Econ 277B: Economic Development II | Tridip Ray
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Final Exam (07 May 2017)

- *There is one question, with several parts; you have to answer all of them. You have* 3 *hours to write this exam.*

The Set-up

- Consider a credit market consisting of risk-neutral entrepreneurs (for example, farmers, households, or small firms), banks (who provide formal finance), and moneylenders (who provide informal finance). The entrepreneur is endowed with observable wealth $w_E \geq 0$. She has access to a deterministic production function, $Q(I)$, where I is the investment volume. The production function is concave, twice continuously differentiable, and satisfies $Q(0) = 0$ and $Q'(0) = \infty$. In a perfect credit market with interest rate r, the entrepreneur would like to attain first-best investment given by $Q'(I^*(r)) = 1 + r$. However, she lacks sufficient wealth, $w_E < I^*(r)$, and thus turns to the bank and/or the moneylender for the remaining funds. We assume that the entrepreneur accepts the first available contract if indifferent between the contracts offered.

- While banks have an excess supply of funds, credit is limited as the entrepreneur is unable to commit to invest all available resources into her project. Specifically, we assume that she may use (part of) the assets to generate nonverifiable private benefits. Non-diligent behaviour resulting in diversion of funds denotes any activity that is less productive than investment, for example, using available resources for consumption or financial saving. The diversion activity yields benefit $\phi < 1$ for every unit diverted. Creditor vulnerability is captured by ϕ (where a higher ϕ implies weaker legal protection of banks).While investment is unverifiable, the outcome of the entrepreneur's project in terms of output and/or sales revenue may be verified. The entrepreneur thus faces the following trade-off: either she invests and realizes the net benefit of production after repaying the bank (and possibly the moneylender), or she profits directly from diverting the bank funds (the entrepreneur still pays the moneylender if she has taken an informal loan). In the case of partial diversion, any remaining returns are repaid to the bank in full. The bank does not to derive any benefit from resources that are diverted.

- Informal lenders are endowed with observable wealth $w_M \geq 0$ and have a monitoring advantage over banks such that credit granted is fully invested. To keep the model tractable, we restrict informal lenders' occupational choice to lending. For simplicity, monitoring cost is assumed to zero. The moneylender's superior knowledge of local borrowers grants him exclusivity. In the absence of contracting problems between the moneylender and the entrepreneur, the moneylender maximizes the joint surplus derived from the investment project and divides the proceeds using Nash Bargaining. A contract is given by a pair $(B, R) \in \mathbb{R}_+^2$,where B is the amount borrowed by the entrepreneur and R the repayment obligation. Finally, if the moneylender requires additional funding he turns to a bank.

- Following the same logic as above, we assume that the moneylender cannot commit to lend his bank loan and that diversion yields private benefits equivalent of $\phi < 1$ for every unit diverted. While lending is unverifiable, the outcome of the moneylender's operation may be verified. The moneylender thus faces the following trade-off: either he lends the bank credit to the entrepreneur, realizing the net-lending profit after compensating the bank, or he benefits directly from diverting the bank loan.

- Banks have access to unlimited funds at a constant unit cost of zero. They offer a contract (L_i, D_i), where L_i is the loan and D_i the interest payment, with subscripts $i \in \{E, M\}$ indicating entrepreneur (E) and moneylender (M).When ϕ is equal to zero, legal protection of banks is perfect and even a penniless entrepreneur and/or moneylender could raise an amount supporting first-best investment. To make the problem interesting, we assume that

$$\phi > \underline{\phi} \equiv \frac{Q(I^*(0)) - I^*(0)}{I^*(0)}. \qquad \text{(Assumption 1)}$$

  In words, the marginal benefit of diversion yields higher utility than the average rate of return to first-best investment at zero rate of interest [henceforth $I^*(0) = I^*$].

- To distinguish formal from informal finance, we assume that banks are unable to condition their contracts on the moneylender's contract offer. The timing is as follows. (1) Banks offer a contract, (L_i, D_i), to the entrepreneur and the moneylender, respectively.

(2) The moneylender offers a contract, (B, R), to the entrepreneur,where R is settled through Nash Bargaining. (3) The moneylender makes his lending/diversion decision. (4) The entrepreneur makes her investment/diversion decision. (5) Repayments are made.

