Math 271: Mathematical Methods — Tridip Ray
Semester I, 2021-22 — ISI, Delhi

First Midterm Exam: Question 1 (08 November 2021)

- Maximum marks: **20**
- Time allotted (including uploading on Moodle): **45 minutes**

- Let x^1, x^2, ..., x^n be a set of linearly independent vectors in $\mathbb{R}^n$. Let y^1, y^2, ..., y^n be a set of vectors defined by

$$y^1 = x^1 + x^2,\ y^2 = x^2 + x^3, ..., y^{n-1} = x^{n-1} + x^n,\ y^n = x^n + x^1.$$

  Is the set of vectors $\{y^1, y^2, ..., y^n\}$ linearly independent? Explain clearly.

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First Midterm Exam: Question 2 (08 November 2021)

- Maximum marks: **25**
- Time allotted (including uploading on Moodle): **55 minutes**

- Let A be a symmetric $n \times n$ matrix. It can be shown that there exists a solution to the following minimization problem:

$$\underset{\{v \in \mathbb{R}^n\}}{\text{Minimize}} \; v^T A v$$
$$\text{subject to} \quad \|v\| = 1.$$

(a) Prove that for any vector $v \in \mathbb{R}^n$ with $\|v\| = 1$, there exists a scalar r such that

$$v^T (A + rI_n) v > 0,$$

where I_n is the $n \times n$ identity matrix.

(b) Prove that there exists a scalar s such that the $n \times n$ matrix $A + sI_n$ is positive definite.

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First Midterm Exam: Question 3 (08 November 2021)

- Maximum marks: **20**
- Time allotted (including uploading on Moodle): **45 minutes**

- Let A be an $n \times n$ matrix, and let $A^1, A^2, ..., A^n$ denote the n column vectors of A. Define an $n \times n$ matrix B as follows:

$$b_{ij} = A^i A^j \qquad i = 1, ..., n;\ j = 1, ..., n$$

where b_{ij} is the element corresponding to the ith row and jth column of matrix B, and $A^i A^j$ denotes the inner product of the vectors A^i and A^j. Suppose A is non-singular. Does it follow that B is non-singular? Explain clearly.

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First Midterm Exam: Question 4 (08 November 2021)

- Maximum marks: **35**
- Time allotted (including uploading on Moodle): **80 minutes**

- Let A be an $m \times n$ matrix, and let c be a vector in $\mathbb{R}^m$. Prove that exactly one of the following two alternatives holds.

  Either the system of equations $Ax = c$ has a solution,

  or, the system of equations $yA = 0$ and $yc = 1$ has a solution.