- Since we assumed investment to be sunk costs, repayment  $d_f$ , has to be 0 when the investment fails, while  $d_s$ , will have to be whatever it takes to cover interest payments in expected terms:

$$d_f = 0$$

$$pd_s + (1 - p)d_f = (1 + r)(k(r) - w)$$

This implies:

$$d_s = (1 + r)(k(r) - w)/p$$

- But incentives to take high effort are now distorted, and ex post (after the contract is signed) the borrower will take high effort if and only if :

$$p[f(k(r)) - d_s] - 1 > q[f(k(r)) - d_s] \quad (7)$$

This implies

$$(p - q)[f(k(r)) - d_s] > 1$$

- The incentive-compatibility equation shows that the more the agent has to borrow (the higher  $k(r) - w$ ), the less the agent benefits from a high probability of success, and the higher the incentive to shirk.
- If the incentive-compatibility condition is not satisfied (i.e. if  $k(r) - w$  is too high), then lenders will anticipate that the agent will shirk and therefore will not invest  $k(r) - w$ : the agent is credit-rationed and cannot make the optimal investment  $k(r)$ .

- Substituting the values of  $d_f$  and  $d_s$ , (7) becomes:

$$(1 + r)(k(r) - w) < [1 + (p - q)f(k(r))]/[r(p - q)/p]$$

or

$$w > k(r) - [pf(k(r)) - p/(p - q)]/(1 + r) \equiv w(r) \quad (8)$$

- If  $w < w(r)$  the incentive-compatibility equation (7) cannot be satisfied for any investment level  $k$ , not even if  $k$  is lower than the first-best investment  $k(r)$ .
- This is because, income net of repayment in case the project succeeds, is maximal for the optimal investment  $k(r)$ , so that incentives to take high effort are lower for any suboptimal investment level.



- It follows that if an agent cannot obtain the required credit for the first-best optimal investment then, the only other option is to make the low investment  $k_0(r)$  and to supply minimal effort  $e = 0$ . As was noted above, agents can always obtain sufficient credit for this low investment.

# Credit Rationing

- The extent to which credit rationing is binding depends however on the current interest rate  $r$ .
- If we assume that  $w(0) < 0$  i.e.

$$pf(k(0)) - k(0) > p/(p - q)$$

which will hold if  $q$  is sufficiently small (say  $q < q_0$ ), then  $w(r) < 0$  for  $r$  sufficiently small (say  $r < r(q)$ ). This means that credit rationing disappears if the interest rate is sufficiently low because the net returns become sufficiently high to give proper incentives to agents with no collateral.

- As  $r$  increases,  $w(r)$  increases, and for any  $q > 0$ , there exists  $r(q) < r^*(q)$  such that if the interest rate  $r$  is above  $r(q)$  then  $w(r)$  is positive. i.e. credit rationing becomes binding for those agents whose initial wealth is below some positive cutoff level  $w(r)$ .

# Credit Rationing

We summarise these static properties of credit rationing in the following proposition:

## Proposition 2

$A(0)$  implies that there exists  $q_0 > 0$ ,  $q_0 < p$ , such that for any  $q$  such that  $0 < q < q_0$ , there exists  $r(q) \in (0, r^*(q))$  such that:

- If  $r \leq r(q)$ , there is no credit rationing:  $\forall w_i$ , dynasty  $i$  can obtain sufficient credit to make the first best investment  $k(r)$ .
- If  $r(q) < r < r^*(q)$ , there is some credit rationing:  $\exists w(r) > 0$  such that if  $w_i < w(r)$ , dynasty  $i$  is credit rationed and can only make the low investment  $k_0(r)$ ; if  $w_i \geq w(r)$ , dynasty  $i$  can obtain sufficient credit to make the optimal investment  $k(r)$ . Moreover,  $w'(r) > 0$  and  $w(r) \rightarrow 0+$  as  $r \rightarrow r(q)+$ .
- If  $r > r^*(q)$ , then everybody prefers to make the low investment  $k_0(r)$  and can obtain sufficient credit to do so.