1. [10 points: $5+5$] **Benchmark: Informal Moneylender in Isolation**

- We begin by analyzing each financial sector in isolation. If the entrepreneur borrows from the informal sector, the moneylender maximizes the surplus of the investment project, $Q\left(w_E+B\right)-B$. The entrepreneur and the moneylender bargain over how to share the project gains using available resources w_E+B. If they disagree, investment fails and each party is left with her/his wealth or potential loan. The assets represent the disagreement point of each respective agent. In case of agreement, the moneylender offers a contract where the equilibrium repayment, using the Nash Bargaining solution, is

$$R\left(B\right)=\arg\max_{\{t\}}\left\{Q\left(w_E+B\right)-t-w_E\right\}^{\alpha}\left\{t-B\right\}^{1-\alpha},$$

where α represents the degree of competition in the informal sector (competition increases if α is high).

(a) Prove that the Nash Bargaining solution is $R^{*}\left(B\right)=\left(1-\alpha\right)\left[Q\left(w_E+B\right)-w_E\right]+\alpha B$.

(b) We assume that α satisfies $\alpha>\tilde{\alpha}$, where $\tilde{\alpha}$ solves $\alpha\left[Q\left(w_E+B\right)-B\right]+\left(1-\alpha\right)w_E=Q\left(w_E\right)$, with $\alpha\in\left(\tilde{\alpha},1\right)$.

  – Prove that this assumption guarantees that the entrepreneur's *participation constraint* is satisfied.

2. [25 points] **Benchmark: Competitive Bank in Isolation**

- Now consider the bank in isolation. There is free entry in the bank market. Following a Bertrand argument, competition drives equilibrium bank profit to zero. Nonetheless, credit is limited since investment of bank funds cannot be ensured.

(a) [4 points]

Suppose the bank offers the entrepreneur a credit limit L_E and repayment amount $D_E=\left(1+r\right)L_E$.

- Express the entrepreneur's net return when she invests $I = w_E + L_E$.
- Argue that the entrepreneur's *incentive compatibility constraint* is given by $Q(I) - (1+r)L_E \geq \phi(w_E + L_E)$.

- As there is no default in equilibrium, the only equilibrium interest rate consistent with zero profit is $r = 0$. With this the contracting problem reduces to choosing L_E to maximize the entrepreneur's net return subject to the incentive compatibility constraint.

(b) [6 points]: Formulate the contracting problem as a constrained optimization problem and write down all the relevant first-order conditions carefully.

- To characterize the solution to the above problem we will use the following two results.
  - **Lemma 1:** $Q'(w_E + L_E) - (1+\phi) < 0$.
  - **Lemma 2:** There exists a unique threshold $w_E^c(\phi) > 0$ such that $Q(w_E + L_E) - L_E - \phi(w_E + L_E) = 0$, and $w_E + L_E = I^*$ for $w_E = w_E^c(\phi)$.

(c) [10 points]: Use the first-order conditions derived above to prove the following proposition.

For all $\phi > \underline{\phi}$ there is a threshold $w_E^c > 0$ such that entrepreneurs with $w_E < w_E^c$ invest $I < I^*$ while entrepreneurs with $w_E \geq w_E^c$ invest I^*.

(d) [5 points]: Demonstrate clearly that for entrepreneurs with $w_E < w_E^c$, credit (L_E) and investment (I) increase in w_E.

3. [65 points] **Formal and Informal Finance**

- Now we consider coexistence of formal bank and informal moneylender. Financial sector coexistence not only allows poor borrowers to raise funds from two sources, but it also permits informal lenders to access banks. This introduces additional trade-offs. On the one hand, (agency-free) informal credit improves the incentives of the entrepreneur as informal finance increases the residual return to the entrepreneur's project, with the end effect equivalent to a boost in internal funds. On the other hand, banks now have to consider the possibility of diversion on part of the entrepreneur and the moneylender.

- Suppose the bank offers credit limits L_E to the entrepreneur and L_M to the moneylender. Also, the moneylender offers a contract (B, R) to the entrepreneur.

(a) [6 points]: Explain clearly that the repayment (to the moneylender) using the Nash Bargaining solution is given by $R^*(B) = (1-\alpha)[Q(w_E + L_E + B) - L_E - w_E] + \alpha B$.

