Homework # 2 *Statistics 134, Bandyopadhyay, Spring 2014*

1.3.10 a) $P(\text{exactly 2 of } A,\, B,\, C) = P(ABC^c) + P(AB^cC) + P(A^cBC)$

$$= (P(AB) - P(ABC)) + (P(AC) - P(ABC)) + (P(BC) - P(ABC))$$
$$= P(AB) + P(AC) + P(BC) - 3P(ABC)$$

b) $P(\text{exactly 1 of } A,\, B,\, C) = P(AB^cC^c) + P(A^cB^cC) + P(A^cB^cC)$

$$= P(A) - P(A \cap (B \cup C)) + P(B) - P(B \cap (A \cup C)) + P(C) - P(C \cap (A \cup B))$$
$$= \dots$$
$$= P(A) + P(B) + P(C) - 2\,(P(AB) + P(AC) + P(BC)) + 3P(ABC)$$

c) $P(\text{exactly none}) = 1 - P(A \cup B \cup C)$

$$= 1 - (P(A) + P(B) + P(C)) + (P(AB) + P(AC) + P(BC)) - P(ABC)$$

1.3.14 Use the identity $P(A \cup B) = P(A) + P(B) - P(AB)$ and the fact that $P(A \cup B) \leq 1$.

1.4.6 $P(\text{second spade} \mid \text{first black})$

$$= \frac{P(\text{first black and second spade})}{P(\text{first black})}$$
$$= \frac{P(\text{first spade and second spade}) + P(\text{first club and second spade})}{P(\text{first black})}$$
$$= \frac{(13/52)(12/51) + (13/52)(13/51)}{(26/32)} = \frac{25}{102}.$$

Or you may use a symmetry argument as follows:
$P(\text{second black} \mid \text{first black}) = 25/51$,
and $P(\text{second spade} \mid \text{first black}) = P(\text{second club} \mid \text{first black})$
by symmetry. Therefore $P(\text{second spade} \mid \text{first black}) = 25/102$.
Discussion. The frequency interpretation is that over the long run, out of every 102 deals yielding a black card first, about 25 will yield a spade second.

1.4.8 Assume n cards and all $2n$ faces are equally likely to show on top.
$P(\text{white on bottom} \mid \text{black on top})$

$$= \frac{P(\text{white on bottom and black on top})}{P(\text{black on top})} = \frac{50\% \times \frac{1}{2}}{50\% \times \frac{1}{2} + 20\%} = 5/9$$

1.5.6 a) The experimenter is assuming that before the experiment, H_1, H_2, and H_3 are equally likely. That is, prior probabilities are given by $P(H_1) = P(H_2) = P(H_3) = 1/3$.

b) No, because the above assumption is being made. Since the prior probabilities are subjective, so are the posterior probabilities.

c) Prior probabilities: $P(H_1) = .5$, $P(H_2) = .45$, $P(H_3) = .05$.
Likelihoods of A: $P(A|H_1) = .1$, $P(A|H_2) = .01$, $P(A|H_3) = .39$.
[Now all these probabilities have a long run frequency interpretation, so the posterior probabilities will as well.]
Posterior probabilities are proportional to: .05, .0045, .0195. So H_3 is no longer the most likely hypothesis; H_1 is, and $P(H_3|A) = \frac{.0195}{.05+.0045+.0195} = .263$.