# Credit Rationing

- In the short-run higher interest rates are always bad for net borrowers and good for net lenders, both with first-best credit and credit rationing.
- With first-best credit however, the aggregate effect depends only on the aggregate credit position: the GNP of an open economy that is a net lender at the current world rate  $r_t$ , ( $W_t > k(r_t)$ ) would rise following a positive interest rate shock ( $dY_t = (W_t - k(r_t))dr_t$ ).
- With credit-rationing, this may not be so: the GNP of the same economy can fall since a higher rate brings the quality of credit allocation further away from the first best:  
 $(dY_t = (W_t - k(r_t))dr_t - G'_t(w(r_t))w'(r_t)(y(r_t) - y_0(r_t))dr_t$ , where the second term measures the output drop due to the increased fraction of credit-constrained agents; the aggregate effect can therefore be negative even if the country is a net lender.
- These are however static effects, not taking into account the dynamic effects on capital accumulation and future equilibrium interest rates.

# Dynamics With Credit Rationing

- At each period  $t$ , given an initial distribution  $G_t(w)$ , the equilibrium interest rate  $r_t = r(G_t)$  is given by the equality of capital demand and capital supply, where capital demand is possibly constrained by credit rationing.
- If aggregate wealth at period  $t$ ,  $W_t$  is sufficiently high, then the equilibrium interest rate  $r_t$ , will be lower than  $r(q)$  and nobody will be credit-constrained in equilibrium and the equilibrium interest rate depends only on the aggregate wealth  $W_t$ :

$$pf'(W_t) < 1 + r(q) \rightarrow 1 + r(G_t) = pf'(W_t) \quad (9)$$

- However, if  $W_t$  is lower so that  $pf'(W_t) > 1 + r(q)$  then there has to be some credit rationing in equilibrium, and the (unique) equilibrium interest rate  $r_t = r(G_t)$  is determined by:

$$pf'(W_t) > 1 + r(q) \rightarrow r_t = r(G_t) : G_t(w(r_t))k_0(r_t) + (1 - G_t(w(r_t)))k(r_t) \quad (10)$$

# Dynamics With Credit Rationing

- In that case the equilibrium interest rate is no longer determined by the marginal product of capital, simply because, the latter varies across production units, depending on whether they are credit-constrained or not.
- The entire distribution of wealth now matters and not only aggregate wealth. This makes the dynamics of the wealth distribution and the interest rate substantially more complicated than in the no-credit-rationing case.
- If  $W_t$  is so low that  $qf'(W_t) > 1 + r^*(q)$ , then everybody turns to the low-effort investment  $k_0(r_t)$ , the equilibrium interest rate is given by  $1 + r_t = qf'(W_t)$  and there is no credit rationing and interest rate is equated to expected marginal product of capital. So this case is less interesting.

# Dynamics With Credit Rationing

- Given the equilibrium interest rate at time  $r_t = r(G_t)$ , individual transitions  $w_{it+1}(w_{it})$  are the same as in the first-best world for the fraction of the population which is not credit-constrained at time  $t$  (i.e. those dynasties with  $w_{it} > w(r_t)$ ). The new individual transitions for those dynasties which are credit-constrained ( $w_{it} < w(r_t)$ ) are given by:

$$w_{it+1}(w_{it}) = \begin{cases} s[f(k_0(r)) + (1+r)(w - k_0(r))/q] & \text{with probability } q \\ 0 & \text{with probability } 1-q \end{cases}$$

# Dynamics With Credit Rationing

We represent these transition functions on Figure 3 for an equilibrium interest rate  $r_t$  for which there is some credit-rationing ( $r(q) < r_t < r^*(q)$ ).

