Econ 277A: Economic Development I — Tridip Ray
Semester II, 2023-24 — ISI, Delhi

Final Exam (05 May 2024)

- Answer all the questions. You have 3 hours to write this exam.

1. **[5 marks]**

   Explain in words how *weak* lender protection rights (that is, lenders cannot seize collateral easily in the event of a default) or *weak* enforcement of credit contracts can affect working of the credit markets: interest rates, access to credit, particularly for poor borrowers who have less collateral to post.

- In what follows we develop a model to analyze comprehensively the impact of stronger credit enforcement on working of the credit markets. Consider an economy populated by risk-neutral borrowers, differentiated by (collaterizable) fixed assets W, distributed according to a cumulative distribution function (c.d.f.) G over support $\left[\underline{\Omega}, \overline{\Omega}\right]$. Each borrower seeks to invest in a project of size $I \geq 0$. This requires up-front investments of $\beta \cdot I$, where $0 < \beta < 1$ is constant. The project generates returns of $y \cdot f(I)$, where $y \in \{y_s, y_f\}$ is a borrower-specific productivity shock and f is an increasing, continuously differentiable, S-shaped function with $\frac{f(I)}{I}$ rising until $I = b$ and falling thereafter, for some $b \geq 0$. Hence $f'(I)$ is rising over some initial range $(0, b')$ and falling thereafter, where $b' < b$. We assume that the borrower does not have any liquid wealth to pay for the up-front investments. The probability of success ($y = y_s$) is given and denoted by e. It is useful to introduce

$$\overline{y} \equiv e \cdot y_s + (1-e) \cdot y_f.$$

- **Credit Contracts:**

  A loan contract stipulates the amount borrowed ($\beta \cdot I$), and the amount T_k to be repaid in state $k \in \{s, f\}$. For simplicity, the realization of the state is costlessly verifiable. We assume *contracts are complete* in the sense that the repayment obligation T_k can vary with the state $k \in \{s, f\}$. One can think of the payment T_s as corresponding to the stated interest rate that the borrower is expected to repay in the event of success. In the event of failure (state f), the borrower defaults on the obligation and this is followed by a mutually agreed upon adjustment of the borrower's repayment in accordance with his/her ability to pay. The two parties can anticipate in advance what this adjustment will be.

Each borrower has the option of not honouring the loan agreement ex post. For simplicity, we suppose that the borrower either decides to repay the entire repayment obligation or none of it. Should the borrower default, lenders can take the borrower to court, and thereafter expect to seize a fraction (θ) of ex post assets owned by the borrower. Ex post assets equal $W + \nu \cdot y_k \cdot f(I)$, where $1 - \nu$ is the fraction of the firm's returns diverted by the entrepreneur. We treat ν as a parameter and assume that it is small; in particular, that it satisfies the condition

$$\nu < \frac{\beta}{\overline{y} \cdot \theta \cdot f'(b')}.$$

This limits the extent to which the returns from the project itself can serve as collateral; the borrower's assets remain the primary source of collateral.

The enforcement institution is represented by θ, incorporating delays and/or uncertainties in the legal process. Enforcement is affected by judicial reforms such as debt recovery tribunals. The main focus of the model is thus on the effects of raising θ.

2. [**6 marks:** $1 + 1 + 2 + 2$]

   (a) Explain that if the entrepreneur honours the loan agreement, then in state $k \in \{s, f\}$ he obtains ex post utility

   $$W + y_k \cdot f(I) - T_k.$$

   (b) Note that when the borrower defaults, lenders can take him to court. Assume that $d > 0$ is an additional deadweight loss incurred by the borrower if he defaults (for example, reputation loss or legal costs). Explain that the ex post utility of the borrower in case of a default (in state $k \in \{s, f\}$) is given by

   $$(1 - \theta) \cdot [W + \nu \cdot y_k \cdot f(I)] + (1 - \nu) \cdot y_k \cdot f(I) - d.$$

   (c) Derive, with a clear explanation, the borrower's *incentive compatibility constraint* of honouring the loan agreement in state $k \in \{s, f\}$.

