Homework # 3 *Statistics 134, Bandyopadhyay, Spring 2014*

1.4.10 a) Let A_i be the event that the ith source works.
$P(\text{zero work}) = P(A_1^c A_2^c) = 0.6 \times 0.5 = 0.3$
$P(\text{exactly one works}) = P(A_1 A_2^c) + P(A_1^c A_2) = 0.4 \times 0.5 + 0.6 \times 0.5 = 0.5$
$P(\text{both work}) = P(A_1 A_2) = 0.4 \times 0.5 = 0.2$

b) $P(\text{enough power}) = 0.6 \times 0.5 + 1 \times 0.2 = 0.5$

1.6.6 a) $p_8 = p_9 = \cdots = 0.$ (Never need to roll more than seven times)
$p_1 = 0$
$p_2 = 1/6$
$p_3 = (5/6) \cdot (2/6)$ (2nd different from first, third the same as first or second)
$p_4 = (5/6) \cdot (4/6) \cdot (3/6)$
$p_5 = (5/6) \cdot (4/6) \cdot (3/6) \cdot (4/6)$
$p_6 = (5/6) \cdot (4/6) \cdot (3/6) \cdot (2/6) \cdot (5/6)$
$p_7 = (5/6) \cdot (4/6) \cdot (3/6) \cdot (2/6) \cdot (1/6) \cdot 1$

b) $p_1 + \cdots + p_{10} = 1$: you must stop before the tenth roll, and the events determining p_1, p_2, etc., are mutually exclusive.

c) Of course you can compute them and add them up. Here's another way. In general, let A_i be the event that the first i rolls are different, then $p_i = P(A_{i-1}) - P(A_i)$ for $i = 2, \ldots, 7$, with $P(A_1) = 1$, and $P(A_7) = 0$. Adding them up, you can easily check that the sum is 1.

1.6.8 a) $P(B_{12} \text{ and } B_{23}) = P(\text{all three have the same birthdates}) = \frac{365}{365} \cdot \frac{1}{365} \cdot \frac{1}{365} = \frac{1}{(365)^2}$.
$P(B_{12}) = \frac{365}{365} \cdot \frac{1}{365} = \frac{1}{365} = P(B_{23})$.
So $P(B_{12} \text{ and } B_{23}) = \frac{1}{(365)^2} = \frac{1}{365} \cdot \frac{1}{365} = P(B_{12})P(B_{23})$.
Therefore, B_{12} and B_{23} are independent!

b) No. If you tell me B_{12} and B_{23} have occurred, then all three have the same birthday, so B_{13} also has occurred. That is, $P(B_{12}B_{23}B_{31}) = \frac{1}{365^2} \neq \frac{1}{365^3} = P(B_{12})P(B_{23})P(B_{31})$.

c) Yes, each pair is independent by the same reason as a).

2.1.4

$$P(\text{2 sixes in first five rolls} | \text{3 sixes in all eight rolls})$$
$$= \frac{P(\text{2 sixes in first 5, and 3 sixes in all eight})}{P(\text{3 sixes in all eight})}$$
$$= \frac{P(\text{2 sixes in first five, and 1 six in next three})}{P(\text{3 sixes in all eight})}$$
$$= \frac{\binom{5}{2}(1/6)^2(5/6)^3 \cdot \binom{3}{1}(1/6)(5/6)^2}{\binom{8}{3}(1/6)^3(5/6)^5} = \frac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}} = \frac{10 \times 3}{56} = 0.535714$$

2.5.12 There are $\binom{52}{5} = 2,598,960$ distinct poker hands.

a) There are 52 cards in a pack: $A, 2, 3, ..., 10, J, Q, K$ in each of the four suits: spades, clubs, diamonds and hearts. A straight is five cards in sequence. Assume that an A can be at the begining or the end of a sequence, but never in the middle. That is $A, 2, 3, 4, 5$ and $10, J, Q, K, A$ are legitimate straights, but $K, A, 2, 3, 4$ (a "round–the–corner" straight) is not. So there are are 10 possible starting points for the straight flush, (A through 10) and 4 suits for it to be in, giving a total of 40 hands.

[This answer will be different under different asssumptions about what constitutes a straight. For example if you don't allow $A, 2, 3, 4, 5$, then the numerator is $4 \times 9 = 36$. If you allow wrap-around straights then the numerator is $4 \times 13 = 52$.]

Thus $P(\text{straight flush}) = \frac{40}{\binom{52}{5}} = 0.0000154$.

b) $P(\text{four of a kind}) = \frac{13 \times 12 \times \binom{4}{4}\binom{4}{1}}{\binom{52}{5}} = \frac{624}{2598960} = 0.000240$

c) $P(\text{full house}) = \frac{13 \times 12 \times \binom{4}{3} \times \binom{4}{2}}{\binom{52}{5}} = \frac{3744}{2598960} = 0.00144$

d) This answer depends on the definition of straight, again. $P(\text{flush}) = P(\text{all same suit}) - P(\text{straight flush})$
$= \frac{4 \times \binom{13}{5} - 4 \times 10}{\binom{52}{5}} = \frac{5108}{2598960} = 0.00197$

e) And again, this depends on your definition of straight. $P(\text{straight}) = P(\text{5 consec. ranks}) - P(\text{straight flush}) = \frac{10 \times \binom{4}{1}^5 - 10 \times 4}{\binom{52}{5}} = \frac{10200}{2598960} = 0.00392$

f) $P(\text{three of a kind}) = \frac{13 \times \binom{4}{3} \times \binom{12}{2} \times \binom{4}{1} \times \binom{4}{1}}{\binom{52}{5}} = \frac{54912}{2598960} = 0.0211$

g) $P(\text{two pairs}) = \frac{\binom{13}{2} \times \binom{4}{2} \times \binom{4}{2} \times 11 \times \binom{4}{1}}{\binom{52}{5}} = \frac{123552}{2598960} = 0.0475$

h) $P(\text{one pair}) = \frac{13 \times \binom{4}{2} \times \binom{12}{3} \times \binom{4}{1} \times \binom{4}{1} \times \binom{4}{1}}{\binom{52}{5}} = \frac{1098240}{2598960} = 0.423$

i) Since the events are mutually exclusive, the probability of none of the above is $1 - \text{sum of (a) through (h)} = 0.501$.