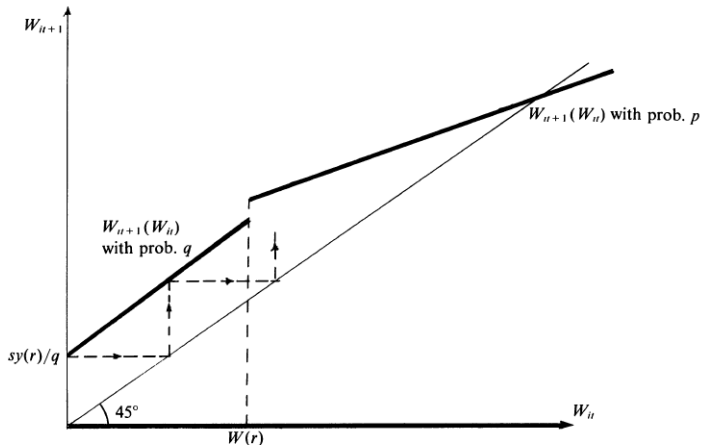


FIGURE 3



# Dynamics With Credit Rationing

- We have a non-linear aggregate transition function  $G_{t+1}(G_t)$ .
- If the economy starts with some initial distribution of wealth  $G_0$ , this defines an infinite sequence of wealth distributions and equilibrium interest rates  $(G_t, r_t)_{t \geq 0}$ .
- We are interested in the long-run steady-states of this dynamic system i.e. in the set of  $(G_\infty, r_\infty)$  such that  $G_{t+1}(G_\infty) = G_\infty$ , and  $r_\infty = r(G_\infty)$ .
- In the same way as in the first-best case, for any possible long-run interest rate  $r_\infty$ , individual transition functions define a linear, globally ergodic Markov process converging toward a unique stationary distribution  $G_{r_\infty}(w)$ .
- It follows that an interest rate  $r_\infty$  can be self-sustaining iff  $r_\infty$  is equal to the equilibrium interest rate  $r(G_{r_\infty})$  associated to its stationary distribution  $G_{r_\infty}$ .

## Proposition 3

Assume (A0) and  $0 < q < q_0$ . To each possible stationary interest rate  $r_\infty \in [0, r^*(q)]$  corresponds a unique stationary, ergodic distribution  $G_{r_\infty}(w)$ . Then  $r_\infty$  is a long-run steady-state interest rate of the dynamic system  $(G_{t+1}(G_t), r_t = r(G_t))$  defined above iff  $r_\infty = r(G_{r_\infty})$ .

# Dynamics With Credit Rationing

- If the long-run interest rate  $r_\infty^*$  associated with unconstrained accumulation is sufficiently low that no credit constraint ever appears (i.e.  $r_\infty^* < r(q)$ ), then high aggregate wealth, low equilibrium interest rates and no credit rationing will be self sustaining, so that the no-credit-rationing steady state  $(G_\infty^*, r_\infty^*)$  analyzed in the first best will also be a steady-state of the second-best economy. This will be so if the saving rate  $s$  is high enough.
- Along with this no-credit-rationing steady state and for the same parameter values there can co-exist another possible long-run steady state associated with a higher interest rate  $r_\infty^{**} > r(q) > r_\infty^*$ , and another stationary distribution  $G_\infty^{**}(w)$  with a positive steady-state fraction  $G_\infty^{**}(w(r_\infty^{**}))$  of credit-constrained agents.
- For this to be true, two key conditions must hold.

# Dynamics With Credit Rationing

- **First**, it must be the case that the steady-state fraction of credit-constrained borrowers  $G_r(w(r))$  increases sufficiently as the interest rate  $r$  goes up.
- In general, a higher interest rate has the effect of making both upward mobility and downward mobility less likely: it is more difficult to escape the credit-rationing interval  $[0, w(r)]$  :both the cutoff  $w(r)$  is higher and because credit-constrained agents are net borrowers, and at the same time it is more difficult to fall into this interval because the wealthy have high interest incomes even if their investment project fails).
- The net effect on  $G_r(w(r))$  will be positive if the first effect dominates, i.e. if the credit-constraint effect dominates the interest-income effect.

