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Common value auctions with asymmetric bidder information

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Abstract

We study a first price auction with two asymmetric bidders. A unique generically asymmetric equilibrium exists under some conditions. Aggressive bidding is not necessarily associated with inferiority of information. Reduction in asymmetry can have an ambiguous effect on revenue.

Keywords: Common value auctions; Asymmetric bidders

JEL classification: D44; D82

1. Introduction

It is usually presumed in studies of first price common value auctions that bidders have symmetric information structures, i.e., the signal generating process is common across bidders. However, bidders may differ in terms of experience, analytical ability, access to information, etc., implying that bidders can possess information processes with differing precision levels.

This essay differs from previous related articles (Hausch, 1987; Kagel and Levin, 1999; Laskowski and Slonim, 1999, which allow bidders' information processes to be independent, with one bidder accessing a signal generating process with higher precision) by allowing information precision to be

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multi-dimensional and bidders to be ambiguously ranked in terms of precision levels.[1] Further, all bidders possess valuable private information without any being perfectly informed. Our paper is most closely related to Hausch (1987), who imposes a strong restriction on the nature of asymmetry across bidders.[2] The absence of any such informational restrictions allows a fuller analysis of the impact of asymmetric information.

We analyze a simple model where bidders' signals are conditionally independent, and have different precision levels. Under some conditions, a unique, generically asymmetric, mixed strategy equilibrium exists. In asymmetric equilibrium, one bidder submits higher average bids than the other; however, aggressive bidding does not necessarily flow from the inferiority of information. The revenue impact of a reduction in the degree of asymmetry between the bidders is in general ambiguous.

2. Model

Consider a first price common value auction with two bidders.[3] The object can be of type H (with prior probability α), with value v_H, or L, with value $-v_L$, with $v_i>0$, $i=H, L$. Thus, a low value object entails losses. For example, a mineral tract may reveal its value to a winner only after significant investments have been made, which are not recoupled if the tract is unproductive.

Each bidder also receives a conditionally independent private signal. A bidder is armed with a test which assigns the object to one of two categories h and l. Let $p_j(y|Y)$, $j=1, 2$ denote the probability the object is assigned to category $y=h, l$ by bidder j given that it is truly of type $Y=H, L$. Let $p_j(h|H)=p_j$; $p_j(h|L)=q_j$, $j=1, 2$. Bidder asymmetry implies that $p_1\neq p_2$ or $q_1\neq q_2$. We have

$$\text{Assumption 1}: 1>p_j>0.5>q_j>0, \qquad j=1,2.$$

$1-p_j$ is the *false rejection rate* for H objects and q_j is the *false acceptance rate* for H objects. A test with a higher p and a lower q is more precise. Let $\vec{\pi}_j=(p_j,q_j)$, $P_j(h)=\alpha p_j+(1-\alpha)q_j$, $\Pi=\frac{P_1(h)}{P_2(h)}$. $P(h)$ is the probability a test yields the signal h.

Bidders submit sealed bids and the highest bidder wins. Ties are broken randomly. We assume a bidder who receives the l signal never submits a bid (because it is then likely that the object has value $-v_L$). We can then restrict attention to the behavior of bidders with the h signal. The assumption is imposed for tractability and can be motivated by situations where winners have to incur substantial preliminary development investments before the true value is revealed.

$$\text{Assumption 2}: \alpha(1-p_j)v_H<(1-\alpha)(1-q_j)v_L, \qquad j=1,2$$

[1] The study of auctions with asymmetric bidders is better developed for the private values case: see, for example, Maskin and Riley (2000a,b) and the references cited therein.

[2] Kagel and Levin (1999) and Laskowski and Slonim (1999) analyze bidding with asymmetrically informed bidders. Their model differs substantially from ours. Moreover, Kagel and Levin (1999) derive their result under the assumptions that the true value is uniformly distributed and that one of the bidders is perfectly informed with a zero margin of error. The result of Laskowski and Slonim (1999) is derived under the assumption of 'translation-invariant' bid functions, i.e., for any player the difference between her signal and her bid is constant.

[3] All results extend to auctions with one superiorly informed bidder and a finite number of inferiorly informed bidders who are symmetric.