(d) The lender's *participation constraint* ensures her an expected return per rupee loaned equal to the going rate, denoted by π. Derive, with a clear explanation, that the lender's participation constraint is given by

$$e \cdot T_s + (1 - e) \cdot T_f \geq \beta I (1 + \pi).$$

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- **Supply of Loans:**

We consider a 'competitive' supply of loans, represented by an upward sloping supply curve $L_s(\pi)$ of loanable funds. We assume that for there to be some supply of credit, lenders must be assured a return that is at least as large as a non-negative lower bound α, that is,

$$L_s(\pi) = \begin{cases} 0 & \text{if } \pi < \alpha, \\ > 0 & \text{if } \pi \geq \alpha. \end{cases}$$

We assume that

$$\overline{y} \cdot \frac{f(b)}{b} > \beta (1 + \alpha).$$

3. **[2 marks]** Interpret the above assumption.

- **Demand for Loans:**

  - As a benchmark, we start with the *first-best* demand, call it I^F.

4. **[4 marks]** Define the optimization problem the solution of which determines the first-best demand. How does I^F respond to small changes in θ and π? Explain clearly.

  - However, the first-best is not always implementable due to the no-default incentive constraint. The 'demand' for credit is modelled as the solution to an optimal loan contracting problem, where the expected utility of a borrower is maximized subject to his repayment incentive constraint and the lender's participation constraint.

5. **[3 marks]** Formulate this optimal loan contracting problem for a borrower with assets W. (Note that a loan contract stipulates the amount borrowed ($\beta \cdot I$), and the amount T_k to be repaid in state $k \in \{s, f\}$. Since β is constant, effectively the choice variables for the contracting problem are I, T_s and T_f.)

– Given θ and π, for a borrower with assets W, an incentive compatible demand for loans, $I(W, \theta, \pi)$, is the solution to the above problem for I. *Aggregate incentive compatible demand* for loans is then given as $L_d(\theta, \pi) = \int I(W, \theta, \pi)\, dG(W)$.

6. **[8 marks]** Given θ and π, prove that for a borrower with assets W there exists a *project size ceiling* (maximum implementable project size), denoted by $I^H(W, \theta, \pi)$, which solves

$$\beta I \cdot (1 + \pi) - \theta \cdot \nu \overline{y} f(I) = \theta \cdot W + d.$$

   Establish rigorously how $I^H(W, \theta, \pi)$ changes with small changes in W, θ and π. Do the directions of change make intuitive sense? Explain clearly.

7. **[3 marks]** Define *first-best asset threshold*, $W^F(\theta, \pi) \equiv \frac{\beta I^F (1+\pi) - d}{\theta} - \nu \overline{y} f\left(I^F\right)$. Interpret the first-best asset threshold. How does it respond to small changes in θ and π? Explain clearly.

8. **[3 marks]** Define *minimum viable project size*, $I^L(\pi)$, as the *smallest* solution to $\overline{y} \cdot \frac{f(I)}{I} = \beta \cdot (1 + \pi)$. Interpret the minimum project size. How does it respond to small changes in θ and π? Explain clearly.

9. **[5 marks]** Define *minimum viable asset threshold*, $W^L(\theta, \pi)$, which solves $I^H(W, \theta, \pi) = I^L(\pi)$. Interpret the minimum viable asset threshold. How does it respond to small changes in θ and π? Explain clearly.

10. **[6 marks]** Complete (with clear explanations) the following which shows the *incentive compatible demand* for loans for a borrower with assets W:

$$I(W, \theta, \pi) = \begin{cases} ? & \text{if } W < W^L(\theta, \pi), \\ ? & \text{if } W^L(\theta, \pi) \leq W \leq W^F(\theta, \pi), \\ ? & \text{if } W > W^F(\theta, \pi). \end{cases}$$

11. **[8 marks]** *Aggregate incentive compatible demand* for loans is $L_d(\theta, \pi) = \int I(W, \theta, \pi)\, dG(W)$. Given θ, explain clearly how L_d varies with the rate of return (π). Illustrate the relationship between L_d and π in a diagram by plotting L_d on x-axis and π on y-axis. How does this relationship change with an increase in θ? Explain clearly and illustrate diagrammatically.