- Because of risk neutrality and the way we modelled individual transitions, the second effect does not operate: wealthy agents always have a fixed probability  $(1-p)$  of going bankrupt and this does not depend on the current interest rate.
- It follows that  $G_r(w(r))$  increases with  $r$ , because of the credit-constraints effect.

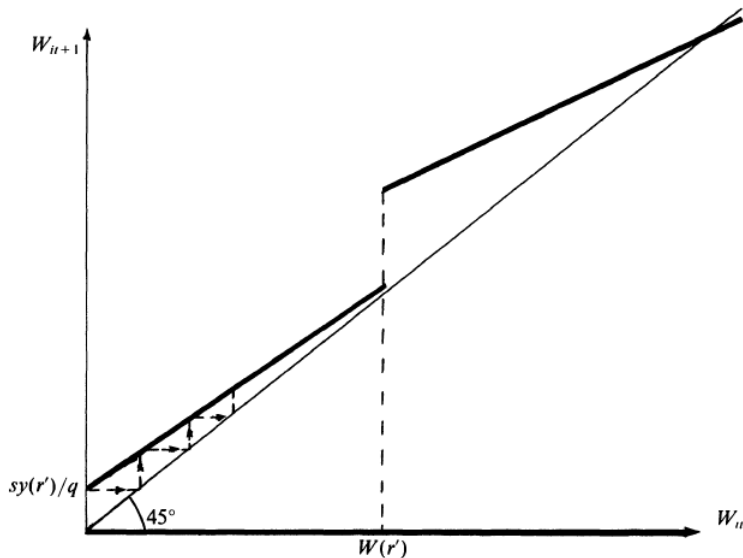


FIGURE 4

Individual transitions with credit-rationing [ $r(q) < r < r^*(q)$ ]

# Dynamics With Credit Rationing

- In Figure 4, we represent the same transitions as in Figure 3 but for a higher rate  $r' > r$ : it takes two consecutive successful investment periods to escape from credit rationing in Figure 3, whereas it takes much more time in Figure 4: implying a higher steady-state fraction of credit-constrained individuals.
- Given the way we modelled credit rationing, this effect will be particularly strong if  $q$  is small :as  $q$  tends to zero credit-constrained dynasties can make only arbitrarily small investments until they reach the threshold  $w(r)$  .  $G_r(w(r))$  goes to 1.
- $q$  measures the outside option of credit constrained people ,providing us with a simple intuitive indicator of the toughness of credit rationing.

# Dynamics With Credit Rationing

- **Second**, it must be the case that the existence of a higher fraction of credit-constrained dynasties tends to push the equilibrium interest rate up.
- This, together with the first condition above, will make high interest rates self-sustaining, provided that the saving rate is not too high (but higher than the minimum saving rate making low interest rates and no credit-rationing self-sustaining).



# Dynamics With Credit Rationing

- Credit-constrained agents supply less capital than other agents. For a given rate  $r$ , they accumulate  $sqf(k_0(r))$ , while others accumulate  $spf(k(r))$ .
- But they also demand less capital:  $k_0(r) < k(r)$ .
- A higher fraction of credit constrained agents will tend to push the equilibrium rate up if the first effect dominates.
- For this, it is sufficient to assume (A1), namely:  $f(k)/kf'(k)$  increases with  $k$ .
- Under these conditions one can prove that there will exist at least one interest rate  $r_{\infty}^{**} > r(q)$  such that  $r_{\infty}^{**}$  and its associated stationary distribution  $G_{\infty}^{**}$  constitute a steady state.

