Midterm Exam (10 March 2024)

- Answer all the questions. You have 3 hours to write this exam.

1. [50 marks]

   A farming family owns land of size A (acres), and farms it with labour L, using the production function

   $$Y = 100L^{\frac{1}{2}}A^{\frac{1}{2}}.$$

   The farm has access to 4 total units of family labour (you can think of 4 as the family size), which it divides as finely as it wishes between working *on* the farm and *off* the farm. Off-farm employment yields a wage of 100 per unit.

   The farm can also hire *in* labour, again at the wage cost of 100 per unit. But unlike family labour, hired labour has to be supervised, and for this the farm has to hire a supervisor at a cost of 225. Once paid, the supervisor can costlessly supervise all hired labour.

   (a) [15 marks]

   Find out, with a clear explanation, a threshold value of owned land size, call it A_1, such that if the family has less than A_1 acres of land ($A \leq A_1$), it will be a *labourer-cum-cultivator* family. That is, the family will devote an appropriate amount of family labour to the farm, hire in no additional labour, and hire out the remaining family labour for off-farm employment. Determine the amount of family labour devoted to the family farm as a function of the owned land size A.

   (b) [20 marks]

   Prove that there exists another threshold value of owned land size, call it A_2, such that if the family has between A_1 and A_2 acres of land ($A_1 < A < A_2$), it will operate as a *self-cultivator* family. That is, the family operates as a full family farm, with all its family members working full time on it, but will not hire in any labour.

   Above A_2 acres of owned land ($A \geq A_2$), the family operates as a *small capitalist* farm:

it hires a supervisor and hires labour from outside the family. Determine the farm's demand for hired labour as a function of the owned land size A.

(c) [15 marks]

How would your answers to parts (a) - (b) change (if at all) if there were no fixed costs to supervision, and if hired labour costs 25 *per unit* to supervise instead?

2. [50 marks]

- Consider transactions in rural land markets under the following economic environment.

    - Production Function:
    $$Q = e + \theta,$$
    where Q is output, e is the cultivator's effort, and θ is a zero-mean random shock.

    - Preferences: All agents are risk-neutral.

    - Endowments: The landlord owns a plot of land, monetary wealth W_L and no labour; the tenant owns no land, monetary wealth W_T, and 1 unit of labour.

    - Markets: Land, labour and goods markets are perfectly competitive. This means, if the landowner wants he could buy the tenant's labour services at the market wage rate w, and if the labour-owner wanted he could lease in the landlord's land at the market rental rate ρ. This implies that the reservation (expected) utility of the landlord is ρ, that is, $U_L \geq \underline{U}_L = \rho$, and the reservation (expected) utility of the tenant is w, that is, $U_T \geq \underline{U}_T = w$.

    - Cost of Effort: We distinguish between labour time and effort. The tenant faces the following cost to exert effort:
    $$c\left(e\right) = \frac{1}{2}ce^2, \ c > 0.$$

    - We consider the following parameter restriction: $w + \rho < \dfrac{3}{8c}$.

    - Contracting: Effort, e, is neither observable nor monitorable. Hence the tenant would need incentives to put in effort. We restrict ourselves to *linear contracts*, that is, the tenant's income, y_T, is a linear function of output:
    $$y_T = s \cdot Q - R, \ \text{where} \ 0 \leq s \leq 1.$$

    Let us summarize this contract as (s, R), where s is referred to as the 'share' component and R as the 'fixed-rent' component.

– Limited Liability Constraint: The fixed component of the rent has to be paid in advance and this amount is bounded above by the tenant's wealth W_T:

$$R \leq W_T.$$

(a) [6 marks]

Under the contract (s, R), find out the expected utilities of the landlord and the tenant, the total surplus, the first-best effort level and the maximized value of total surplus. Are there gains to be made from trade between the landlord and the tenant? Explain clearly.

(b) [15 marks]

Suppose that the landlord has all the bargaining power.

(i) Formulate the optimal contracting problem when the landlord has all the bargaining power.

(ii) Analyzing the contracting problem show that the optimal contract, (s^*, R^*), has the following structure:

- $\left(s^* = \dfrac{1}{2}, \ R^* = W_T\right)$ for $\rho - \dfrac{1}{4c} \leq W_T \leq \dfrac{1}{8c} - w$;
- $\left(s^* = \sqrt{2c\,(w + W_T)}, \ R^* = W_T\right)$ for $\dfrac{1}{8c} - w < W_T < \dfrac{1}{2c} - w$;
- $\left(s^* = 1, \ R^* = \dfrac{1}{2c} - w\right)$ for $W_T \geq \dfrac{1}{2c} - w$.

(c) [17 marks]

Now suppose that the tenant has all the bargaining power.

(i) Formulate the optimal contracting problem when the tenant has all the bargaining power.

(ii) Analyze the contracting problem rigorously and characterize completely the structure of the optimal contract, (s^{**}, R^{**}).

(d) [12 marks]

Compare the efficiency of the optimal contracts under the above two alternative bargaining protocols. In particular, could you identify a few parameter ranges where you can clearly rank the above two alternative bargaining protocols in terms of efficiency? Explain clearly.