The assumption implies that a monopolist bidder with the l signal cannot break even if she submits a 0 bid. Given A1 and A2, the following conditions are sufficient to ensure that a bidder with the h signal submits a bid and earns a positive return (if $P_i(h)=P_j(h)$, A3b reduces to A3a):

$$\text{Assumption 3a}: \alpha p_j(1-p_i)v_H > (1-\alpha)q_j(1-q_i)v_L, \qquad i,j=1,2;\ i\neq j$$

$$\text{b}: \alpha p_j\left[P_i(h)-p_iP_j(h)\right]v_H > (1-\alpha)q_j\left[P_i(h)-q_iP_j(h)\right]v_L, \qquad i,j=1,2;\ i\neq j$$

3. Analysis

Let $v_i^F(b)$ denote the expected payoff to bidder i from bidding b when her test yields signal h. $w_i(b|Y)$ is the probability that bidder i wins the object with bid b conditional on the true type of the object being $Y=H$, L. Then

$$v_i^F(b) = \frac{\alpha p_i}{P_i(h)}(v_H-b)w_i(b|H) - \frac{(1-\alpha)q_i}{P_i(h)}(v_L+b)w_i(b|L), \qquad i=1,2 \tag{1}$$

In discrete information first-price auctions with a random tie-break rule, it is well known that an equilibrium with pure bidding strategies does not exist (see Wang, 1991). We show that a unique generically asymmetric mixed strategy equilibrium exists in the bidding game. Equilibrium can be written as $(B_1,F_1(b),B_2,F_2(b))$, where B_i and F_i, $i=1$, 2 are, respectively, the set of bids and a distribution function over this set which bidder i uses to announce bids, conditional on private information h. We therefore have $w_i(b|L)=q_jF_j(b)+(1-q_j)$ and $w_i(b|H)=p_jF_j(b)+(1-p_j)$. Substituting into (1), we obtain v_i^F and use the expression to derive equilibrium in the bidding game.

$$v_i^F\left(b,F_j(b)\right) = \frac{\alpha p_i}{P_i(h)}\left[p_jF_j(b)+\left(1-p_j\right)\right](v_H-b) - \frac{(1-\alpha)q_i}{P_i(h)}\left[q_jF_j(b)+\left(1-q_j\right)\right](v_L+b), \quad i\neq j. \tag{2}$$

Proposition 1. *Given A1 through A3, a unique equilibrium in mixed bidding strategies exists. The equilibrium is symmetric if and only if $P_1(h)=P_2(h)$, in which case bidders randomize over an interval $[0,b^*]$, using a continuous distribution function, where $b^*=\frac{\alpha p_1p_2v_H-(1-\alpha)q_1q_2v_L}{P(h)}$. Otherwise, if $P_i(h)>P_j(h)$, the equilibrium is asymmetric. Bidder j randomizes over the interval $(0,b_i^*)$, using a continuous distribution function F_j, where $b_i^*=\frac{\alpha p_1p_2v_H-(1-\alpha)q_1q_2v_L}{P_i(h)}$. Bidder i randomizes over the interval $[0,b_i^*]$, using a continuous distribution function F_i over $(0, b_i^*)$ and bids 0 with probability $\left(1-\frac{P_j(h)}{P_i(h)}\right)$ and $F_i(b) > F_j(b)$ for any $b \in (0, b_i^*)$.*

Proof. Let $\inf B_i=\underline{b}_i$ and $\sup B_i=\bar{b}_i$. It is easy to show that $\underline{b}_1=\underline{b}_2=0$ and $\bar{b}_1=\bar{b}_2$. Let $\bar{b}_1=\bar{b}_2=\bar{b}$. Further, an equilibrium distribution function cannot have any 'holes' in $(0,\bar{b})$ or an atom at any $b\in(0,\bar{b})$. Also, if at least one of B_1 and B_2 is closed above, then an equilibrium distribution function cannot have an atom at $\bar{b}$. Thus, let B_1 and B_2 be closed above, and let F_1 and F_2 be atomless at $\bar{b}$. There are three possibilities:

(I) B_1 and B_2 are closed below, and F_1 and F_2 are atomless on $[0,\bar{b}]$; (II) B_1 is closed below, B_2 is open below and F_1 has an atom at 0; and (III) B_1 is open below, B_2 is closed below and F_2 has an atom at 0.

We first study possibility (I). We have $B_1 = B_2 = [0,\bar{b}]$.