- **Market Equilibrium:**

We solve for the market equilibrium so as to determine the equilibrium rate of return

(π). We consider a competitive market for loan contracts and use a standard Walrasian equilibrium notion, where the rate of return is determined by the equality of aggregate supply of loans and incentive-constrained demand for loans aggregating across all borrowers. We would like to understand the effects of an increase in θ.

- **Perfectly Elastic Loan Supply Function:**

  Consider the case where the loan supply function $L_s(\pi)$ is perfectly elastic at $\pi = \alpha$.

12. **[10 marks]** Suppose θ increases.

    (a) What happens to the equilibrium rate of return (π)? Explain clearly and illustrate in a diagram with L_d and L_s on x-axis, and π on y-axis.

    (b) Explain clearly what happens to (i) the proportion of borrowers excluded from the market, (ii) the loan size of borrowers who were previously credit-constrained, (iii) the loan size of borrowers who were previously not credit-constrained, and (iv) the lenders? How would you Pareto rank the new equilibrium with the old equilibrium?

    (c) Comment on the distributional impact of the increase in θ.

13. **[3 marks]** From now on we consider the case where the supply of loans is *inelastic* to some degree. To understand the basic intuition for the next set of questions, explain in words the total effect of an increase in θ through the *partial equilibrium* effect of a change in the aggregate demand function and the resulting *general equilibrium* effect on the rate of return π.

- **Nearly Perfect Elasticity of Loan Supply:**

  Suppose elasticity of the loan supply function at any $\pi > \alpha$ is finite but bounded below by some $\underline{\varepsilon}$ where $\underline{\varepsilon}$ is sufficiently large. We consider the effects of an increase in θ.

14. **[10 marks]**

    (a) Argue that the change in the equilibrium rate of return can be made arbitrarily small if $\underline{\varepsilon}$ is sufficiently large.

    (b) When $\underline{\varepsilon}$ is sufficiently large, explain clearly what happens to (i) the proportion of borrowers excluded from the market, (ii) the loan size of borrowers who were previously credit-constrained, (iii) the loan size of borrowers who were previously not credit-constrained, and (iv) the lenders?

(c) Comment on the distributional impact of the increase in θ.

- **Perfectly Inelastic Loan Supply:**

  Now turn to the other extreme where the supply of loanable funds is perfectly inelastic. To see the results most clearly, we make two modifications in the model. First, we assume that $\nu = 0$, that is, only the borrowers' initial assets (W) serve as collateral. Second, we assume that the upper bound of the wealth distribution is low enough that no borrower attains the first-best project scale.

15. [**21 marks:** $1+5+5+5+5$] Suppose θ increases.

    (a) Show that the project ceiling for a borrower with wealth W is $I^H(W, \theta, \pi) = \frac{\theta W + d}{\beta(1+\pi)}$.

    (b) Suppose θ rises to θ' and suppose the corresponding equilibrium rate of return rises from π to π'. Then prove that if the project ceiling does not fall for some borrower with wealth W, then it must rise and is larger for all borrowers with higher wealth $W' > W$. [That is, prove that if $\triangle(W) \equiv I^H(W, \theta', \pi') - I^H(W, \theta, \pi) \geq 0$, then $\triangle(W') > \triangle(W) \geq 0$.]

    (c) Prove that the proportion of borrowers that are excluded must rise.

    (d) Argue that there must exist a cut-off wealth level $\widehat{W}$ such that the credit level of borrowers with that wealth level is unaffected. Argue that there must be a *regressive redistribution* of credit across borrowers.

    (e) Now let us allow for a wide enough support of the wealth distribution so that the largest firms are *not* credit-constrained. It can be shown that, in response to an increase in θ, the proportion of borrowers that are excluded rises and there exists $\widehat{W}$ such that the credit level of borrowers with $W < \widehat{W}$ falls. Suppose that the wealth distribution is such that most borrowers are in the intermediate-size category, that is, have $W > \widehat{W}$ but are credit-constrained (before the rise in θ).

    Argue that in this case the increase in θ will result in a *progressive redistribution* of credit across borrowers.

16. [**3 marks**]

    Given the above analysis, would you like to revise your answer to Question #1 above? Explain clearly.