(b) [6 points]: Explain clearly that the *entrepreneur's incentive compatibility constraint* is given by $\alpha [Q(w_E + L_E + w_M + L_M) - L_E - L_M - w_M] + (1-\alpha) w_E \geq \phi(w_E + L_E)$.

(c) [6 points]: Explain clearly that the *informal moneylender's incentive compatibility constraint* is given by $(1-\alpha)[Q(w_E + L_E + w_M + L_M) - L_E - L_M - w_E] + \alpha w_M \geq \phi(w_M + L_M)$.

- So the contracting problem reduces to choosing L_E and L_M to maximize the entrepreneur's net return subject to the incentive compatibility constraints of the entrepreneur and the moneylender.

(d) [12 points]: Formulate the contracting problem as a constrained optimization problem and write down all the relevant first-order conditions carefully.

- To characterize the solution of the contracting problem we proceed as follows.
  - We only consider entrepreneurs with $w_E < w_E^c$ as we have found out from part 2 that entrepreneurs with $w_E \geq w_E^c$ achieve the first-best level of investment I^* by borrowing exclusively from the bank.
  - Also, since $w_E < w_E^c$, the entrepreneur's incentive compatibility constraint is binding.
  - When the moneylender is wealthy enough so that he does not need to borrow from the bank, that is, $L_M = 0$, then competition with the formal bank sector implies that he makes zero profit.

(e) [5 points]: Argue that when $L_M = 0$ the entrepreneur's incentive compatibility constraint becomes $Q(w_E + L_E + w_M) - L_E - w_M = \phi(w_E + L_E)$.

- We will use the following result.
  - **Lemma 3:** There exist unique thresholds $\underline{w}_M^c(\alpha, \phi)$ and $\bar{w}_M^c(\alpha, \phi)$ such that:

(i) $\alpha\left[Q\left(w_E+L_E+w_M+L_M\right)-L_E-L_M-w_M\right]+(1-\alpha)\,w_E=\phi\left(w_E+L_E\right)$,
$(1-\alpha)\left[Q\left(w_E+L_E+w_M+L_M\right)-L_E-L_M-w_E\right]+\alpha w_M=\phi\left(w_M+L_M\right)$,
and $w_E+L_E+w_M+L_M=I^*$ for $w_M=\underline{w}_M^c\left(\alpha,\phi\right)$;

(ii) $Q\left(w_E+L_E+w_M\right)-L_E-w_M=\phi\left(w_E+L_E\right)$, and $w_E+L_E+w_M=I^*$ for $w_M=\bar{w}_M^c\left(\alpha,\phi\right)$;

(iii) $0<\underline{w}_M^c\left(\alpha,\phi\right)<\bar{w}_M^c\left(\alpha,\phi\right)$.

(f) [15 points]: Use the first-order conditions derived above to prove the following proposition.

When $w_E<w_E^c$, entrepreneurs borrow from a bank and a bank-financed moneylender and invest $I<I^*$ if $w_M<\underline{w}_M^c\left(\alpha,\phi\right)$, and invest $I=I^*$ if $w_M\in\left[\underline{w}_M^c\left(\alpha,\phi\right),\bar{w}_M^c\left(\alpha,\phi\right)\right]$.

(g) [5 points]: When $w_E<w_E^c$ and $w_M<\underline{w}_M^c\left(\alpha,\phi\right)$, it can be shown that $\dfrac{\partial L_E}{\partial w_M}>0$, $\dfrac{\partial L_M}{\partial w_M}>0$, $\dfrac{\partial L_E}{\partial w_E}>0$, and $\dfrac{\partial L_M}{\partial w_E}\geq 0$.

- Sketch out briefly the steps involved in the derivations (*you do not need to do the derivations*).

(h) [10 points]: When $w_E<w_E^c$ and $w_M\in\left[\underline{w}_M^c\left(\alpha,\phi\right),\bar{w}_M^c\left(\alpha,\phi\right)\right]$, derive that $\dfrac{\partial L_E}{\partial w_M}=0$, $\dfrac{\partial L_M}{\partial w_M}=-1$, $\dfrac{\partial L_E}{\partial w_E}=\dfrac{1-\phi}{\phi}>0$, and $\dfrac{\partial L_M}{\partial w_E}=-\dfrac{1}{\phi}<0$.