Using (2), we can derive bidder payoffs. Let $\alpha_i(H|h) = \frac{\alpha p_i}{P_i(h)}, \alpha_i(L|h) = \frac{(1-\alpha)q_i}{P_i(h)}$.

$$v_i^F\big(b, F_j(b)\big) = \alpha_i(H|h)\big[1 - p_j\big(1 - F_j\big)\big](v_H - b) - \alpha_i(L|h)\big[1 - q_j\big(1 - F_j\big)\big](v_L + b), \qquad i \neq j$$

In equilibrium, $F(0)=0$. Let bidder i's payoff at $b=0$ be denoted $\bar{v}_i^F$. Notice $\bar{v}_i^F > 0$, $i=1, 2$, by A3a. A bidder's payoff is constant over $[0,\bar{b}]$. We define $\bar{b}_1$ and $\bar{b}_2$:

$$v_i^F\left(\bar{b}_i, 1\right) = \bar{v}_i^F, \quad i = 1,2. \text{ Therefore, } \bar{b}_i = \alpha_i(H|h)p_j v_H - \alpha_i(L|h)q_j v_L, \qquad i \neq j$$

In equilibrium, $\bar{b}_1 = \bar{b}_2 = \bar{b}$. Thus, $\bar{b}_1 = \bar{b}_2 \Leftrightarrow P_1(h) = P_2(h)$. Suppose therefore $P_1(h) = P_2(h) = P(h)$ (say). It is easy to show that $\bar{b} = \frac{\alpha p_1 p_2 v_H - (1-\alpha) q_1 q_2 v_L}{P(h)} \in (0, v_H)$. Since $v_i^F(b, F_j(b)) = \bar{v}_i^F$, we obtain the equilibrium distribution functions F_1 and F_2.

$$F_i(b) = \frac{b(1 - C_i)}{A_i - bC_i}$$

$$A_i = \alpha_j(H|h)p_i v_H - \alpha_j(L|h)q_i v_L, C_i = \alpha_j(H|h)p_i + \alpha_j(L|h)q_i, \qquad i,j = 1,2; \; i \neq j$$

It is easy to see that $F_i(b)$, $i=1, 2$ is continuously differentiable and strictly increasing. For $P_1(h) = P_2(h)$, since $\bar{v}_i^F > 0$, $i=1, 2$, equilibrium is thus established, as bidders earn the same payoff for all $b \in [0,\bar{b}]$, and cannot gain by deviating and bidding more than $\bar{b}$. Further, the equilibrium is symmetric. To see that, consider any $b \in (0,\bar{b})$.

$$F_1(b) - F_2(b) = \frac{bD}{\alpha p_1 p_2 (v_H - b) - (1-\alpha) q_1 q_2 (v_L + b)},$$

where $D = \alpha(p_2 - p_1) - (1-\alpha)(q_1 - q_2)$.

But $P_1(h) = P_2(h) \Leftrightarrow \alpha(p_2 - p_1) = (1-\alpha)(q_1 - q_2)$. Hence, $F_1(b) = F_2(b)$ for all $b \in (0,\bar{b}]$.

We now study possibility (II): B_1 is closed below, B_2 is open below and F_1 has an atom at 0. Let $B_1 = [0,\bar{b}]$, $B_2 = (0,\bar{b}]$.

If bidder 1 bids $b=0$, she obtains the object if and only if bidder 2 does not submit a bid. Since a bidder always bids unless she receives the signal l, the probability that bidder 1 obtains the object, given that she has received the signal h, is $(1 - p_2)$ if the object is truly of type H and $(1 - q_2)$ if the object is truly of type L. Thus, bidder 1's payoff on receiving signal h, and bidding 0, is

$$\bar{v}_i^F = \alpha_1(H|h)(1 - p_2)v_H - \alpha_1(L|h)(1 - q_2)v_L$$

Since bidder 1's equilibrium payoff is constant over $[0,\bar{b}]$, $v_1^F(\bar{b},1)=\bar{v}_1^F$. *Also,* $\bar{v}_1^F(b,F_2(b))=\bar{v}_1^F$. Hence:

$$\bar{b}=\frac{\alpha p_1 p_2 v_H-(1-\alpha)q_1 q_2 v_L}{P_1(h)}$$

$$F_2(b)=\frac{b[\alpha p_1(1-p_2)+(1-\alpha)q_1(1-q_2)]}{[\alpha p_1 p_2(v_H-b)-(1-\alpha)q_1 q_2(v_L+b)]}$$

$F_2(b)$ is strictly increasing in b and is continuously differentiable. We now turn to bidder 2. Given she has received signal h, suppose she bids $\bar{b}$. Clearly, she always wins the object. Let her payoff be denoted by $\tilde{v}_2^F$.

$$\begin{aligned}\tilde{v}_2^F&=\alpha_2(H|h)\left(v_H-\bar{b}\right)-\alpha_2(L|h)\left(v_L+\bar{b}\right)\\&=\bar{v}_2^F+[\alpha_2(H|h)-\alpha_1(H|h)]v_H-[\alpha_2(L|h)-\alpha_1(L|h)]v_L\end{aligned}$$

