Statistics 134 (Lecture - 2), Fall 2002

Practice Midterm (Time : 50 minutes)

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NOTE : There are three problems total of 30 points. Show your work and write explanation when needed.

1. Suppose a box contains 70 red balls and 30 green balls. You do sampling **with replacement** of 100 balls. Let X be the number of green balls in the first 40 draws, and Y be the number of green balls in the next 60 draws. Let $T = X + Y$ be the total number of green balls in the sample.

   (a) What are the distributions of X, Y and T ? [**2 points**]

   (b) Are X and Y independent ? [**1 points**]

   (c) Fix $0 \leq t \leq 100$, find $\mathbf{P}\Big(X = k \Big| T = t\Big)$ for $0 \leq k \leq t$. [**5 points**]

   (d) Are X and T independent ? Why ? [**2 points**]

2. Cards are dealt one after the other from a well shuffled deck of 52 cards until the **first King** appears. Find a formula for $p(n) :=$ probability that exactly n cards are dealt. [**5 points**]

3. There are n balls labeled $1, 2, 3, \ldots, n$, and n boxes also labeled $1, 2, 3, \ldots, n$. Balls are being placed in the boxes at random so that, any ball can go into any box, and a box may contain more than one ball.

   (a) Find $\mathbf{P}($ none of the n boxes are empty $)$. [**2 points**]

   (b) Say that i^{th} ball is correctly placed if it goes to the i^{th} box. Let A_i be the event that i^{th} ball is correctly placed. Find $\mathbf{P}(A_i)$ for $1 \leq i \leq n$. [**2 points**]

   (c) Let X be the total number of correct placement of balls, find the distribution of X. [**5 points**]

   (d) Find $\mathbf{E}[X]$ and $\mathbf{Var}(X)$. [**2 points**]

   (e) If $n = 10,000$ calculate approximately $\mathbf{P}(X < 2)$, $\mathbf{P}(X = 2)$ and $\mathbf{P}(X > 2)$. [**4 points**]