## Proposition 4

Assume (A0) and (A1). Then there exists  $s_1(q), s_2(q)$  such that  $0 < s_0(q) < s_1(q) < s_2(q)$ ,  $q_1$  such that  $0 < q_1 < p$ , such that if  $s_1(q) < s < s_2(q)$  and  $0 < q < q_1$ , there exists at least two steady states  $(r_\infty^*, G_\infty^*)$  and  $(r_\infty^{**}, G_\infty^{**})$  of the dynamic system  $(G_{t+1}(G_t), r_t = r(G_t))$ , with  $r_\infty^* < r(q) < r_\infty^{**}$ .

# Dynamics With Credit Rationing

We can summarize the intuition for this multiplicity in the following way.

- Starting from the high-accumulation, low-interest-rate steady state  $(r_{\infty}^*, G_{\infty}^*)$ , a positive shock on the interest rate can be self-sustaining if it pushes sufficiently many agents in the credit-rationing region for a sufficiently long time, so that capital accumulation is sufficiently depressed to make high interest rates self-sustaining.
- The key conditions for this to happen is that credit rationing has large negative consequences for capital accumulation, which in our model is captured by a very low  $q$ .
- These multiple steady states can always be ranked in aggregate terms: steady states associated to higher interest rates have lower aggregate capital stock and aggregate output. This is simply because under (A1), steady-state multiplicity requires the steady-state fraction  $G_r(w(r))$  of credit-constrained individuals to be increasing with  $r$ .

## Proposition 5

Assume (A0) and (A1). Then if there exist multiple steady states  $(G_{\infty i}(w), r_{\infty i})$  for  $r_{\infty 1} < r_{\infty 2} < \dots < r_{\infty n}$ , then aggregate capital stock  $W_{\infty i}$  and output levels  $Y_{\infty i}$  associated to these steady-states are inversely related to the interest rate:

$$W_{\infty 1} > W_{\infty 2} > \dots > W_{\infty n}$$

$$Y_{\infty 1} > Y_{\infty 2} > \dots > Y_{\infty n}$$

# Dynamics With Credit Rationing

- On the other hand, if the second channel doesn't work i.e. if the net effect of credit-constrained individuals on the interest rate is negative (i.e. if their lower capital demand  $k_0(r) < k(r)$  outweighs their lower capital accumulation  $spf(k_0(r)) < spf(k(r))$ ), which in the particular context of our model means that (A1) does not hold, then opposite phenomena could in general happen.
- The existence of multiple steady-state interest rates would then require the steady-state fraction of credit-constrained agents  $G_r(w(r))$  to be sufficiently decreasing with  $r$  (opposite of channel 1)
- This could arise if credit-constrained agents are net lenders [ $k_0(r) > w(r)$ ] or if the interest income effect is stronger than the credit-constraint effect, so that higher interest rates make credit rationing a more transitory state in individual trajectories.
- One could then obtain high steady state interest rates associated to high output and high wealth.

# Concluding Comments

- There is no room for long-run growth in our constant returns accumulation model.
- However, if long-run growth is positively related to aggregate investment through some economy-wide externality (as in Romer(1986)), then the low-interest-rate, high-wealth-mobility steady state would exhibit faster growth than the high-interest-rate, low-wealth-mobility steady state.

# Concluding Comments

- Since stationary distributions associated to higher rates are typically more unequal (there are more credit-constrained poor and the very rich accumulate more), countries with more unequal wealth distribution would grow less, assuming that national credit markets are imperfectly integrated (so that different countries can be in different steady-states).
- We showed how capital accumulation itself is determined by the pattern of credit allocation of the previous period, and that this interaction between credit constraints and capital accumulation can give rise to multiple equilibrium paths even with a perfectly convex Solow type technology.
- Therefore credit rationing is not only a powerful transmission mechanism , but can also have long-run consequences: policy shocks reducing real rates temporarily from  $r_{\infty}^{**}$  to  $r_{\infty}^*$  can be self-sustaining through the induced effects on accumulation and the distribution of new wealth and credit (and conversely).

# The End