$\tilde{v}_2^F>0$, by A3b. Further, $v_2^F(b,F_1(b))=\tilde{v}_2^F$. Thus

$$F_1(b)=\frac{\alpha p_2\left[b(1-p_1)+p_1 v_H\left(1-\frac{1}{\Pi}\right)\right]+(1-\alpha)q_2\left[b(1-q_1)-q_1 v_L\left(1-\frac{1}{\Pi}\right)\right]}{[\alpha p_1 p_2(v_H-b)-(1-\alpha)q_1 q_2(v_L+b)]}$$

where $\Pi=\frac{P_1(h)}{P_2(h)}$. Thus, $F_1(b)$ is strictly increasing and continuously differentiable. We see that $F_2(\bar{b})=F_1(\bar{b})=1$. Also, $F_2(0)=0$. By hypothesis, $F_1(0)>0$. We have

$$F_1(0)>0\Longleftrightarrow\left(1-\frac{1}{\Pi}\right)[\alpha p_1 p_2 v_H-(1-\alpha)q_1 q_2 v_L]>0$$

A1 and A3a together imply that $\alpha p_i v_H>(1-\alpha)q_i v_L$. Moreover, it follows from A1 that $\alpha p_i v_H-(1-\alpha)q_i v_L>0\Rightarrow\alpha p_1 p_2 v_H-(1-\alpha)q_1 q_2 v_L>0$, $i=1, 2$. Thus, $F_1(0)>0\Leftrightarrow\Pi>1\Leftrightarrow P_1(h)>P_2(h)$. Hence, for $P_1(h)>P_2(h)$, since bidders earn positive payoffs, equilibrium is established. Bidder 2 randomizes over $(0,\bar{b}]$, using a continuous distribution function. Bidder 1 randomizes over $[0,\bar{b}]$, using a continuous distribution function over $(0,\bar{b}]$, and an atom at 0. Each bidder earns the same payoff for all bids in her equilibrium support and cannot gain by deviating.

To conclude, consider any $b\in(0,\bar{b})$. Let $P_1(h)>P_2(h)$.

$$F_1(b)>F_2(b)\Longleftrightarrow\left(1-\frac{1}{\Pi}\right)[\alpha p_1 p_2 v_H-(1-\alpha)q_1 q_2 v_L]+b[\alpha(p_2-p_1)-(1-\alpha)(q_1-q_2)]>0$$

As $b\rightarrow0$, clearly $F_1(b)>F_2(b)$, as $\Pi>1$. Also, at $b=\bar{b}$, $F_1(b)=F_2(b)$. Therefore, $F_1(b)>F_2(b)$ for any $b\in(0,\bar{b})$.

We are therefore left with possibility (III). Following the discussion above, it is easy to show that such an equilibrium exists if and only if $\Pi<1\Leftrightarrow P_1(h)<P_2(h)$. The details are omitted for brevity. □

Thus, a unique equilibrium exists. The equilibrium is symmetric if and only if $P_1(h)=P_2(h)$. To compare the results of Hausch (1987) with ours, define $p_i(y|z)$, $i=1, 2$ as the probability that bidder i receives signal y, conditional on bidder j receiving signal z; y, $z=h$, l. Hausch (1987) restricts his asymmetric setting by assuming $p_1(y|z)=p_2(y|z)$, for all y, z and derives the existence of a symmetric mixed strategy equilibrium.

Consider $p_i(h|h)$, $i=1, 2$. By conditional independence, we have $p_i(h|h)=[\alpha p_1 p_2+(1-\alpha)q_1 q_2]/[P_j(h)]$. Hence, $p_1(h|h)=p_2(h|h)$ if and only if $P_1(h)=P_2(h)$. It can similarly be shown that $p_1(l|l)=p_2(l|l)$ if and only if $P_1(h)=P_2(h)$. Consider now $p_1(h|l)$ and $p_2(h|l)$. We see that $p_i(h|l)=[\alpha p_i(1-p_j)+(1-\alpha)q_i(1-q_j)]/[1-P_j(h)]$. It is easy to show that $p_1(h|l)=p_2(h|l)$ if and only if $P_1(h)=P_2(h)$ and $p_1(l|h)=p_2(l|h)$ if and only if $P_1(h)=P_2(h)$. Summing up, the assumption that $p_1(y|z)=p_2(y|z)$, for all y, z, is equivalent in our model to assuming $P_1(h)=P_2(h)$. Given this non-generic condition, we would have a symmetric equilibrium.[4]

Previous analyses have noted (see Engelbrecht-Wiggans et al., 1983) that, if bidders are not symmetric, the phenomenon of 'aggressive bidding' may arise, with the less informed bidder on an average submitting higher bids than the more informed bidder. In our model, we see that, if $P_1(h)=P_2(h)$, the two bidders submit the same average bids, irrespective of whether one is more informed than the other. Now suppose $P_i(h)>P_j(h)$, $i, j=1, 2$; $i\neq j$. The above proposition shows that j's bid stochastically dominates i's bid, i.e., aggressive bidding is generated in this environment as well. However, it is no longer uniquely associated with inferiority of information. To see that, suppose $p_i=p_j$ and $q_i>q_j$ so that j is better informed than i. Clearly, $P_i(h)>P_j(h)$ and hence j submits a higher bid on average. Equilibrium is also consistent with the notion that a less informed bidder engages in aggressive bidding, as for example in the case when $q_i=q_j$ and $p_i>p_j$.

How does informational asymmetry between the bidders affect auction revenue? Previous results have suggested that reduction of asymmetries between the bidders increases expected revenue for the seller (see, for example, Milgrom and Weber, 1982). In our model, however, it is easy to show that a reduction of informational asymmetry has an ambiguous effect on auction revenue.

To analyze the effect of a change in the degree of asymmetry, fix $\vec{\pi}_1=(p_1,q_1)$ and examine the impact of a change from $\vec{\pi}_2=(p_2,q_2)$ to $\vec{\pi}_2^*=(p_2^*,q_2^*)$. Let $\bar{p}=\max(p_1,p_2)$ and $\underline{p}=\min(p_1,p_2)$. Define $\bar{q}$ and $\underline{q}$ similarly. We assume that $p_2^*\in(\underline{p},\bar{p})$ and $q_2^*\in(\underline{q},\bar{q})$. Let $R(R^*)$ denote ex ante expected revenue in the auction when the precision levels of bidder 2's test are given by $\vec{\pi}_2=(p_2,q_2)\left(\vec{\pi}_2^*=(p_2^*,q_2^*)\right)$. Also, define $P_2^*(h)=\alpha p_2^*+(1-\alpha)q_2^*$.

Suppose $P_1(h)=P_2(h)=P_2^*(h)$. It is straightforward to show that $R^*-R=\alpha(p_2^*-p_2)(p_1 v_H+q_1 v_L)$. Thus, $R^*>R$ if and only if $p_2^*>p_2$ (or $q_2>q_2^*$). Hence, if asymmetry is reduced through an increase in the information precision of the less informed bidder, revenue is raised.

Retaining the assumption $P_2(h)=P_2^*(h)$, similar results also obtain if $P_1(h)\neq P_2(h)$. Suppose now $P_2(h)\neq P_2^*(h)$. In general, the revenue impact of a change in the degree of informational asymmetry is ambiguous and could be positive or negative. For example, if $P_1(h)<\min(P_2(h), P_2^*(h))$, then

$$R^*-R=\alpha p_1(p_2^*\Pi^*-p_2\Pi)v_H+(1-\alpha)q_1(q_2\Pi-q_2^*\Pi^*)v_L$$

where $\Pi^*=P_1(h)P_2^*(h)$. R^* could therefore be greater or less than R. A similar ambiguity arises if $P_1(h)>\max(P_2(h),P_2^*(h))$ or $P_2^*(h)\geq P_1(h)>P_2(h)$ or if $P_2^*(h)\leq P_1(h)<P_2(h)$.

[4] If $p_1=p_2$ and $q_1=q_2$, $P_1(h)=P_2(h)$ trivially. The unique equilibrium is then symmetric.

4. Conclusions

In common value auctions, bidders are usually assumed to be symmetric. This paper has attempted to extend the study of sealed-bid first price auctions when the distributions of the signals differ across bidders. In a simple binary model, we characterize mixed-strategy equilibrium and show that it is generically asymmetric. Aggressive bidding can occur and it may result from, though is not necessarily associated with, inferiority of information. While both bidders can obtain positive payoffs, if a bidder is unambiguously superiorly informed, her payoff is higher. A change in the degree of asymmetry between bidders in general has an ambiguous effect on revenue.

While our model structure is simple, it has some advantages over previous studies. Both bidders are allowed to draw conditionally independent imperfect signals and bidders may not be unambiguously ranked in terms of precision levels. Furthermore, we impose no ex ante restrictions on equilibrium bid distributions or on the nature of asymmetry. Extensions of this basic structure are left for future work.

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